Radiative decay of the ψ' into $\gamma\pi^+\pi^-$,γK^+K^- and $\gamma K^0_S K^0_S$

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Abstract

The 3.6×10^6 ψ'data collected by the BES detector has been used to study the ψ' radiative decay into $\gamma\pi^+\pi^-$,$\gamma\mathrm{K}^+\mathrm{K}^-$ and $\gamma\mathrm{K}^0_S\mathrm{K}^0_S$ at the region where the $\pi^+\pi^-$,$\mathrm{K}^+\mathrm{K}^-$ and $\mathrm{K}^0_S\mathrm{K}^0_S$ invariant masses are smaller than 3 GeV. The $\mathrm{f}_2(1270)$ and $\mathrm{f}_J(1710)$ signals have been seen and the branch fraction of $\psi' \to \gamma f_2(1270)$ and $\psi' \to \gamma f_J(1710) \to \gamma K\bar{K}$ have been measured. The results shows that these radiative decay modes are not suppressed from the "15% rule".

It is believed that the dominant processes of J/ψ and ψ' hadronic decay are through three gluons and the strong coupling constant a_s doesn't change significantly from the J/ψ mass to the ψ' mass. The lowest order perturbative QCD predicts the "15% rule" based on this assumption:

$$Q = \frac{B(\psi' \to X)}{B(J/\psi \to X)} = \frac{B(\psi' \to ggg)}{B(J/\psi \to ggg)} = \frac{B(\psi' \to l^+l^-)}{B(J/\psi \to l^+l^-)} = (14.6 \pm 2.2)\%$$

where X is an arbitrary non-charmonium state. Results from the Mark II experiment and the BES experiment show that most of the ψ' decay channels follow the theoretical prediction. But the ψ' decay into vector-pseudoscalar modes such as $\psi' \to \rho\pi$ and K^*K, as well as ψ' decay into vector-tensor modes are suppressed from the "15% rule".

The ψ' radiative decay modes are similiar to its hadronic decay modes with one of the gluon being replaced by a photon. And it is believed that the "15% rule" should also hold for radiative decay modes[1]. In this studying, we use the 93/94/95 ψ' data sets collected by BES to study the $\psi' \to \gamma\pi^+\pi^-$,$\gamma K^+K^-$ and $\gamma K^0_S K^0_S$ with $\pi^+\pi^-$,K^+K^- and $K^0_S K^0_S$ invariant mass below 3 GeV.

1 $\psi' \to \gamma\pi^+\pi^-$

The event selection for this channel starts from DST data with 2 charged tracks. The following criteia are imposed to each event:

1. Two charged tracks with $\sum Q_i = 0$ and the "mfit" parameter are 2 for both tracks.
2. At least one photon hit at Barrel Shower Counter and the energy deposited in it should be larger than 10 MeV.
3. The opening angle of these two charged tracks is smaller than 175^o. $\mid Z_1 - Z_2 \mid < 5cm$. A vertex can be found from these two charged tracks and the Z coordinate of the vertex satisfy $\mid Z_V \mid < 15cm$.

4. For each charged track, we asked $|\cos(\theta)| < 0.85$; $R_{xy} < 2cm$ and $Z < 15cm$; In order to remove $e^+e^- \to (\gamma)e^+e^-$ process, we required that for each track with momentum larger than 700 MeV, $E/P < 0.75$ should be satisfied and for both tracks, $(E_+/P_+ - 1)^2 + (E_-/P_- - 1)^2 > 0.4$ should be satisfied. Also, to remove $\mu^+\mu^-$ events, we require that there should not be more than 3 layers being hit in Muon detector.

5. The opening angle between the radiative γ and any charged tracks should be larger than 15^o.

6. In order to seperate π from K, we combine the information from barrel TOF, dE/dx and kinematic fit togather and require that:

$$P_{(\chi^2_{TOF}+\chi^2_{dE/dx}+\chi^2_{KF})}(\gamma\pi^+\pi^-) > P_{(\chi^2_{TOF}+\chi^2_{dE/dx}+\chi^2_{KF})}(\gamma K^+K^-)$$

$$P_{(\chi^2_{TOF}+\chi^2_{dE/dx}+\chi^2_{KF})}(\gamma\pi^+\pi^-) > 0.01$$

7. $P^2_{t\gamma} = 4P^2_{miss}\sin^2\frac{\theta}{2} < 0.002$ in order to remove $\psi' \to \pi^0\pi^+\pi^-$ background.

   Kinematic fit at here also serve the purpose of selecting good photon and refining the mass resolution.

The $\pi^+\pi^-$ invariant mass is shown in Fig. 1(a). While the $f_2(1270)$ signal is very obvious, there are also strong QED background presented. The initial state radiation process $e^+e^- \to \gamma\rho^0 \to \gamma\pi^+\pi^-$ contributes to the ρ^0 signal in the plot and $e^+e^- \to (\gamma)\mu^+\mu^-$ process contributes to the continuous background.

In order to remove these QED background, we fed the τ scan data collected by the BES detector into our program. The result is shown in Fig 1(b). We will scale these two histograms to remove the QED background in ψ' data. In order to do this, we used the number of events with $\pi^+\pi^-$ invariant mass lower than 900 MeV on the two plots to extract the scaling factor ϖ, which is ~ 1.34. This factor is consistant with the ratio obtained from the luminositys of ψ' data and τ data ($1.2 \sim 1.5$). The result is shown in Fig. 2(a). Each ψ' event will be assigned a weight of 1 and each τ event will be assigned a weight $-\varpi$ in the likelihood fitting so that the QED background can be removed.

In order to fit the histogram in Fig. 2(a), we adopted a special form of relativistic Breit-Wigner function to descript the rather broad signal in ψ' radiative decay into two identical pseudo-scalars[2]:

$$\frac{d\sigma}{dm^2} = C|T|^2\frac{P}{m_{\psi'}}\frac{Q}{m} \tag{1}$$

where:

- C is a constant that will be computed numerically based on normalization.
- m is the invariant mass of the two pseudo-scalars.
- P is the momentum of the $\pi^+\pi^-$ in ψ' rest frame.
- Q is the momentum of the $\pi^{+/-}$ in $\pi^+\pi^-$'s rest frame.
- T is the invariant amplitude describing the production and decay of the $f_2(1270)$.

If we think that T has the form:

$$T = \frac{Q^s}{m_0^2 - m^2 - im_0\Gamma_{total}(E)}$$

where s is the spin of the resonant which is 2 for $f_2(1270)(J^{PC} = 2^{++})$. Consider that $f_2(1270) \to \pi\pi$ and $K\bar{K}$ made 90% of its total decay width, we can make a good approximation on the energy dependent width $\Gamma_{total}(E)$:

$$\begin{aligned}\Gamma_{total}(E) &= \Gamma_0 \frac{m_0}{m} \times \\ &\frac{B(f_2 \to \pi\pi)\left[\frac{Q_\pi(m)}{Q_\pi(m_0)}\right]^{(2s+1)} + \theta(m_K)B(f_2 \to K\bar{K})\left[\frac{Q_K(m)}{Q_K(m_0)}\right]^{(2s+1)}}{B(f_2 \to \pi\pi) + B(f_2 \to K\bar{K})}\end{aligned} \tag{2}$$

$$\theta(m_K) = \theta(m - (m_K + m_{\bar{K}})) = \begin{array}{l} 0 \text{ if } m < (m_K + m_{\bar{K}}); \\ 1 \text{ if } m > (m_K + m_{\bar{K}}); \end{array}$$

where θ is step function, m_0 and Γ_0 are the mass and width of $f_2(1270)$ on PDG.

The background is described by modified Legendre polynomial functions:

$$\frac{1}{m_{\max} - m_{\min}} \sum_{i=0}^{N_{poly}} b_i p_i(x) P(m) Q(m) \tag{3}$$

$$x = 2\frac{m - m_{\min}}{m_{\max} - m_{\min}} - 1;\ x \in [-1, 1]$$

where $p_i(x)$'s are the Legendre polymornial. b_i's are coefficients and $b_0 = 1$ for normalization if P and Q are not presented at here. The reason P and Q are here is to keep the Lorentz invariant phase space behavior. The background function's normalization is done numerically.

With the mass and width of $f_2(1270)$ fixed at PDG values, we fit the histogram with a D-wave Breit-Wigner function and it gives us 139.1 ± 18.0 events. The efficiency of this analysis is 25.1% calculated from the uniform phase space Monte Carlo simulation with D-wave Breit-Wigner included, as shown in Fig. 2(b). Assuming $B(f_2(1270) \to \pi^+\pi^-) : B(f_2(1270) \to \pi^0\pi^0) = 2 : 1$ according to the isospin, we obtain:

$$B(\psi' \to \gamma f_2(1270)) = (2.68 \pm 0.35 \pm 0.53) \times 10^{-4}$$

where the first error is statistical and the second error is systematic. Table 1 lists the number of events survived various cuts for both ψ' data and Monte Carlo data. Table 2 lists the major sources of systematic error. In Fig.2(a) there is also a potential signal of $f_J(1700) \to \pi^+\pi^-$ but it is probabaly too weak to say anything on it.

Comparing our result of $B(\psi' \to \gamma f_2(1270))$ with $B(J/\psi \to \gamma f_2(1270)) = (13.8 \pm 1.4) \times 10^{-4}$ on PDG, we get the ratio of these two branching fractions:

$$Q = \frac{B(\psi' \to \gamma f_2(1270))}{B(J/\psi \to \gamma f_2(1270))} = (19.4 \pm 2.5 \pm 3.8)\%$$

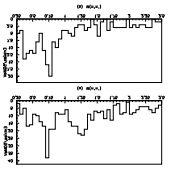

Figure 1: Invariant mass of $\pi^+\pi^-$in: (a) ψ'data; (b)τ scan data

Cuts	Data	Monte Carlo
Number of events come in	4786432	28610
$\sum Q_i = 0$; $N_\gamma > 0$; mfit=2	1062523	21082
γ hit at BSC, $E_\gamma > 10MeV$	971176	18620
$\lvert Z_1 - Z_2\rvert < 5cm$; $\lvert Z_V\rvert < 15cm$; $\theta_{\pi^+\pi^-} < 175^o$	402323	18178
Total μ layers< 3; $\lvert\cos(\theta_\pi)\rvert < 0.85$; $R_{xy} < 2cm$; $Z < 15cm$; $E/P < 0.75$; $(E_+/P_+ - 1)^2 + (E_-/P_- - 1)^2 > 0.4$	222326	16447
$P_{(\chi^2_{TOF}+\chi^2_{dE/dx}+\chi^2_{KF})}(\gamma\pi^+\pi^-) > P_{(\chi^2_{TOF}+\chi^2_{dE/dx}+\chi^2_{KF})}(\gamma K^+K^-)$ $P_{(\chi^2_{TOF}+\chi^2_{dE/dx}+\chi^2_{KF})}(\gamma\pi^+\pi^-) > 0.01$	1783	13085
$\theta_{\gamma\pi} > 15^o$	1715	13010
$P^2_{t\gamma} < 0.002$	613	7481

Table 1: Number of $\gamma\pi^+\pi^-$ events survive various cuts

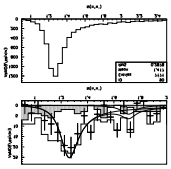

Figure 2: Invariant mass of $\pi^+\pi^-$ in (a) ψ' data (normal histogram) and τ scan data (in shadowed histogram). The data points with error bar are the difference of these two after normalization (b) Monte Carlo data.

Major cuts	$B(\psi' \to \gamma f_2(1270))$ difference(%)
$P_{\chi^2}(\gamma\pi^+\pi^-) > 0.01 \to 0.05$	1.5%
$\lvert Z_V \rvert < 15cm \to 10cm$	0.8%
$\theta_{\gamma\pi} > 15^o \to 10^o$	4.1%
$(E_+/P_+ - 1)^2 + (E_-/P_- - 1)^2 > 0.4 \to 0.6$	8.2%
$P_{t\gamma}^2 < 0.002 \to 0.003$	15.3%
Fitting Region from $0.3 \sim 2.5$ GeV to $0.3 \sim 2.0$	6.3%
Number of ψ' events	6%
Total	19.9%

Table 2: System Error estimation of $B(\psi' \to \gamma f_2(1270))$

The PDG value of $B(J/\psi \to \gamma f_2(1270))$ is actually obtained from a slightly improperly handled analysis. If we use the Mark III's result of $B(J/\psi \to \gamma f_2(1270)) = (15.6 \pm 1.4) \times 10^{-4}$[3], we then have:

$$Q = \frac{B(\psi' \to \gamma f_2(1270))}{B(J/\psi \to \gamma f_2(1270))} = (17.2 \pm 2.2 \pm 3.4)\%$$

The $B(\psi' \to \gamma f_2(1270))$ we measured clearly shows that "*The 15% rule hold for ψ' radiative decay into $\gamma f_2(1270)$*".

2 $\psi' \to \gamma K^+K^-$

The event selection of this mode is almost identical to that we used in $\gamma\pi^+\pi^-$ mode, except that we require

$$P_{(\chi^2_{TOF}+\chi^2_{dE/dx}+\chi^2_{KF})}(\gamma K^+K^-) > P_{(\chi^2_{TOF}+\chi^2_{dE/dx}+\chi^2_{KF})}(\gamma\pi^+\pi^-)$$

$$P_{(\chi^2_{TOF}+\chi^2_{dE/dx}+\chi^2_{KF})}(\gamma K^+K^-) > 0.01$$

The $P^2_{t\gamma} < 0.002$ also hasn't been used in this mode. Fig. 3(a) and (b) show the invariant mass disbribution of K^+K^- in ψ' data and in τ scan data respectively. The initial state radiation process $e^+e^- \to \gamma\phi \to \gamma K^+K^-$ is clearly shown in the two plots. The ratio of the number of ϕ events in these two plots is ~ 1.30. Fig. 4(a) shows the mass plot of K^+K^- in ψ' data, τ scan data (shadowed histogram) and the difference of the two (data points with error bars) after normalization. Fig. 4(b) is the signal from a modified phase space Monte Carlo which tells us the efficiency of 33.6%. We employed the same negative likelihood technique we used in $\gamma\pi^+\pi^-$ analysis to remove the QED background.

We fit $f_J(1710)$ signal in Fig. 4(a) by using a similiar relativistic Breit-Wigner that we used in the $\gamma\pi^+\pi^-$ studying. The J^{PC} of $f_J(1710)$ is $even^{++}$ on PDG. We assume the spin of the $f_J(1710)$ is zero. By fixing the mass and width of $f_J(1710)$ to the PDG value, the fit gives us (76.1 ± 12.0) events and a branching fraction of:

$$B(\psi' \to \gamma f_J(1710) \to \gamma K\bar{K}) = (1.24 \pm 0.20 \pm 0.28) \times 10^{-4}$$

The number of events survive each cut are listed in Table 3 and the systematic error sources are listed in Table 4. Compare to the PDG value of $B(J/\psi \to \gamma f_J(1710) \to \gamma K\bar{K}) = (9.7 \pm 1.2) \times 10^{-3}$, we have the ratio of the two branching fractions:

$$Q = \frac{B(\psi' \to \gamma f_J(1710) \to \gamma K\bar{K})}{B(J/\psi \to \gamma f_J(1710) \to \gamma K\bar{K})} = (12.8 \pm 2.6 \pm 3.3)\%$$

Again, this result shows that "*The 15% rule hold for ψ' radiative decay into $\gamma f_J(1710)$*".

In additon, bump at $\sim 1.3GeV$ region on Fig. 4(a) might indicate $\psi' \to \gamma f_2(1270) \to \gamma K^+K^-$ signal. But we don't have a good fit over it. Bump at $\sim 1.5GeV$ region indicates there are events either from $\psi' \to \gamma f_2'(1525) \to \gamma K^+K^-$ or from the contermination of $\psi' \to \gamma f_2(1270) \to \gamma\pi^+\pi^-$ with both $\pi^+\pi^-$ been misidentified at K^+K^-. To estimate the contermination of this

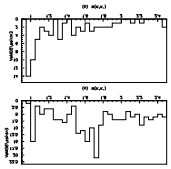

Figure 3: Invariant mass of K^+K^- in (a) ψ' data (b) τ scan data

Cuts	Data	Monte Carlo
Number of events come in	4786432	14305
$\sum Q_i = 0$; $N_\gamma > 0$; mfit=2	1062523	9165
γ hit at BSC, $E_\gamma > 10MeV$	971176	8176
$\lvert Z_1 - Z_2\rvert < 5cm$; $\lvert Z_V\rvert < 15cm$; $\theta_{\pi^+\pi^-} < 175^o$	402323	7277
Total μ layers< 3;$\lvert\cos(\theta_K)\rvert < 0.85$; $R_{xy} < 2cm$; $Z < 15cm$; $E/P < 0.75$; $(E_+/P_+ - 1)^2 + (E_-/P_- - 1)^2 > 0.4$;	222326	6033
$P_{(\chi^2_{TOF}+\chi^2_{dE/dx}+\chi^2_{KF})}(\gamma K^+K^-) > P_{(\chi^2_{TOF}+\chi^2_{dE/dx}+\chi^2_{KF})}(\gamma\pi^+\pi^-)$ $P_{(\chi^2_{TOF}+\chi^2_{dE/dx}+\chi^2_{KF})}(\gamma K^+K^-) > 0.01$	1243	4818
$\theta_{\gamma\pi} > 15^o$	1237	4802

Table 3: Number of γK^+K^- events survive various cuts

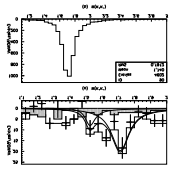

Figure 4: Invariant mass of K^+K^- in (a) ψ' data (normal histogram) and τ scan data (in shadowed histogram). The data points with error bars are the difference of these two after normalization. (b) Monte Carlo data.

Major cuts	$B(\psi' \to \gamma f_J(1710))$ difference(%)
$P_{\chi^2}(\gamma K^+K^-) > 0.01 \to 0.05$	8.1%
$\lvert Z_V \rvert < 15cm \to 10cm$	4.8%
$\theta_{\gamma\pi} > 15^o \to 10^o$	2.4%
$(E_+/P_+ - 1)^2 + (E_-/P_- - 1)^2 > 0.4 \to 0.6$	0%
Fitting Region $1.0 \sim 2.5$ GeV to $1.0 \sim 2.0$	4.8%
Assuming spin 2 for $f_J(1710)$	18.5%
Number of ψ' events	6%
Total	22.7%

Table 4: Systematic error estimation of $B(\psi' \to \gamma f_J(1710))$

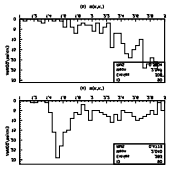

Figure 5: Invariant mass of K^+K^- in (a) $\psi' \to \gamma f_2(1270) \to \gamma\pi^+\pi^-$ Monte Carlo (b) $\psi' \to \gamma\pi^+\pi^-$ Monte Carlo

misidentification, we fed the $\psi' \to \gamma f_2(1270) \to \gamma\pi^+\pi^-$ Monte Carlo data to our γK^+K^- analysis program. The result is shown in Fig. 5(a). The contermination is estimated to be 1%, or 10 events in mass region of $1 \sim 3\ GeV$ in ψ' data according to the branching fraction of $\psi' \to \gamma f_2(1270)$ we just obtained. They are peaked at $\sim 1.5GeV$ region. At this stage, it is hard to say what is really at $\sim 1.5GeV$ region. The invariant mass of K^+K^- from a Monte Carlo simulated $\psi' \to \gamma\pi^+\pi^-$ process is also shown in Fig. 5(b).

3 $\psi' \to \gamma K^0_S K^0_S$

The events selection for this channel started from DST data with four charged tracks and is similiar to that we have done previously:

1. Four charged tracks with $\sum Q_i = 0$ and the "mfit" parameters are all 2.
2. At least one photon hit at Barrel Shower Counter and the energy deposite in it should be larger than 10 MeV.
3. For each charged track, we asked $|\cos(\theta)| < 0.85$; $R_{xy} < 2cm$; $Z < 15cm$; In order to remove electrons and positions, we required that $E/P < 0.75$ and for any tracks with momentum larger than 1.0GeV.

Cuts	Data	Monte Carlo
Number of events come in	2370479	14305
$\sum Q_i = 0$; $N_\gamma > 0$; mfit=2	569865	6479
γ hit at BSC, $E_\gamma > 10MeV$	561215	5178
$\lvert\cos(\theta_\pi)\rvert < 0.85$; $R_{xy} < 2cm$; $Z < 15cm$; $E/P < 0.75$	407819	4455
$P_{\chi^2}(\gamma\pi^+\pi^-\pi^+\pi^-) > 0.01$	11900	3351
$\lvert m_{\pi^+\pi^-} - m_{K^0_S}\rvert < 20MeV$; VEEFND finds both vertexes	74	2580

Table 5: Numer of $\gamma\pi^+\pi^-\pi^+\pi^-$ events survive various cuts

Major cuts	$B(\psi' \to \gamma f_J(1710))$ difference(%)
$P_{\chi^2}(\gamma\pi^+\pi^-\pi^+\pi^-) > 0.01 \to 0.05$	20%
$\lvert m_{\pi^+\pi^-} - m_{K^0_S}\rvert < 20MeV \to 10MeV$	28%
Number of Vee $\geq 2 \to 1$	38%
Number of ψ' events	6%
Total	51.6%

Table 6: Systematic error estimation for $B(\psi' \to \gamma K^0_s K^0_s)$

4. 4-C fits with all good photons as radiative photon candidate in all $\gamma\pi^+_1\pi^-_2\pi^+_3\pi^-_4$ combinations are done and we then select the combination with smallest $\sqrt{(m_{\pi^+_1\pi^-_2} - m_{K^0_S})^2 + (m_{\pi^+_3\pi^-_4} - m_{K^0_S})^2}$. We required that probability of this χ^2 is larger than 0.01.

5. Various plots are shown in Fig. 6. Fitting Fig. 6(b) with a gauss function gives us the mass resolution of $m_{K^0_S}$ equals $9.0MeV$. So we apply another cut: for each pair of $\pi^+\pi^-$, we required $|m_{\pi^+\pi^-} - m_{K^0_S}| < 20MeV$.

6. Both $\pi^+\pi^-$ pairs are identified by VEEFND.

Fig 8 show the invarianat mass of $\pi^+\pi^-\pi^+\pi^-$. The phase space Monte Carlo (Fig 8(b)) gives us the efficiency of the program is 18.0%. A simple S-wave Breit-Wigner fit over Fig 8(a) with resonance parameters fixed at PDG value of $f_J(1710)$ gives us 6.8 ± 3.1 events. Assuming $B(f_J(1710) \to K\bar{K}) : B(f_J(1710) \to K^0\bar{K}^0) : B(f_J(1710) \to K^0_S K^0_S) = 8 : 4 : 1$, we calculate the branching fraction:

$$B(\psi' \to \gamma f_J(1710) \to \gamma K\bar{K}) = (1.75 \pm 0.80 \pm 0.90) \times 10^{-4}$$

or

$$B(\psi' \to \gamma f_J(1710) \to \gamma K\bar{K}) < 3.03 \times 10^{-4} \text{ (at 90\% C.L.)}$$

The number of events surviving various cuts for real data and Monte Carlo data are listed in Table 5 and the major sources of systematic error are listed in Table 6. The result obtained from this channel is consistant with the result obtained from γK^+K^-.

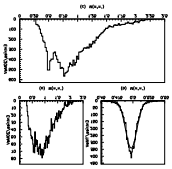

Figure 6: Invariant mass of $\pi^+\pi^-$in (a) $\gamma\pi^+\pi^-\pi^+\pi^-$ Monte Carlo (b) $\gamma f_J(1710) \to \gamma K_S^0 K_S^0 \to \gamma\pi^+\pi^-\pi^+\pi^-$ Monte Carlo (c) ψ' data.

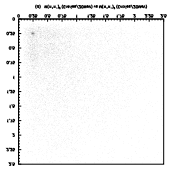

Figure 7: $M^2_{\pi^+\pi^-}$ vs. $M^2_{\pi^+\pi^-}$ in ψ' data.

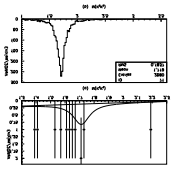

Figure 8: Invariant mass of $\pi^+\pi^-\pi^+\pi^-$ in (a) ψ' data (b) $\psi' \to \gamma\pi^+\pi^-\pi^+\pi^-$ phase space Monte Carlo.

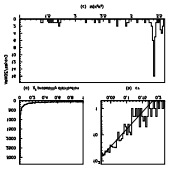

Figure 9: From ψ' data: (a) χ^2 distribution. (b) lifetime of K^0_S, $c\tau = 3.28cm$ (3) Invariant mass distribution of $K^0_S K^0_S$.

4 Summary

In this analysis, we used the $\sim 3.6 \times 10^6$ ψ' data collected by the BES detector [4]and we studied the radiative decay processes of ψ' into $\gamma\pi^+\pi^-$,γK^+K^- and $\gamma K_S^0 K_S^0$. Signals of $f_2(1270)$ and $f_J(1710)$ have been seen in $\psi' \to \gamma\pi^+\pi^-$ and γK^+K^- channels and the branching fractions of ψ' radiative decay into them have been measured as:

$$B(\psi' \to \gamma f_2(1270)) = (2.68 \pm 0.35 \pm 0.53) \times 10^{-4}$$

$$B(\psi' \to \gamma f_J(1710) \to \gamma K\bar{K}) = (1.24 \pm 0.20 \pm 0.28) \times 10^{-4}$$

From the $\gamma K_S^0 K_S^0$ we also obtained a consistant result:

$$B(\psi' \to \gamma f_J(1710) \to \gamma K\bar{K}) = (1.75 \pm 0.80 \pm 0.90) \times 10^{-4}$$

Due to the strong QED background and lack of statistics, we have not tried to determinate the spin of $f_J(1710)$.

The ratioes of these branching fractions and those corresponding branching fractions in J/ψ decay are:

$$Q = \frac{B(\psi' \to \gamma f_2(1270))}{B(J/\psi \to \gamma f_2(1270))} = (19.4 \pm 2.5 \pm 3.8)\%$$

$$Q = \frac{B(\psi' \to \gamma f_J(1710) \to \gamma K\bar{K})}{B(J/\psi \to \gamma f_J(1710) \to \gamma K\bar{K})} = (12.8 \pm 2.6 \pm 3.3)\%$$

These results show that the theoretical prediction by the lowerst order QCD are followed in these radiative decay modes.

It should point out that although not fully accomplished by analysis memo, Li Xinhua and Bill Dunwoodie had made pioneer work in these area and their idea is adopted in this studying[5][6].

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