THE MEASUREMENT OF BRANCHING RATIO $D^+ \to \mu^+ \nu_\mu$

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Abstract

The Beijing Spectrometer (BES) experiment has studied the purely leptonic decays of the D^+ meson produced in $e^+e^- \to D^*D$ at a center mass energy of 4.03 GeV. A search was performed for $D^+ \to \mu\nu$ recoiling from $\pi^0 D^+(\to K^-\pi^+\pi^+, K^0\pi^+)$ and $\pi^+ D^0(\to K^-\pi^+, K^-\pi^+\pi^+\pi^-, K^0\pi^+\pi^-)$. A single event candidate was found in the reaction, $e^+e^- \to D^{+*}D^-$, where, $D^{*+} \to \pi^+ D^0$ with the D^0 decaying into $K^-\pi^+$ and the recoiling decay is $D^- \to \mu\nu$. This gives the branching ratio of $D^+ \to \mu^+\nu_\mu = 0.08^{+0.16+0.05}_{-0.05-0.02}\%$.

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1. Introduction

The pure leptonic decays of the D meson proceed via the annihilation of the charm and anti-down quarks into a virtual W boson. The decay rate of this process is determined by the wavefunction overlap of the two quarks at the origin and is parametrized as the meson decay constant f_D. The decay width of the D^+ can be written as:

$$\Gamma(D^+ \to \ell^+\nu) = \frac{G_F^2|V_{cd}|^2}{8\pi} f_D^2 m_D m_\ell^2 (1 - \frac{m_\ell^2}{m_D^2})^2 \qquad (1)$$

where G_F is the Fermi constant, V_{cd} is the c$\rightarrow$d CKM matrix element, m_D is the D mass, and m_ℓ is the lepton mass.

There are many theoretical models to predict f_{D_s} and f_D(Cabbibo-suppressed decay), varying from 90 to 350 MeV. So far, there are a few experimental measurement on f_{D_s}, such as WA75, CLEO and BES. But no measurement has been made on f_D(MARK III only set a branching ratio upper limit 0.07%).

In this memo we will report the first evidence of decay $D^+ \to \mu^+\nu$. The data sample used was collected by the BES detector at the Beijing e^+e^- Collider(BEPC). The integrated luminosity is 22.3 $(pb)^{-1}$ (from large angle Bhabha scattering events) and the center of mass energy is 4.03 GeV. There is a large cross-section ($\sim$2.4 nb) for the D*D decay mode at this energy.

2. Particle Identification

The BES particle identification system includes both dE/dx information(from Main Drift Chamber) and Time Of Flight information. They are used to identify pions, kaons and leptons. The PID consistency requirement is imposed by calculating the χ^2 distribution between the measured and predicted TOF and dE/dx values:

$$\chi^2 = \chi^2{}_{tof} + \chi^2{}_{dE/dx} \tag{2}$$

$$\chi^2{}_{tof} = \frac{(TOF_{meas} - TOF_{pred})^2}{\sigma^2{}_{tof}} \tag{3}$$

$$\chi^2{}_{tof} = \frac{(dE/dx_{meas} - dE/dx_{pred})^2}{\sigma^2{}_{dE/dx}} \tag{4}$$

The confidence level(CL) is constructed by calculating the probability of the χ^2 distribution for each particle hypothesis(Pion, Kaon, Proton and leptons). A 1% cut(3σ) is used in this analysis.

The normalized likelihood ratio(Weight) is also defined to separate the pion and kaon:

$$Weight(K) = \frac{L(K)}{L(K) + L(\pi)} \tag{5}$$

$$L(K) = e^{-\frac{\chi^2{}_{tof}(K)}{2}} \times e^{-\frac{\chi^2{}_{dEdx}(K)}{2}} \tag{6}$$

$$L(\pi) = e^{-\frac{\chi^2{}_{tof}(\pi)}{2}} \times e^{-\frac{\chi^2{}_{dEdx}(\pi)}{2}} \quad (7)$$

The Weight(K) is close to 1 for well identified kaons and close to 0 for pions. The weight(K) $>$ 50 % cut is imposed for the Kaon hypothesis in this analysis.

The muon chamber information is used along with dE/dx and TOF for the μ identification. We require each μ candidate have at least two muon hits with well associated charged track(standard deviation less than 5 σ between the muon counter position and the drift chamber projected position).

The shower counter information is used to define the isolated photon. Each isolated photon has at least 60 MeV shower energy and 18 degree separation angle from the closest charged track.

3. Event Quality Criterion

In order to get the "good quality" events, we need to impose cuts on each event. They are listed here.

dEdx: the ratio of dEdx hits to possible hits in each event larger than 60%.

track fit quality: only tracks with a good helix fit are used (MFIT=2 or -19 or -9).

vertex: each track's closest approach to the origin should be smaller than 0.01m in xy plane and 0.15m in z direction(not required for the K^0 channel).

fiducial angle: $\mid cos\theta \mid \leq 0.85$ for each charged track.

4. D^+ and D^0 Single Tags

The single tagging of $D^\pm$ and D^0 from $e^+e^- \to D^{+*}D^-$, $D^{+*} \to D^+\pi^0$ or $D^0\pi^+$ decay system is used to set up the data sample of hadronic decay of D^+ and D^0 so that we can look at the recoil system and search for the muon decay of charged D meson. In the Monte Carlo study, the slow charged π from D^{+*} is not reconstructed and consequently this track is ignored in this analysis.

In the single-tag analysis, we studied two $D^\pm$ decay modes ($K^-\pi^+\pi^+, K^0\pi^+$) and three D^0 decay modes ($K^-\pi^+$, $K^-\pi^+\pi^+\pi^-, K^0\pi^+\pi^-$). The cuts used are summarized as follows. We require 1% confidence level cut on both pions and kaons. The weight of kaon to pion is larger than 50% for kaon hypothesis. The momentum range of the reconstructed D is from 410 MeV to 670 MeV to ensure it is from $e^+e^- \to D^{+*}D^-$ decay mode. For the ${K_s}^0$ channel, the invariant mass cut of $\pi^+\pi^-$ is 498 $\pm$ 30 MeV(3.5 σ of ${K_s}^0$ mass) and the decay time is larger than 0 ns. The invariant mass of each decay mode has been plotted in Fig.1(a) through (e). The observed number of events is listed in TABLE I.

Only the $D^+ \to K^-\pi^+\pi^+$ inclusive decay mode is used to calculate the Luminosity$\times$Cross-section of $D^{+*}D^-$ ($L \times \sigma(D^{+*}D^-)$). The result will be used in the calculating of branching ratio $D^- \to \mu^-\bar{\nu_\mu}$.

The number of observed events equals,

$$L\sigma(D^{+*}D^{-}) \times \varepsilon \times Br(D^{+} \rightarrow K^{-}\pi^{+}\pi^{+})$$
$$\times(1 + Br(D^{+*} \rightarrow \pi^{0}D^{+}) + Br(D^{*+} \rightarrow \gamma D^{+})). \qquad (8)$$

The variables and their values are listed in TABLE II.

5. Search for $D^{-} \rightarrow \mu^{-}\bar{\nu_{\mu}}$

On the basis of the single tag study, we searched for the muon decays by looking at the recoil system of the tagged hadronic decays.

The momentum of the reconstructed D tag is 430 - 590 MeV for D^0 and 440 - 610 MeV for $D^{\pm}$ according to the Monte Carlo study. The muon momentum cut is from 700 MeV to 1,250 MeV. For the particle identification, we imposed 1% confidence level cut on each pion, kaon and muon candidate and weight(Kaon to Pion) larger than 50% for Kaon candidate. The difference between the invariant mass of $\pi^{+}\pi^{-}$ and the ${K_s}^0$(498 MeV) less than 30 MeV for ${K_s}^0$ decays. The decay time of ${K_s}^0$ has to be larger than 0 ns. Other cuts depending on the decay modes are also listed here:

$D^0 \rightarrow K^{-}\pi^{+}, D^{-} \rightarrow \mu^{-}\bar{\nu_{\mu}}$: 3 charged tracks; the difference between the invariant mass of $K^{-}\pi^{+}$ and the D^0 mass less than 87 MeV(3 σ); no isolated photon

$D^0 \rightarrow K^{-}\pi^{+}\pi^{+}\pi^{-}, D^{-} \rightarrow \mu^{-}\bar{\nu_{\mu}}$: 5 charged tracks; the difference between the invariant mass of $K^{-}\pi^{+}\pi^{+}\pi^{-}$ and the D^0 mass less than 54 MeV(3 σ); no isolated photon

$D^0 \to K^0\pi^+\pi^-, K^0 \to \pi^+\pi^-; D^- \to \mu^-\bar{\nu_\mu}$: 5 charged tracks; the difference between the invariant mass of $\pi^+\pi^-\pi^+\pi^-$ and the D^0 mass less than 63 MeV(3 σ); no isolated photon

$D^+ \to K^-\pi^+\pi^+, D^- \to \mu^-\bar{\nu_\mu}$: 4 charged tracks; the difference between the invariant mass of $K^-\pi^+$ and the D^0 mass less than 66 MeV(3 σ); at most 2 photons and the maxium photon energy less than 400 MeV(from π^0 decay)

$D^+ \to K^0\pi^+, D^- \to \mu^-\bar{\nu_\mu}$: 4 charged tracks; the difference between the invariant mass of $\pi^+\pi^-\pi^+$ and the D^+ mass less than 81 MeV(3 σ); at most 2 photons and the maxium photon energy less than 400 MeV(from π^0 decay)

After the above selections are applied to the D^*D event sample, there are only a few remaining data events in the hadronic modes, $K\pi\pi$, $K\pi\pi\pi$ and $K\pi$. Two-dimensional plots of muon momentum(Y axis) vs the square of missing mass(X axis) are shown in Fig.2 for the three decay modes which have these events The data events appear as solid black dots in Fig. 2. Monte Carlo simulations for signal and backgrounds were performed. In Fig. 2, the left contour is the Monte Carlo signal(D muon decay) region and the right contour is the Monte Carlo background(D semileptonic decay) region. The signal and background regions are well separated. One signal candidate is observed in $K^-\pi^+$ tag mode(Fig.2(b)) in the left signal region. The properties of the candidate are listed in TABLE IV and the event display is shown in Fig.3. The event is very clean with 2 hits in the muon counter for the muon track, no extra photons and pion and kaon tracks that are well identified by the dEdX and TOF systems.

6. Branching Ratio of $D^- \rightarrow \mu^- \bar{\nu}$

For the calculation of the $Br(D^- \rightarrow \mu^- \bar{\nu})$, we use the following formulas, where ε is the efficiency to detect $D^+ \rightarrow$ hadronic tag and $D^- \rightarrow \mu^- \bar{\nu}$

For D^0 decay mode i:

$$\begin{aligned}
\#\ of\ obs.\ events &= Br(D^- \rightarrow \mu^- \bar{\nu}) \times Br(D^{+*} \rightarrow \pi^+ D^0) \\
&\quad \times L\sigma(D^{+*} D^-)\varepsilon \times Br(D^0 \rightarrow Hadrons) \\
&= Br(D^- \rightarrow \mu^- \bar{\nu}) \times N_i \qquad (9)
\end{aligned}$$

For D^+ decay mode i:

$$\begin{aligned}
\#\ of\ obs.\ events &= Br(D^- \rightarrow \mu^- \bar{\nu}) \times Br(D^{+*} \rightarrow \pi^0 D^+) \\
&\quad \times 2L\sigma(D^{+*} D^-)\varepsilon \times Br(D^+ \rightarrow Hadrons) \\
&= Br(D^- \rightarrow \mu^- \bar{\nu}) \times N_i \qquad (10)
\end{aligned}$$

The Poisson Distribution Function $P_i(Br \times N_i + bg_i, n)$ for decay mode i is:

$$P_i(Br \times N_i + bg_i, n) = \frac{(Br \times N_i + bg_i)^n}{n!} \times e^{-(Br \times N_i + bg_i)}$$

n is the number of the observed events, bg_i is the predicted background for the decay mode i, Br is the $D^- \rightarrow \mu^- \bar{\nu}$ branching ratio and N_i is the predicted number of observable $D^+ \rightarrow$ hadronic tag and $D^- \rightarrow \mu^- \bar{\nu}$ exclusive events.

The combined Poisson Distribution Function(LK) is then constructed(numbers are from TABLE III):

$$\begin{aligned} LK &= \prod P_i(Br \times N_i + bg_i, n) \\ &= (Br \times N_1 + bg_1) \times e^{(Br\times N_1 + bg_1)} \\ &\quad \times e^{(Br\times N_2 + bg_2)} \times e^{(Br\times N_3 + bg_3)} \times e^{(Br\times N_4 + bg_4)} \times e^{(Br\times N_5 + bg_5)} \\ &= e^{-\Sigma bg_i} \times (Br \times N_1 + bg_1) \times e^{-\Sigma Br\times N_i} \\ &= 0.82(268.9Br + 0.03)e^{-1039.7Br} \qquad (11) \end{aligned}$$

The Fig.4 is the combined Poisson Distribution Function plot. By integrating the likelihood function we obtain a 90% upper limit for the branching ratio of

$$Br(D^- \to \mu^- \bar{\nu}) < 0.34\%. \qquad (12)$$

If we assume that the single event is a $D^- \to \mu^- \bar{\nu}$ decay, the resulting branching ratio from the likelihood curve with fit errors is, Br($D^- \to \mu^- \bar{\nu}$)=$0.08^{+0.16}_{-0.05}$ %. The one standard deviation fit errors on the branching ratio are set by 68% of the area between the peak of the likelihood (0.08) and infinity for the plus error and 68% of the area between the 0 and 0.08 for the minus error.

The systematic errors are estimated by varying the event selection criteria, the background estimates by 50% and the value of $L \times \sigma$ by 1 standard deviation. Adding these errors linearly, the value for the systematic errors are obtained. The final value for the branching ratio is

$$Br(D^- \to \mu^- \bar{\nu}) = 0.08^{+0.16+0.05}_{-0.05-0.02}\%. \qquad (13)$$

where the first errors are due to the fit and the second are the systematic errors. With the 0.08% branching ratio, we can predict the number of events we should see for each decay mode. This is listed in table III and is consistent with what we observe.

By using equation 1 and relations,

$$Br(D^- \to \mu^- \bar{\nu}) = \frac{\tau_D}{\hbar}\Gamma(D^- \to \mu^- \bar{\nu}) \tag{14}$$

$$\begin{aligned}\Gamma(D^+ \to \mu^+ \nu) &= \frac{G_F^2|V_{cd}|^2}{8\pi} f_D^2 m_D m_\mu^2 (1 - \frac{m_\mu^2}{m_D^2})^2 \\ &= constant \times f_D^2\end{aligned} \tag{15}$$

$$constant = \frac{G_F^2|V_{cd}|^2}{8\pi} m_D m_\mu^2 (1 - \frac{m_\mu^2}{m_D^2})^2$$

the D meson decay constant f_D is estimated. The 90% C.L. upper limit for f_D is

$$f_D < 5.9 \times 10^2 \text{MeV} \tag{16}$$

and for a single event signal, it is

$$300^{+180+80}_{-150-40} \text{ MeV}. \tag{17}$$

The first error is the statistical, determined from the likelihood function, and the second error is the systematic error caused by uncertainties in the efficiency, the cross section and background estimation.

7. Background Study

In order to estimate the background feed down from other D decay modes, we generated 5 million Monte Carlo $D^{+*}D^-$ events(cocktail vs cocktail) to determine the possible background contributions in the signal region. The three biggest background decay modes and their contributions in the signal region are listed in TABLE V.

Also in a high statistic study, we generated 1 million $D^{+*}D^-$ events for each tag mode ($D^0 \to K^-\pi^+$, $K^-\pi^+\pi^+\pi^-$, $K^0\pi^+\pi^-$ vs $D^- \to$ cocktail and $D^+ \to K^-\pi^+\pi^+$, $K^0\pi^+$ vs $D^- \to$ cocktail) and checked each background channel. We got very similar background contributions.

The biggest background from D_S decay is ${D_S}^+ \to K^+K^-\pi^+$ which can be mistreated as either $K^-\pi^+\pi^+$ or $\pi^+\pi^-\pi^+$ final stat because of the K π misidentification. We estimated these contributions by generating 0.5 million Monte Carlo ${D_S}^+{D_S}^-$ events where one D_S decays to muon neutrino and the other decays to cocktail. The resulting background contributions are also in TABLE V.

As a further check we studied outside the signal area (missing mass square larger than 0.7) where we observed two event in Fig.2(a) ($K^-\pi^+\pi^+$ mode), two events in Fig.2(b) ($K^-\pi^+$ mode) and one event in Fig.2(c) ($K^-\pi^+\pi^+\pi^-$ mode). By using equation(9) and (10), we calculated how many semi-leptonic decay events which are the major contribution we should see in this region. The results are listed in TABLE VI. They are consistent with the number of observed events.

8. The K_L Detecting Efficiency

The K_L is involved in the background decays and MC simulation can not give detecting efficiency because it does not decay in the detector. But sometimes K_L does interact with the shower counter material to generate photon(s). In this case, it can be detected. So we need to use the real data set to find this detecting efficiency and put it into the background estimation. Since BES has collected about 3 million ψ' events, we can analysis and detect the K_L detecting efficiency. The decay mode we used is $\psi' \to \pi^+\pi^- J/\psi$, $J/\psi \to K_S K_L$, $K_S \to \pi^+\pi^-$.

The decay event selections are listed here. Four charged tracks are required, two positive and two negtive. Without particle ID cut, we assume all 4 tracks are pions. the recoil J/*psi* invariant mass cut is from 3.072 to 3.120 GeV and the K_S invariant mass is 0.468 - 0.528 GeV. The momentum of K_S and K_L is from 1.25 to 1.65 GeV. The angle between K_S momentum and K_L momentum(missing momentum) is larger than 135 degree to ensure they are back to back. We require isolated photon number ≤ 2 to reduce the background. When #photon ≥ 1, angle between missing momentum and photon(s) is smaller than 10 degree and total photon energy is greater than 1.2 GeV. For #photon=2, angle between two photons should be less than 15 degree.

The Fig.5 is the invariant mass distribution of J/ψ. The peak value is 3.095 GeV and the σ is 8 MeV. The Fig.6 is the invariant mass plot of K_S. The signal is peaked at 498 MeV with 11 MeV width. The missing mass(K_L mass) plot is in Fig.7 where we can see the signal events around 500 MeV. We use the following formula to estimate how many decay events we should see and get 17.3 events,

$$\begin{aligned} N &= L\sigma \times \varepsilon \times Br(\psi' \to \pi^+\pi^-\psi) \\ &\quad \times Br(\psi \to K_S K_L) \times Br(K_S \to \pi^+\pi^-) \\ &= 17.3 \end{aligned} \tag{18}$$

where $L\sigma$=3 million, ε(efficiency)=24%, $Br(\psi' \to \pi^+\pi^-\psi)$=32.4%, $Br(\psi \to K_S K_L)$=0.01%, and $Br(K_S \to \pi^+\pi^-)$=68.8%.

In Fig.7, there are total 14.3 events in 400 MeV to 600 MeV mass region after extracted from the background. It is consistent with the calculation. The observed events can be divided into two groups, One with photon(s)(N_1) and the other without photon(N_2). The ${K_L}^0$ detecting efficiency is defined as the ratio of the N_1 divided by the total number of events(N_1+N_2). With $N_1 = 5.3 \pm 2.3$ events and $N_2 = 7.5 \pm 2.7$ events, ${K_L}^0$ detecting efficiency $= \frac{N_1}{N_1+N_2} = (41 \pm 14)\%$.

The ${K_L}^0$ momentum distribution in ψ' decays is from 1.25 to 1.65 GeV. But in the D meson semileptonic decay, the ${K_L}^0$ momentum distribution is from 0.5 to 0.9 GeV. So we need to rescale this efficiency to this momentum region. By using the isospin conservation, we can generate the following interaction relationship:

$$\begin{aligned} \sigma(K_L n) &= \frac{1}{2}(\sigma(K^0 n) + \sigma(\bar{K}^0 n)) = \frac{1}{2}(\sigma(K^+ p) + \sigma(K^- p)) \\ \sigma(K_L p) &= \frac{1}{2}(\sigma(K^0 p) + \sigma(\bar{K}^0 p)) = \frac{1}{2}(\sigma(K^+ n) + \sigma(K^- n)) \\ \sigma(K_L) &= \frac{1}{2}(\sigma(K_L n) + \sigma(K_L p)) \end{aligned} \tag{19}$$

The total cross section distribution of interaction $K^+p(n)$ and $K^-p(n)$ can be found in the PDG. The values are summarized in TABLEV VII. The scaling factor is the ratio of the $\sigma(K_L)$ at momentum of 0.7 GeV devided by the $\sigma(K_L)$ at momentum of 1.45 GeV. The value equals 0.92. So the ${K_L}^0$ detecting efficiency will be scaled down by 0.92. The final efficiency is $38 \pm 14\%$.

9. CONCLUSION

1. We found one decay event of $D^+ \rightarrow \mu^+\nu$.

2. The measured Branching Ratio of the decay $D^+ \rightarrow \mu^+\nu$ is $0.08^{+0.16+0.05}_{-0.05-0.02}\%$.

3. The estimated background in the signal region are small and the estimated background outside the signal region are consistent with the number of observed events.

Table of the Inclusive decays				
Decay Mode	Final State	Obs. events	$\sigma(data)$MeV	σ(MC)MeV
$D^0 \to K^-\pi^+$	$K^-\pi^+$	2,348±83	28±1	31±1
$D^0 \to K^-\pi^+\pi^+\pi^-$	$K^-\pi^+\pi^+\pi^-$	827±56	17±1	19±1
$D^0 \to K^0\pi^+\pi^-$	$\pi^+\pi^-\pi^+\pi^-$	180±31	23±4	21±1
$D^+ \to K^-\pi^+\pi^+$	$K^-\pi^+\pi^+$	1,409±66	21±1	23±1
$D^+ \to K^0\pi^+$	$\pi^+\pi^-\pi^+$	122±17	27(fixed)	27±1

TABLE I. Single-tagged D^0 and $D^\pm$ events

Table of the cross section		
Variable	Value	source
N(Observed Events)	$1,409 \pm 66$	fit from data(input)
ε(Efficiency)	$(22.6 \pm 0.4)\%$	m.c. simulation(input)
$Br(D^+ \to K^-\pi^+\pi^+)$	$(9.1 \pm 0.6)\%$	PDG(input)
$Br(D^{+*} \to \pi^0 D^+)$	$(30.8 \pm 0.8)\%$	PDG(input)
Integrated Luminosity(L)	$22.3 \pm 0.9(pb)^{-1}$	measured(input)
$L\sigma(D^{+*}D^-)$	$52,379 \pm 4,372$	calculated
$\sigma(D^{+*}D^-)$	$2.35 \pm 0.22(nb)$	calculated

TABLE II. Calculation of $\sigma(D^{+*}D^-)$

Table of the Branching Ratio					
i	Decay Mode	N_i	Obs./expected($0.07\% N_i$) Event	Efficiency(%)	BG(bg_i)
1	$K^-\pi^+$	268.9	1/0.19	18.8±0.3	0.03
2	$K^-\pi^+\pi^+\pi^-$	268.7	0/0.19	9.3± 0.3	0.03
3	$K^0\pi^+\pi^-$	30.2	0/0.02	3.2± 0.2	0.01
4	$K^-\pi^+\pi^+$	447.6	0/0.31	15.6± 0.3	0.12
5	$K^0\pi^+$	24.3	0/0.02	5.5± 0.2	0.01
	total	1,039.7	1/0.73		0.20

TABLE III. Double-tagged $D^{\pm *}D^\mp$ events

Table of the decay candidate			
Tagged D Decay	D Mass(MeV)	Muon Momentum(MeV/c)	$M^2{}_{miss}(GeV^2)$
$D^0 \to K^-\pi^+$	1,850	1,048	0.533

TABLE IV. D^+ Leptonic Decay Candidates

Table of the Background in the signal region				
Mode/BG	$D^- \to \pi^- K^0$	$D^- \to \mu^-\bar{\nu}K^0$	$D^- \to \mu^-\bar{\nu}\pi^0$	$D_S{}^+ \to K^+K^-\pi^+$
$D^0 \to K^-\pi^+$	0.02	0.01	0.00	0.00
$D^0 \to K^-\pi^+\pi^+\pi^-$	0.02	0.00	0.01	0.0
$D^0 \to K^0\pi^+\pi^-$	0.01	0.00	0.00	0.0
$D^+ \to K^-\pi^+\pi^+$	0.04	0.02	0.04	0.02
$D^+ \to K^0\pi^+$	0.00	0.00	0.00	0.01

TABLE V. Background in the Signal Region

Table of the BG check outside signal region			
Decay Mode	Efficiency(%)	Expected events	Observed events
$K^-\pi^+\pi^+$ vs $K^0\mu^+\nu$	1.1	2.2	2
$K^-\pi^+$ vs $K^0\mu^+\nu$	1.5	1.5	2
$K^-\pi^+\pi^+\pi^-$ vs $K^0\mu^+\nu$	0.8	1.5	1
total		5.2	5

TABLE VI. Background outside the Signal Region

Table of the Cross Section					
K_L Momentum\Mode	K^-p(mb)	K^+p(mb)	K^-n(mb)	K^+n(mb)	K_L(mb)
1.45 GeV	31.0	17.5	25.0	18.0	21.25
0.70 GeV	32.0	12.5	20.0	13.5	19.5

TABLE VII. Interaction Cross Section

Figure 1: Invariant Mass Plot of Each Decay Mode

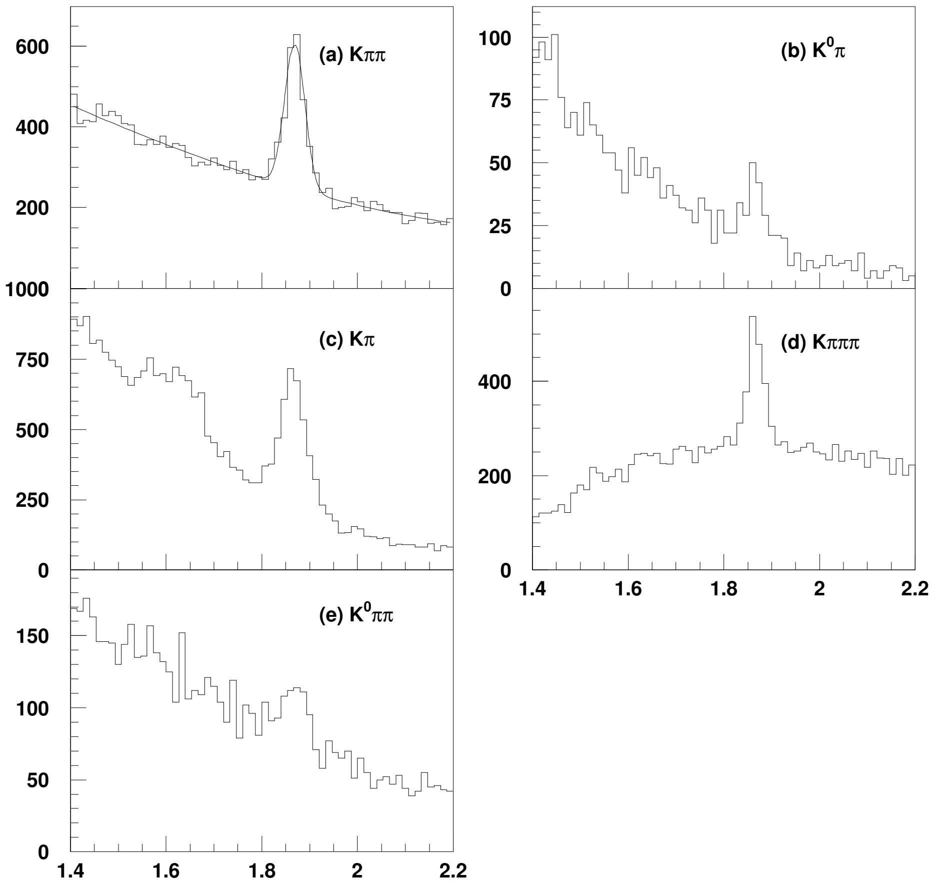

Figure 2: Muon Momentum vs $M^2{}_{miss}$

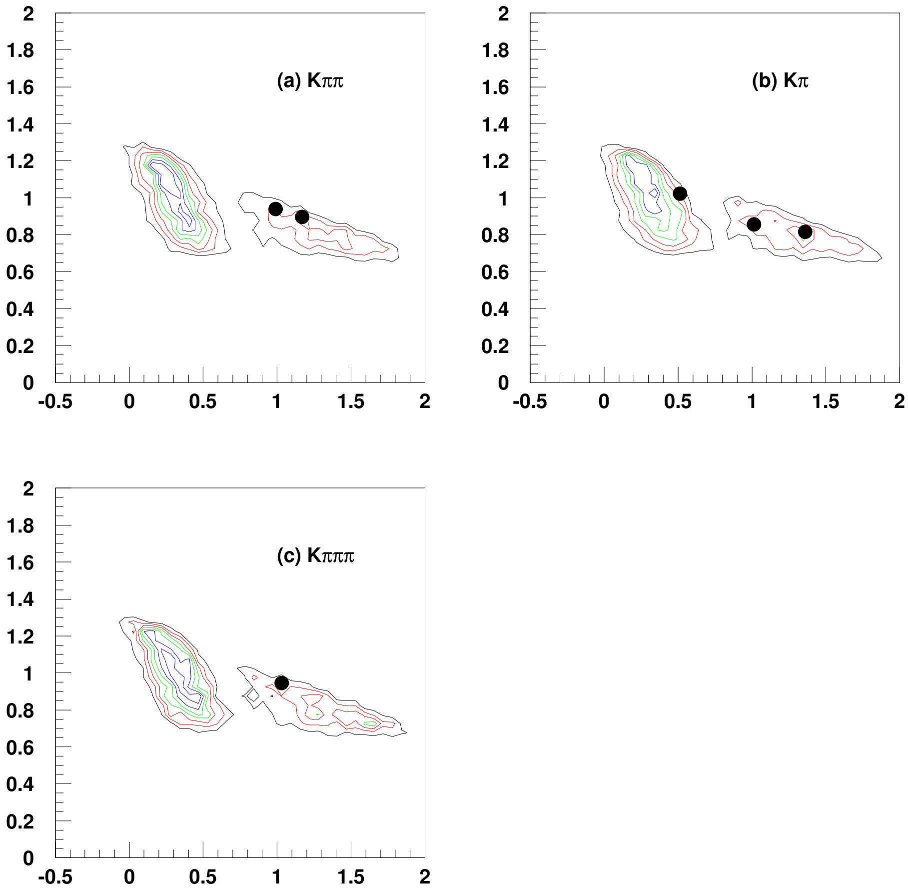

Figure 3: $D^- \to \mu^- \bar{\nu}$ candidate1 display

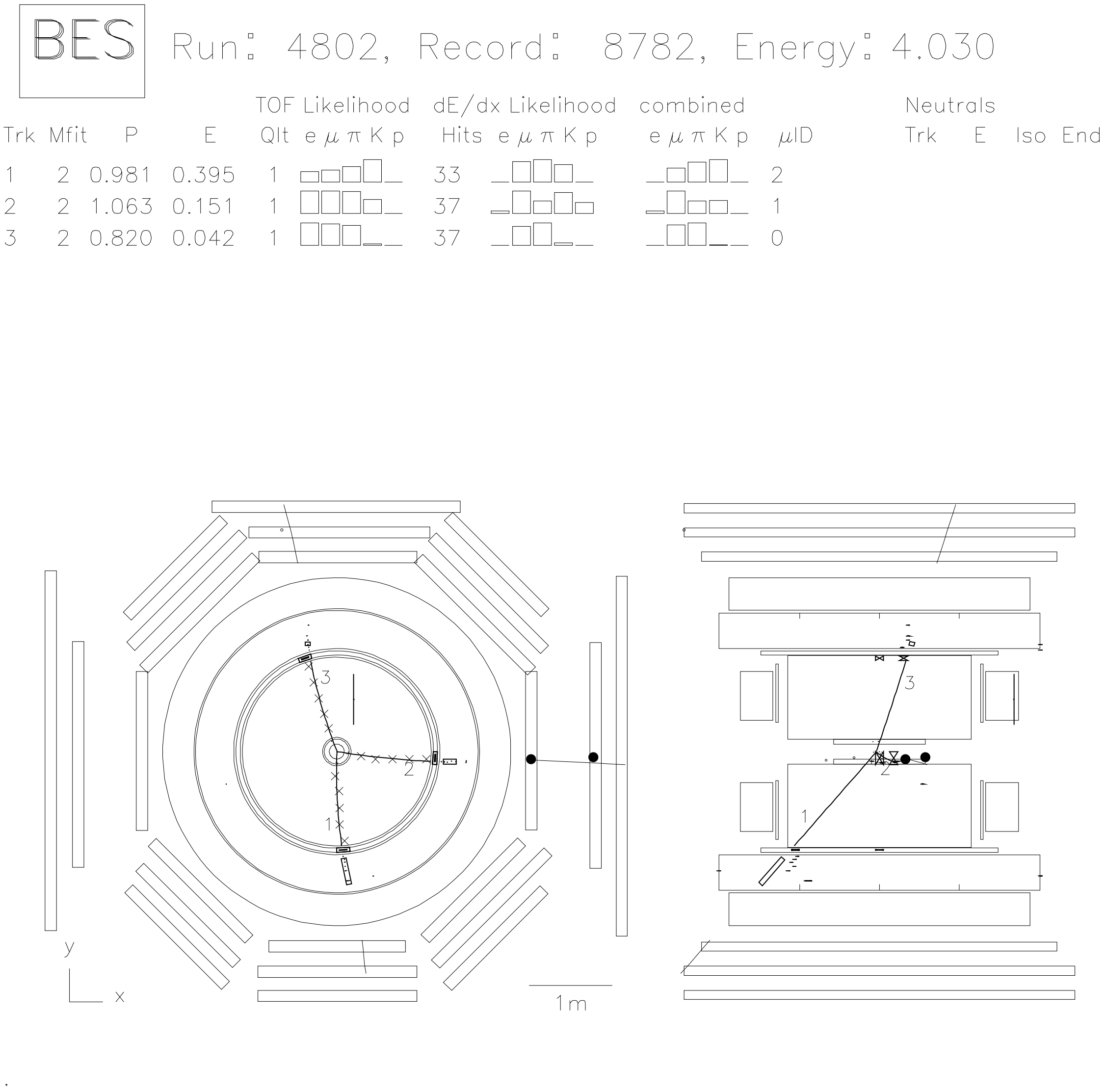

Figure 4: Likelihood function of BR($D^- \rightarrow \mu^- \bar{\nu}$)

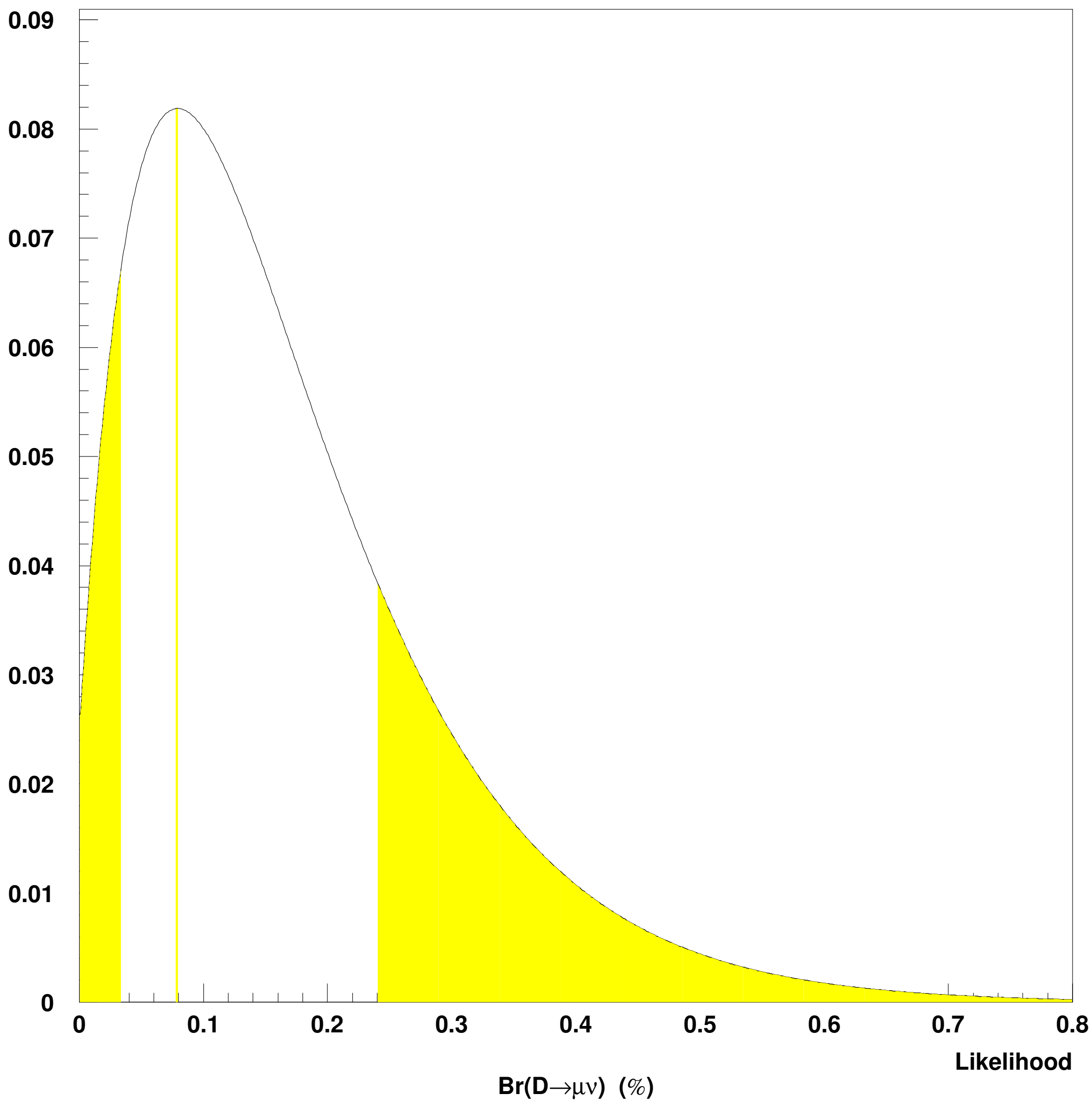

Figure 5: Invariant Mass Plot of J/ψ

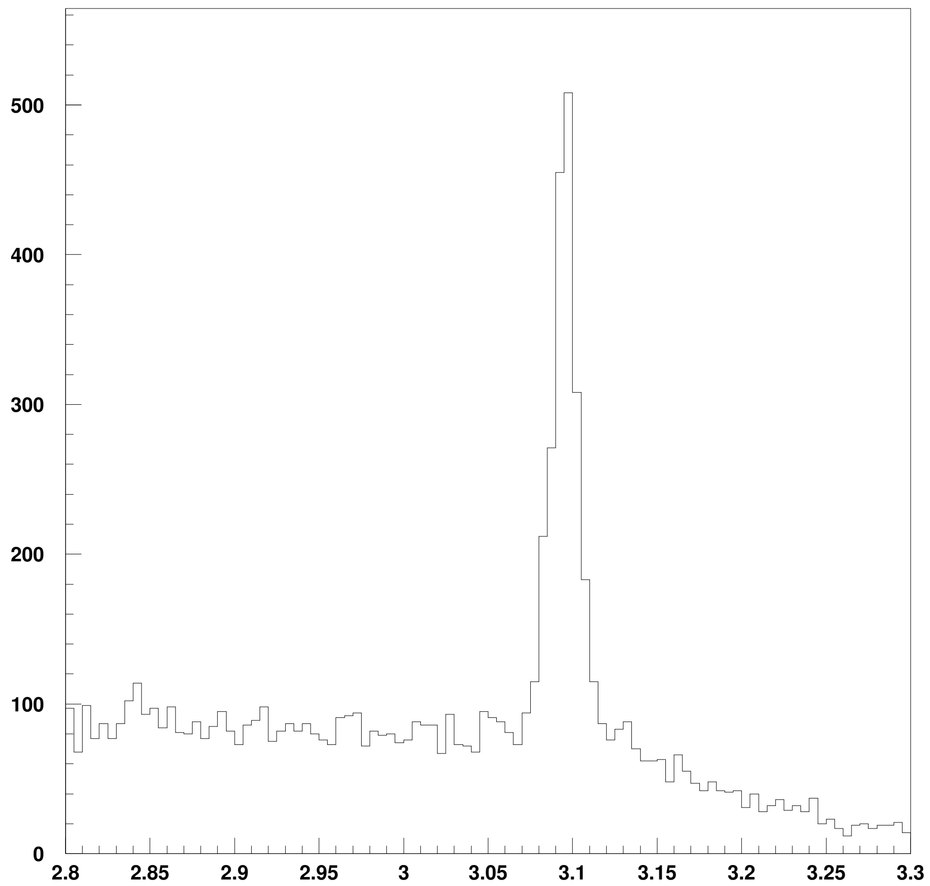

Figure 6: Invariant Mass Plot of K_S

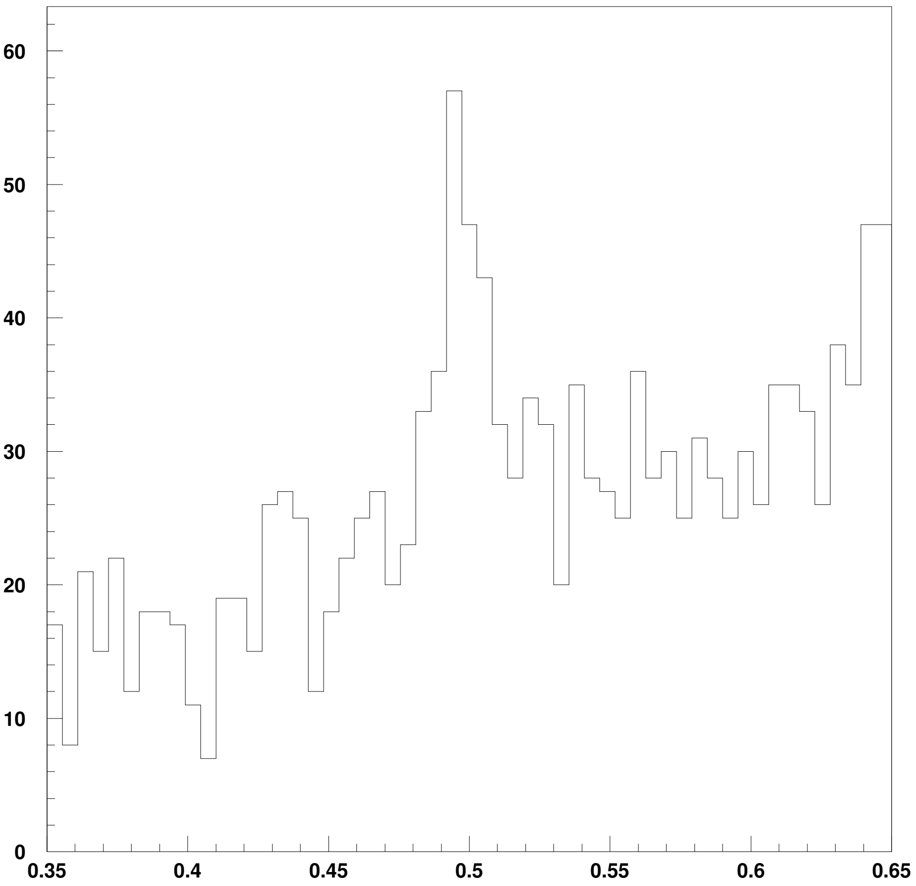

Figure 7: Invariant Mass Plot of K_L