After long delay, we have a little bit new ideas about the search of gluonium state from BES scan data around J/ψ. In our previous work, we did not find the second, wider resonance, beside the normal J/ψ narrow peak. But, as pointed out by Dr. Hou, this result does not mean that the gluonium state is failed to explain the $\rho\pi$ puzzle. According to Dr. Hou's formulation, the scan around J/ψ can hardly find, if at all, the possible gluonium state there. Then, instead, we want to see, what kind of restriction our seaching result could impose on the proposed gluonium state.

In the denominator of Eq.(8) of our paper, the $\sigma_{J/\psi}$ should be replaced by $\sigma_{c\overline{c}}$, i.e., the physical J/ψ state should be replaced by pure charmonium state. While $|c\overline{c}>=\cos\theta|J\psi>-\sin\theta|O>\equiv c|J\psi>-s|O>$; Here θ is the mixing angle.

$$\begin{aligned}\Gamma(c\overline{c}\to\rho\pi) &= c^2\Gamma(J/\psi\to\rho\pi)+s^2\Gamma(O\to\rho\pi)-2sc\sqrt{\Gamma(J/\psi\to\rho\pi)\Gamma(O\to\rho\pi)}\\ &= (c\sqrt{\Gamma(J/\psi\to\rho\pi)}-s\sqrt{\Gamma(O\to\rho\pi)})^2. \qquad (1)\end{aligned}$$

Then Eq.(8) in our paper should be replaced by

$$\frac{\Gamma(\psi'\to\rho\pi)/\Gamma(\psi'\to total)}{\Gamma(c\overline{c}\to\rho\pi)/\Gamma(J/\psi\to total)}=0.147 \qquad (2)$$

By substitute Eq.(1) into Eq.(2), we get

$$\frac{\Gamma(\psi'\to\rho\pi)}{(c\sqrt{\Gamma(J/\psi\to\rho\pi)}-s\sqrt{\Gamma(O\to\rho\pi)})^2}=\frac{\Gamma(\psi'\to total)}{\Gamma(J/\psi\to total)}\cdot 0.147 \qquad (3)$$

Experimentally

$$\frac{\Gamma(\psi'\to\rho\pi)/\Gamma(\psi'\to total)}{\sigma(W)/\sigma(e^+e^-\to J/\psi\to total)}=Q_{\rho\pi} \qquad (4)$$

Here $\sigma(W)=\sigma_{J/\psi}+\sigma_{O+I}$ is expressed by Eq.(8) in our paper. Then by dividing Eq.(3) by Eq.(4), we get

$$\frac{\sigma(W)/\sigma(e^+e^-\to J/\psi\to total)}{(c\sqrt{\Gamma(J/\psi\to\rho\pi)}-s\sqrt{\Gamma(O\to\rho\pi)^2}}=\frac{0.147}{\Gamma_(J/\psi\to\rho\pi)Q_{\rho\pi}} \qquad (5)$$

Write

$$\sigma(W)\equiv\sigma_{J/\psi}+\sigma_{O+I}=\sigma_{J/\psi}(1+\frac{\sigma_{O+I}}{\sigma_{J/\psi}}) \qquad (6)$$

and

$$\sigma_{J/\psi}/\sigma(e^+e^- \to J/\psi \to total) = \frac{\Gamma(J/\psi \to \rho\pi)}{\Gamma(J/\psi \to total)} \tag{7}$$

Finally we have

$$(1 + \frac{\sigma_{O+I}}{\sigma_{J/\psi}}) \frac{\Gamma(J/\psi \to \rho\pi)}{(c\sqrt{\Gamma(J/\psi \to \rho\pi)} - s\sqrt{\Gamma(O \to \rho\pi)})^2} = \frac{0.147}{Q_{\rho\pi}} \tag{8}$$

The lower limit of $\frac{0.147}{Q_{\rho\pi}}$ is 52; while the factor $(1 + \frac{\sigma_{O+I}}{\sigma_{J/\psi}})$ is determined by this experiment to be

$$\frac{\sigma_{O+I}}{\sigma_{J/\psi}} < 0.098(90\% C.L.) \tag{9}$$

So Eq.(8) can give a restriction between $\sin\theta$ and $\Gamma(O \to \rho\pi)$. Although alone it may not be very useful. But if combined with other theoretical or experimental constraints on, e.g. θ, it could be a strong restriction to the theory.