BES Analysis Memo

Do the BES results preclude the vector glueball explanation for the $\rho\pi$-$K^*\overline{K}$ puzzle?

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-ABSTRACT-

No.

1 Background

1.1 The $\rho\pi$-$K^*\overline{K}$ puzzle

The hadronic decays of the ψ' should not be interesting. It is the first radial excitation of the triplet $c\bar{c}$ bound state and, since it is very similar to the J/ψ at the parton level, it was originally expected that its hadronic decays should be reasonable extrapolations from the corresponding J/ψ decays. This has turned out not to be the case; studies of $\psi' \rightarrow$ hadrons have uncovered a number of peculiar patterns. Foremost among these peculiarities is the so-called $\rho\pi$-$K^*\overline{K}$ puzzle.

1.1.1 The 15% rule for hadronic ψ' decays

In the context of perturbative quantum chromodynamics (QCD), the hadronic decays of both the J/ψ and the ψ' are expected to proceed primarily via an annihilation into three gluons followed by the hadronization of these gluons into physically observable hadrons. The similarity between the parton-level final states has led to the conjecture that ratio of the J/ψ and ψ' decay branching fractions into any exclusive final state X_h is given by the ratio of the square of the wave function at the origin of the constituent $c\bar{c}$ quark state, which is well determined from the dilepton decay rates:

$$\frac{Br(\psi' \rightarrow X_h)}{Br(J/\psi \rightarrow X_h)} = \frac{Br(\psi' \rightarrow e^+e^-)}{Br(J/\psi \rightarrow e^+e^-)} = 0.147 \pm 0.023. \tag{1}$$

This conjecture is sometimes referred to as the 15% rule. Although this conjecture seems to work reasonably well for a number of decay channels, it fails badly in ψ' decays to two-body vector- plus pseudoscalar-meson (VP) final states.

1.1.2 $\psi' \to VP$ decays

In 1983, the MARK II collaboration reported that the $\rho\pi$ and $K^*(890)\overline{K}$ final states, two important hadronic decay modes of the J/ψ, were not observed in ψ' decays [1]. Subsequently, the BES collaboration has reported that the suppression of the $\rho\pi$ and $K^*\overline{K}$ decays of the ψ' decays is quite severe: upper limits for the two modes correspond to supressions relative to 15%-rule expectations by factors of more than 50 and 30, respectively [2]. This is the $\rho\pi$ $K^*\overline{K}$ puzzle [3].

2 The vector-glueball explanation

In 1987 it was noted by Brodsky, LePage and Tuan (BLT) that, since $J/\psi \to VP$ decays violate hadron helicity conservation (HHC), it is not the ψ' decays that are suppressed, but, rather the J/ψ decays that are anomalously enhanced [3]. An enhancement mechanism involving quantum mechanical mixing of the J/ψ with a vector glueball (a three-gluon state called the Omicron, $\mathcal{O}$) had previously suggested by Hou and Soni [4].

2.1 $c\bar{c}$-glueball mixing

We consider the physical $|J/\psi>$ and $|\mathcal{O}>$ states to be quantum mechanical mixtures of the pure $|c\bar{c}>$ and $|3g>$ parton states:

$$\begin{array}{c} |J/\psi> \\ |\mathcal{O}> \end{array} = \left(\begin{array}{cc} m_1 & \delta \\ \delta & m_2 \end{array} \right) \begin{array}{c} |c\bar{c}> \\ |3g> \end{array}$$

here $m_1 =< c\bar{c}|\mathcal{H}|c\bar{c}>$, $m_2 =< 3g|\mathcal{H}|3g>$, and $\delta =< c\bar{c}|\mathcal{H}|3g>$ where $\mathcal{H}$ is the Hamiltonian operator. The off-diagonal term δ corresponds to an OZI-suppressed transition and is, thus, expected to be small compared to the diagonal terms.

The physical states are expressed in terms of the underlying parton states as follows

$$\begin{array}{rcccl} |J\psi> & = & \cos\theta|c\bar{c}> & + & \sin\theta|3g> \\ |\mathcal{O}> & = & \cos\theta|3g> & - & \sin\theta|c\bar{c}>, \end{array} \qquad (2)$$

where θ, the mixing angle, can be expressed in terms of the masses of the physical states as

$$\tan 2\theta = \frac{2\delta}{\sqrt{(M_{J/\psi} - M_{\mathcal{O}})^2 - 4\delta^2}}.$$

(The minimum value of $(M_{J/\psi} - M_{\mathcal{O}})^2$ is $4\delta^2$.) The mixing angle decreases with increasing mass difference and, to get enough $c\bar{c}(1S)$-$3g$ mixing relative to that for $c\bar{c}(2S)$-$3g$ to account for the VP decay rate discrepancy, $M_{\mathcal{O}}$ should be within $\sim \pm 80$ MeV of $M_{J/\psi}$ [3]. Unlike the case for the J/ψ, the hadronic decays of the $\mathcal{O}$ are OZI-allowed and its total width is expected to be typical of massive hadronic resonances, i.e. $10 \leq \Gamma^{\mathcal{O}}_{tot} \leq 160$ MeV [3, 4, 5].

2.2 Decays of the mixed J/ψ and $\mathcal{O}$ states

For the parton state decay matrix elements we have

$$< \rho\pi|\mathcal{M}|c\bar{c} >= 0 \quad \text{and} \quad < e^+e^-|\mathcal{M}|3g >= 0. \tag{3}$$

The former is a consequence of HHC and the latter because the gluons are electrically neutral. Thus, we have the following relations for J/ψ and $\mathcal{O}$ decay widths:

$$\begin{array}{rcllcl} \Gamma^{J/\psi}_{ee} & = & \kappa_{ee}\cos^2\theta|< e^+e^-|\mathcal{M}|c\bar{c} > |^2 & \simeq & 5 \text{ keV} \\ \Gamma^{J/\psi}_{\rho\pi} & = & \kappa_{\rho\pi}\sin^2\theta|< \rho\pi|\mathcal{M}|3g > |^2 & \simeq & 1 \text{ keV} \\ \Gamma^{\mathcal{O}}_{ee} & = & \kappa_{ee}\sin^2\theta|< e^+e^-|\mathcal{M}|c\bar{c} > |^2 & \simeq & 5 \text{ keV} \times \tan^2\theta \\ \Gamma^{\mathcal{O}}_{\rho\pi} & = & \kappa_{\rho\pi}\cos^2\theta|< \rho\pi|\mathcal{M}|3g > |^2 & \simeq & 1 \text{ keV} \times \cot^2\theta, \end{array} \tag{4}$$

where the κ's contain phase space factors, etc., and, since the J/ψ and $\mathcal{O}$ masses are nearly the same, should nearly common to both states. The numerical values for the J/ψ widths are from the PDG tables [6].

2.3 s-channel production of J/ψ's and $\mathcal{O}$'s

The amplitude for $e^+e^- \to \rho\pi$, $\mathcal{A}$, is the sum of $\mathcal{O}$ and J/ψ terms:

$$\mathcal{A} = \frac{B}{s - M^2_{J/\psi} - iM_{J/\psi}\Gamma_{J/\psi}} + \frac{D}{s - M^2_{\mathcal{O}} - iM_{\mathcal{O}}\Gamma_{\mathcal{O}}} \tag{5}$$

where $|B|^2 = 12\pi\Gamma^{J/\psi}_{ee}\Gamma^{J/\psi}_{\rho\pi}$ and $|D|^2 = 12\pi\Gamma^{\mathcal{O}}_{ee}\Gamma^{\mathcal{O}}_{\rho\pi}$. From the relations in eqns. 3 above, it follows that the magnitudes of B and D are equal:

$$|B| = \sin\theta\cos\theta\sqrt{\kappa_{ee}\kappa_{\rho\pi}}| < ee|\mathcal{M}|c\bar{c} > || < \rho\pi|\mathcal{M}|3g > | = |D| \tag{6}$$

It is important to note that the model does not leave us free to vary $|D|$ relative to $|B|$.

2.4 Expectations for ψ' decays

Since the point of the vector glueball explanation is that the ψ' does not mix with the $\mathcal{O}$, we expect it to be a relatively pure $c\bar{c}(2S)$ state. In that case, the simplest conjecture is that the $\psi' \to \pi\pi J/\psi$ transition produces a pure $c\bar{c}(1S)$ state, which in terms of the physical states is

$$|c\bar{c}(1S)>= \cos\theta|J/\psi> - \sin\theta|\mathcal{O}> .$$

If there are no direct $\psi' \to \pi\pi 3g$ decays, we have the following simple relation (for a derivation, see the appendix to this note):

$$\frac{\mathcal{B}(\psi' \to \pi\pi\mathcal{O}; \mathcal{O} \to \rho\pi)}{\mathcal{B}(\psi' \to \pi\pi J/\psi; J/\psi \to \rho\pi)} \simeq \tan^2\theta \frac{\mathcal{B}(\mathcal{O} \to \rho\pi)}{\mathcal{B}(J/\psi \to \rho\pi)} = \frac{\Gamma_{tot}^{J/\psi}}{\Gamma_{tot}^{\mathcal{O}}}. \tag{7}$$

This follows from the relation $\tan^2\theta = \Gamma_{\rho\pi}^{J/\psi}/\Gamma_{\rho\pi}^{\mathcal{O}}$, extracted from eqns. 3 above, and from the definition $\mathcal{B}(X \to \rho\pi) \equiv \Gamma_{\rho\pi}^{X}/\Gamma_{tot}^{X}$. Thus, if, as expected, $\Gamma_{tot}^{\mathcal{O}} \geq 10$ MeV, the most we can expect for the number of $\mathcal{O} \to \rho\pi$ events originating from $c\bar{c}$-$3g$ mixing is about 1% of the $J/\psi \to \rho\pi$ events. This corresponds to the narrowest $\mathcal{O}$; the event rate decreases for a wider $\mathcal{O}$.

3 BES measurements

The data relevant to the discussion of this note were obtained in a 238 nb^{-1} scan over a 40 MeV cm energy region spanning the J/ψ resonance and a sample of $\simeq$3.6 million ψ' decays.

3.1 Scan of the J/ψ resonance

In ref. [7], the $e^+e^- \to \rho\pi$ cross section cross section, measured in a scan near the J/ψ, is analyzed in the context of a possible $\mathcal{O}$ contribution with an $\mathcal{O}$ mass within ± 80 MeV of $M_{J/\psi}$ and with a total width in the range $2 \leq \Gamma_{tot}^{\mathcal{O}} \leq 160$ MeV. No distortions of the J/ψ line shape are seen and a cross section limit of $\sigma_{\mathcal{O}+I}/\sigma_{J/\psi} < 0.098$, where $\sigma_{\mathcal{O}+I}$ is the combined effect of direct $e^+e^- \to \mathcal{O}$ production and its interference with J/ψ production, is reported.

In ref. [7], the $3g$ part of the mixed-J/ψ state is ignored and the magnitudes and relative phases of B and D are left as free parameters. In that analysis, the anomalous $\rho\pi$ production is attributed to the physical $\mathcal{O}$ state, and, since the $\rho\pi$ peak is narrow (consistent with the energy resolution), the claim is made that we can rule out any $\mathcal{O}$ explanation with $\Gamma_{\mathcal{O}} > 2$ MeV. However, this ignores the $3g$ part of the physical J/ψ state, which is the mechanism proposed in refs. [3] and [4].

I made χ^2 fits of the $e^+e^- \to \rho\pi$ cross sections from the scan data to the function

$$\sigma(W) = \frac{\lambda_0}{\sqrt{2\pi}\Delta}\int |\mathcal{A}|^2 \exp(\frac{(W'-W)^2}{2\Delta^2}dW',$$

where $\mathcal{A}$ is the amplitude given in eqn. 5 above, $W = \sqrt{s}$ and Δ, which I fix at 0.99 MeV, is the BES CM energy spread. (I also included, in an approximate way, the effects of the radiative tail.) Here λ_0 is the only free parameter. Using the J/ψ term only, i.e. setting the $\mathcal{O}$ contribution to zero, gives a reasonable fit to the scan data: $\chi^2 = 30.5/28\ dof$. (The fact that the J/ψ mass is much narrower than the beam energy spread ensures that any effects of the interference term in $\mathcal{A}$ are undetectable. Thus the possibility of a relative phase between B and D is irrelevant.)

Including the $\mathcal{O}$ contribution at the level specified by eqn. 5 and with a width $\Gamma_{\mathcal{O}} \geq 3$ MeV has no major effect on the quality of the fit for any value of $M_{\mathcal{O}}$. For narrower widths a potential $\mathcal{O}$ contribution starts to show up around $M_{\mathcal{O}} \simeq 3.080$ GeV, near the one data point that has a particularly large integrated luminosity. Figure 1 shows the contribution of an $\mathcal{O}$ with mass $M_{\mathcal{O}} = 3.080$ GeV and with width $\Gamma_{\mathcal{O}} = 3$ MeV. This is the $\mathcal{O}$ mass that gives the highest χ^2 value for $\Gamma_{\mathcal{O}} = 3$ MeV, namely $\chi^2 = 35.7/28\ dof$, which is still a marginally acceptable fit. For larger $\Gamma_{\mathcal{O}}$ values, the $\mathcal{O}$ signal is smaller; the data easily accomodates a $\Gamma_{\mathcal{O}} > 3.0$ MeV $\mathcal{O}$ of any mass. Thus, the implication in ref. [7] that the scan measurements rule out a BLT-type $\mathcal{O}$ with $\Gamma_{\mathcal{O}} \geq 2$ MeV is unfounded. We can only rule out some mass regions for an Omicron with $\Gamma_{\mathcal{O}} < 3$ MeV.

The lack of sensitivity can be understood from the fact that the integrated Breit-Wigner denominator is proportional to Γ_{tot}^{-1}. Thus a measure of the relative $\mathcal{O}/J/\psi$ contribution is $\Gamma_{J/\psi}/\Gamma_{\mathcal{O}}$. For $\Gamma_{\mathcal{O}} = 3$ MeV, the total $\mathcal{O}$ contribution is $\simeq 3\%$ that of the J/ψ, which is ~ 4 events, spread over about as many bins.

3.2 $\psi' \to \pi^+\pi^-\rho\pi$ decays

We selected a sample of $\psi' \to \pi^+\pi^-\rho\pi$ events by subjecting events with four charged tracks with zero net charge and at least two neutrals to a five-constraint fit to the hypothesis

$$\psi' \to \pi^+\pi^-\pi^+\pi^-\pi^0; \qquad \pi^0 \to \gamma\gamma.$$

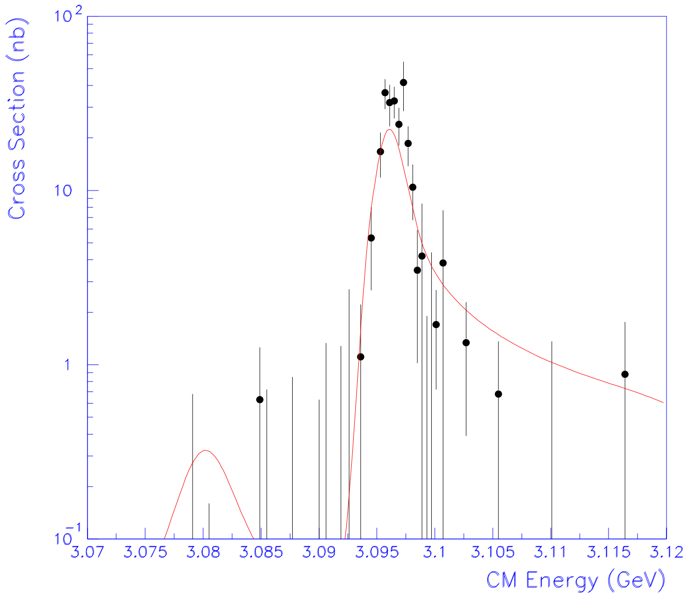

Figure 1: The best fit to the $e^+e^- \to \rho\pi$ scan data when an $\mathcal{O}$ of mass 3.080 GeV and total width of 3.0 MeV is included. The contribution of a wider $\mathcal{O}$ would be smaller.

We further require at least one $\pi^+\pi-$ or $\pi^\pm\pi^0$ pair to have an invariant mass within ±150 MeV of $m_{\rho(770)}$. We determine the $\rho\pi$ mass by computing the recoil mass from each $\pi^+\pi^-$ pair in the event that is not associated with the selected ρ. The $\rho\pi$ mass spectrum, shown in Fig. 2, shows a large peak with 1437 ± 40 events at the J/ψ. For the recoil mass determination, we use the *unfit* values for the charged-pion momenta, this allows us to look for possible $\mathcal{O}$-induced distortions to the $J/\psi \to \rho\pi$ lineshape by comparing to the precisely determined $J/\psi \to \mu^+\mu^-$ lineshape, also determined from the unfit π^+ and π^- momenta. In this way for each sample, the line shape is determined from the measured values of $\pi^+\pi^-$ pairs with the same angular and momentum distributions and systematic experimental effects.

The $\rho\pi$ mass distribution of Fig. 2 is well described by the J/ψ lineshape determined from the $\mu^+\mu^-$ sample ($\chi^2/dof = 207/198$). No distortion indicating the presence of an $\mathcal{O}$ is apparent.

From eqn. 7 above, we expect $\sim 60 \times \frac{2\ \mathrm{MeV}}{\Gamma_{\mathcal{O}}}$ events. We are, thus, only sensitive to an $\mathcal{O}$ with width less than our experimental resolution, which is determined from a fit to recoil $\mu^+\mu^-$ mass spectrum to be $FWHM \simeq 15$ MeV. The data rule out a narrow peak with more than $\simeq 30$ events over the intervals $2.9 \leq M_{\mathcal{O}} \leq 3.085$ GeV

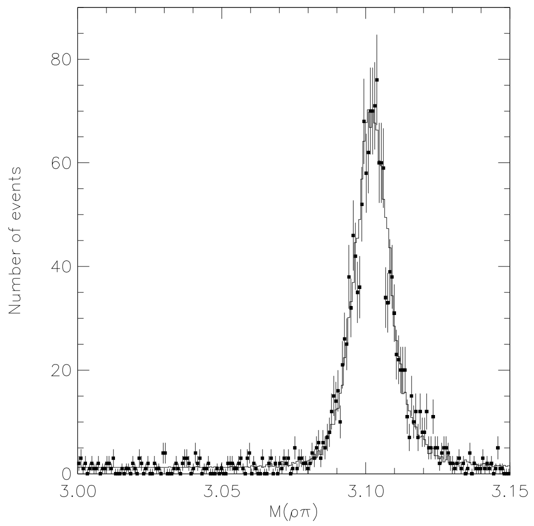

Figure 2: The $\rho\pi$ mass distribution from $\psi' \to \pi^+\pi^-\rho\pi$ events. The histogram is the scaled $\mu^+\mu^-$ mass distribution from $\psi' \to \pi^+\pi^-\mu^+\mu^-$ decays plus a flat background.

and $3.12 \leq M_{\mathcal{O}} \leq 3.25$ GeV, allowing us rule out an $\mathcal{O}$ with $\Gamma_{\mathcal{O}} < 4$ MeV in these two mass regions.

4 Conclusions

The $\rho\pi$ line shape from BES measurements of the $e^+e^- \to \rho\pi$ cross section near the J/ψ resonance and the decays $\psi' \to \pi^+\pi^-\rho\pi$ are analyzed in the context of the vector glueball model of Hou and Soni [4] and Brodsky, LePage and Tuan [3]. It is found that the measurements do not place any restrictions on a vector glueball with $\Gamma_{\mathcal{O}} > 4$ MeV. Since it is expected that $\Gamma_{\mathcal{O}} > 10$ MeV [3, 4, 5], the BES measurements do not constrain the model in any significant way. The vector glueball hypothesis remains a compelling explanation of the $\rho\pi$-$K^*\overline{K}$ puzzle.

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A Derivation of the ψ' decay rate formula

I derived eqn. 7 intuitively. Here I try a more rigorous derivation. I assume that initially, i.e. at $t = 0$, the $\psi' \to \pi\pi J/\psi$ transition selects a pure $|c\bar{c}(1S) >$ state, which then evolves in time as

$$|c\bar{c}(1S) > (t) = \cos\theta |J/\psi > e^{[-iM_{J/\psi} - \Gamma_{J/\psi}/2]t} - \sin\theta |\mathcal{O} > e^{[-iM_{\mathcal{O}} - \Gamma_{\mathcal{O}}/2]t}.$$

The amplitude of the final $\rho\pi$ state, $c_{\rho\pi(M)}(t)$, is the solution to

$$\frac{dc_{\rho\pi(M)}(t)}{dt} = e^{iMt} < \rho\pi|\mathcal{M}|c\bar{c} > (t), \tag{8}$$

where M is the $\rho\pi$ mass. Using the above expression for $|c\bar{c} > (t)$ and eqns. 2 and 3 above, we have

$$\frac{dc_{\rho\pi(M)}(t)}{dt} = \mathcal{M}_0 \left(e^{\left[i(M - M_{J/\psi}) - \Gamma_{J/\psi}/2\right]t} - e^{[i(M - M_{\mathcal{O}}) - \Gamma_{\mathcal{O}}/2]t} \right), \tag{9}$$

where $\mathcal{M}_0 = \sin\theta\cos\theta < \rho\pi|\mathcal{M}|c\bar{c} > (t = 0)$. Note that $\frac{dc_{\rho\pi(M)}(t)}{dt}(t = 0) = 0$, as required by eqn. 3 above. Integrating this equation with the boundary condition $c_{\rho\pi(M)}(0) = 0$ and taking the limit $t \to \infty$ gives

$$c_{\rho\pi(M)}(t \to \infty) = \mathcal{M}_0 \left[\frac{1}{\Gamma_{J/\psi}/2 - i(M - M_{J/\psi})} - \frac{1}{\Gamma_{\mathcal{O}}/2 - i(M - M_{\mathcal{O}})} \right], \tag{10}$$

which is similar to the amplitude for s-channel production (eqn. 5, above). The absolute value squared of this gives the expected $\rho\pi$ mass distribution:

$$\begin{aligned} |c_{\rho\pi(M)}(t \to \infty)|^2 \;\; = \;\; & |\mathcal{M}_0|^2 \left(\tfrac{1}{(M - M_{J/\psi})^2 + \Gamma^2_{J/\psi}/4} + \tfrac{1}{(M - M_{\mathcal{O}})^2 + \Gamma^2_{\mathcal{O}}/4} \right. \\ & \left. - \tfrac{1}{2} \tfrac{\Gamma_{\mathcal{O}}\Gamma_{J/\psi} + 4(M - M_{\mathcal{O}})(M - M_{J/\psi})}{[(M - M_{J/\psi})^2 + \Gamma^2_{J/\psi}/4][(M - M_{\mathcal{O}})^2 + \Gamma^2_{\mathcal{O}}/4]} \right). \end{aligned} \tag{11}$$

The first two terms are the J/ψ and $\mathcal{O}$ contributions and, since the integral of each term is proportional to Γ^{-1}_{tot}, these terms, taken alone, give eqn. 7 above. Here, as in the case for s-channel production, the fact that $\Gamma_{J/\psi}$ is much smaller than $\Gamma_{\mathcal{O}}$ and the experimental resolution ensures that the effects of the interference term are negligible.