PWA Analysis of BES Data on $J/\psi \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$

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Abstract

The BES data on $J/\psi \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$ has been analyzed by using the partial wave amplitude analysis method. We fit with resonances having $J^{PC} = 2^{++}$ at 1275 MeV, 0^{-+} at 1385 MeV, 0^{++} at 1505 MeV, 2^{++} at 1565 and 1697 MeV, 0^{++} at 1775 MeV, 2^{++} at 2010 MeV, 0^{++} at 2100 MeV and 2^{++} at 2220 MeV. The 0^{++} resonances decay dominantly to $\sigma\sigma$, while 2^{++} resonances at high mass region can decay to $f_2(1270)\sigma$, $\sigma\sigma$ and $\rho\rho$, and 2^{++} at low mass region decay dominantly to $\rho\rho$.

Introduction

MARK III and DM2 measured their data on $J/\psi \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$ in 1986 and 1989 respectively[1, 2]. They found dominant component of the spectrum below 2 GeV is due to $J/\psi \rightarrow \gamma\rho\rho$ with spin-parity 0^- of the $\rho\rho$ system in which the two ρ's appear in a relative P-wave. MARK III claimed a pseudoscalar resonance at 1.55 GeV/c^2. This was supported by DM2, and in addition they claimed that the structures at 1.80 and 2.10 GeV/c^2 were also $J^{PC} = 0^{-+}$.

By the way, the E760 data presents three structures in the $\eta\eta$ spectrum[3], which are very similar to those observed in $J/\psi \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$. However, the quantum number 0^{-+} is forbidden to $\eta\eta$. This throws into question the J^P values assigned by DM2.

Both MARK III and DM2 analyzed their data in term of $J/\psi \rightarrow \gamma\rho\rho \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$, but they didn't consider $J/\psi \rightarrow \gamma\sigma\sigma \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$. In 1995, D. Bugg *et al.* reanalyzed MARK III data on $J/\psi \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$[4]. They proposed that the $\sigma\sigma$ could be a decay mode in addition to $\rho\rho$, where σ is $\pi - \pi$ S-wave. Inclusion of this decay mode results in the observation of two $I = 0$ scalar resonances decaying exclusively via $\sigma\sigma$. Those states have mass in the region $M = 1500$ and 2100 MeV/c^2. An additional scalar state is required at $M = 1750$ MeV/c^2 and decays dominantly to $\sigma\sigma$, but also shows significant decays via $\rho\rho$.

We have analyzed the BES data on $J/\psi \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$ by using the partial wave amplitude analysis method provided by D. Bugg. The full Monte Carlo simulation, which was not used by D. Bugg in his reanalysis of Mark III data, is employed in BES analysis, for the proper Monte Carlo simulation for Mark III data is no longer available at that time. The main background channel $J/\psi \rightarrow \pi^0\pi^+\pi^-\pi^+\pi^-$ has been much carefully studied in order to determine the background in $J/\psi \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-$. What we are interested in is to make clear if all states observed by Bugg can be confirmed by BES and any states missed in Bugg's reanalysis can be added in BES analysis.

Events Selection

Candidates for the decay $J/\psi \to \gamma\pi^+\pi^-\pi^+\pi^-$ are selected by requiring exactly four charged tracks in the drift chamber with zero total charge and exactly one photon. The minimum energy of 50 MeV is required for photon. The number of neutral clusters is one or two, if there are two neutral clusters in a event, require that one is photon, the other is fake photon with energy less than 50 MeV. Then the four-constraint kinematic fits are performed by assuming the final states as $\gamma\gamma 4\pi$, $\gamma\gamma 4K$, $\gamma\gamma 2K2\pi$, $\gamma\gamma 2p2\pi$, $\gamma 4\pi$, $\gamma 4K$, $\gamma 2K2\pi$, $\gamma 2p2\pi$, 4π, $4K$, $2K2\pi$, and $2p2\pi$ respectively for each event. The value of $\chi^2(\gamma 4\pi)$ is required to be the smallest one and probability of $\chi^2(\gamma 4\pi)$ is larger than 5% to improve the resolution and to select the good photon. Further cuts are as following. To remove the main background $J/\psi \to \pi^0\pi^+\pi^-\pi^+\pi^-$, $P(\chi^2(\gamma 4\pi))$ larger than $P(\chi^2(\gamma\gamma_{miss}4\pi))$ is required, where γ_{miss} means this photon is missing, if the two photons in $J/\psi \to \gamma\gamma_{miss}\pi^+\pi^-\pi^+\pi^-$ are from a π^0 decay, the probability $P(\chi^2(\gamma\gamma_{miss}4\pi))$ should be less than 2%; $\mid U_{miss} \mid = \mid E_{miss} - P_{miss} \mid < 0.09$ GeV/c^2 is needed to reject the events with multi-photon and charged kaons, here, E_{miss} and P_{miss} are, respectively, the missing energy and missing momentum of all charged particles, and they are calculated by assuming the charged particles are $\pi^+\pi^-\pi^+\pi^-$; The transverse momentum of the 4π system $P_{t\gamma}^2 = 4 \mid P_{miss} \mid^2 \sin^2(\theta_{m\gamma}/2) < 0.004$ (GeV/c)^2 is required to remove the background $J/\psi \to \pi^0\pi^+\pi^-\pi^+\pi^-$, where $\theta_{m\gamma}$ is the angle between the missing momentum and the photon direction; The background $J/\psi \to \omega\pi^+\pi^-$ events are eliminated by the cut $\mid M_{\pi^+\pi^-\pi^0} - M_\omega \mid > 25$ MeV, in the $\pi^0\pi^+\pi^-\pi^+\pi^-$ hypothesis with only one photon detected and the π^0 is associated to the missing momentum; To remove the small background due to $\gamma K_s^0 K_s^0$ events, a double cut on the two $\pi^+\pi^-$ invariant mass is performed, $\mid M_{\pi^+\pi^-} - M_{K_s^0} \mid > 15$ MeV; The BES data shows a small signal of $f_1(1285)$, this is not of concern in this analysis, in order to remove it, an additional cut was applied to rule out the events within the mass region between 1.24 and 1.32 GeV/c^2.

Figure 1 shows the 2π invariant mass distribution, the strong signals of ρ, σ and $f_2(1270)$ are seen. The 4π invariant mass distribution of $J/\psi \to \gamma(\pi^0)\pi^+\pi^-\pi^+\pi^-$ is shown in Figure 2, which presents peaks at approximately 1.50, 1.75 and 2.10 GeV/c^2.

Analysis Method

The maximum likelihood method is employed to fit the data. The probability density function $f(x_i;\alpha)$ with x_i as a set of independently measured quantities and α as a set of unknown parameters is given as below

$$f(x_i;\alpha) = \frac{d\sigma}{d\Phi}/\sigma,$$

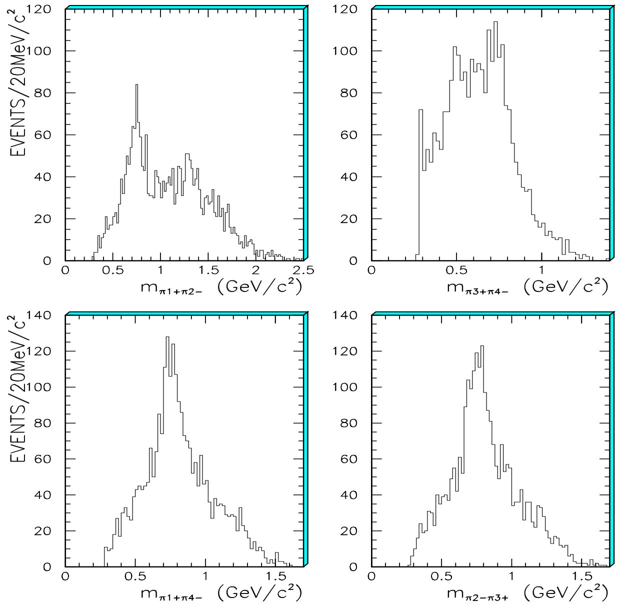

Figure 1: The 2π mass spectrum from BES data on $J/\psi \to \gamma(\pi^0)\pi^+\pi^-\pi^+\pi^-$ note, $P(\pi_1^+) > P(\pi_3^+)$ and $P(\pi_2^-) > P(\pi_4^-)$

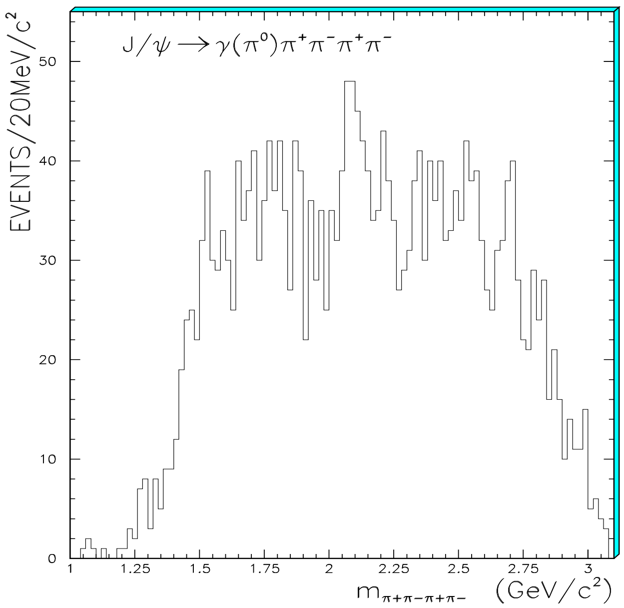

Figure 2: The 4π mass spectrum from BES data on $J/\psi \to \gamma(\pi^0)\pi^+\pi^-\pi^+\pi^-$

where the total cross section $\sigma = \int | A_{total} |^2 \, d\Phi$ is calculated by using the Monte Carlo integration. The differential cross section is given by

$$\frac{d\sigma}{d\Phi} = | A^{0-} + A^{0+} + A^{2+} + \cdots |^2 + BG,$$

here, BG is the background term, A^{0-}, A^{0+}, A^{2+} are the amplitudes for pseudoscalar, scalar and tensor components respectively.

Fit is made to the data in order to maximize the value of

$$\mathcal{L} = \prod_{i=1}^{N_{event}} f(x_i; \alpha).$$

Actually, $S = -\ln \mathcal{L}$ is minimized in the fit.

The technique employed in determining the background function BG is as following. We have generated 1.35 million $J/\psi \to \pi^0\pi^+\pi^-\pi^+\pi^-$ M.C. events. The events that can pass the $\gamma 4\pi$ data selection program are regarded as the simulation for background. Fit the 4π mass distribution of this background by using the function $b(m) \times (4\pi$ phase space of $J/\psi \to \gamma\pi^+\pi^-\pi^+\pi^-)$, then the background function $b(m)$ can be obtained. Due to the difference between the M.C. and real data of $J/\psi \to \pi^0\pi^+\pi^-\pi^+\pi^-$, a slight correction to the background function is needed. The correction function is obtained in this way, fit the 4π mass spectrum of real data of $J/\psi \to \pi^0\pi^+\pi^-\pi^+\pi^-$ by using the function $p(m)\times$ (4π phase space of $J/\psi \to \pi^0\pi^+\pi^-\pi^+\pi^-)$, then $p(m)$ is the correction function.

So the background function BG reads

$$\begin{aligned} BG &= a \times b(m) \times p(m) \\ &= a(e^{-0.0856(3.499-m)^2})^2 \times (e^{-0.1020(3.462-m)^2})^2. \end{aligned}$$

According to Monte Carlo simulation, the background events account for 31% of the real data events in $J/\psi \to \gamma(\pi^0)\pi^+\pi^-\pi^+\pi^-$ channel. So coefficient a is set to 1.30, which depends on about 31% contribution of the background.

Analysis and Results

Possible resonances and possible decay modes for each resonance have been included in the fit. The resonances and decay modes which make real improvements in log likelihood function have been listed in Table 1.

The masses and widths of $f_0(1500)$, $f_0(1750)$, $f_0(2100)$, $f_2(2220)$, $f_2(2010)$, $f_2(1710)$, $f_2(1560)$ and $\eta(1440)$ are well defined. The mass and width of $f_2(1270)$ is not settled down well. One possible reason is that the statistics of real data in $f_2(1270)$ mass region is very low and the $f_1(1285)$ has been removed. So it is felt preferable to constrain the mass and width of the $f_2(1270)$ to agree with the PDG value[5]. Though the optimum mass for a 2^+ resonance lies at 1470 MeV, probably this 2^+ resonance is $f_2(1560)$. The physical explanation is that $f_2(1560)$ is sitting at the $\rho\rho$ and $\omega\omega$ threshold and can decay into 4π according PDG[6]. It is reasonable that $f_2(1560)$ presents in $J/\psi \to \gamma\pi^+\pi^-\pi^+\pi^-$ channel. In the final fit, the mass of $f_2(1560)$ has been fixed at 1565 MeV, which is extremely well determined by the Crystal

Table 1: Resonances fitted to the BES data and decay modes used in the final fit. L is the orbital angular momentum between the two final state particles following the decay of the resonance.

		Measured		Used in final fit		decays	L or ${}^{2S+1}L_2$
J^P	Resonance	Mass (MeV)	Γ (MeV)	Mass (MeV)	Γ (MeV)		
2^+	$f_2(1270)$	1310^{+150}_{-40}		1275	185	$\rho\rho$	${}^1D_2$ ${}^3D_2$
0^-	$\eta(1420)$	1385^{+20}_{-20}	190^{+15}_{-15}	1385	190	$\rho\rho$	$L=1$
0^+	$f_0(1500)$	1505^{+15}_{-20}	105^{+20}_{-20}	1505	120	$\sigma\sigma$	$L=0$
2^+	$f_2(1560)$	1470^{+25}_{-25}	180^{+70}_{-60}	1565	150	$\rho\rho$	${}^5S_2$ ${}^1D_2$ ${}^3D_2$
2^+	$f_2(1710)$	1690^{+25}_{-25}	360^{+250}_{-220}	1697	175	$\rho\rho$	${}^5S_2$
0^+	$f_0(1750)$	1775^{+25}_{-25}	190^{+80}_{-60}	1775	190	$\sigma\sigma$	$L=0$
2^+	$f_2(2010)$	2010^{+50}_{-70}	350^{+100}_{-70}	2010	350	$f_2(1270)\sigma$	${}^5S_2$
						$\sigma\sigma$	${}^1D_2$ ${}^3D_2$
						$\rho\rho$	${}^1D_2$ ${}^3D_2$
0^+	$f_0(2100)$	2055^{+35}_{-30}	205^{+220}_{-80}	2100	205	$\sigma\sigma$	$L=0$
2^+	$f_2(2220)$	2220^{+20}_{-20}	105^{+70}_{-55}	2220	80	$f_2(1270)\sigma$	${}^5S_2$
						$\sigma\sigma$	${}^5S_2$ ${}^3D_2$
						$\rho\rho$	${}^5S_2$ ${}^3D_2$

Barrel and VES data. The possible cause of the optimum mass of $f_2(1560)$ lying at 1470 MeV, is due to the crosstalk between the two neighboring resonance ($f_0(1500)$ and $f_2(1560)$) of different spin in 1500 MeV region.

The final full fit is performed, the 4π mass spectrum for the final full fit is shown in Figure 3. The contributions of various components in the final full fit are shown in Figure 4 and Figure 5.

From Figure 5, it can be seen that the 2^+ resonances at high mass region mainly decay via $f_2(1270)\sigma$ into 4π, and the 2^+ resonances at low mass region mainly via $\rho\rho$ into 4π.

The obtained branching ratios are given in Table 2, and the branching fraction for $M_{4\pi}$ less than 3 GeV/c^2 is

$$B(J/\psi \to \gamma\pi^+\pi^-\pi^+\pi^-) = (6.7 \pm 0.3 \pm 0.8) \times 10^{-3}$$

In order to check whether the final full fit is the best solution for BES data, four tests to final full fit are performed.

The first test is to check the significance level of each resonance. Table 3 summaries the changes in log likelihood function if various components of the fit are dropped and remaining contributions are re-optimized. Table 3 indicate that the $f_2(1270)$, $\eta(1420)$, $f_0(1500)$,

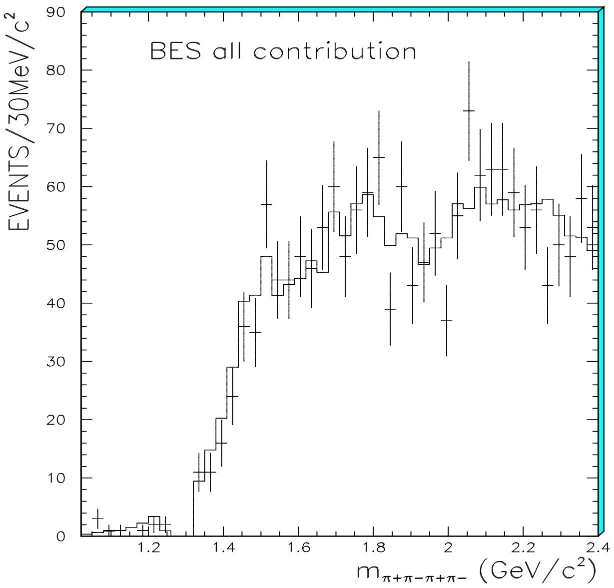

Figure 3: The 4π mass spectrum from the final full fit

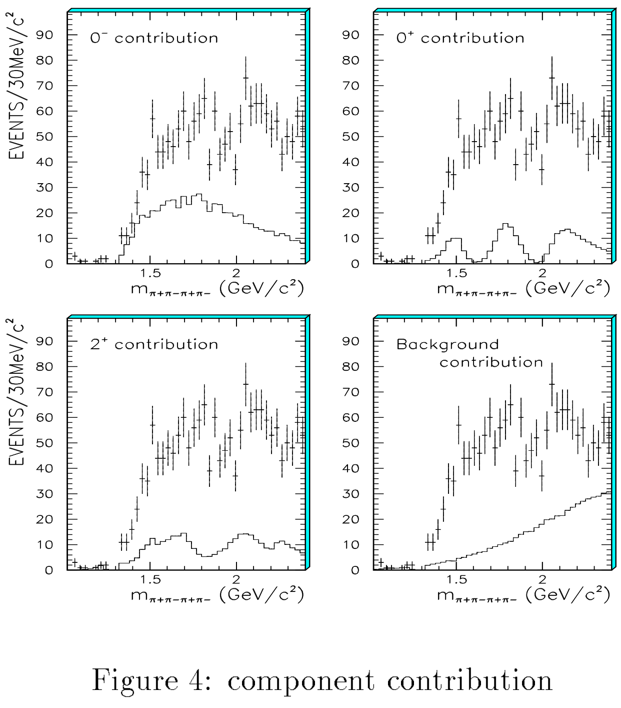

Figure 4: component contribution

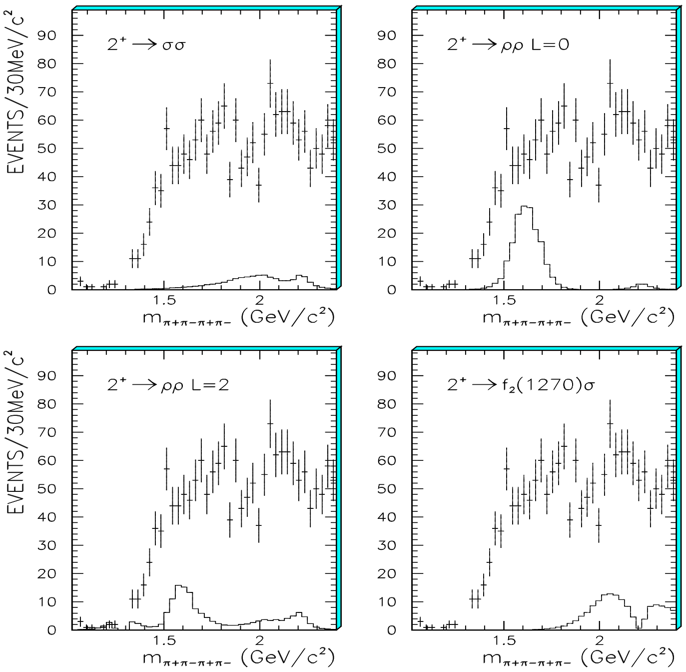

Figure 5: tensor component contribution

Table 2: Branching ratios for various final states in J/ψ decays, integrated up to $M(4\pi) =$ 2400 MeV. Errors are statistical. There is in addition a $\pm 15\%$ normalization error common to all channels.

Process	Branching ratios
$B(J/\psi \to \gamma f_2(1270)) \times B(f_2(1270) \to \pi^+\pi^-\pi^+\pi^-)$	$(2.1 \pm 0.3) \times 10^{-4}$
$B(J/\psi \to \gamma\ \eta(1420)) \times B(\ \eta(1420) \to \pi^+\pi^-\pi^+\pi^-)$	$(2.5 \pm 0.4) \times 10^{-3}$
$B(J/\psi \to \gamma f_0(1500)) \times B(f_0(1500) \to \pi^+\pi^-\pi^+\pi^-)$	$(4.0 \pm 0.6) \times 10^{-4}$
$B(J/\psi \to \gamma f_2(1560)) \times B(f_2(1560) \to \pi^+\pi^-\pi^+\pi^-)$	$(4.3 \pm 0.7) \times 10^{-4}$
$B(J/\psi \to \gamma f_2(1710)) \times B(f_2(1710) \to \pi^+\pi^-\pi^+\pi^-)$	$(5.5 \pm 0.8) \times 10^{-4}$
$B(J/\psi \to \gamma f_0(1750)) \times B(f_0(1750) \to \pi^+\pi^-\pi^+\pi^-)$	$(5.5 \pm 0.8) \times 10^{-4}$
$B(J/\psi \to \gamma f_2(2010)) \times B(f_2(2010) \to \pi^+\pi^-\pi^+\pi^-)$	$(1.0 \pm 0.2) \times 10^{-3}$
$B(J/\psi \to \gamma f_0(2100)) \times B(f_0(2100) \to \pi^+\pi^-\pi^+\pi^-)$	$(6.1 \pm 0.9) \times 10^{-4}$
$B(J/\psi \to \gamma f_2(2220)) \times B(f_2(2220) \to \pi^+\pi^-\pi^+\pi^-)$	$(2.6 \pm 0.4) \times 10^{-4}$

$f_2(1560)$, $f_2(1710)$, $f_0(1750)$, $f_2(2010)$ and $f_0(2100)$ should be included in the fit. Though the signal of $f_2(2220)$ can be seen in the 4π mass spectrum, but the evidence for the presence of this narrow $f_2(2220)$ is slender, for the significance level is only 2.0 σ.

Table 3: Changes ΔS in log likelihood function S when various components of the fit are dropped and remaining contributions are re-optimized

J^P	Resonance	ΔS	Freedom	Significance level
2^+	$f_2(1270)$	+11.22	3	4.0σ
0^-	$\eta(1420)$	+319.58	2	
0^+	$f_0(1500)$	+9.76	2	4.0σ
2^+	$f_2(1560)$	+12.34	4	4.0σ
2^+	$f_2(1710)$	+7.27	2	3.4σ
0^+	$f_0(1750)$	+8.80	2	3.8σ
2^+	$f_2(2010)$	+22.30	7	5.2σ
0^+	$f_0(2100)$	+10.96	2	4.3σ
2^+	$f_2(2220)$	+7.03	7	2.0σ
	background	+75.32		

Table 4: Changes ΔS in log likelihood function when various components are assigned a different J^P and all contributions are re-optimized

J^P	Resonance	0^-	0^+	2^+	4^+
2^+	$f_2(1270)$	+10.2	+5.4		+9.0
0^-	$\eta(1420)$		+313.8	+318.6	+306.3
0^+	$f_0(1500)$	+9.6		+6.5	+8.6
2^+	$f_2(1560)$	+11.1	+10.7		+11.7
2^+	$f_2(1710)$	+5.7	+4.9		+6.9
0^+	$f_0(1750)$	+7.7		+4.3	+8.8
2^+	$f_2(2010)$	+21.5	+21.7		+22.2
0^+	$f_0(2100)$	+8.3		+0.3	+9.8
2^+	$f_2(2220)$	+4.2	+4.9		+6.3

The second test is to check the J^P assignment of each resonance. Making alternative J^P assignment for each resonance is clearly rejected except for $f_0(2100)$. The results are shown in Table 4. When 2^+ is assigned to $f_0(2100)$ with 8 free parameters, the log likelihood function S is comparable to that of the 0^+ assignment with 2 free parameters. This is reasonable, because the 2^+ structure at 2.0 GeV is very wide. But the requirement that a 0^+ resonance $f_0(2100)$ be present in the fit is highlighted by the slice fit and the fact that its inclusion gives an improvement in S of 10.96.

The third test is to scan an additional structure. Scanning an additional structure in 0^-, 0^+, 2^+, or 4^+ with 2, 4, 6 and 4 free more paremeters respectively indicates that no distinctive structures are found. The plots are drawn in Figure 6.

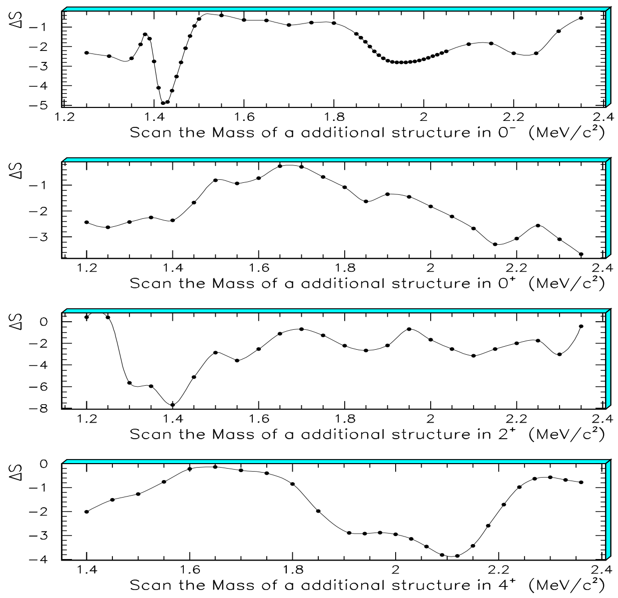

Figure 6: Scan an additional structure

From Figure 6, it can be seen that Scanning an additional structure in 0^- with two more parameters indicates that in 1420 MeV region, there seems to be another 0^- resonance, but the improvement over a single resonance does not look enough to justify two resonances, by the way, even if the additional resonance really exits, the improvements is quite small; Scanning an additional structure in 2^+ with six more parameters indicates that there is a dip in 1400 MeV, but the improvement by six parameters is also small. We don't believe an additional 2^+ resonance exists.

The fourth test is to fit the data in slice. The data is fitted in each mass bin. 10 bins are used with 100 MeV width for each bin and extended from 1350 to 2350 MeV. The comparisons of the full fit and the slice fit are plotted in Figure 7.

The solid line presents the full fit results, while the dot stands for the slice analysis results. They are in agreement with each other. From the slice analysis, the tensor and scalar contribution of both slice fit and full fit confirms that around 2000 MeV region, tensor contribution is prominent.

In conclusion, it can be seen that the final full fit is the best solution to BES data.

Discussion and Conclusion

The BES results are consistent with the reanalysis results of MARK III data. But the BES data seems more definite in favor of 2^+ at high mass region. Possible cause is that a better

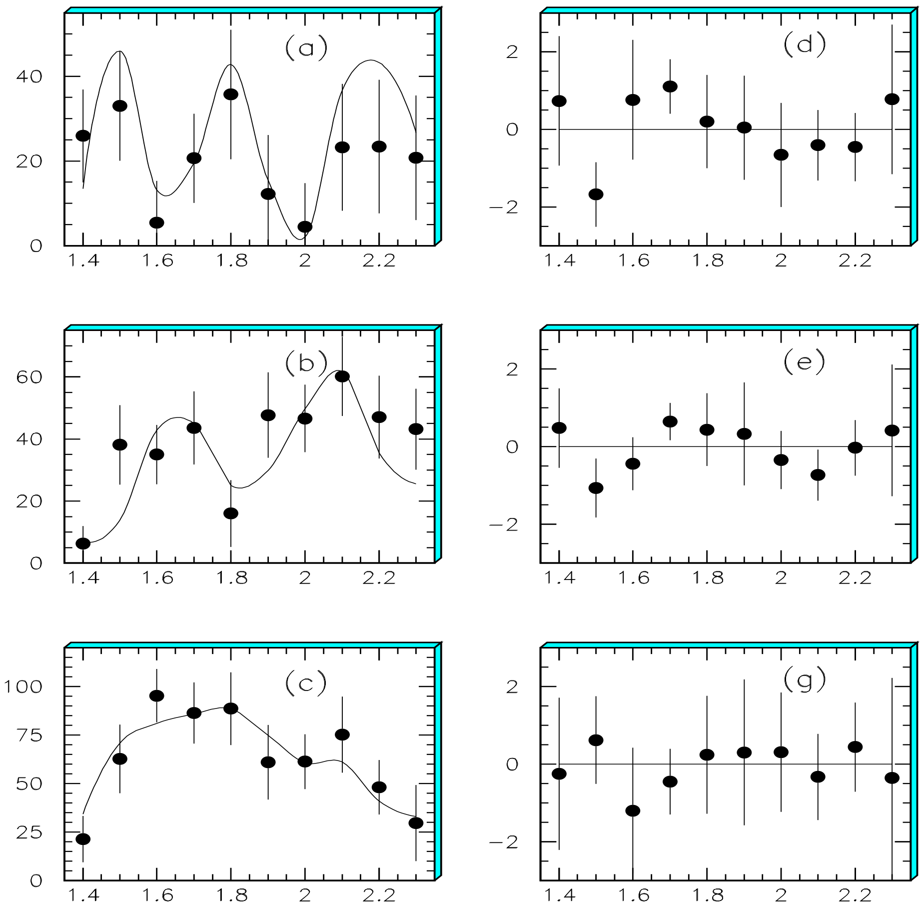

Figure 7: Numbers of events in 100 MeV slices of $M(4\pi)$ (data point) from (a) 0^+, (b) 2^+ and (c) 0^- and the full fit (curves); the discrepancy in phase difference (d) $\phi(0^+) - \phi(0^-)$, (e) $\phi(2^+) - \phi(0^-)$ and (f) $\phi(2^+) - \phi(0^+)$ between the data and fit(radians); the value plotted is that for each slice minus the value from the fit.

background function for BES data is available and a full Monte Carlo simulation is employed in BES analysis. A very important conclusion is that a 2^+ contribution is definitely needed over a broad mass region around 2 GeV, for it is just located at the mass region where the 2^+ glueball is expected, and J/ψ radiative decays are supposed to be one of the best places to search for the glueballs. Recently, the WA102 group found a broad 2^+ resonance at 1930 MeV in their 4π mass spectrum [7], this agrees with our observation.

The BES data on $J/\psi \to \gamma\pi^+\pi^-\pi^+\pi^-$ shows two resonances in 1.7 GeV region which were identified as $f_J(1710)$: a scalar with mass 1775 MeV and width 190 MeV, and a tensor with mass 1697 MeV and width 175 MeV. This agree with the results of BES data on $J/\psi \to \gamma K^+K^-$ channel[8].

There are some tentative evidences for the existence of a fairly narrow $f_2(2220)$, but they are not conclusive. Whether this $f_2(2220)$ is really $\xi(2230)$ needs further studies.

We find that the branching ratios of J/ψ radiative decaying into $f_0(1500)$ and $f_2(2220)$ (if the $f_2(2220)$ here is really the $\xi(2230)$) in $J/\psi \to \gamma X \to \gamma\pi^+\pi^-\pi^+\pi^-$ channel are larger than those in $\gamma 2\pi$ channel[9, 10] . This is very interesting.

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