2.4 Spin kinematics

The initial spin state of J/ψ is manifested in the angular distribution of the decay leptons in the J/ψ rest frame. The final spin state of e^+e^- must be an eigenstate of angular momentum of quantum numbers $J = 1$ and $J_\theta = \pm 1$ if the spin is measured, where θ (same as $\theta_{1'}$ in the previous section) denotes the direction of momentum vector of e^+(see Figure 6 (a)).[†]

The initial spin state of J/ψ is in general a mixed state[‡] of $J_z = \pm 1$ and 0, with weights $w_{\pm 1,0}$. Eigen states of J_z and J_θ(angular momentum in $\theta = 0$ direction and θ direction) are related by a rotation transformation. Therefore, for a given initial spin state described by $w_{\pm 1,0}$(polarization state), the angular distribution of e^+ in $J_\theta = \pm 1$ spin state is given by

$$\begin{aligned} f^{(\pm 1)}(\theta) &= w_1 \left(d^1_{\pm 1,1}\right)^2 + w_{-1}\left(d^1_{\pm 1,-1}\right)^2 + w_0^2\left(d^1_{\pm 1,0}\right)^2 \\ &= w_1\left(\frac{1 \pm \cos\theta}{2}\right)^2 + w_{-1}\left(\frac{1 \mp \cos\theta}{2}\right)^2 + w_0 \frac{\sin^2\theta}{2} \quad , \end{aligned} \tag{21}$$

where $w_{\pm 1,0}$ are constrained by the normalization requirement

$$w_1 + w_{-1} + w_0 = 1 \quad . \tag{22}$$

Since the final spin is definite but not measured in the experiment, the observed angular distribution is

$$f(\theta) = \frac{1 + \alpha\cos^2\theta}{2 + 2\alpha/3} \tag{23}$$

$$\alpha = \frac{1 - 3w_0}{1 + w_0} \quad . \tag{24}$$

Three special cases are shown in Figure 6 (b):

[†] We use Gottfried-Jackson frame in this discussion. ϕ is trivial and dropped for the reason explained in the next footnote.

[‡] Since beams are unpolarized and ϕ rotation about the z axis is not measured, the phases $e^{i\phi}$ of different J_z state,which can be changed by a rotation about z axis, are uncorrelated.

$\alpha = 1,\ w_0 = 0$	J/ψ is polarized along $\pm z$ axis
$\alpha = -1,\ w_0 = 1$	J/ψ is polarized perpendicular to $\pm z$ axis
$\alpha = 0,\ w_0 = 1/3$	J/ψ is unpolarized

Note that the value of α depends on the choice of z-axis. For example, if $\alpha = 1.0$ in Gottfried-Jackson frame, then $\alpha < 1.0$ in the helicity frame. Values of α in different frames are related by a rotation.

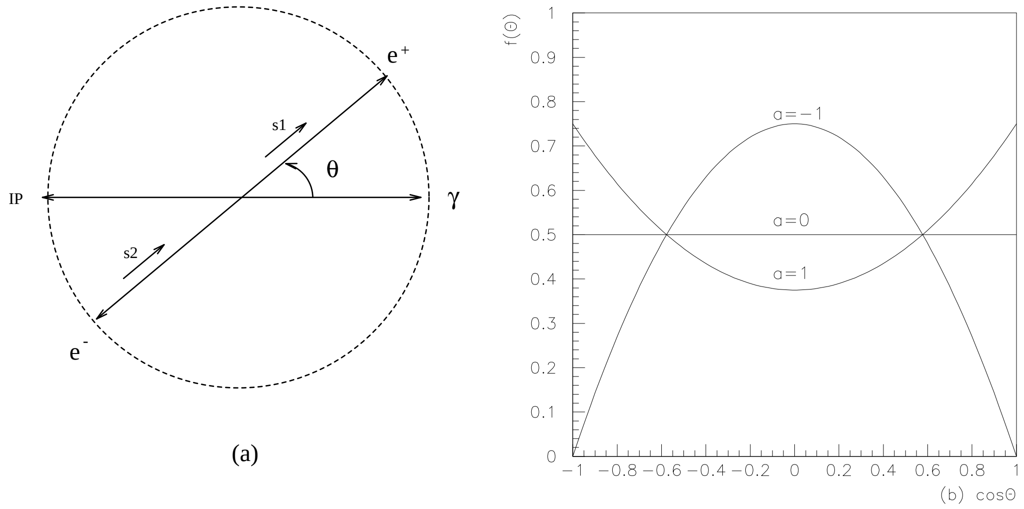

Figure 6: (a) J_θ may take either $+1$, as illustrated, or -1. (b) $\cos\theta$ distribution of e^+ for three polarization states $\alpha = 1, 0, -1$.