Multiplicity Distribution in Lund Model

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1. Multiplicity Distribution

We calculate the multiplicity distribution of primary hadrons in e^+e^- collision based on the Lund area law.

Using single particle transverse mass $m_\perp$ and rapidity y, the particle's momentum may be represented as

$$p_\pm = m_\perp(exp(+y), exp(-y)). \tag{1}$$

The n-particle phase-space factor reads

$$\begin{aligned} d\Psi &= [\prod_{j=1}^{n} \hat{N} dp_j \delta(p_j^2 - m_j^2)]\delta(\sum_{j=1}^{n} p_j - P_0) \\ &\simeq \frac{\hat{N}^2}{s} \prod_{j=2}^{n-1} \hat{N} dy_j. \end{aligned} \tag{2}$$

Assume that the remaining particles are placed in rapidity between $[-\Delta y/2, +\Delta y/2]$ with

$$\Delta y = ln\frac{s}{s_0}, \tag{3}$$

s_0 is some suitable scale. The phase space factor may be integrated analyticly,

$$\Psi = \int d\Psi = \frac{\hat{N}^n(\Delta y)^{n-2}}{s(n-2)!} \tag{4}$$

In case of all particles have the same transverse mass, the fragmentation area A of

string way be written as

$$A = m_\perp^2 \sum_{j=1}^{n} \sum_{i=j}^{n} exp(y_i - y_j). \qquad (5)$$

Define the dimensionless n-particle partition function

$$\begin{aligned} Z_n &= s \int d\Psi \cdot exp(-bA) \\ &\simeq \frac{(c_1 \Delta y)^n}{n!} exp(-\frac{b m_\perp^2 c_2 n^2}{\Delta y}), \end{aligned} \qquad (6)$$

which is related to the multiplicity distribution P_n by

$$P_n = \frac{Z_n}{\Sigma Z_n}. \qquad (7)$$

Binominal distribution

$$P(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n} \qquad (8)$$

form a family of distributions depending on the values of parameters N and p. To determine c_1 and c_2 in Z_n from the N and p parameters in binomial distribution, we begin by using Stirlings approximation

$$N! \simeq \sqrt{2\pi} \sqrt{N} e^{-N} N^N \qquad (9)$$

to write the difference

$$ln(P_n) - ln(P(n)). \qquad (10)$$

Express $n = \mu + \delta n$ and expand above formula around $\delta n = 0$ up to terms of order

δn^2. **Equating the series coefficients to zero, we obtain**

$$\begin{aligned} c_1 \Delta y &= Np\ exp\left[\frac{1-2p+2Np-2Np^2}{4N^2(1-p)^2}\right], \\ \frac{bm_\perp^2 c_2}{\Delta y} &= \frac{1}{2N(1-p)}, \end{aligned} \qquad (11)$$

then we have $(\mu = Np)$

$$P_n \simeq \frac{\mu^n}{n!} \cdot exp\left[\frac{-n^2+2\mu n}{2N(1-p)}\right]. \qquad (12)$$

In practice, we rewrite P_n **in a more symmetric form** $(c_0, c_1 > 0, c_2 < 0)$

$$P_n = \frac{\mu^n}{n!} \cdot exp[c_0 + c_1(n-\mu) + c_2(n-\mu)^2]. \qquad (13)$$

The energy-dependence of μ **may be expressed as two forms**

$$\mu = a + b \cdot exp(c\sqrt{s/s_0}), \qquad (14)$$

or

$$\mu = a + b\ ln(s/s_0) + c\ ln^2(s/s_0). \qquad (15)$$

All of the parameters used in above multiplicity distribution have to be determined by BES data phenomenologically.

2. Exclusive probability

The probability to obtain a particular state with the n particles is given by

$$R_n = C(VPS)(SUD)Z_n(m_{\perp 1}, \cdots, m_{\perp n}; s). \quad (16)$$

The functions R_n are the same independent of the rank-ordering of the particles.

To obtain the right channel-distribution we will sum over all the different orderings and strings in Eq. (16).

- C **is the combinatorial number (stemming from the quark contents of the possible strings-there are in general more than one such ordered "quark-string" possible to make the particular mesons and even sometimes the possibility to make the same meson(s) at different places in the same string);**
- (VPS) **are the vector to pseudoscalar rates;**
- (SUD) **the strange to up and down probability.**

3. Summary

Multiplicity distribution (13) of primary hadron stemed from Lund area law gives a reasonable description for continuous state production in $2 - 5$ GeV energy.