BES Analysis Memo
MEMO # 1: **release 0.1**
χ_c Decays to High Multiplicity $\pi^+\pi^-$ Pairs

Gary S. Varner
University of Hawaii
varner@uhhepi.phys.hawaii.edu

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Abstract

An analysis is performed of χ_c decays to high multiplicity $\pi^+\pi^-$ pairs. Numerous techniques for removing γ conversion background from $\psi' \rightarrow n(\pi^+\pi^-)\pi^0$ were evaluated, necessitated by the low momentum spectrum of mimimum mass pairs for such a large number of $m_{\pi\pi}$ combinations. Re-analysis of the $3(\pi^+\pi^-)$ channel indicates that the dominant decay is not a direct decay to $6\pi^\pm$, but mediated by at least 1 intermediate ρ^0. First observation of $\chi_{c0,1,2}$ decays to the $4(\pi^+\pi^-)$ final state is reported. Upper limits are reported for $\chi_{c0,1,2}$ decays to the $5(\pi^+\pi^-)$ final state.

1 Introduction

Utilizing the world's largest sample of resonant ψ' decays, as measured by the BES collaboration during 1993-1995, a detailed study of the hadronic decay modes of the χ_c states has been undertaken. Compared with the data set for the Mark I detector[1], roughly an order of magnitude larger data set has been obtained.

Taking advantage of these larger statistics, many new modes have been observed[2], while others have been refined. In particular, the analysis of hadronic decay modes with a high multiplicity of $(\pi^+\pi^-)$ pairs in the final state has benefitted from larger statistics.

While reconstruction of the χ_c states is contaminated with background from direct ψ' decays, the predominant source of this background is due to $\psi' \rightarrow \pi^0 n(\pi^+\pi^-)$. In these background events, one of the γ's from the π^0 decay converts in the beampipe/detector and is mis-identified as a $\pi^+\pi^-$ pair, while the other γ is reconstructed as if from a radiative ψ' decay. Techniques for and results from removing such background are discussed below.

2 Event Selection

The entire BES ψ' data set has been used for this analysis, which corresponds to $(3.67 \pm 0.03 \pm 0.36) \times 10^6$ produced ψ's[3]. For the determination of the number of $\chi_{c_{0,1,2}}$ produced from the radiative decays of the ψ's, results from the Crystal Ball detector[4] are used. This choice is dictated by the much better γ detection efficiency and uniformity of this non-magnetic detector measurement. The values used are tabulated below:

Mode	$Value \pm \sigma_{stat} \pm \sigma_{syst}$
$\psi' \rightarrow \gamma + \chi_{c_0}$	$9.9 \pm 0.5 \pm 0.8$ %
$\psi' \rightarrow \gamma + \chi_{c_1}$	$9.0 \pm 0.5 \pm 0.7$ %
$\psi' \rightarrow \gamma + \chi_{c_2}$	$8.0 \pm 0.5 \pm 0.7$ %

Analysis proceeds on events in the event summaries which contain 5 or more charged tracks (5PRN). Events are selected with exactly the number of charged tracks sought, with zero net charge, and with at least one of the γ's within the detector barrel region $|cos\theta| < 0.8$. Events are rejected where the highest momentum charged track pair are identified as muons or where these tracks leave excessive energy $E_{BSC} > 0.8GeV$. For the remaining events, kinematical fitting is performed in two separate ways, and the results of the two fits are compared for consistency.

2.1 Fitting

A full ψ' kinematic fit and a partial $n(\pi^+\pi^-)$ kinematic fit is performed for each event. For these tasks the BES standard routines *NEWSQUID* and *SQUIDJR*, respectively, are used. For a $\gamma\, n(\pi^+\pi^-)$ final state, the *NEWSQUID* fit is a (4)C-fit (including the "best" γ in the event), while the *SQUIDJR* fit is a (3)C-fit making use of only the charged pions. The motivation for considering the two fits separately is to compare the performance of the charged tracking fit both with and without reconstruction of the neutral energy in an event, since the γ reconstruction efficiency of BES is relatively modest[5] and not well characterized[6].

For the full kinematic fit, in the case of events with more than one accepted photon candidate, the fits are applied for each γ candidate and the one that gives the best fit is chosen. In the partial reconstruction, the mass of the resonance is varied from $2.9 - 3.7 GeV$, and the resonance mass giving the best χ^2 fit is accepted.

For each event, the confidence level of each kinematic fit is required to be better than 5%. It is interesting to note that the partial reconstruction almost always selected a mass giving $\sim 100\%$ confidence level to the fit.

2.2 General Cuts

Below are listed the general cuts used for analyzing all of the high-multiplicity $\pi^+\pi^-$ modes. In addition to the constraints mentioned briefly above, specific cuts are applied as tabulated below:

Cut Parameter	Value	Comment
mfit	2, -9, -19	use only tracks with good fits
# of $\mu^\pm$	< 2	remove μ pairs as seen by the muon chambers
E_{BSC}	< 0.8 GeV	remove e^+e^- via EM showers in the Barrel Shower Counter (BSC)
$\|cos\theta\|$	< 0.8	acceptance cut on charged, neutral particles
$\|m_{rec}$ - $m_{\psi'}\|$	> 25 MeV	remove ψ' background
$\|m_{rec} - m_{\pi\pi}$ - $m_{J/\psi}\|$	> 25 MeV	remove decays involving J/ψ's

Making use of these generic cuts, a separate π^0 rejection cut is applied to each of the modes subsequently considered. The general strategy for π^0 rejection is considered below, with the removal of background in the $3(\pi^+\pi^-)$ case given as an example. In addition, an example of the criterion used to select the cuts for the $4(\pi^+\pi^-)$ case is provided graphically.

2.3 π^0 Rejection

In the high multiplicity $n(\pi^+\pi^-)$ modes, a modest and rather uniform background is seen as in Figure 1a. To attempt to remove these background events, cuts on the details of the event kinematics and topology, such as the following items were tried:

- Straightforward cuts
  - $M_{\pi\pi}^{min}$ - Minimum $\pi\pi$ mass
  - $cos(\pi\pi)_{max}$ - $\pi\pi$ opening angle cut
- Slight variants
  - $M_{\pi\pi}^{min}$ for $cos(\pi\pi)_{max}$
  - $cos(\pi\pi)_{max}$ for $M_{\pi\pi}^{min}$
- Other techniques
  - Use Drift Chamber $\frac{dE}{dx}$
  - Fox-Wolfram analysis
- π^0 Reconstruction
  - Calc. *vs.* Measured $cos(\gamma, \pi^+\pi^-)$
  - Apply for both $cos(\pi\pi)_{max}$, $M_{\pi\pi}^{min}$
  - $\Delta(\vec{p_{\pi^+}} - \vec{p_{\pi^-}})$ for $cos(\pi\pi)_{max}$, $M_{\pi\pi}^{min}$

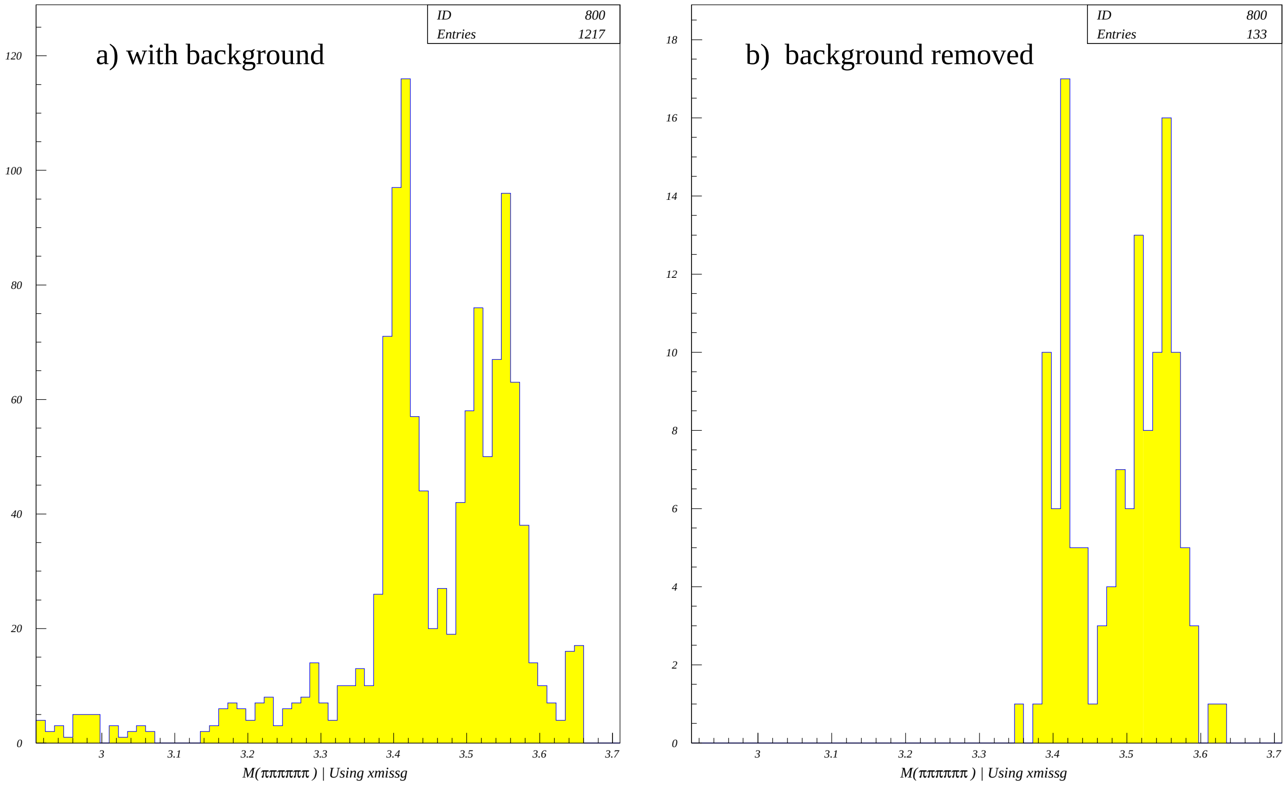


Figure 1: a) Figure showing the χ_c spectrum of events for decays to the $3(\pi^+\pi^-)$ mode with a uniform background due to π^0 γ conversions. In b) showing the same χ_c spectrum as that for a) with cuts to remove π^0 γ conversion background, leaving clean χ_c signatures.

In Figure 1b the success of these combined cuts are shown, with a clear removal of all of the low and high mass background around the χ_c peaks. Each of the techniques itemized above were tried with rather mixed results. A detailed analysis of these parameters for simulation data, as well as for in peak and out of peak data indicate little discriminatory power for all but one parameter. This is at first suprising, given that one might expect the minimum mass pair cut to be more effective in removing background. Especially in light of the fact that the conversion electron pair would be expected to be reconstructed as a low mass π pair. However, this cut criterion also cuts most of the good events, as the probability of a low mass combination, with such a large number of permutations, is sizeable.

An extensive and ultimately unproductive line of attack was to attempt to "reconstruct" the π^0 from the candidate radiative γ and a back-calculated conversion γ, in terms of the candidate fake $\pi^+\pi^-$ pairs. Again, because of the large number of combinations, the imprecision in the tracking due to ignorance of the conversion point, and the low momentum and spherical nature of the event, little discrimination between real and background events is obtained.

2.4 Choice of Cut Parameter Value

Eventually a moderately successful cut parameter was found. Based on the fact that the γ conversion e^+e^- pair are correlated, the properties of this correlation were analyzed. While a cut on the opening angle is somewhat effective, in fact because of hard elastic multiple scattering and track reconstruction errors, the effectiveness is limited. However, if the difference in momentum of the two tracks (or equivalently, the energy) of the candidate conversion pair are considered, a more effective cut parameter is realized. The physical motivation of this cut is that γ conversion $e^\pm$ should carry away roughly equal momentum, even in the case of subsequent hard elastic multiple-scattering.

In Figure 2, a comparison of a number of event cut parameters is shown for the case of events in the background and in the peaks of the χ_c mass distribution for the $4(\pi^+\pi^-)$ final state. In each data set, the cut on the momentum difference for the minimum mass π pair is identified. Because this cut isn't completely effective by itself, it is used with a combination of looser cuts, justified by their minimal impact on Monte-Carlo events, while providing some background suppression. Because the kinematics vary somewhat with the number of final state $\pi^+\pi^-$ pairs, the specific cut values used are documented in each analysis section that follows.

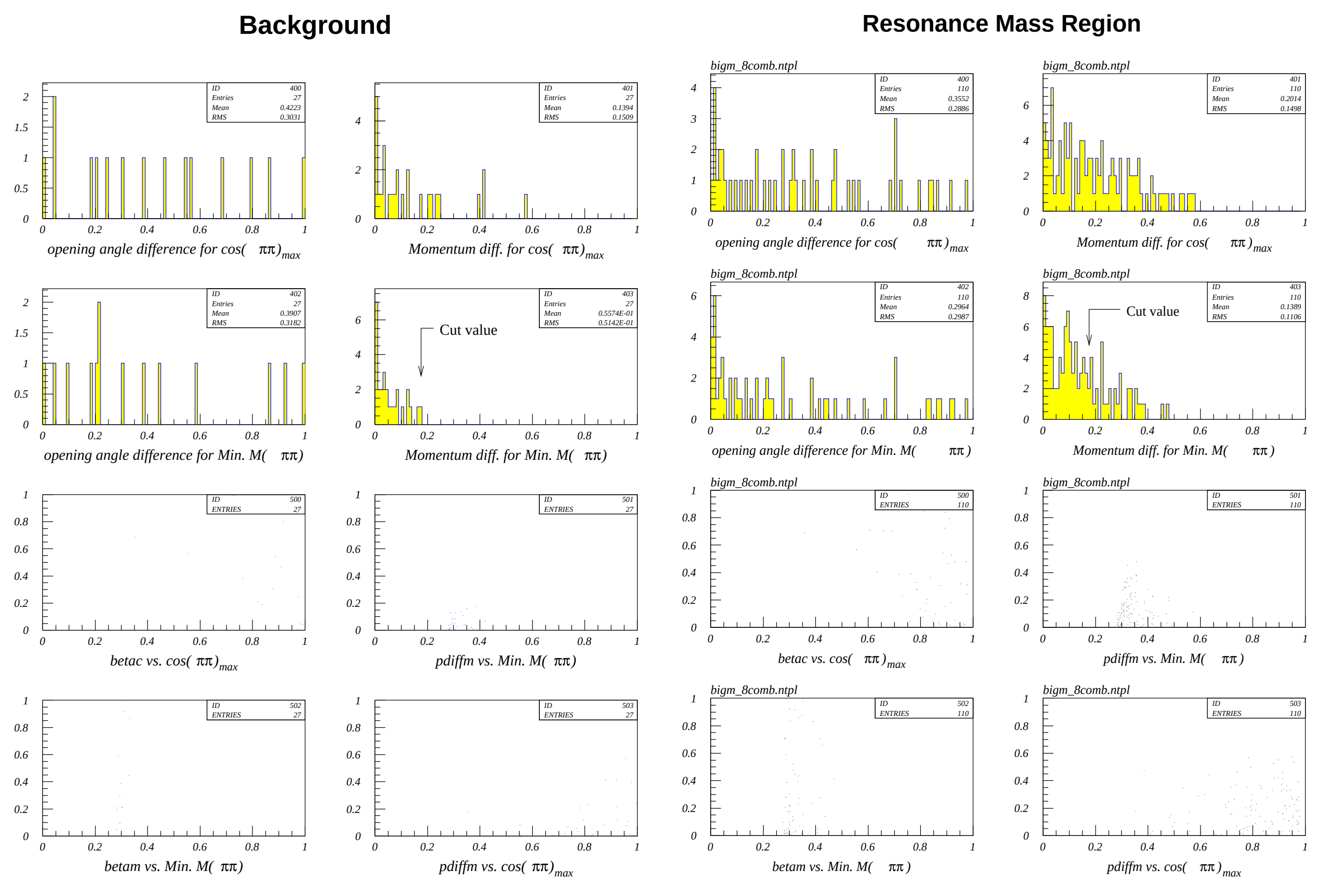


Figure 2: Plots from the background and peak of the reconstructed χ_c mass spectrum for $4(\pi^+\pi^-)$ decays, showing the efficiency of cuts to remove π^0 γ conversion background. Cutting on the momentum difference between the minimum $\pi^+\pi^-$ mass pair is the most effective discriminator, as indicated by the label "Cut value". The physical motivation of this cut is that the γ conversion electron and positron should carry away roughly equal momentum, even in the case of subsequent hard multiple-scattering.

3 $3(\pi^+\pi^-)$ Results

In the preliminary analysis, after applying cuts to remove background, drastically different values of the branching ratio for direct $\chi_{cJ} \to 3(\pi^+\pi^-)$ were obtained with respect to previous values. After carefully checking, no apparent error in the process was detected. A proper description of the kinematics of the χ_c decay should remain constant, within errors, as the signal-to-noise ratio of the proper decay channel is varied.

This revelation led to a reinvigoration of a previous attempt to determine the ρ^0 intermediate state content of the $3(\pi^+\pi^-)$ observed final states. Further investigation revealed that the combined cuts to remove background are a tool for selecting the proper $m_{\pi\pi}$ momentum multivariate distributions. While something like a hypothetical higher dimensional Dalitz plot would probably reveal the same internal correlations, the background cut parameters are a simultaneously useful tool for improving the signal-to-noise ratio.

After considering the various intermediate state hypothesis, it was decided that an analysis should consider the possibility of 0, 1 and 2 ρ^0's. It should be noted that the χ_{cJ} decays to $3\rho^0$ are Isospin forbidden[7]. To investigate how the branching ratios "scale" for each of the intermediate state hypotheses, three background levels were considered. These correspond to roughly 0, 50, and 100% π^0 γ conversion background removal. In Figure 3 are shown the χ_{cJ} mass spectra, with gaussian fits and a background fit (as appropriate) for each value of the background removal.

To obtain these background cuts, the values used are as shown in the table below. Only differential values are shown, as common cut values are described previously. A fit to the χ_c spectrum, after each of these cuts, is shown in Figure 3. As was claimed previously, the low and high mass tails may be fit nicely with a uniform background. Figure 4 contains the tabulated values, including the results of detector acceptance simulation, used to calculate the branching ratios for the three background cases considered.

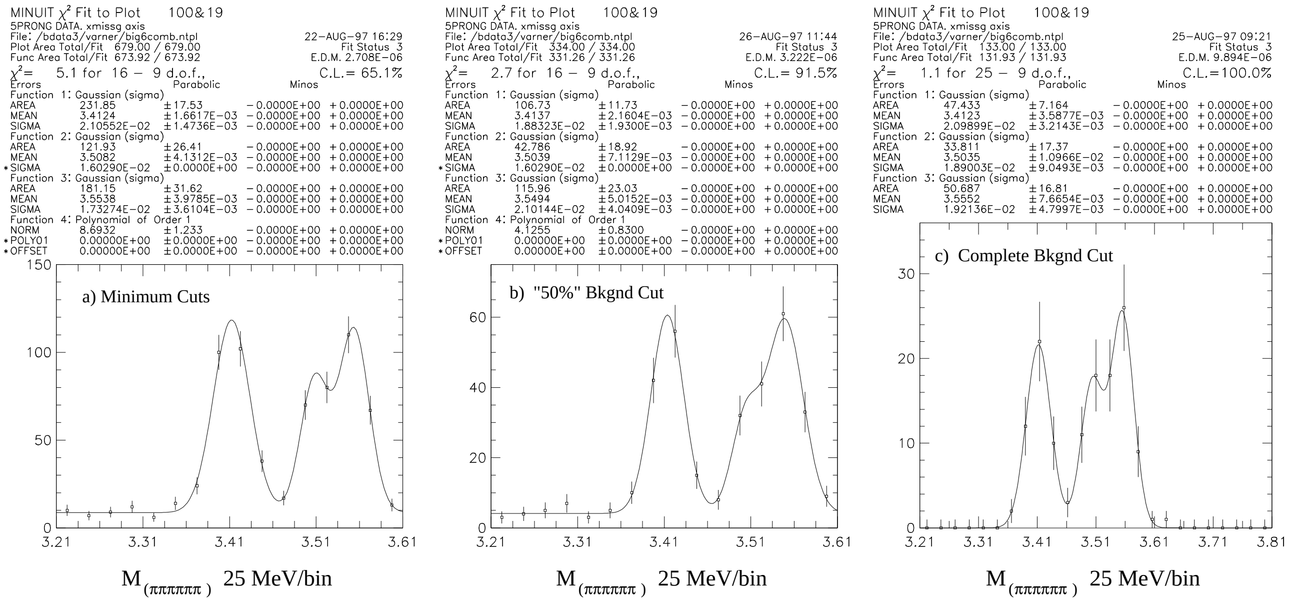


Figure 3: Data with gaussian fits to the χ_c mass peaks for 3 different background cut scenarios: a) no π^0 cut; b) approximately a 50% background cut; c) a complete removal of background from π^0 γ conversion.

a) 3(π+π−) Analysis **0**ρ "0% Bkgnd Cut"

	<Value>	σ stat	σ sys		MC σ sys:
N(ψ')	3.67E+06	3.00E+04	3.60E+05		20%

	γ+χc0			γ+χc1			γ+χc2		
B(ψ' -> X)	9.9%	0.5%	0.8%	9.0%	0.5%	0.7%	8.0%	0.5%	0.7%
ηdet	4.20%	0.07%	0.84%	4.75%	0.12%	0.95%	4.39%	0.12%	0.88%
N(χc)	363330	18589	46176	330300	18548	41349	293600	18506	38593
N(χc) det	15260	821	3616	15689	967	3702	12889	886	3085
Nevts	230	17	0	136	15	0	164	17	0
B(X->6(π+/−)	1.51%	0.14%	0.36%	0.87%	0.11%	0.20%	1.27%	0.16%	0.30%
	1.5%	0.1%	0.4%	0.9%	0.1%	0.2%	1.3%	0.2%	0.3%
combined Errors:	1.51%	0.38%		0.87%	0.23%		1.27%	0.34%	

b) 3(π+π−) Analysis **0**ρ "50% Bkgnd Cut"

	<Value>	σ stat	σ sys		MC σ sys:
N(ψ')	3.67E+06	3.00E+04	3.60E+05		20%

	γ+χc0			γ+χc1			γ+χc2		
B(ψ' -> X)	9.9%	0.5%	0.8%	9.0%	0.5%	0.7%	8.0%	0.5%	0.7%
ηdet	1.86%	0.04%	0.37%	2.15%	0.08%	0.43%	1.93%	0.08%	0.39%
N(χc)	363330	18589	46176	330300	18548	41349	293600	18506	38593
N(χc) det	6758	375	1601	7101	478	1676	5666	427	1356
Nevts	105	11	0	62	12	0	98	14	0
B(X->6(π+/−))	1.56%	0.19%	0.37%	0.87%	0.18%	0.20%	1.74%	0.29%	0.42%
	1.6%	0.2%	0.4%	0.9%	0.2%	0.2%	1.7%	0.3%	0.4%
combined Errors:	1.56%	0.42%		0.87%	0.27%		1.74%	0.51%	

c) 3(π+π−) Analysis **0**ρ "100% Pi^0 Cut"

	<Value>	σ stat	σ sys		MC σ sys:
N(ψ')	3.67E+06	3.00E+04	3.60E+05		20%

	γ+χc0			γ+χc1			γ+χc2		
B(ψ' -> X)	9.9%	0.5%	0.8%	9.0%	0.5%	0.7%	8.0%	0.5%	0.7%
ηdet	0.38%	0.02%	0.08%	0.48%	0.03%	0.10%	0.43%	0.03%	0.09%
N(χc)	363330	18589	46176	330300	18548	41349	293600	18506	38593
N(χc) det	1381	101	327	1585	133	374	1262	119	302
Nevts	44	7	0	33	7	0	44	8	0
B(X->6(π+/−))	3.20%	0.54%	0.76%	2.07%	0.48%	0.49%	3.47%	0.70%	0.83%
	3.2%	0.5%	0.8%	2.1%	0.5%	0.5%	3.5%	0.7%	0.8%
combined Errors:	3.20%	0.93%		2.07%	0.68%		3.47%	1.09%	

Figure 4: An array of tables showing the detector acceptance (η_{det}) and obtained branching ratios for different background cut values for direct $6\pi^{\pm}$ production (0 intermediate ρ^0's): (a) no π^0 background cut; (b) $\sim$ 50% cut; and (c) complete background removal. Observe that these branching ratios are not constant versus background cut.

$3(\pi^+\pi^-)$ Cut Values

Cut %	Cut Parameter	Value	Comment
0%			
	$\Delta p, \Delta m$	$\leq 200 MeV$	γ kinematics match reconstructed recoil resonance
50%			
	$m_{\pi\pi}(cos_{max}(\pi\pi))$	$\geq 480 MeV$	cut on mass of the minimum opening angle pair (max. value)
	$\Delta p_{m_{\pi\pi}}$	$\geq 140 MeV$	momentum difference of minimum mass pair
100%			
	$m_{\pi\pi}(cos_{max}(\pi\pi))$	$\geq 480 MeV$	cut on mass of the minimum opening angle pair (max. value)
	$\Delta p_{m_{\pi\pi}}$	$\geq 140 MeV$	momentum difference of minimum mass pair
	$\Delta p_{m_{\pi\pi}}$	$\geq 250 MeV$	for $cos_{max}(\pi\pi) \geq 0.8$
	β_c	≥ 0.02	opening angle difference: calculated versus best-fit π^0 γ pair
	β_m	≥ 0.03	same cut as β_c, applied to specifically to the minimum $m_{\pi\pi}$

In Figure 5 are listed the analysis tables as given in Figure 4, except now considering the insertion of $1\rho^0$ and $2\rho^0$ final states. Note that the values for certain χ_{c_J} are more stable versus background cut in these cases, and that the detection efficiency for these events increases slightly with the number of intermediate ρ's.

a)
3(π+π−) Analysis
1ρ
"0% Bkgnd Cut"

	<Value>	σ stat	σ sys	MC σ sys:
N(ψ')	3.67E+06	3.00E+04	3.60E+05	20%

	γ+χc0			γ+χc1			γ+χc2		
B(ψ' -> X)	9.9%	0.5%	0.8%	9.0%	0.5%	0.7%	8.0%	0.5%	0.7%
ηdet	4.65%	0.40%	0.93%	5.66%	0.40%	1.13%	4.95%	0.39%	0.99%
N(χc)	363330	18589	46176	330300	18548	41349	293600	18506	38593
N(χc) det	16895	1691	4003	18695	1687	4411	14533	1466	3478
Nevts	230	17	0	136	15	0	164	17	0
B(X->1ρ2(π+−))	1.36%	0.17%	0.32%	0.73%	0.10%	0.17%	1.13%	0.16%	0.27%
	1.4%	0.2%	0.3%	0.7%	0.1%	0.2%	1.1%	0.2%	0.3%
combined Errors:	1.36%	0.36%		0.73%	0.20%		1.13%	0.32%	

b)
3(π+π−) Analysis
1ρ
"50% Bkgnd Cut"

	<Value>	σ stat	σ sys	MC σ sys:
N(ψ')	3.67E+06	3.00E+04	3.60E+05	20%

	γ+χc0			γ+χc1			γ+χc2		
B(ψ' -> X)	9.9%	0.5%	0.8%	9.0%	0.5%	0.7%	8.0%	0.5%	0.7%
ηdet	2.31%	0.15%	0.46%	2.86%	0.30%	0.57%	2.66%	0.29%	0.53%
N(χc)	363330	18589	46176	330300	18548	41349	293600	18506	38593
N(χc) det	8393	694	1989	9447	1124	2229	7810	984	1869
Nevts	105	11	0	62	12	0	98	14	0
B(X->1ρ2(π+−))	1.25%	0.17%	0.30%	0.65%	0.15%	0.15%	1.26%	0.24%	0.30%
	1.3%	0.2%	0.3%	0.7%	0.1%	0.2%	1.3%	0.2%	0.3%
combined Errors:	1.25%	0.34%		0.65%	0.21%		1.26%	0.39%	

c)
3(π+π−) Analysis
1ρ
"100% Pi^0 Cut"

	<Value>	σ stat	σ sys	MC σ sys:
N(ψ')	3.67E+06	3.00E+04	3.60E+05	20%

	γ+χc0			γ+χc1			γ+χc2		
B(ψ' -> X)	9.9%	0.5%	0.8%	9.0%	0.5%	0.7%	8.0%	0.5%	0.7%
ηdet	0.66%	0.08%	0.13%	0.99%	0.22%	0.20%	1.02%	0.21%	0.20%
N(χc)	363330	18589	46176	330300	18548	41349	293600	18506	38593
N(χc) det	2398	315	568	3270	750	772	2995	645	717
Nevts	44	7	0	33	7	0	44	8	0
B(X->1ρ2(π+−))	1.84%	0.37%	0.44%	1.00%	0.32%	0.24%	1.46%	0.41%	0.35%
	1.8%	0.4%	0.4%	1.0%	0.3%	0.2%	1.5%	0.4%	0.3%
combined Errors:	1.84%	0.57%		1.00%	0.39%		1.46%	0.54%	

d)
3(π+π−) Analysis
2ρ
"0% Bkgnd Cut"

	<Value>	σ stat	σ sys	MC σ sys:
N(ψ')	3.67E+06	3.00E+04	3.60E+05	20%

	γ+χc0			γ+χc1			γ+χc2		
B(ψ' -> X)	9.9%	0.5%	0.8%	9.0%	0.5%	0.7%	8.0%	0.5%	0.7%
ηdet	5.77%	0.24%	1.15%	5.77%	0.41%	1.15%	6.01%	0.41%	1.20%
N(χc)	363330	18589	46176	330300	18548	41349	293600	18506	38593
N(χc) det	20964	1382	4968	19058	1726	4497	17645	1639	4223
Nevts	230	17	0	136	15	0	164	17	0
B(X->2ρ(π+−))	1.10%	0.11%	0.26%	0.71%	0.10%	0.17%	0.93%	0.13%	0.22%
	1.1%	0.1%	0.3%	0.7%	0.1%	0.2%	0.9%	0.1%	0.2%
combined Errors:	1.10%	0.28%		0.71%	0.20%		0.93%	0.26%	

e)
3(π+π−) Analysis
2ρ
"50% Bkgnd Cut"

	<Value>	σ stat	σ sys	MC σ sys:
N(ψ')	3.67E+06	3.00E+04	3.60E+05	20%

	γ+χc0			γ+χc1			γ+χc2		
B(ψ' -> X)	9.9%	0.5%	0.8%	9.0%	0.5%	0.7%	8.0%	0.5%	0.7%
ηdet	3.04%	0.18%	0.61%	3.28%	0.30%	0.66%	3.21%	0.29%	0.64%
N(χc)	363330	18589	46176	330300	18548	41349	293600	18506	38593
N(χc) det	11045	864	2617	10834	1163	2556	9425	1038	2256
Nevts	105	11	0	62	12	0	98	14	0
B(X->2ρ(π+−))	0.95%	0.13%	0.23%	0.57%	0.13%	0.13%	1.04%	0.19%	0.25%
	1.0%	0.1%	0.2%	0.6%	0.1%	0.1%	1.0%	0.2%	0.2%
combined Errors:	0.95%	0.26%		0.57%	0.18%		1.04%	0.32%	

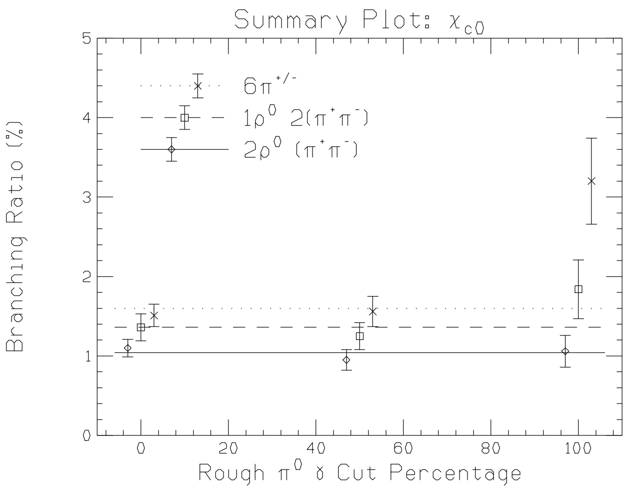
f)
3(π+π−) Analysis
2ρ
"100% Pi^0 Cut"

	<Value>	σ stat	σ sys	MC σ sys:
N(ψ')	3.67E+06	3.00E+04	3.60E+05	20%

	γ+χc0			γ+χc1			γ+χc2		
B(ψ' -> X)	9.9%	0.5%	0.8%	9.0%	0.5%	0.7%	8.0%	0.5%	0.7%
ηdet	1.15%	0.11%	0.23%	1.35%	0.15%	0.27%	1.04%	0.14%	0.21%
N(χc)	363330	18589	46176	330300	18548	41349	293600	18506	38593
N(χc) det	4178	453	990	4459	555	1052	3053	454	731
Nevts	44	7	0	33	7	0	44	8	0
B(X->2ρ(π+−))	1.06%	0.20%	0.25%	0.74%	0.18%	0.17%	1.43%	0.33%	0.34%
	1.1%	0.2%	0.3%	0.7%	0.2%	0.2%	1.4%	0.3%	0.3%
combined Errors:	1.06%	0.32%		0.74%	0.25%		1.43%	0.48%	

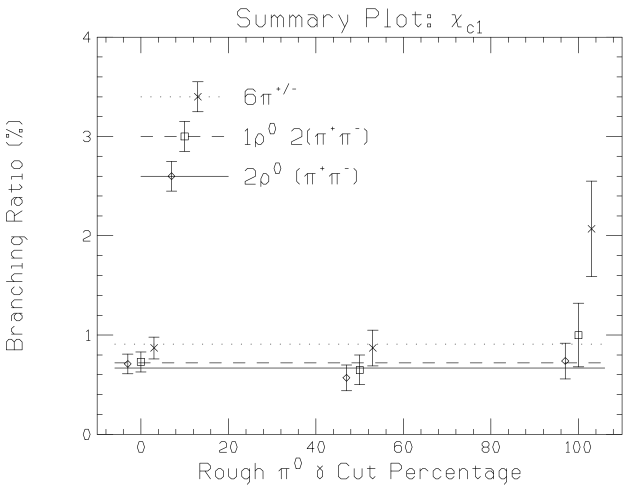
Figure 5: A two-fold array of tables similar to those of Figure 4. On the left, (a) - (c) is the tabulation for 1 intermediate ρ^0; on the right, (d) - (f) are the values for 2 intermediate ρ^0. It is interesting to note that the detection efficiency increases slightly with the number of intermediate ρ^0's.

A grand summary of summaries is given in Figure 6. This busy figure shows graphically the quality of a "fit" to the branching ratios for each of the χ_{c_J} cases on the left, and a quantitative analysis in tabular form on the right. It should be noted that for none of the χ_c decays is the direct decay to $6\pi^{\pm}$ a reasonable fit versus background cut. Because the background cut points are completely correlated in these fits, $\chi^2/d.o.f.$ values appear much better than they are - that is, much better than unity. Probably they should be rescaled, however it is the relative values which are important, as only statistical errors have been given, systematic errors are assumed common to all fit points.



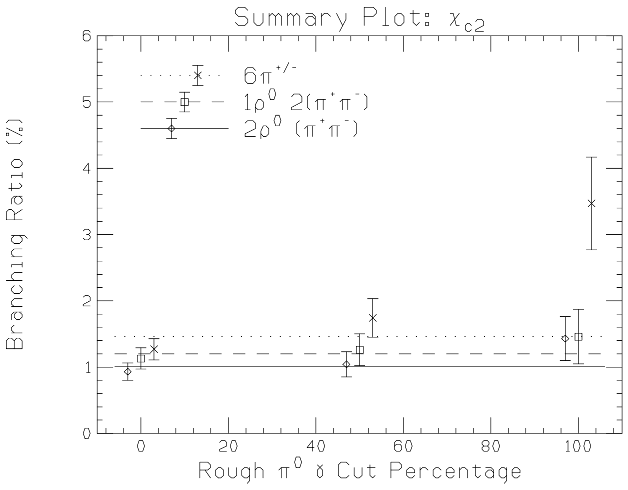
χc0 fits

		0 ρ		1 ρ		2 ρ	
	Cut %	μ	σ	μ	σ	μ	σ
	0	1.51	0.14	1.36	0.17	1.1	0.11
Wi			51.02		34.60		82.64
	50	1.56	0.19	1.25	0.17	0.95	0.13
			27.70		34.60		59.17
	100	3.20	0.54	1.84	0.37	1.06	0.2
			3.43		7.30		25.00
Weighted	Average	1.60	0.11	1.36	0.11	1.04	0.08
	χ2	9.24		2.10		0.79	
	χ2 / (N–1)	4.62		1.05		0.39	



χc1 fits

		0 ρ		1 ρ		2 ρ	
	Cut %	μ	σ	μ	σ	μ	σ
	0	0.87	0.11	0.73	0.10	0.71	0.10
Wi			82.64		100.00		100.00
	50	0.87	0.18	0.65	0.15	0.57	0.13
			30.86		44.44		59.17
	100	2.07	0.48	1.00	0.32	0.74	0.18
			4.34		9.77		30.86
Weighted	Average	0.91	0.09	0.72	0.08	0.67	0.07
	χ2	6.02		0.99		0.90	
	χ2 / (N–1)	3.01		0.50		0.45	



χc2 fits

		0 ρ		1 ρ		2 ρ	
	Cut %	μ	σ	μ	σ	μ	σ
	0	1.27	0.16	1.13	0.16	0.93	0.13
Wi			39.06		39.06		59.17
	50	1.74	0.29	1.26	0.24	1.04	0.19
			11.89		17.36		27.70
	100	3.47	0.70	1.46	0.41	1.43	0.33
			2.04		5.95		9.18
Weighted	Average	1.46	0.14	1.20	0.13	1.01	0.10
	χ2	10.59		0.66		2.02	
	χ2 / (N–1)	5.29		0.33		1.01	

Figure 6: Summary of χ_{cJ} decays to a $3(\pi^+\pi^-)$ final state. On the left, a graphical representation of the calculated branching ratio versus a cut on π^0 background, for each of the intermediate ρ hypothesis, and for each of the χ_{cJ} cases. On the right, tables show the details of the fits, summarized in a $\chi^2/d.o.f.$ for the various viable intermediary ρ^0 resonance hypotheses. Note that in no case is the direct $6\pi^\pm$ decay a tolerable fit. Errors are statistical only - systematic errors are assumed to be common to all data. Because these "measurements" are completely correlated, the $\chi^2/d.o.f.$ is overestimated, as indicated by values less than unity, and should probably the rescaled.

4 $4(\pi^+\pi^-)$ Results

The cut values are summarized below:

$4(\pi^+\pi^-)$ Cut Values

Cut Parameter	Value	Comment
$\Delta p, \Delta m$	$\leq 100 MeV$	γ kinematics match reconstructed recoil resonance
$m_{\pi\pi}(cos_{max}(\pi\pi))$	$\geq 480 MeV$	cut on mass of the minimum opening angle pair (max. value)
$\Delta p_{m_{\pi\pi}}$	$\geq 180 MeV$	momentum difference of minimum mass pair

The resultant data and detector simulation results with these cuts are shown in Figure 7. It should be noted that when statistics less than 10^5 simulated decays were used, large statistical fluctuations were observed, a clear indication of the effect of having such a small detector acceptance.

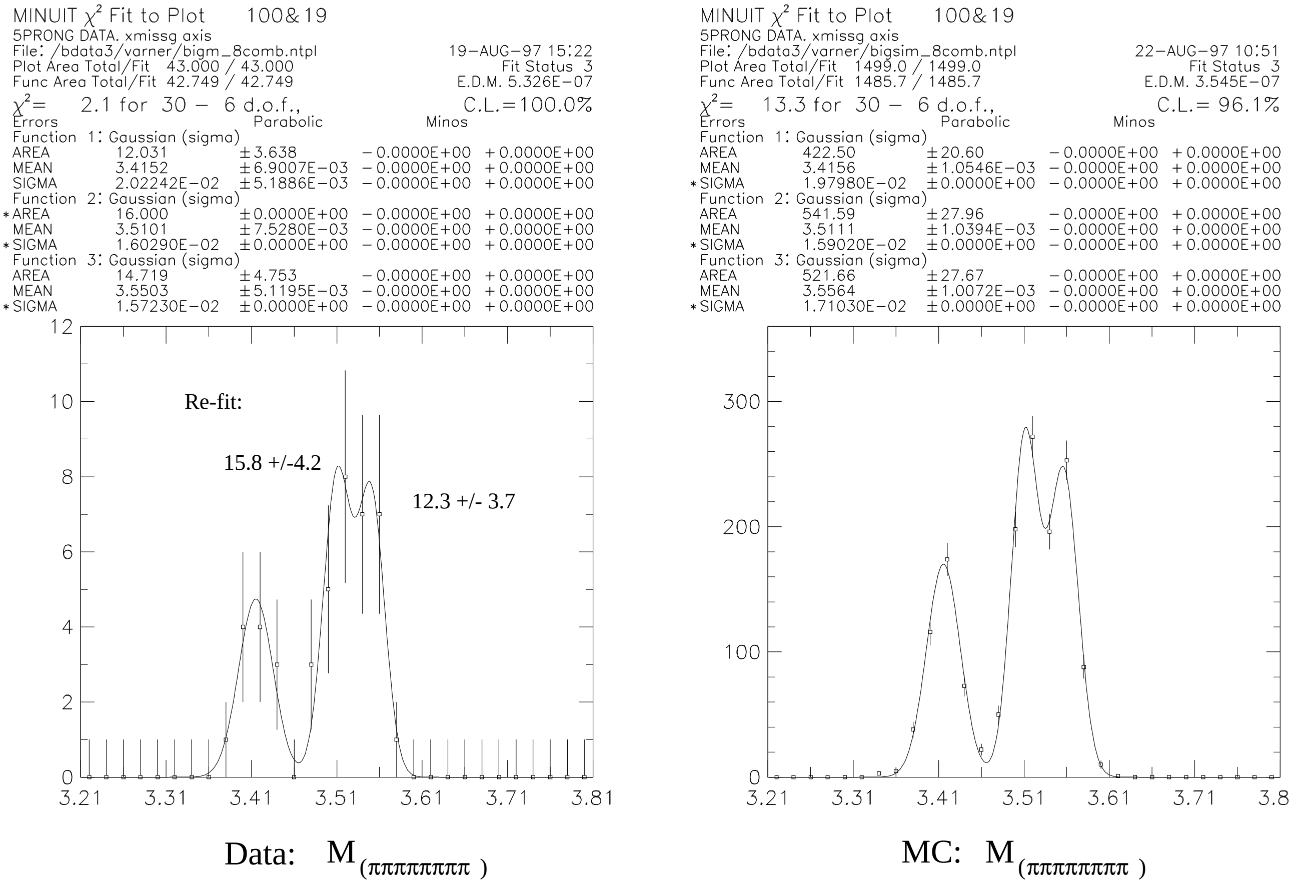


Figure 7: On the left is the fit to the data to extract the number of events, and on the right is the fit to extract the detection efficiency for $\chi_{c_J} \to 4(\pi^+\pi^-)$. The simulation is based on a 10^5 event sample, so these low acceptance modes correspond to detection efficiencies on the order of half a percent. Note that a re-fit was done to the data allowing the area of the χ_{c_1} to float, leading to the slightly different numbers shown.

The full calculation through to branching fractions is shown in Figure 8. It should be noted that in analogy with the $3(\pi^+\pi^-)$ case, these decays are almost certainly not a direct decay to $8\pi^{\pm}$, and consist of intermediate resonances. However, due to a lack of statistics and cleverness on the part of the author, a similar exercize to that for the $3(\pi^+\pi^-)$ has not been undertaken.

Perhaps startlingly, these calculated branching ratios are almost the same as those for the $3(\pi^+\pi^-)$ case, within errors. Clearly the construction of any theoretical model describing the details of the hadronization process in the decays of these χ_c states must be able to account for this interesting observation.

4(π+π–) Analysis
no intermediate

	<Value>	σ stat	σ sys		
N(ψ')	3.67E+06	3.00E+04	3.60E+05	Fit σ sys:	5%
				MC σ sys:	24.6%

	γ+χc0			γ+χc1			γ+χc2			
B(ψ' -> X)	9.9%	0.5%	0.8%	9.0%	0.5%	0.7%	8.0%	0.5%	0.7%	Cball #'s
ηdet	0.42%	0.02%	0.10%	0.54%	0.03%	0.13%	0.52%	0.03%	0.13%	
N(χc)	363330	18589	46176	330300	18548	41349	293600	18506	38593	
N(χc) det	1533	109	425	1790	137	494	1533	127	427	
Nevts	12.0	3.6	0.6	15.8	4.2	0.8	12.3	3.7	0.6	
B(X->8(π+/–))	0.78%	0.24%	0.22%	0.88%	0.24%	0.25%	0.80%	0.25%	0.23%	
	0.8%	0.2%	0.2%	0.9%	0.2%	0.2%	0.8%	0.3%	0.2%	
combined Errors:	0.78%	0.33%		0.88%	0.35%		0.80%	0.34%		

Figure 8: Summary table for the decays to $4(\pi^+\pi^-)$ final states. Note that the detection efficiencies are quite low, so that while previous branching ratio measurements reported in the PDG may be more sensitive, the product of BR times detection efficiency precluded their previous observation.

5 $5(\pi^+\pi^-)$ Limits

Having developed the machinery for dealing with high multiplicity pion final states, it is only natural to extend the analysis as far as statistics permit. Indeed, as the numbers below indicate, with the ψ' data set considered in this analysis, $\chi_{c_J} \to 5(\pi^+\pi^-)$ is the end of the road.

The cut values are summarized in the table directly below:

$5(\pi^+\pi^-)$ Cut Values

Cut Parameter	Value	Comment
β_c	≥ 0.02	opening angle difference: calculated versus best-fit π^0 γ pair
β_m	≥ 0.03	same cut as β_c, applied to specifically to the minimum $m_{\pi\pi}$
$\Delta p_{m_{\pi\pi}}$	$\geq 180 MeV$	momentum difference of minimum mass pair

Utilizing these cuts, a total of 3 candidate events are shown in Figure 9. In addition, the results of the detector acceptance are shown to the right. Note that this acceptance is higher than that for the $4(\pi^+\pi^-)$ case, but that a more restrictive cut on the π^0 γ conversions would remove all candidate events, so this represents a fairly unrestrictive upper bound.

In Figure 10 is the tabulation of the limits calculated in this analysis. While the statistics, or equivalently, the BES detector efficiency for reconstructing such a large number of tracks is a limiting factor, and these limits are not that stringent, nevertheless they are significantly below a naive estimation of the branching ratio based on extrapolating (energetically) to the incremental creation of an additional $\pi^+\pi^-$ pair, as is done in the table below. A simple "linear" extrapolation, based on phase-space arguments has been used. Numbers are rounded-off for convenience.

Naive Estimate of $5(\pi^+\pi^-)$ Branching Ratio

Decay Mode	$\mathbf{B}(\chi_{\mathbf{c_0}})$	$\mathbf{B}(\chi_{\mathbf{c_1}})$	$\mathbf{B}(\chi_{\mathbf{c_2}})$	**Q-value** (χ_{c_0}) $[MeV]$
$2(\pi^+\pi^-)$	$1.6 \pm 0.3\%$	$0.5 \pm 0.2\%$	$1.0 \pm 0.2\%$	~ 2859
$3(\pi^+\pi^-)$	$1.0 \pm 0.3\%$	$0.7 \pm 0.3\%$	$1.2 \pm 0.3\%$	~ 2581
$4(\pi^+\pi^-)$	$0.8 \pm 0.3\%$	$0.9 \pm 0.3\%$	$0.8 \pm 0.3\%$	~ 2303
Est. $5(\pi^+\pi^-)$	$0.5 \pm 0.3\%$	$0.5 \pm 0.3\%$	$0.8 \pm 0.3\%$	~ 2025

This naive estimate simply shows that the details of the long-distance terms (non-perturbative) in the expression for hadronization in the χ_c decays are inconsistent with a naive expectation. However, just as with the case for the $4(\pi^+\pi^-)$, any proper theory describing final state interactions must be able to explain this limit naturally.

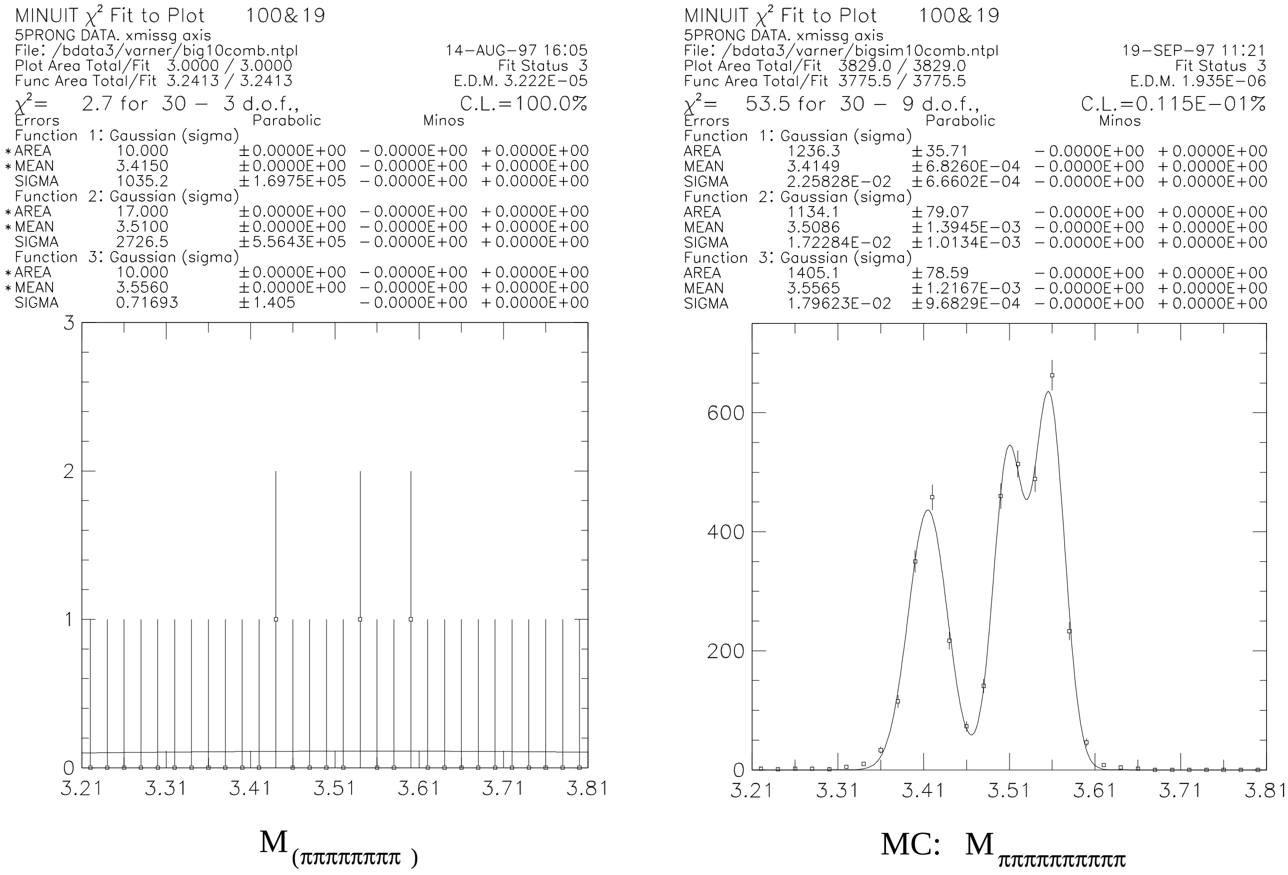


Figure 9: Search for $\chi_{cJ} \to 5(\pi^+\pi^-)$. On the left are the 3 candidate events. It should be noted that more severe cuts, comparable to π^0 γ conversion removal in the $4(\pi^+\pi^-)$ case, would reject all candidate events, and so the resulting limit is not very strigent, but an interesting one (see text).

5(π+π–) Analysis
Limits only

	<Value>	σ stat	σ sys		
N(ψ')	3.67E+06	3.00E+04	3.60E+05	Fit σ sys:	5%
				MC σ sys:	28.6%

	γ+χc0			γ+χc1			γ+χc2		
B(ψ' -> X)	9.9%	0.5%	0.8%	9.0%	0.5%	0.7%	8.0%	0.5%	0.7%
ηdet	1.24%	0.04%	0.35%	1.13%	0.08%	0.32%	1.41%	0.08%	0.40%
N(χc)	363330	18589	46176	330300	18548	41349	293600	18506	38593
N(χc) det	4505	272	1410	3732	337	1165	4140	351	1303
Nevts	3.9	0	0.2	3.9	0	0.2	3.9	0	0.2
B(X->10(π+/–)	0.09%	0.01%	0.03%	0.10%	0.01%	0.03%	0.09%	0.01%	0.03%
	0.1%	0.0%	0.0%	0.1%	0.0%	0.0%	0.1%	0.0%	0.0%
combined Errors:	0.09%	0.03%		0.10%	0.03%		0.09%	0.03%	
	< 0.1%			< 0.1%			< 0.1%		

Figure 10: Summary table of limits on the decay to $5(\pi^+\pi^-)$ final states. Note that the statistics, or equivalently, the detection efficiencies are the limiting factor, and these limits are not very strigent, but are notably lower than the naive expection (energetically) for the incremental creation of an additional $\pi^+\pi^-$ pair.

6 Summary

The results of this analysis may be summarized as follows:

1. $3(\pi^+\pi^-) Decays$ - intermediate resonances

   - $\chi_{c_0} \longrightarrow 2\rho^0(\pi^+\pi^-)$ is the dominant decay with BR $= 1.04 \pm 0.08 \pm 0.31\%$
   - $\chi_{c_1} \longrightarrow 1\rho^0 2(\pi^+\pi^-)$ and $2\rho^0(\pi^+\pi^-)$ admixture, with an average BR $= 0.70 \pm 0.08 \pm 0.32\%$
   - $\chi_{c_2} \longrightarrow 1\rho^0(\pi^+\pi^-)$ is the dominant decay with BR $= 1.20 \pm 0.13 \pm 0.28\%$

   By analyzing the details of these intermediate resonances, it may be possible to obtain a deeper understanding of the color-octet contribution to χ_c hadronic decays[7].

2. $4(\pi^+\pi^-) Decays$ - first observations

   - $B(\chi_{c_0} \longrightarrow 4(\pi^+\pi^-)) = 0.78 \pm 0.24 \pm 0.22\%$
   - $B(\chi_{c_1} \longrightarrow 4(\pi^+\pi^-)) = 0.88 \pm 0.24 \pm 0.25\%$
   - $B(\chi_{c_2} \longrightarrow 4(\pi^+\pi^-)) = 0.80 \pm 0.25 \pm 0.23\%$

   These signals are very clean. Unfortunately, statistics limit the search for intermediate resonances, and this decay mode is almost certainly mediated by them. It is interesting to note that the branching ratio for $\chi_{c_1} \rightarrow 4(\pi^+\pi^-)$ is larger than that for the $3(\pi^+\pi^-)$ case, though it may be said that for all of the χ_c decays, the BR's for the $3(\pi^+\pi^-)$ and $4(\pi^+\pi^-)$ are the same within statistics.

3. $5(\pi^+\pi^-) Decays$ - limits

   - $B(\chi_{c_0} \longrightarrow 5(\pi^+\pi^-)) \leq 0.09\%$
   - $B(\chi_{c_1} \longrightarrow 5(\pi^+\pi^-)) \leq 0.10\%$
   - $B(\chi_{c_2} \longrightarrow 5(\pi^+\pi^-)) \leq 0.09\%$

   By comparison with the detection efficiencies after π^0 cut for the $4(\pi^+\pi^-)$ case, these are not a very stringent limits. However, these limits are smaller than a simple phase space extrapolation to the incremental creation of another $\pi^+\pi^-$ pair, and are a boundary condition on any model of final state interactions describing the hadronization process in these decays.

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