N	estimated number of observed events
$\mathcal{L}_{ep}$	estimated integrated ep luminosity in 1993 $\sim 500\text{nb}^{-1}$
$F_{\gamma/e}$	fraction of $\mathcal{L}_{ep}$ that becomes photon flux, ~ 0.16 within the geometrical coverage($W = 30 - 170\text{GeV}$).
$\sigma_{\gamma p}$	J/ψ photoproduction cross section, estimated to be 20-100nb.
B	branching ratio, 6.0% for $J/\psi \to e^+e^-$, 6.0% for $J/\psi \to \mu^+\mu^-$
ϵ	detector efficiency, use geometrical acceptance ~ 0.5

The estimated number of observed e^+e^- events is then $N \sim$ 50-250 for 1993 runs. Cross section measurement was feasible.

2.8 On J/ψ polarization measurement

As explained in Section 2.4, the decay angular distribution is

$$f_0(x;\alpha) = \frac{1+\alpha x^2}{\int\limits_{-1}^{1}(1+\alpha x^2)\,dx} \tag{33}$$

where x stands for $\cos\theta_{\text{decay}}$, and α is an undetermined parameter. The observed angular distribution $f(x,y;\alpha)$ in a detector with acceptance function $\epsilon(x,y)$, where y stands for a complete set of kinematical variables describing J/ψ production, is

$$\begin{aligned} f(x,y;\alpha) &= \frac{(1+\alpha x^2)\,\epsilon(x,y)}{\int(1+\alpha x^2)\,\epsilon(x,y)\,d\sigma_{(x,y)}} \\ &= \frac{1+\alpha x^2}{1+\alpha\langle x^2\rangle}\frac{\epsilon(x,y)}{\int\epsilon(x,y)\,d\sigma_{(x,y)}} \end{aligned} \tag{34}$$

where

$$\langle x^2\rangle = \frac{\int x^2\epsilon(x,y)\,d\sigma_{(x,y)}}{\int\epsilon(x,y)\,d\sigma_{(x,y)}} \tag{35}$$

How many events do we need to determine α with uncertainty $\Delta\alpha$? This can be studied by the maximum likelihood(ML) method.[14]

In ML method, α is determined by minimizing $w(\alpha)$

$$\begin{aligned} w &= -\sum_{i=1}^{N} \ln f\left(x_i, y_i; \alpha\right) \\ &= -\sum_{i=1}^{N} \ln \frac{1+\alpha x_i^2}{1+\alpha\langle x^2\rangle} \end{aligned} \tag{36}$$

$$-\sum_{i=1}^{N} \frac{\partial \ln f\left(x_i, y_i; \alpha\right)}{\partial \alpha} = 0 \tag{37}$$

and the standard deviation $\Delta\alpha$ can be either calculated using data by

$$\Delta\alpha = \left(-\frac{\partial^2 w}{\partial \alpha^2}\right)^{-1/2} \tag{38}$$

or predicted without data by

$$\Delta\alpha = \frac{1}{\sqrt{N}} \left(\int f\left(x, y; \alpha\right) \left(\frac{\partial \ln f\left(x, y; \alpha\right)}{\partial \alpha}\right)^2 d\sigma_{(x,y)}\right)^{-1/2} \tag{39}$$

Substituting Eq. (34) into Eq. (37) and Eq. (39) to get

$$-\frac{1}{N}\sum_{i=1}^{N} \frac{x_i^2}{1+\alpha x_i^2} + \frac{\langle x^2\rangle}{1+\alpha\langle x^2\rangle} = 0 \tag{40}$$

$$\Delta\alpha = \sqrt{\frac{N_0\left(\alpha\right)}{N}} \tag{41}$$

$$\begin{aligned} N_0\left(\alpha\right) &= \left(\langle \frac{1+\alpha x^2}{1+\alpha\langle x^2\rangle} \left(\frac{x^2}{1+\alpha x^2} - \frac{\langle x^2\rangle}{1+\alpha\langle x^2\rangle}\right)^2 \rangle\right)^{-1} \\ &= \left(1+\alpha\langle x^2\rangle\right) \left(\langle \frac{x^4}{1+\alpha x^2}\rangle - \frac{\langle x^2\rangle^2}{1+\alpha\langle x^2\rangle}\right)^{-1} \end{aligned} \tag{42}$$

$\langle x^2 \rangle$ and $\langle \frac{x^4}{1+\alpha x^2} \rangle$ can be evaluated using a Monte Carlo data sample generated with isotropic decay.

$$\langle x^2 \rangle = \frac{1}{N_{\text{sel}}} \sum_{i=1}^{N_{\text{sel}}} x_i^2 \tag{43}$$

$$\langle \frac{x^4}{1+\alpha x^2} \rangle = \frac{1}{N_{\text{sel}}} \sum_{i=1}^{N_{\text{sel}}} \frac{x_i^4}{1+\alpha x_i^2} \tag{44}$$

Using Monte Carlo events(generated by EPJPSI13) and the standard elastic J/ψ selection cuts, it is found that $\langle x^2 \rangle = 0.220$ and $\langle \frac{x^4}{1+0.95x^2} \rangle = 0.0667$ for $\alpha = 0.95$, and from Eq. (42) $N_0(0.95) = 45$.

Now consider a simple case. If J/ψ mesons are produced at rest, and the detector detects leptons within $(\theta_0, \pi - \theta_0)$ with uniform efficiency, then

$$\langle x^2 \rangle = \frac{1}{2x_0} \int_{-x_0}^{x_0} x^2 dx \tag{45}$$

$$\langle \frac{x^4}{1+\alpha x^2} \rangle = \frac{1}{2x_0} \int_{-x_0}^{x_0} \frac{x^4}{1+\alpha x^2} dx \tag{46}$$

where $x_0 = \cos\theta_0$. Figure 9(a) and (b) show N_0 as function of α and θ_0. Figure 9 (a) shows that even for a detector without holes($\theta_0 = 0°$), $\alpha = 0.95$ case requires $36/(\Delta\alpha)^2$ events. In general large α requires more statistics. Figure 9 (b) shows that N_0 increases rapidly with θ_0 for $\theta_0 > 30°$. This simple case is not very far from reality. For example, for $\alpha = 0.95$, $\theta_0 = 35°$(this corresponds to $-1.15 < y_{\text{rap}} < 1.15$), it is found that $\langle x^2 \rangle = 0.224$, $\langle \frac{x^4}{1+0.95x^2} \rangle = 0.0625$ and $N_0 = 57$, comparable to what was found using the more realistic Monte Carlo data.

A more direct approach is to plot $w(\alpha)$ curve versus α(Eq. (36)). At one standard deviation from the minimum point, w increases by 0.5. Since there is only one integration $\langle x^2 \rangle$ which is a constant independent of α, $w(\alpha)$ plot is extremely simple. The result is again that N_0 is about 50 for $\alpha \approx 1.0$.

Consistent results are obtained from all these methods. In order to measure $\alpha \approx 1.0$ with error $\Delta\alpha$ less than 0.5, more than 250 J/ψ events are needed. In the 1993 runs, 50-250 J/ψ events were expected (about 100 events were collected). Therefore, α could not be well determined($\Delta\alpha > 0.7$) for the 1993 runs.

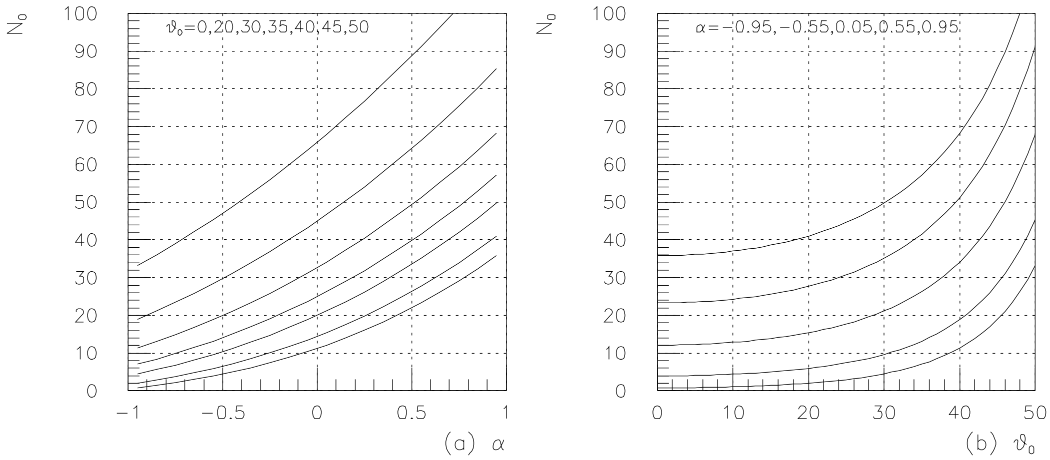

Figure 9: Minimum number of events needed for polarization measurement. Assuming J/ψ mesons are produced at rest and the detector detects leptons within $(\theta_0, \pi - \theta_0)$ with uniform efficiency. (a) N_0 as function of α for hole size of 0,20,30,35,40,45,50 degrees(upward from bottom). (b) N_0 as function of θ_0 for α =-0.95, -0.55, 0.05, 0.55, 0.95(upward from bottom).