beam proton and electron escape through the forward and rear beampipes, respectively.[†]

2.2 Kinematics

In this section, studies of kinematics for elastic photoproduction of J/ψ is made in context of general J/ψ production. In our experiment in 1993, without detection of the beam electron and proton, the kinematical measurements are incomplete, as shown in the next subsection. This complicates the relation between the Lorentz invariant variables of theoretical interest and laboratory measurements. In this section, a thorough investigation of the kinematics is made to clarify what assumptions and approximations are essential for kinematical reconstruction of J/ψ photoproduction at ZEUS and the reconstruction formulae are derived and presented in a most precise form.

2.2.1 Degrees of freedom and kinematical variables

A final state is defined by a set of kinematic variables. A differential cross section, which is the integrand in the following equation,

$$\sigma = \int \frac{d^n\sigma}{dx_1 dx_2 \cdots dx_n} dx_1 dx_2 \cdots dx_n \tag{1}$$

is the probability for each final state$(x_1, x_2, \cdots, x_n)$ to occur. It is preferred to write the differential cross section with respect to a set of Lorentz invariant variables. These Lorentz invariant variables can be reconstructed from 4-momenta of the final particles measured in the laboratory.

Following symbols are used for 4-momenta of particles (refer to Figure 5):

[†] Due to finite resolution of the tracking chamber, and energy loss by Bremsstrahlung radiation in the case of e^+e^- decay, the measured invariant mass is a continuous distribution peaked near 3.1GeV.

e	4-momentum of the incoming beam electron
p	4-momentum of the incoming beam proton
e'	4-momentum of the outgoing electron
p'	4-momentum of the outgoing proton
q	4-momentum of the virtual photon
ψ	4-momentum of J/ψ

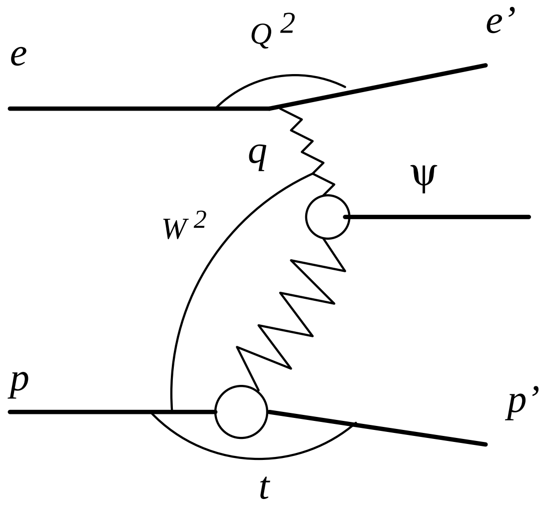

Figure 5: Illustration of kinematic variables.

A final state of elastic(inelastic) J/ψ production, before decay, consists of three prongs with three(two) known masses and four constraints from conservation of 4-momentum of the system, and hence has five(six) degrees of freedom. Not all of these five(six) degrees of freedom have the same dynamical importance, though. The differential cross section may be independent of or insensitive to certain variables due to certain symmetry or approximate symmetry. First, the (nominally) zero crossing angle of the electron and proton beams results in a rotational symmetry about the beam axis so that the differential cross section must be symmetric in ϕ, which is the azimuthal angle of the final states(e', p', J/ψ)

about the ep beam axis. This ϕ variable is thus trivial and only four(five) variables need to be specified for elastic(inelastic) J/ψ production. Next, we explore features of photoproduction. In photoproduction regime, the deflection angle of the beam electron is approximately zero(even less than electron beam emittance and was not measured in our experiment) and consequently, an approximate rotational symmetry about the γp beam axis exists so that the angle between $(\vec{\psi}, \vec{p'})$ plane and $(\vec{q}, \vec{p})$ plane is not measurable. Therefore only two(three) variables are left to be specified for elastic(inelastic) J/ψ photoproduction. This is not a surprise when we compare the zero crossing angle γp scattering with elastic(inelastic) zero crossing angle ep scattering, where cross section depends on two(three) variables s and Q^2(s, Q^2 and x).

Commonly used Lorentz invariant variables for J/ψ production in general (not only for photoproduction), of which only three are independent,[†] are defined by following equations:

$$Q^2 = -q^2 = 2e \cdot e' - 2m_e^2 > 0 \tag{2}$$

$$s_{\gamma p} = W^2 = (p+q)^2 = 2p \cdot q + m_p^2 - Q^2 \tag{3}$$

$$y = \frac{2p \cdot q}{\tilde{s}} \tag{4}$$

$$t = \left(p - p'\right)^2 = (\psi - q)^2 = m_{J/\psi}^2 - 2q \cdot \psi - Q^2 < 0 \tag{5}$$

$$z = \frac{p \cdot \psi}{p \cdot q} \tag{6}$$

where $\tilde{s} = 2p \cdot e$.

$s_{\gamma p}$ is the square of the center of mass energy of γp system. Dependence of elastic photoproduction cross section on this variable has been of major interest. Variable t measures how hard the proton is scattered, and the differential cross section is most sensitive to this variable. Q^2 is a measure of the virtuality of the photon, as can be clearly seen in its definition. y is the fraction of electron energy carried by the virtual photon as seen in the proton rest frame(or in any

[†] They can all be expressed as functions of $e \cdot e'$, $p \cdot q$ and $q \cdot \psi$.

colliding frame in the case of photoproduction). z is the fraction of photon energy transferred to J/ψ as seen in the proton rest frame. The z variable is historically known as "inelasticity", because in inelastic J/ψ production, $z < 1$ and in elastic J/ψ production $z \approx 1$. It should be noted that many previous measurements of elastic J/ψ photoproduction cross section were based on $Z > 0.95$ criterion which did not distinguish elastic and diffractive J/ψ events.

Since only three of these variables are independent, they are not sufficient to describe J/ψ production in most general case with six degrees of freedom. However, that is not our concern here. For J/ψ photoproduction, dynamically important degrees of freedom reduce to three as discussed before. For example, (s_γ, t, z) is a complete set of kinematical variables for J/ψ photoduction. For elastic J/ψ photoproduction, only two degrees of freedom are dynamically important. The set (s_γ, t), for example, is complete. y can be expressed as a function of $(s_{\gamma p}, t)$ with $Q^2 \approx 0$ assumption, and z can be expressed as a function of $(s_{\gamma p}, t)$ with additional elastic assumption($m_{p'} = m_p$), as shown in the following equations.

$$y = \frac{s_{\gamma p} - m_p^2}{\tilde{s}} \tag{7}$$

$$z = 1 - \frac{m_{p'}^2 - m_p^2 - t}{W^2 - m_p^2} \tag{8a}$$

$$= 1 - \frac{(-t)}{s_{\gamma p} - m_p^2} \tag{8b}$$

2.2.2 Reconstruction of kinematical variables

If the kinematical measuerements are complete, all Lorentz variables of theoretical interest can be reconstructed from 4-momentum measurement in the laboratory(some equations can be found inAppendix A).

For photoproduction, kinematical reconstruction is made on assumption of $Q^2 \approx 0$. In our experiment, by requiring $\theta_{e'} > 176°$ and both decay leptons be detected in the central tracking detector, Q^2 is upbounded at about $4\mathrm{GeV}^2$, as will be shown in Section 2.5. Since in photoproduction regime, cross section is dynamically dominated by very small Q^2(as also shown in Section 2.5), $Q^2 \approx 0$ approximation is quite valid.

Three measurements are required to reconstruct $(s_{\gamma p}, t, y, z)$(of which three are independent). With photoproduction assumption $\cos\theta_{e'} \approx -1$, $(p \cdot \psi, q \cdot \psi, p \cdot q)$ can be expressed in terms of HERA laboratory measurables E_γ, E and p_z of J/ψ, as follows,

$$p \cdot \psi = E_p \left(E - p_z\right)_\psi \tag{9}$$

$$q \cdot \psi = E_\gamma \left(E + p_z\right)_\psi \tag{10}$$

$$p \cdot q = 2E_p E_\gamma \tag{11}$$

We then substitute these into Eq. (3-6) to get

$$s_{\gamma p} = W^2 = 4E_p E_\gamma + m_p^2 \tag{12}$$

$$|t| = -t = 2E_\gamma \left(E + p_z\right)_\psi - m_{J/\psi}^2 \tag{13}$$

$$y = \frac{E_\gamma}{E_e} \tag{14}$$

$$z = \frac{\left(E - p_z\right)_\psi}{2E_\gamma} \tag{15}$$

$(E+p_z)_\psi$ and $(E-p_z)_\psi$ can be derived from (E,θ) of J/ψ measurements. E_γ was not measured in the 1993 runs.† Therefore, the 1993 measurements were incomplete for J/ψ photoproduction.

For elastic J/ψ photoproduction(with two degrees of freedom), only two measurements are needed. With elastic assumption $m_{p'} = m_p$, E_γ can be solved from Eq. (8b) and Eq. (12-15), and kinematics can be reconstructed using Eq. (17a-20b)

$$E_\gamma = \frac{\frac{(E-p_z)_\psi}{2} - \frac{m_{J/\psi}^2}{4E_p}}{1 - \frac{(E+p_z)_\psi}{2E_p}} \tag{16a}$$

$$\approx \frac{(E-p_z)_\psi}{2} \tag{16b}$$

In summary, for inelastic J/ψ photoproduction, if energy loss of the beam electron, which equals the photon energy E_γ, is measured, kinematics can be reconstructed using Eq. (12-15), where the only assumption is $Q^2 \approx 0$ or equivalently $\cos\theta_{e'} \approx -1$; For elastic($m_{p'} = m_p$) J/ψ photoproduction($Q^2 \approx 0$), kinematics can be reconstructed using Eq. (17a-20b), where further approximations denoted by $\approx$ have been widely used but are not essential.

$$s_{\gamma p} = W^2 = 2E_p \frac{(E-p_z)_\psi - \frac{m_{J/\psi}^2}{2E_p}}{1 - \frac{(E+p_z)_\psi}{2E_p}} \tag{17a}$$

$$\approx 2E_p (E-p_z)_\psi \tag{17b}$$

$$|t| = -t = \frac{(E-p_z)_\psi (E+p_z)_\psi - \frac{m_{J/\psi}^2}{2E_p}(E+p_z)_\psi}{1 - \frac{(E+p_z)_\psi}{2E_p}} - m_{J/\psi}^2 \tag{18a}$$

$$\approx \vec{pt}_{J/\psi}^{\,2} \tag{18b}$$

$$y = \frac{\frac{(E-p_z)_\psi}{2E_e} - \frac{m_{J/\psi}^2}{s}}{1 - \frac{(E+p_z)_\psi}{2E_p}} \tag{19a}$$

† There is almost no overlap between EDET(luminosity electron detector) coverage $\sqrt{s_{\gamma p}} = 167 - 194\text{GeV}$ and CTD coverage for elastic J/ψ $\sqrt{s_{\gamma p}} = 30 - 170\text{GeV}$.

$$\approx \frac{(E-p_z)_\psi}{2E_e} \tag{19b}$$

$$z = \frac{1-\frac{(E+p_z)_\psi}{2E_p}}{1-\frac{m^2_{J/\psi}}{1+m^2_{J/\psi}}\frac{(E+p_z)_\psi}{2E_p}} \tag{20a}$$

$$\approx 1 \tag{20b}$$

2.3 Reconstruction of the J/ψ meson

The J/ψ is reconstructed through the measurement of the two decay leptons. The decay process $J/\psi \to l^+l^-$ is most conveniently described in the J/ψ rest frame where l^+l^- move back to back, each with energy of $\frac{1}{2}m_{J/\psi}$, and the angle of one lepton in J/ψ rest frame, e.g., $(\cos\theta_{1'}, \phi_{1'})$ of l^+, defines the decay kinematic state. Choice of z' axis($\theta_{1'} = 0$) is not unique. Helicity frame takes $(-\vec{p'}_{CM})$ as the z' axis. Gottfried-Jackson frame takes $\vec{q}_{CM}$(photon direction) as the z' axis. Subscript CM denotes the vector as measured in J/ψ rest frame. An interesting question is how the spin state of J/ψ is correlated to that of the photon. Spin state of J/ψ can be determined from the angular distribution of the two leptons as will be explained later in this chapter. Since beam electron and proton are not measured in our experiment, we simply choose the electron beam direction as seen in J/ψ rest frame, as z' axis(this is close to Gottfried-Jackson frame).

Detection of the l^+ and l^- provides six measurements but only four nontrivial results because of the invariant mass constraint $m_{l^+l^-} = m_{J/\psi}$ and an undetermined rotation angle of l^+l^- about ep beam axis when e' and p' are not detected. The four nontrivial results are $(E \pm p_z)_\psi$ and $(\cos\theta_{1'}, \phi_{1'})$. $(\cos\theta_{1'}, \phi_{1'})$ are not relevant to the production cross section, but is closely related to the spin state of J/ψ. The issue will be discussed in the next section.