The Radiative Correction in the Measurement of pion Form Factors

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1 Introduction

The treatment of QED radiative corrections in the continuous region reveals another interesting aspect of radiative corrections. Here like in the problem of resonances, and τ mass measurement, the radiative corrections include vertex correction and initial state radiation (bremstrahlung). Each of them is infrared-divergent. But the sum of the two is finite, with the physical meaning of higher order terms. The correction depends in a sensitive way on the experimental conditions, i.e., on the criteria of the event selection. In the pion form factor measurement, the final state interactions are included in the definition of the form factor, i.e. theoretical models which calcalute the pion form factor should incorporate all the final state interactions. Only initial state radiative correction is treated in the experiments.

Also, unlike in R measurement, here the contributions from the 1^{--} resonances are NOT subtracted from the observed cross section. This is because here the theoretical mechanism of pion form facor is dominated by these resonant vector mesons (VMD). The effects of these resonances are the very thing we want to learn from pion form factors.

2 The formulae

The expression for first order radiative correction is given by G.Bonneau and F.Martin on Nucl.Phys **B27**(1971)381

$$\begin{aligned}
\sigma &= \sigma(4E^2)[1+\frac{2\alpha}{\pi}\{(-1+2\log\frac{2E}{m_e}) \\
&\quad (\log\frac{A}{E}+\frac{13}{12} \\
&\quad +\int_A^{q_{max}}\frac{dq_0}{q_0}(1-\frac{q_0}{E}+\frac{q_0^2}{2E^2})\frac{\sigma_0[4E(E-q_0)]}{\sigma_0(4E^2)}) \\
&\quad -\frac{17}{36}+\frac{1}{6}\pi^2\}]
\end{aligned}$$

where $\sigma_0(s)$ is the Born cross section at s; q_{max} is the maximum energy of the emitted photon; m_e is the electron mass; E is the energy per beam.

In the above formula, the result does not depend on the value of A. Usually it is taken to be the order of 1% of E. If the contributions from higher order Feynman diagrams are included, then the radiative corrected cross section is expressed in the general form

$$\sigma(s) \ = \int_0^\varepsilon dx \ \tilde{\sigma}(s(1-x))F(x,s), (1.1)$$

where $\sqrt{s}$ is the C.M. energy of the colliding beam,

$$\tilde{\sigma}(s) = \sigma_0(s)\frac{1}{|1-\Pi(s)|^2} \tag{1}$$

$\sigma_0(s)$ is the cross section in Born approximation, $\frac{1}{|1-\Pi(s)|^2}$ is the vacuum polarization; the expression

$$F(x,s) \ = \ \beta x^{\beta-1}\delta^{V+S} + \delta^H, \tag{2}$$

where

$$\beta \ = \ \frac{2\alpha}{\pi}(\ln\frac{s}{m_e^2} - 1), \tag{3}$$

$$\delta^{V+S} \ = \ 1 + \tfrac{3}{4}\beta + \tfrac{\alpha}{\pi}(\tfrac{\pi^2}{3} - \tfrac{1}{2})$$
$$+\beta^2(\tfrac{9}{32} - \tfrac{\pi^2}{12}),$$

$$\delta^H \ = \ \delta_1^H + \ \delta_2^H, \tag{4}$$

$$\delta_1^H \ = \ -\beta(1-\frac{x}{2}), \tag{5}$$

$$\delta_2^H \ = \ \tfrac{1}{8}\beta^2[4(2-x)\ln\tfrac{1}{x}$$
$$-\tfrac{1+3(1-x)^2}{x}\ln(1-x) - 6 + x].$$

This formula is slightly more complicated than the first order correction, but it is still easy for programming and numerical handling on computer. For the case of pion form factor, the deviation between the two formulae is roughly the order of α.

In the second formula, unlike in the expression of the first order correction, the x does NOT bear the physics meaning of the fraction of the energy carried away by photons (from one beam). Nevertheless, for small value of x, it approximately equal to that fraction. While in the measurement of pion form facotors, among the event selection criteria, the condition $|E_{2\pi} - E_{cm}| < 0.08GeV$ or $|E_{2K} - E_{cm}| < 0.08GeV$ are imposed. So the photon energy cannot exceed $0.08GeV$, which is small compared to the $E_{cm} = 2.6GeV$. Corresponding to this experimental condition, the upper limit of the integration above should be $0.08GeV/E_{beam}$. Notice that the dominant contribution of bremstrahlung comes from the events with photons flying along the direction of the colliding e^+e^- beams (collinear divergence). These photons can not be detected any way.

In the above formulae, both the first order ones, as well as the one including higher order contributions, the radiative correction depends on the behavior of the Born cross section at lower energies. In the cases of resonances or τ production near threshold, this behavior is known theoretically. But for pion form factor, this behavior can only come from experiments, i.e. the measurement of pion form factor at lower energies. Here a linear interpolation is performed between the energy which the measurement is performed ($2.6GeV$), and the ones at lower energy points published by other experimental groups. Strictly speaking, to find the pion form factor at the measured energy is a fitting process. The Born cross section is varied, to make the cross section with higher oder terms equal to what observed by the experiment.

3 The Result and Discussions

The observed cross section by Dr. Tong Guoliang and Dr. Xu Guofa is

$$\sigma_{\pi^+\pi^-,K^+K^-} = 0.11nb \tag{6}$$

Here the upper limit of the integration $\varepsilon = 0.08GeV/1.3GeV$, so the concerned energy range is between

$$\sqrt{2.6^2(1-\varepsilon)}GeV = 2.51GeV \tag{7}$$

and $2.6GeV$. Input the next available $\pi^+\pi^-$ or K^+K^- cross section which is at $2.025GeV$ and equal to $0.55nb$, and interpolating the cross section between $2.51GeV$ and $2.6GeV$, the cross section after considering radiative correction

$$\sigma_{\pi^+\pi^-,K^+K^-} = 0.1201nb \tag{8}$$

with higher order corrections included, or

$$\sigma_{\pi^+\pi^-,K^+K^-} = 0.1197nb \tag{9}$$

with only the first order (α^3) correction. The relative small discrepancy between first order result and higher order one reflects the fact that in the energy range concerned, the variation of the Born cross section is rather smooth. This is very different from the resonances, or τ production near threshold.

Here the radiative corrected cross section is smaller than the Born cross section. The factor is 0.92. This is due to that in the event selection criteria, a stringent cut $|E_{2\pi} - E_{cm}| < 0.08GeV$ or $|E_{2K} - E_{cm}| < 0.08GeV$ is imposed. This makes the allowed phase space for the bremstrahlung to be small. Or in terms of mathematical language, the upper limit of the integration $\varepsilon = 0.0615$. The interval of the integration is from 0 to 0.0615. Whereas the maximum phase space allowed by kinematics gives $\varepsilon = 1 - m_\pi^2/s = 0.9971$. In that case the radiative correction is substantially larger than 1.

For example, if the photon cut is $0.8GeV$ instead, then the radiative correction factor would be 1.44; whereas if the photon cut is down to $0.008GeV$, then the radiative correction factor would be 0.75.

This is because theoretically the process $e^+e^- \to \pi^+\pi^-$ cannot be seperated from $e^+e^- \to \pi^+\pi^-\gamma$. If the event selection allows photons carrying more energy to come through, then the cross section is bigger. On the contrary, if the selection is more stringent, fewer events pass through, then the cross section is smaller.