Cross Sections and Electromagnetic Form Factors of Hadrons

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@ 7th BES Annual Meeting, Datong, Shanxi

June 19 ,1998

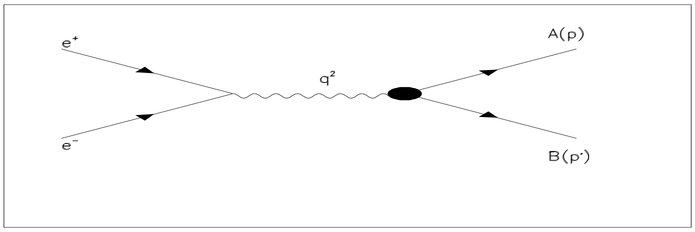
I. Introduction

Low-energy e^+e^--machine are the **ideal tool** to investigate the **time-like region of the electromagnetic form factors(F.F.) of low-mass hadrons**, i.e., of the lowest spin mesonic and baryonic states constructed only with u-, d- and s-quarks. The initial e^+e^- pair converts into an exclusive final state containing two particles(A and B, with four-momenta p and p') through the exchange of a virtual single-photon of squared four-momentum $q^2 = s = (p+p')^2 \geq (m_A + m_B)^2$ as show in Fig(1).

The amplitude associated to this process is

$$M = \frac{e^2}{q^2}\bar{v}\gamma^\mu u < A(p)\bar{B}(p')|J_\mu^{em}|0 > \tag{1}$$

where the first factors contain the QED point-like vertex and the photon propagator, whereas the final one, written in terms of the hadronic electromagnetic (e.m.) current and represented by the blob in Fig.(1), incorporates the effects of strong interactions and hadronic structure in one or a few form factors, $F(q^2)$.



Feynman Diagram for $e^+e^- \longrightarrow \gamma \longrightarrow AB$

From the well-known conservation rules, the two-particle final state is forced to have the photonic quantum numbers too, i.e., J=1,P=C=-1 and isospin I=1 or 0. These quantum numbers can be achieved either with A=B or with $A \neq B$, thus allowing to distinguish between "**elastic (or static)**" form factors and "**transition (or dynamical)**" ones.

Our present understanding of the strong interractions governing quark dynamics is based on the QCD lagrangian from which no reliable information can be extracted except for asymptotic Q^2-values. In this region, quark counting rule predictions can be deduced. In the remaining q^2-regions, accessible to low-energy e^+e^- machines, no valid theoretical predictions are available and several phenomenological models have been proposed. Examples are the **Vector Meson Dominance(VMD) model** (where the similarities between vector-mesons and photons are exploited) and constituents are quark models with their underlying SU(3) symmetry.

Now VMD has been developed in two main directions. One consists in extending the sum not only lowest-lying V-mesons but also to their high-mass recurrencies or radial excitations. A second line of improvement consists in introducing SU(3)-breaking effects.

1) The e^+e^- annihilation to hadrons Investigations of the **electromagnetic form factors** of hadrons provide important information about their structure.

Since 1988, the $\rho(1600)$ was replaced by two new resonances- $\rho(1450)$, and $\rho(1700)$ in PDG, Erkal[1] had pointed out this possibility with a theoretical analysis on the consistency of 2π and 4π electromagnetic form factors.

The experimental status are included in table 1 and table 2.

Table 1. Study of Electromagnetic Form Factor

in e^+e^- Annihilation (Past)

Date	Group	channel	$\sqrt{S}$(GeV)	Comment
1972	Novosibirsk	$\pi^+\pi^-$	< 1.4	
1980	Frascati	$\pi^+\pi^-$	1.2-1.6	
1988	DM2(DCI)	$\pi^+\pi^-$	1.35-2.4	*
1973	Frascati ADONE	$p\bar{p}$	2.1	
1979	DM1(DCI)	$p\bar{p}$	1.925-2.180	(400nb^{-1})
1983	DM2(DCI)	$p\bar{p}$	1.975-2.250	(500nb^{-1})
1994	FENICE	$p\bar{p}$	1.90-2.44	
1976	DASP(DORIS)	$\pi^+\pi^-$ K^+K^- $p\bar{p}$ $K^\pm K^{*\mp}$(890)	J/ψ, ψ'	
1985	MARK III	$\pi^+\pi^-$ K^+K^- $K^0_S K^0_L$ $K^0_S K^0_S$	J/ψ	84 $\pi^+\pi^-$ evt.
1988	DM2(DCI)	K^+K^-	1.35-2.4	

* ρ(1600) was divided into ρ(1450) and ρ(1700)

Table 2. Study of Electromagnetic Form Factor in e^+e^- Annihilation (Future)

Date	Group	$\sqrt{S}$(GeV)
The end of 1998 ?	BES	2.0-3.1
1999?	KLOE	1.02 -1.5
The end of 1998	VEPP-4M	1.5-3.1

Table 3 shown the fitted results of $\rho'_1 - \rho'_2$, ω'_1, and ϕ'_1.

Table 3. The Fit Parameters of Resonances

resonance	mass(MeV) width(MeV)	PDG
ρ'_1	1370 $(\pi^+\pi^-)$	1465
ρ'_2	1900 $(\pi^+\pi^-)$	1700
ω'_1	1670(K^+K^-) 160	1420 174
ϕ'_1	1680 (K^+K^-) 180	1680 150

Detailed experimental data on the pion form factor in timelike region permit highly accurate determinations of the parameters of the ρ**-meson and its radial recurrencies such as** $\rho(1450)$**,**

$\rho(1700)$. There are intentions to study the energy range of e^+e^- annihilation in the interval of the center-of-mass energy from $\sqrt{s} = 1.5$ GeV up to 5 GeV. This raises the question of comparison of the results of existing analysis of the diagonal and transition form factors of light hadrons in the energy range between 1 and 2 GeV [2,3] with the data at J/ψ. see Fig.(2).

This is a good opportunity for BEPC to reveal and confirm the hypothesis.

The proton electromagnetic form factors have been widely investigated in the **space-like region** [4]. Several measurements have been performed also in the **time-like region** at s values between the nucleon-antinucleon threshold ($s = (2M_p)^2 = 3.52 GeV^2$) and $s = 13 GeV^2$ [5,6,7,8]. As well as in the space-like region, but approaching a different proportionality constant value.

2) The measurement is also very important for the study of the SU(3) breaking effects.

II. Expressions for the cross section

Let us give the expressions for the cross section of the reactions of interest taking into account the mixing of the resonances $\rho(770) - \rho_1' - \rho_2'$, $\omega(782) - \omega_1' - \omega_2'$, or $\phi(1020) - \phi_1' - \phi_2'$. Here also give the expressions for the cross section of $e^+e^- \to p\bar{p}$.

According to final states, the cross section are expressed through form factors as follows[2,3,9].

1). **If** $f = \pi$ **or K**, then

$$\sigma(e^+e^- \to f^+f^-) = \frac{8\pi\alpha^2}{3s^{5/2}}|F_f(s)|^2\mathbf{P}_f, \tag{2}$$

where $\mathbf{P}_f$ is the modulus of the 3-momentum of the hadron f in the center-of-mass system of e^+e^- beams, whose total energy is $\sqrt{s}$.

$\mathbf{P}_f \equiv \mathbf{P}_f(s) = \frac{2}{3s}q^3_{\pi\pi}, \frac{1}{6\pi s}q^3_{KK}$

$q_{ij} \equiv q(M, m_i, m_j) = \frac{1}{2M}\sqrt{[M^2 - (m_i - m_j)^2][M^2 - (m_i + m_j)^2]}$

The **form factors** of $e^+e^- \to \pi^+\pi^-$ can be expressed as

$$F_f(s) = \left(\frac{m_\rho^2}{f_\rho}, \frac{m_{\rho_1'}^2}{f_{\rho_1'}}, \frac{m_{\rho_2'}^2}{f_{\rho_2'}}\right) G^{-1}(s) \begin{pmatrix} g_{\rho f} \\ g_{\rho_1' f} \\ g_{\rho_2' f} \end{pmatrix}, \tag{3}$$

The **form factors** of $e^+e^- \to K^+K^-$ can be expressed as

$$\begin{aligned} F_f(s) \quad = \quad & \sum_{i=\rho,\omega,\varphi} \left(g_{\gamma V_i}, g_{\gamma V_i'1}, g_{\gamma V_i'2}\right) G_{V_i}^{-1}(s) \begin{pmatrix} g_{V_i f} \\ g_{V_i'1f} \\ g_{V_i'2f} \end{pmatrix} \\ & + \left(g_{\gamma\omega}, g_{\gamma\omega_1'}, g_{\gamma\omega_2'}\right) G^{(1)}_{\text{OZI}}(s) \begin{pmatrix} g_{\varphi f}/D_\varphi \\ g_{\varphi_1' f}/D_{\varphi_1'} \\ g_{\varphi_2' f}/D_{\varphi_2'} \end{pmatrix}, \\ & + \left(g_{\gamma\varphi}, g_{\gamma\varphi_1'}, g_{\gamma\varphi_2'}\right) G^{(2)}_{\text{OZI}}(s) \begin{pmatrix} g_{\omega f}/D_\omega \\ g_{\omega_1' f}/D_{\omega_1'} \\ g_{\omega_2' f}/D_{\omega_2'} \end{pmatrix}, \end{aligned} \tag{4}$$

2). If the final states are **VP**, where **V,P=**$\omega\pi^0(\rho^0\eta)$, then

$$\sigma(e^+e^- \to VP) = \frac{4\pi\alpha^2}{3s^{3/2}}|F_{VP}(s)|^2 p_{VP}^3, \tag{5}$$

where p_{VP} is the modulus of the 3-momentum of the hadron V (or P) in the center-of-mass system of e^+e^- beams. The form factor for VP final states is given by Eq. (3).

3) If the final states are $2\pi^+2\pi^-$, then

$$\sigma(e^+e^- \to 2\pi^+2\pi^-) = \frac{4\pi\alpha^2}{s^{3/2}}|F_{\rho^0\pi^+\pi^-}(s)|^2 W_{\pi^+\pi^-\pi^+\pi^-}(s), \tag{6}$$

where $W_{\pi^+\pi^-\pi^+\pi^-}(s)$ is given by Eq. (2.10) in Phys. Rev.D**55**, 2663 (1997). The form factor for the production of $2\pi^+2\pi^-$ was given by Eq.(3) in this paper. 4) For the reaction $e^+e^- \to p\bar{p}$, the cross section can be expressed through the **form factor** $|G^p|$:

$$\sigma(e^+e^- \to p\bar{p}) = \frac{4\pi\alpha^2\beta}{3s}|G^p(s)|^2(1 + \frac{2M_p^2}{s})^2, \tag{7}$$

where M_p is the proton mass , β is the proton velocity and $|G^p|$ is the form factor in the hypothesis that $|G_E^p| = |G_M^p|$ also at $s \neq 4M_p^2$.

III. Preliminary analysis of

$e^+e^- \to \pi^+\pi^-$ **and** $p\bar{p}$ **at** J/ψ

$J/\psi \to \pi^+\pi^-$ **event selection.**

1. Two back-to-back (acollinearity less than 6^o) oppositely charged tracks.
2. $1.75 > p > 1.34$ GeV/c.
3. $N_{good\gamma} < 1$
4. TOF $W_\pi > W_K$.

After above selection, 176 $J/\psi \to \pi^+\pi^-$ candidates have been got among $7.8 \times 10^6 J/\psi$ events.

$J/\psi \to p\bar{p}$ **event selection.**

1. Two back-to-back oppositely charged tracks.
2. $p < 1.4$ GeV/c.
3. $N_{good\gamma} < 1$
4. $\Delta|P_x| < 0.25$ GeV/c .
   $\Delta|P_y| < 0.35$ GeV/c .
   $\Delta|P_z| < 0.25$ GeV/c .

   After above selection, 11471 $J/\psi \to p\bar{p}$ candidates have been got among $7.8 \times 10^6 J/\psi$ events.

Finally the following parameter have been obtained

$Br(J/\psi \to \pi^+\pi^-) \sim 2.1 \times 10^{-4}$
$Br(J/\psi \to p\bar{p}) \sim (1.9 \pm 0.3 \pm 0.5) \times 10^{-4}$
$|F_\pi|^2 \sim 1.5 \times 10^{-2}$

IV. Our suggestion

The **existing data** on the decays of J/ψ into some hadrons shows that some new resonance structure may be present in the mass range between 2 GeV and the J/ψ mass.

The data of e^+e^- decaying to hadrons in the energe range **beween 2 and 5 GeV will be taken for the R-value measurement on BES**. Many more events will be taken in the energe points between 3 and 5 GeV than in the points between 2 and 3 Gev.

But the events below 3 GeV will give more information about the resonances and the electromagnetic form factors of light hadrons.

The R-value measured between 2 and 3 GeV also gives more contribution to the calculating of muon anomal magnet moment a_μ and the electromagnetic coupling constant α than that between 3 and 5 GeV. Following tables are the contribution and the error (consider the measure relative error of R-value is 15% and 5%) in each energe region.

Table 4. $a_\mu * 10^{10}$

Energe range	contribution to to a_μ	abs. error when R err 15%	abs. error when R err 5%
(2.0,3.0)	20.8	3.1	1.04
(3.0,5.0)	14.7	2.2	0.73
Total	702		

Table 5. $\triangle\alpha_{had} * 10^4$

Energe range	contribution to to $\triangle\alpha_{had}$	abs. error when R err 15%	abs. error when R err 5%
(2.0,3.0)	15.07	2.26	0.75
(3.0,5.0)	26.36	3.95	1.31
Total	280		

And it is said that the luminosity of BEPC may enlarge twice in the energy range below 2.6 GeV if the measuring time of the beam is long enough. So we suggest that we should get more hadron events in the energe points of table 6 in the process of the R-value measurement on BEPC.

V. Discussions

The comparison with existing data on the e^+e^- annihilation shows that some new resonance structures may be present in the mass range between 2GeV and 5GeV. Searching them may help in a better understanding of the **mass spectrum in both the simple and a more sophisticated quarks models**.

The proton electromagnetic form factors have been widely investigated in the **space-like region** [4]. The form factors of the light hadrons are also measured in the **time-like region**.

VI. Acknowledgements

We are greatly indebted to Prof. N.N.Achasov , Prof. LI Bingan and Prof. ZHANG Chuang.

Fig. 1 Feynman Diagram for $e^+e^- \to \gamma \to A\bar{B}$

Fig. 2 The pion form factor

Fig. 3 The proton electromagnetic form factor

Fig. 4 The Kaon form factor

References

1.Erkal and Olsson, Z.Phys. C31(1986)615

2.N.N.Achasov and A.A.Kozhevnikov,
Phys. Rev. D55(1997)2663

3.N.N.Achasov and A.A.Kozhevnikov,
Phys. Rev. D57(1998)4334

4. R. Hofstadter, Electron scattering and nuclear and
nucleon structure, (W.A.Benjamin,1963)

5. M.Castellano et al., Nuovo Cimento A41(1973)1
G.Bassompierre et al., Phys. Lett. B64(1976)475
B.Delcourt et al., Phys. Lett. B86(1979)395
D.Bisello et al., Nucl. Phys. B224(1983)379

6. M.Conversi et al., Nuovo Cimento 40(1965)690
D.L.Hartill et al., Phys. Rev. 184(1969)1415
C.Baglin et al., Phys. Lett. B163(1985)400
G.Bardin et al., Phys. Lett. B255(1991)149;
Phys. Lett. B257(1991)514

7. T.A.Armstrong et al., Phys. Rev. Lett. 70(1992)1212

8. G.Bardin et al., Nucl. Phys. B411(1994)3

9. A.Antonelli et al., Phys. Lett. B334(1994)431

Table 6. Suggested Scan Energy Points

Ecm(GeV)	2.00	**2.05**	**2.10**	**2.20**	2.40	2.60
Expected Events	5+3 (7)*	5+2 (5)*	5+1 (4)*	5+1	5+2	5+3
Expected Peak $Luminosity^+$ ($10^{30}cm^{-2}s^{-1}$)	0.28	0.32	0.36	0.46	0.73	1.10
Expected Integral $Luminosity$ (nb^{-1})	167	86	70	91	208	385
Expected Running Time (hours)	275	90	89	91	133	162

* The numbers in the parenthesis are the events from DM2.

\+ The expected luminosity could be increased $20\% - 30\%$

The total running time is about $31(50)^@$ **DAYS !** and the total running time for these three extra energy points we required is only about $12(20)^@$ **DAYS !**

@ Considering the difference between BEPC £ and the BES £.

So the requirement for £ is possible!

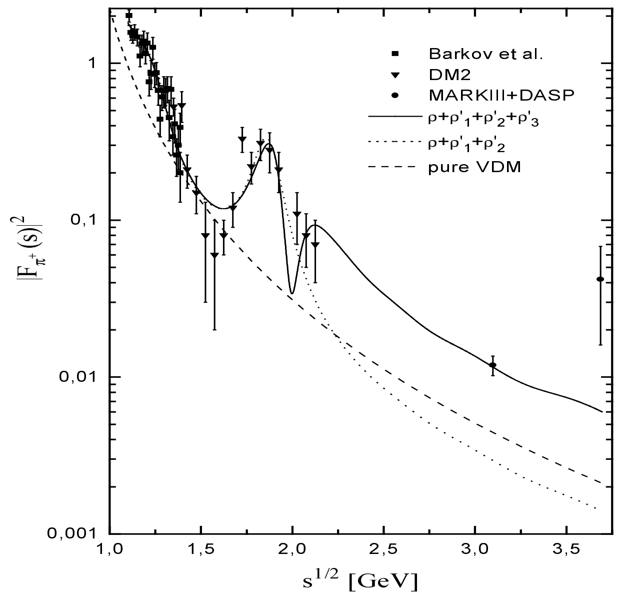


Fig. 2

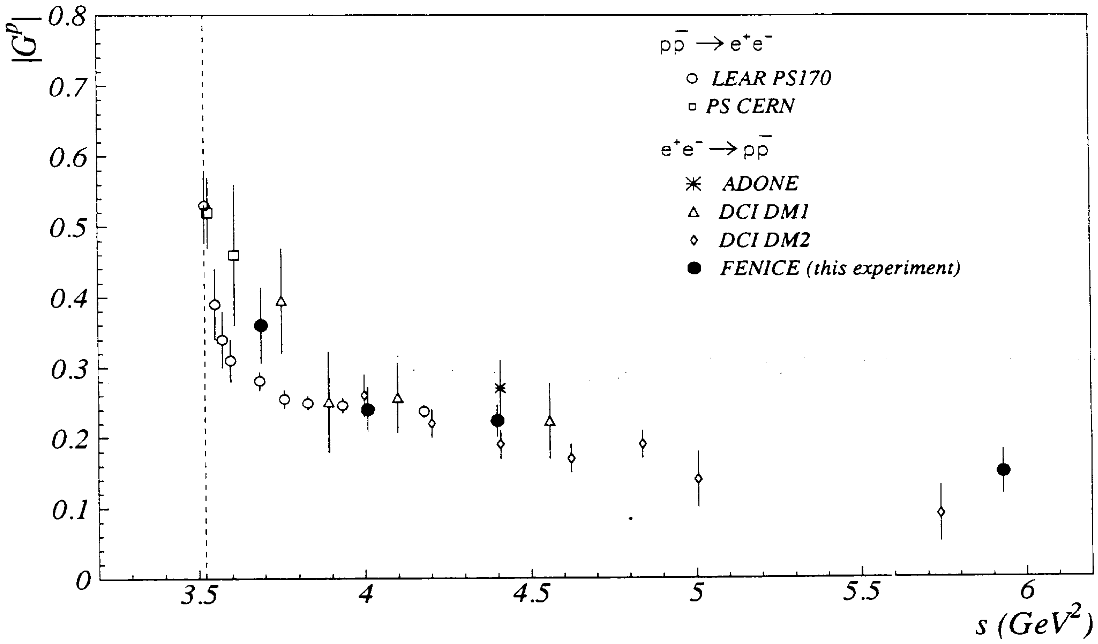


FIG. 3 – The proton electromagnetic form factor as a function of $\sqrt{s}$. The dashed line corresponds to the nucleon-antinucleon threshold. The FENICE results (full circles) are compared with worldwide data in the same s region.

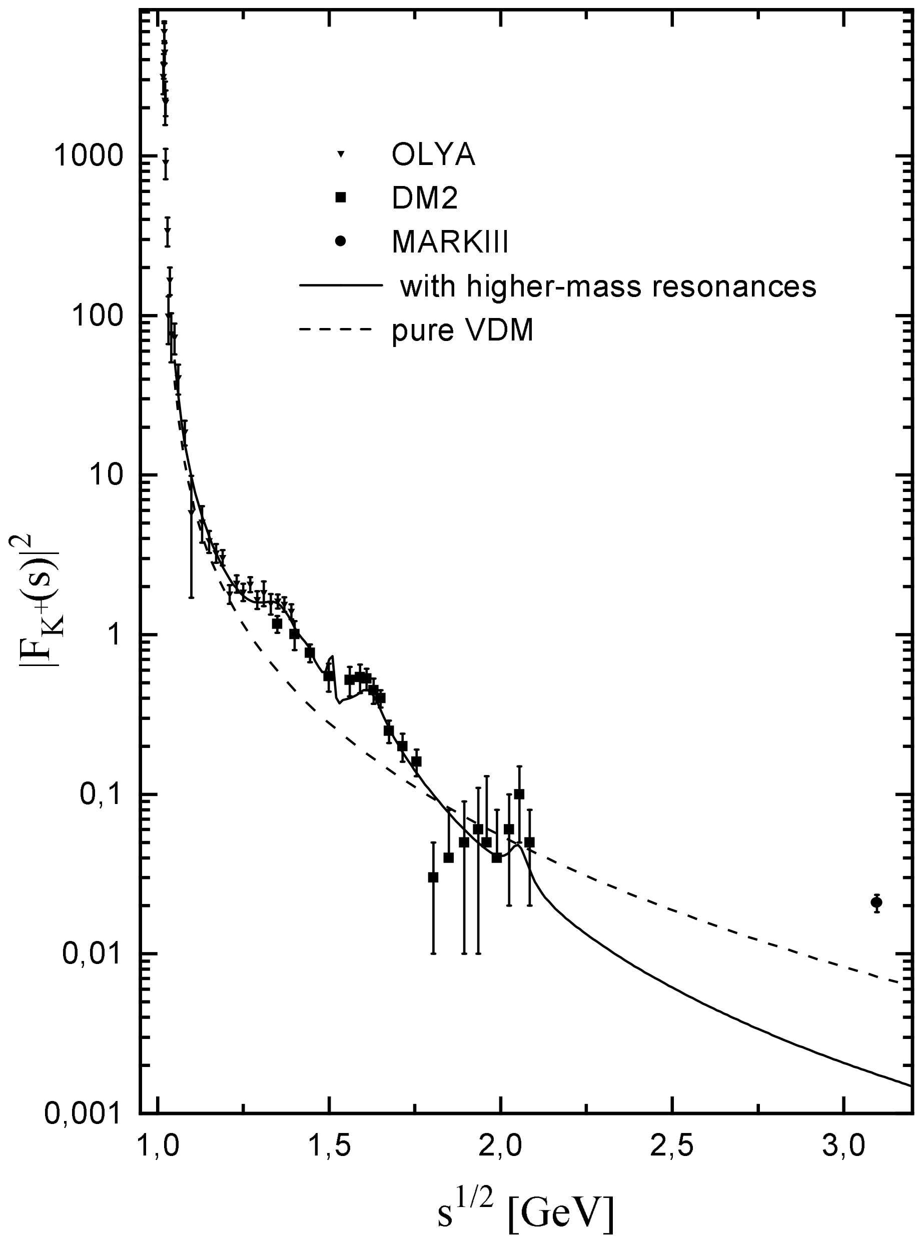

OLYA
DM2
MARKIII
with higher-mass resonances
pure VDM
1000
100
10
1
0,1
0,01
0,001
1,0
1,5
2,0
2,5
3,0
$|F_{K^+}(s)|^2$
$s^{1/2}$ [GeV]

Fig.