The Analysis of $D^0 \to \overline{K^0}\pi^+\pi^-$

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Abstract

Using the data collected by BES detector at BEPC, we have investigated the decay of $D^0 \to \overline{K^0}\pi^+\pi^-$. A fit to the observed Dalitz plot was performed to determine the complex amplitudes of the sub-component modes.We find that (59±8±8)% are $D^0 \to K^{*-}\pi^+$ and (23±4±4)% are $D^0 \to \overline{K^0}\rho^0$ with a relative phase of $(-129\pm13)^\circ$ between channels.

I. Introduction

The Standard Model succesfully describes the weak interactions between pointlike objects as proceeding via the exchange of charged or neutral bosons (i.e. photons, $W^\pm$ and Z^0) between particle currents. The amplitude for this process is

$$A_{SM} = \frac{G_F}{\sqrt{2}} V_{cq} j^\mu J_\mu$$

where V_{cq} is the appropriate CKM matrix elements. On the other hand , weak interactions between composite particles(mesons and baryons) cannot be completely described by Standard Model. Several Models [1,2] have been developed to explain the weak hadronic decay mechanisms of D mesons.

Motivated by spectator model, one may make a factorization assumption for two-body hadronic weak decays of D mesons :

$$A(D \to Mm) = \frac{G_F}{\sqrt{2}} V_{cq} h^\mu H_\mu \qquad (1)$$

Here H_μ is the hadronic current

$$H_\mu = < M \,|J_\mu|\, D >$$

and

$$h^\mu = < m \,|j^\mu|\, 0 >$$

In the model of Bauer,Stech and Wirbel(BSW)[2], the amplitude has two terms: one is proportional

Figure 1: Feynman Diagram of $D^0 \to K^{*-}\pi^+$ and $D^0 \to \overline{K^0}\rho^0$.

to the quantity a_1,which parameterizes the strength of external W-emission, e.g. $D^0 \to K^{*-}\pi^+$, and the other is proportional to a_2,which parameterizes the strength of the internal W-emission,e.g. $D^0 \to \overline{K^0}\rho^0$. Thus,internal W-emission decays are expected to be suppressed relative to external W-emission.

II. Event Selection

Charged pions are required to have a likelihood greater than 1%,according to TOF and dE/dx information. Neutral Kaons are identified through the decay $K_S^0 \to \pi^+\pi^-$, where the $\pi^+\pi^-$ invarant mass is required to lie within 0.02 GeV/c^2 of the K_S^0 mass.The $\pi^+\pi^-$ second vertex is

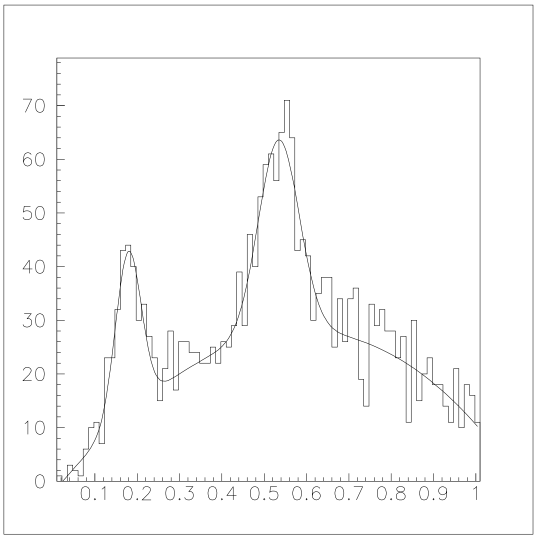

Figure 2: p_X distribution in D^0 mass region.

required to be more than 3 mm from the beam axis,and the vertex position and K_S^0 momentum vector are required to align in xy-plane(An opening angle cut θ with cosθ >0.8 is applied).

In 4.03GeV energy region, D^0 could be from several physics procedures (such as,$D^*\overline{D}$, $D^*\overline{D^*}$,$D\overline{D}$). Figure. 2 show the momentum of $K_S^0\pi^+\pi^-$ combinations (X) distribution in the D^0 mass region; Table. 1 show the result of Monte Carlo study.To reduce background, a $K_S^0\pi^+\pi^-$ combination (X) is acceptable if the momentum lie within

$$50MeV/c < p_X < 260MeV/c$$

or

$$420MeV/c < p_X < 600MeV/c$$

Figure 3 shows the m_X distribution for the remaining candidates.The mass plot is fitted with a Gassian for the signal and a linear polynomial for the background. There are 646±42 events above the background. We define a signal region with

$$1.8285GeV/c^2 < m_X < 1.9061GeV/c^2 \tag{2}$$

for Dalitz plot fit.

For each $K_S^0\pi^+\pi^-$ combination, we perform a kinematic fit using the K_S^0 and D^0 mass constraint. With this type of fit, the kinematic boundaries are strictly respected.

III. The Likelihood Function

For $D^0 \to \overline{K^0}\pi^+\pi^-$ final states, there are several two-body decay modes and nonresonant three-body decay mode which can contribute. We determine the contribution of each decay mode using a maximum likelihood fit.

For each decay mode, we define a complex amplitude in the three-body phase space as:

$$A(D \to rc, r \to ab) = F_D F_r BW(r) M(abc|r)$$

where the Blatt and Weisskopf form factors F_D and F_r take the forms as table 2. We choose

Spin J of Decaying Particle X	Form Factor F_X
0	1
1	$\frac{1}{\sqrt{1+R^2q^2}}$
2	$\frac{1}{\sqrt{9+3R^2q^2+(R^2q^2)^2}}$
3	$\frac{1}{\sqrt{225+45R^2q^2+6(R^2q^2)^2+(R^2q^2)^3}}$
>3	1

Table 1: Blatt and weisskopf Form Factor F_X, where q is the momentum of the decay products in the rest frame of particle X, R is the meson radius parameter and is taken to have the following values[4]:for parent charm meson D ,we choose R=5GeV^{-1}; for intermediate resonance particle r ,we choose R=1.5GeV^{-1} .

relativistic Breit-Wigner propagtor $BW(r)$ as[5]:

$$\frac{-1}{m^2 - m_0^2 + i\Gamma m_0}$$

where Γ is the energy dependence width,given by

$$\Gamma = \Gamma_0(\frac{q}{q_0})^{2J+1}\frac{m_0}{m}\frac{F_X^2(q^2)}{F_X^2(q_0^2)}$$

the relevant value m_0 and Γ_0 are taken from ref.6. The angular momentum part $M(abc|r)$ are given in Zemarch Formalism[7], we wrote it in ab rest frame as table 3. We define a probability density function (p.d.f.) which consists of a coherent sum of these amplitudes ,and fit to dalitz plot to determine the relative fractions and phases of these amplitudes. The likelihood function L is defined as

$$L = \prod_{events} F$$

$J^P(r)$	$M(abc\|r)$	Notation
nonresonant	1	NR
0^+	$1 \times BW$	$S(abc\|r)$
1^-	$(-2p_a p_c)\cos\theta \times BW$	$P(abc\|r)$
2^+	$(-2p_a p_c)^2 \frac{1}{2}(3\cos^2\theta - 1) \times BW$	$D(abc\|r)$
3^-	$(-2p_a p_c)^3 \frac{1}{2}\,(5\cos^3\theta - 3\cos\theta) \times BW$	$F(abc\|r)$

Table 2: Zemarch Amplitudes $M(abc|r)$ for $D \to rc \to (ab)c$.

The p.d.f is written as a superposition of signal and background p.d.f as follows:

$$F = \frac{R_{S/B} F_S + F_B}{R_{S/B} + 1} \tag{3}$$

The ratio of signal to background,$R_{S/B}$,as a function of m_X, is obtainted from the mass plot fit.

The signal p.d.f F_S ,consists of a coherent sum of complex amplitude, weighted by the density of the states in phase space and the detector efficiency, is normalized over phase space weighted by the detector efficiency:

$$F_S = \frac{1}{N_{F_S}} \epsilon \phi \left| \sum_i f_i^M e^{i\alpha_i} A_i \right|^2 \tag{4}$$

The A_i are the complex amplitudes verying over phase space; ϵ is the detector efficiency as function of location in phase space;ϕ is the 3-body phase space function; and N_{F_S} is the overall normalization of the p.d.f. The fraction f_i^M and the phase α_i are varied in the fit to maximize the likelihood.

The background p.d.f is :

$$F_B = \frac{1}{N_{F_B}} \epsilon \phi \sum_i g_i^M B_i \tag{5}$$

The B_i are function which describ the resonant content of the background. They don't interfere with each other. The normalization factor N_{F_B} is evaluated with the same procedure used for F_S. g_i^M are allowed to vary in the fit.

The function ϵ and ϕ don't depend on the f_i^M,g_i^M or α_i,and are omitted in the likelihood function before minimization. If we define $F_S = S\epsilon\phi$ and $F_B = B\epsilon\phi$,then we minimize

$$-2lnL = -2 \sum_{events} ln \left(\frac{R_{S/B} S + B}{R_{S/B} + 1} \right) - 2ln\epsilon\phi \tag{6}$$

The minimization is performed by setting the most prominent channel and letting the relative fraction and phase vary. The fractions f_i can be obtained from

$$f_i = \frac{\left| f_i^M \right|^2 \int |A_i|^2 \, dPS}{\int \left| \sum_i f_i^M e^{i\alpha_i} A_i \right|^2 dPS} \tag{7}$$

and do not,in general,add up to 100% due to the interference between the amplitudes.

IV. Background Parameterization

To determine the $R_{S/B}$ in (3), we form a likelihood function

$$L_M = \prod_{events} \left(\frac{1}{\sqrt{2\pi}\sigma_{M_D}} e^{-\frac{(M_i - M_D)^2}{2\sigma^2_{M_D}}} + f_b (1 + 2\beta_1 M_i) \right) / (1 + f_b) \tag{8}$$

to fit the $K^0_S\pi^+\pi^-$ invarant mass plot (showed in figure 3).The fit results are listed as following:

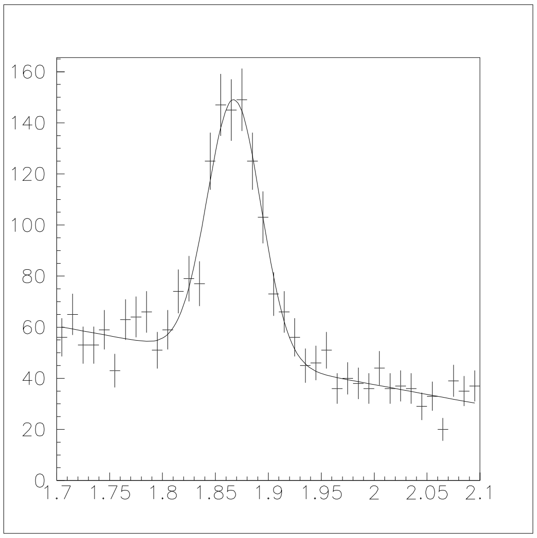

Figure 3: $K^0_S\pi^+\pi^-$ invarant mass distribution.The curve represents a fit using Gassian above a linear background.

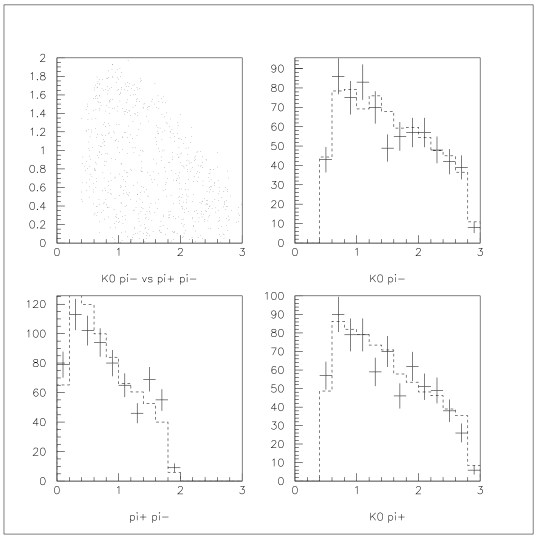

Figure 4: The Dalitz plot,three mass-squared projection for sideband events.

$$M_D = (1.8673 \pm 0.0017) GeV/c^2$$
$$\sigma_{M_D} = (0.0259 \pm 0.0019) GeV/c^2$$
$$f_b = 2.85^{+0.27}_{-0.24}$$
$$\sigma_{f_b} = 0.25$$
$$\beta_1 = -0.199 \pm 0.006$$

In order to allow for the inflation of statistical error bars due to background fluctuations , we all the fitter to adjust the background on an event-by-event basis. This is done by adding an addtional χ^2 term $(f_b - f^0_b)^2/\sigma^2_{f_b}$ to (6). f_b is allowed to vary within its error range during the amplitude fit. This is so-called 'background re-normalization' method and is used for our standard fit.

In figure 3, there is a clear $D^0 \to K^0_S\pi^+\pi^-$ signal above the background.

The signal region is definded as

$$|M - M_D| < 1.5\sigma_{M_D}$$

and the two sidebands are specified as

lower sideband (LSB) :

$$-8\sigma_{M_D} < (M - M_D) < -4\sigma_{M_D}$$

high sideband(HSB):

$$4\sigma_{M_D} < (M - M_D) < 8\sigma_{M_D}$$

According to the description in section V,we form a likelihood function

$$L_{sb} = \prod_{events}^{sideband} (F_D + F_{\overline{D}}) \tag{9}$$

to fit the two sets of sideband simple data. The background fraction(a plat no-resonant term with

Background Mode	3-body NR	K^*	ρ
Fraction	$(88.7 \pm 2.5)\%$	$(7.0 \pm 2.0)\%$	$(4.4 \pm 2.3)\%$

Table 3: Background fraction determined by fitter.

K^* and ρ bands) defined in (5) could be determined via this procedure.

An addtional χ^2 term (the same as the f_b term defined) is added to (6). g_i^Ms are allowed to vary within their error.

V. Fit Results

The modes $K^{*-}\pi^+$,$\overline{K^0}\rho^0$,and three-body nonresonant are used to measure the decay fractions and relative phases. Since the charm quantum number is not determined by K_S^0,we form a total p.d.f. out of the p.d.f. for each hypothesis:

$$F_S = F_{D^0} + F_{\overline{D^0}} \tag{10}$$

The D^0 decay is specified in terms of amplitude cofficients f_1^M, f_3^M and phase α_1,α_3 by

$$A(D^0) = f_1^M e^{i\alpha_1} + BW(K^{*-})P(K_S^0\pi^-\pi^+|K^{*-}) + f_3^M e^{i\alpha_3} BW(\rho^0)P(\pi^+\pi^-K_S^0|\rho^0) \tag{11}$$

For the $\overline{D^0}$ decay, the amplitude is specidied by

$$A(\overline{D^0}) = f_1^M e^{i\alpha_1} + BW(K^{*+})P(K_S^0\pi^+\pi^-|K^{*+}) + f_3^M e^{i\alpha_3} BW(\rho^0)P(\pi^-\pi^+K_S^0|\rho^0) \tag{12}$$

The fit results are summrized in Table 5. In Table 6, the decay fraction into a given mode was compared to results from other experiments. The goodness of fit cannot be read directly from the value of likelihood function. We binned the data into a two dimensional histogram and three projections.The fit results are nomalized to the number of the total events,then are compared to the binned data. A χ^2 test[6]

$$\chi^2 = \sum_i^{bins} \left(\frac{n_i - \mu_i}{\sqrt{n_i}} \right)^2 \tag{13}$$

is evaluated for each histogram. In equation (13),n_i is the number of events in ith bin and μ_i is the

Decay Mode	relative fraction	relative phase	braching ratio(%)
$K^{*-}\pi^+$	$0.59 \pm 0.08 \pm 0.08$	$0°$(fixed)	$4.78 \pm 0.65 \pm 0.71$
$\overline{K^0}\rho^0$	$0.23 \pm 0.04 \pm 0.04$	$(-129 \pm 13)°$	$1.24 \pm 0.22 \pm 0.24$
non-resonant	$0.28 \pm 0.06 \pm 0.06$	$(-111 \pm 11)°$	$1.51 \pm 0.32 \pm 0.35$

Table 4: Fit results,using the world average branching ratio $Br(D^0 \to \overline{K^0}\pi^+\pi^-) = (5.4 \pm 0.5)\%$ we covert the fraction to braching ratio.

	$M_{K^0_S\pi^-}$	$M_{\pi^+\pi^-}$	$M_{K^0_S\pi^+}$	Dalitz plot
χ^2	83.7	31.5	74.9	450.8
Degree of freedom	25	19	25	284

Table 5: χ^2 test results for Dalitz plot, three projections.

total predicted by fit in ith bin.The χ^2 test results are listed in table 6.

The systematic errors are estimated by varying the events selection criteris, the method of background determination,the uncertain of Monte Carlo efficency. The largest source of systematic error comes from the parameterization of the background.

To calculate systematic error from the events selection,we change the window of the momentum cut and signal region defination, K^0_S selection and pion identification ; The Monte Carlo efficency are estimated to be 10% uncertain; The systematic error from the background contribution are determined from a comparison of fits to the signal events using either a flat background or a backgrounds with shape determined from the LSB,HSB and the average of the two sets of sideband. The result are summized in table 7.

In BSW model,the ratio of

$$\frac{B_r(D^0 \to K^{*-}\pi^+)}{B_r(D^0 \to \overline{K^0}\rho^0)} = \frac{5.12a_1^2}{3.14a_2^2} \tag{14}$$

can be used to estimate the ratio of the effective QCD parameter a_1 and a_2. From our result,we find

$$\left|\frac{a_1}{a_2}\right| = 1.54 \pm 0.20$$

which is smaller than the value 2.4 used in [1].

Decay mode	Events Selection	Background	Monte Carlo	total
$K^{*-}\pi^+$	0.01	0.05	0.06	0.08
$\overline{K^0}\rho^0$	0.02	0.03	0.02	0.04
3-body NR	0.01	0.05	0.03	0.06

Table 6: Systematic error of relative fractions estimated from the contribution of different sources.

Table 7: Results from other experiments on the fraction and relative phase of the main contributions to $D^0 \to \overline{K^0_S}\pi^+\pi^-$.

$D^0 \to$	ARGUS	MarkIII	E687(92)	E691	BES
$K^{*-}\pi+$	$(69.5 \pm 3.5)\%$ $0°$(fixed)	$(56 \pm 4 \pm 5)\%$ $0°$(fixed)	$(64 \pm 8 \pm 5)\%$ $0°$(fixed)	$(48 \pm 10 \pm 10)\%$ $(109 \pm 9)°$	$(59 \pm 8 \pm 8)\%$ $0°$(fixed)
$\overline{K^0}\rho^0$	$(21.2 \pm 3.0)\%$ $(-143 \pm 8)°$	$(12 \pm 1 \pm 7)\%$ $(90 \pm 30)°$	$(20 \pm 6 \pm 3)\%$ $(-143 \pm 29)°$	$(22 \pm 5 \pm 3)\%$ $(-123 \pm 12)°$	$(23 \pm 4 \pm 4)\%$ $(-129 \pm 13)°$
NR	$(20.0 \pm 1.7)\%$ $(107 \pm 9)°$	$(33 \pm 5 \pm 10)\%$ incoherent	$(26 \pm 8 \pm 5)\%$ $(-126 \pm 17)°$	$(26 \pm 2 \pm 8)\%$ $0°$(fixed)	$(28 \pm 6 \pm 6)\%$ $(-111 \pm 11)°$

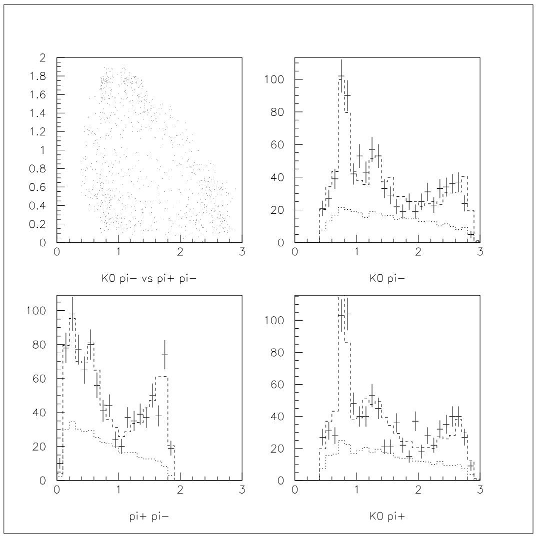

Figure 5: Dalitz plot and mass-squared projections for the decay $D^0 \to K^0_S\pi^+\pi^-$. The data are represented by the poined with error bars,and in each mass-squared projection the upper histogram describes the predicted signal plus background contribution as determined by the fit,and the lower histogram represents the background contribution.