# CP violation in $\psi(2S) \to J/\psi\pi\pi$ processes

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We propose to search for CP-violating effects in the decay $\psi(2S) \to J/\psi\pi\pi$. The scheme has the advantage that one does not need to track two or more CP-conjugate processes. Model independent amplitudes are derived for this purpose. The fact that leading CP violating terms are $\mathcal{O}(k)$ under low energy expansion and the processes are flavor disconnected make the measurement of these CP breaking parameters practical. Our results can be extended to the case of $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$ and $\Upsilon(3S) \to \Upsilon(2S)\pi\pi$ straightforwardly.

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CP violation is a subject attracting much interest. In the Standard Model, it arises as a phase entering the CKM matrix. It is believed that, with the CP violation presented in the Standard Model, it is not possible to generate the observed size of matter-antimatter asymmetry of the Universe [1,2]. However, CP violation has only been observed in neutral-kaon systems till now. The evidence come from the measurements of $\eta_{+-}$, $\eta_{00}$, and the semileptonic decay charge asymmetry for $K_L$. Currently experimental efforts are concentrated on neutral systems such as $K^0$-$\bar{K}^0$, $B^0$-$\bar{B}^0$, and $D^0$-$\bar{D}^0$ complex. Other searches, as summarized by Wolfenstein and Trippe, devide into two classes: (a) Those that involve weak interactions or parity violation. The most sensitive are the searches for an electric dipole moment of the elementary particles such as neutron and electron. (b) Those that involve processes otherwise allowed by the strong or electromagnetic interactions. This includes the search for C violation in $\eta$ decay and searches for T violation in a number of nuclear and electromagnetic reactions [3].

In this paper we suggest a new means to search for CP violation. We propose to measure CP asymmetries in the decay $\psi(2S) \to J/\psi\pi^+\pi^-$ or $\psi(2S) \to J/\psi\pi^0\pi^0$. The process $\psi(2S) \to J/\psi\pi^+\pi^-$ has been discussed based on an effective lagrangian [4,5], the multipole expansion hypothesis [6–8], or current algebra [9]. These discussions are model dependent, and C, P, T invariance are presumed. We derive model independent amplitudes of the two processes. CP invariance requires particular parameters in the decay amplitude vanish. Any

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non-zero value of those particular parameters implies CP violation. One will not need to track two or more channels at the same time. And we need not to compare the phases or decay rate of two or more CP-conjugate processes to get CP violation observables, thus avoid precision loss when we subtracting two very close numbers or information loss because of using total transition rates instead of differential cross sections.

$J/\psi\pi^+\pi^-$ and $J/\psi\pi^0\pi^0$ are the two largest decay modes of $\psi(2S)$, with branching ratios $(30.2 \pm 1.9)\%$ and $(17.9 \pm 1.8)\%$ [3]. The masses for $\psi(2S)$ and $J/\psi$ are $3686.00 \pm 0.09$ MeV and $3096.88 \pm 0.04$ MeV, with a difference of 589 MeV. The energy available for the pions are small and as we will shown, most part of the amplitude are expected to be the contribution of contact interactions. This validates the low energy expansion. The leading CP-violating terms in the model independent amplitudes are of the first order of the soft pion momentum($\mathcal{O}(k)$). BES has enough data to see D-wave contributions [10], which is $\mathcal{O}(k^2)$. Because the processes are flavor disconnected, the strong interactions are suppressed, and we have chances to see CP-violating effects beyond (or even within) the Standard Model in these decays. So it is practical to measure these CP-violating parameters. It will not take long for BEPC to accumulate $10^8$ $\psi(2S)$ events. Now BEPC is scheduled for another upgrade in the near future. After the upgrade it will have the ability to take more than $10^9$ $\psi(2S)$ events and give a high precision test of CP invariance in $\psi(2S) \to J/\psi\pi\pi$ decays.

The Feynman amplitude for the process $\psi(2S) \to J/\psi\pi^+\psi^-$ reads

$$\mathcal{M}_{\lambda,\sigma}(p,q,k_1,k_2) = e^{\mu}_{\psi'}(\vec{p},\lambda)e^{*\nu}_{J/\psi}(\vec{q},\sigma)\Gamma_{\mu\nu}. \tag{1}$$

Here $p, q, k_1, k_2$ are four-momenta of the particles $\psi(2S), J/\psi, \pi^+, \pi^-$. The helicities of $\psi(2S)$ and $J/\psi$ are $\lambda$ and $\sigma$, while $e^{\mu}_{\psi'}(\vec{p},\lambda)$ and $e^{*\nu}_{J/\psi}(\vec{q},\sigma)$ are the corresponding polarization vectors for the two particles. $\Gamma_{\mu\nu}$ is the effective vertex for the process, which contains all details of the decay.

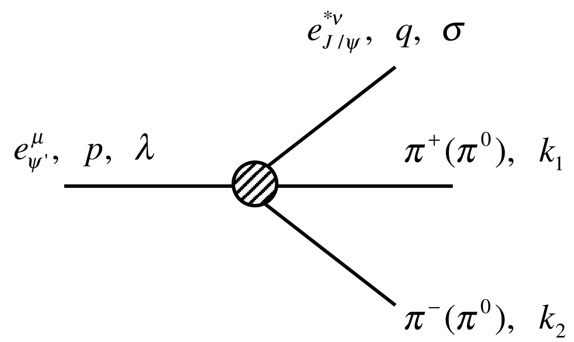


FIG. 1. $\psi(2S)$ decays into $J/\psi$ and two pions.

The effective vertex $\Gamma^{\mu\nu}$ should be constructed from the four-momenta $p^\alpha$, $q^\alpha$, $k_1^\alpha$, $k_2^\alpha$ and isotropic tensors $g^{\alpha\beta}$, $\varepsilon^{\alpha\beta\gamma\delta}$. The effective vertex contains only the zeroth and first order of the antisymmetric tensor because of the identity

$$\varepsilon_{\alpha\beta\gamma\delta}\varepsilon_{\alpha'\beta'\gamma'\delta'} = -\det\begin{pmatrix} g_{\alpha\alpha'} & g_{\beta\alpha'} & g_{\gamma\alpha'} & g_{\delta\alpha'} \\ g_{\alpha\beta'} & g_{\beta\beta'} & g_{\gamma\beta'} & g_{\delta\beta'} \\ g_{\alpha\gamma'} & g_{\beta\gamma'} & g_{\gamma\gamma'} & g_{\delta\gamma'} \\ g_{\alpha\delta'} & g_{\beta\delta'} & g_{\gamma\delta'} & g_{\delta\delta'} \end{pmatrix}. \tag{2}$$

Considering energy-momentum conservation

$$p^\alpha = q^\alpha + k_1^\alpha + k_2^\alpha \tag{3}$$

and the Lorentz conditions for polarization vectors

$$p_\alpha e^\alpha_{\psi'}(\vec{p},\lambda) = 0, \tag{4}$$

$$q_\alpha e^{*\alpha}_{J/\psi}(\vec{q},\sigma) = 0, \tag{5}$$

we find the general form of the effective vertex

$$\begin{aligned} &\Gamma^{\mu\nu}(p,q,k_1,k_2,g^{\alpha\beta},\varepsilon^{\alpha\beta\gamma\delta}) \\ &= c_1 g^{\mu\nu} + c_2 k_1^\mu k_1^\nu + c_3 k_2^\mu k_2^\nu + c_4 k_1^\mu k_2^\nu + c_5 k_2^\mu k_1^\nu + c_6 A_1^{\mu\nu} + c_7 A_2^{\mu\nu} \\ &\quad + c_8 A_3^{\mu\nu} + c_9 Q^\mu k_1^\nu + c_{10} Q^\mu k_2^\nu + c_{11} k_1^\mu Q^\nu + c_{12} k_2^\mu Q^\nu, \end{aligned} \tag{6}$$

where we define

$$A_1^{\alpha\beta} = p_\gamma k_{1\delta}\varepsilon^{\alpha\beta\gamma\delta}, \tag{7}$$

$$A_2^{\alpha\beta} = p_\gamma k_{2\delta}\varepsilon^{\alpha\beta\gamma\delta}, \tag{8}$$

$$A_3^{\alpha\beta} = k_{1\gamma} k_{2\delta}\varepsilon^{\alpha\beta\gamma\delta}, \tag{9}$$

$$Q^\alpha = p_\beta k_{1\gamma} k_{2\delta}\varepsilon^{\alpha\beta\gamma\delta}. \tag{10}$$

The twelve form factors in $\Gamma^{\mu\nu}$ are not independent since

$$Q_\mu k_{1\nu} e^\mu_{\psi'} e^{*\nu}_{J/\psi} \equiv \left(k_{1\mu}Q_\nu + (k_1\cdot k_2)A_{1\mu\nu} - k_1^2 A_{2\mu\nu} + (p\cdot k_1)A_{3\mu\nu}\right) e^\mu_{\psi'} e^{*\nu}_{J/\psi}, \tag{11}$$

$$Q_\mu k_{2\nu} e^\mu_{\psi'} e^{*\nu}_{J/\psi} \equiv \left(k_{2\mu}Q_\nu + k_2^2 A_{1\mu\nu} - (k_1\cdot k_2)A_{2\mu\nu} + (p\cdot k_2)A_{3\mu\nu}\right) e^\mu_{\psi'} e^{*\nu}_{J/\psi}, \tag{12}$$

$$\left(Q_\mu k_{1\nu} + Q_\mu k_{2\nu}\right) e^\mu_{\psi'} e^{*\nu}_{J/\psi} \equiv \left((p\cdot k_2)A_{1\mu\nu} - (p\cdot k_1)A_{2\mu\nu} + p^2 A_{3\mu\nu}\right) e^\mu_{\psi'} e^{*\nu}_{J/\psi}. \tag{13}$$

We can eliminate three terms in the effective vertex without introducing kinematic singularities. After a redefinition of form factors we have nine independent terms,

$$\begin{aligned} &\Gamma^{\mu\nu}(p,q,k_1,k_2,g^{\alpha\beta},\varepsilon^{\alpha\beta\gamma\delta}) \\ &= c_1 g^{\mu\nu} + c_2\left(k_1^\mu k_1^\nu + k_2^\mu k_2^\nu\right) + c_3\left(k_1^\mu k_1^\nu - k_2^\mu k_2^\nu\right) + c_4\left(k_1^\mu k_2^\nu + k_2^\mu k_1^\nu\right) \\ &\quad + c_5\left(k_1^\mu k_2^\nu - k_2^\mu k_1^\nu\right) + c_6\left(A_1^{\mu\nu} + A_2^{\mu\nu}\right) + c_7\left(A_1^{\mu\nu} - A_2^{\mu\nu}\right) \\ &\quad + c_8 A_3^{\mu\nu} + c_9 Q^\mu\left(k_1^\nu - k_2^\nu\right), \end{aligned} \tag{14}$$

with

$$c_i = c_i\left((k_1 \cdot k_2), [p \cdot (k_1 - k_2)]\right) \tag{15}$$

for $i = 1, 2, \cdots, 9$.

Expand the form factors in Taylor series of $(k_1 \cdot k_2)$ and $[p \cdot (k_1 - k_2)]$, and keep those terms up to $\mathcal{O}(k^2)$,

$$\begin{aligned}
&\Gamma^{\mu\nu}(p, q, k_1, k_2, g^{\alpha\beta}, \varepsilon^{\alpha\beta\gamma\delta}) \\
=& \{f_0 + \frac{[p \cdot (k_1 - k_2)]}{M_{\psi'}^2} f_1 + \frac{(k_1 \cdot k_2)}{M_{\psi'}^2} f_2 + \frac{[p \cdot (k_1 - k_2)]^2}{M_{\psi'}^4} f_3\} g^{\mu\nu} \\
&+ \frac{(k_1^\mu k_1^\nu + k_2^\mu k_2^\nu)}{M_{\psi'}^2} f_4 + \frac{(k_1^\mu k_1^\nu - k_2^\mu k_2^\nu)}{M_{\psi'}^2} f_5 \\
&+ \frac{(k_1^\mu k_2^\nu + k_2^\mu k_1^\nu)}{M_{\psi'}^2} f_6 + \frac{(k_1^\mu k_2^\nu - k_2^\mu k_1^\nu)}{M_{\psi'}^2} f_7 \\
&+ i\frac{(A_1^{\mu\nu} + A_2^{\mu\nu})}{M_{\psi'}^2} f_8 + i\frac{(A_1^{\mu\nu} + A_2^{\mu\nu})[p \cdot (k_1 - k_2)]}{M_{\psi'}^4} f_9 \\
&+ i\frac{(A_1^{\mu\nu} - A_2^{\mu\nu})}{M_{\psi'}^2} f_{10} + i\frac{(A_1^{\mu\nu} - A_2^{\mu\nu})[p \cdot (k_1 - k_2)]}{M_{\psi'}^4} f_{11} \\
&+ i\frac{A_3^{\mu\nu}}{M_{\psi'}^2} f_{12},
\end{aligned} \tag{16}$$

where $M_{\psi'}$ is the mass of $\psi(2S)$ and $f_i$ are dimensionless complex constants. The reason why the energy scale should be $M_{\psi'}$, and why we extract a factor "$i$" from $f_8, f_9, \cdots, f_{12}$ will be explained later.

Now let's see what CP invariance can say about the form factors. If CP parity is conserved, then

$$\begin{aligned}
&\langle\psi'|\mathbf{S}|J/\psi\pi^+\pi^-\rangle \\
=& \langle\psi'|(\mathbf{CP})^\dagger(\mathbf{CP})\mathbf{S}(\mathbf{CP})^{-\mathbf{1}}(\mathbf{CP})|J/\psi\pi^+\pi^-\rangle \\
=& \langle\psi'|(\mathbf{CP})^\dagger\mathbf{S}(\mathbf{CP})|J/\psi\pi^+\pi^-\rangle.
\end{aligned} \tag{17}$$

The spin-parity $J^{PC}$ for $\psi(2S)$, $J/\psi$ and pions are $1^{--}$, $1^{--}$ and $0^{-+}$ [3],

$$\mathbf{CP}|\vec{p}, \lambda\rangle = -|-\vec{p}, -\lambda\rangle, \tag{18}$$

$$\mathbf{CP}|\vec{q}, \sigma, \vec{k}_1, \vec{k}_2\rangle = -|-\vec{q}, -\sigma, -\vec{k}_2, -\vec{k}_1\rangle. \tag{19}$$

So conservation of CP require

$$\mathcal{M}_{\lambda,\sigma}(p, q, k_1, k_2) \equiv \mathcal{M}_{-\lambda,-\sigma}(\bar{p}, \bar{q}, \bar{k}_2, \bar{k}_1). \tag{20}$$

Where a vector with a bar indicate its space reflected value, e.g.,

$$\bar{q}^{\alpha} \equiv \mathcal{P}^{\alpha}_{\beta} q^{\beta}, \tag{21}$$

and $\mathcal{P}$ is the space reflection matrix, $(\mathcal{P}^{\alpha}_{\beta}) = \mathrm{diag}\{1, -1, -1, -1\}$. That is

$$\begin{aligned}
& e^{\mu}_{\psi'}(\vec{p}, \lambda) e^{*\nu}_{J/\psi}(\vec{q}, \sigma) \Gamma_{\mu\nu}(p, q, k_1, k_2, g^{\alpha\beta}, \varepsilon^{\alpha\beta\gamma\delta}) \\
&= e^{\mu}_{\psi'}(-\vec{p}, -\lambda) e^{*\nu}_{J/\psi}(-\vec{q}, -\sigma) \Gamma_{\mu\nu}(\bar{p}, \bar{q}, \bar{k}_2, \bar{k}_1, g^{\alpha\beta}, \varepsilon^{\alpha\beta\gamma\delta}) \\
&= \bar{e}^{\mu}_{\psi'}(\vec{p}, \lambda) \bar{e}^{*\nu}_{J/\psi}(\vec{q}, \sigma) \Gamma_{\mu\nu}(\bar{p}, \bar{q}, \bar{k}_2, \bar{k}_1, g^{\alpha\beta}, \varepsilon^{\alpha\beta\gamma\delta}) \\
&= e^{\mu}_{\psi'}(\vec{p}, \lambda) e^{*\nu}_{J/\psi}(\vec{q}, \sigma) \Gamma_{\mu\nu}(p, q, k_2, k_1, g^{\alpha\beta}, -\varepsilon^{\alpha\beta\gamma\delta}).
\end{aligned} \tag{22}$$

Since all form factors in Eq. (16) are independent, it is easy to see that *any non-vanishing value of* $f_1$,$f_5$,$f_7$,$f_8$ *or* $f_{11}$ *implies CP violation.* $f_1/f_0$, $f_5/f_0$, $f_7/f_0$, $f_8/f_0$ and $f_{11}/f_0$ can be taken as CP breaking parameters. Among them, the $f_1$ and $f_8$ terms are $\mathcal{O}(k)$. In fact, the parameters in Eq. (16) can be classified into four types(as shown in Tab. I): (1)$f_0$, $f_2$, $f_3$, $f_4$ and $f_6$ terms keep both C and P symmetry. (2)$f_1$, $f_5$ and $f_7$ terms are C nonconserving but keep P symmetry. (3)$f_8$ and $f_{11}$ terms keep C symmetry but break P. (4)$f_9$, $f_{10}$ and $f_{12}$ terms break both C and P, but keep CP invariance.

If we suppose the amplitude is completely the first order contribution of an effective lagrangian, and the lagrangian is time reversal invariant, the form factors $f_0, f_2, \cdots, f_{12}$ will be relatively real. That's why we extract a factor "$i$" in the abnormal terms. Non-zero relative phases may come from T violation in the effective lagrangian or high order terms(loops and resonance). However, since there is no strong resonance in the process, we can expect the Feynman amplitude to be dominated by contact interaction terms in the effective lagrangian, so that their relative phases will be small. At least this will be true for those terms keeping both CP and isospin symmetry, i.e., the relative phases between $f_0, f_2, f_3, f_4, f_6$ will be very small(but one should not mistake a nonzero relative phase as the signal of CP violation).

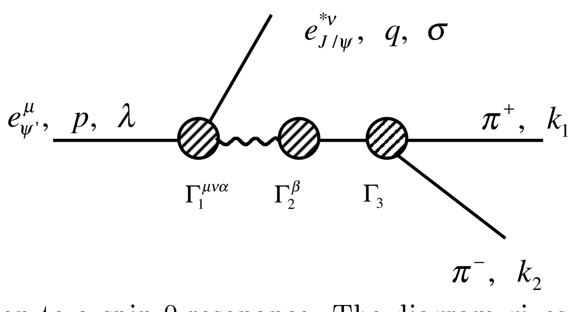


FIG. 2. Gluonic transition to a spin-0 resonance. The diagram gives a zero contribution due to gauge invariance.

The $\psi(2S) \rightarrow J/\psi\pi^+\pi^-$ process is flavor disconnected. In an effective theory, the tree level diagram with a spin-0 resonance(e.g. $f_0$) of $\pi\pi$ is illustrated in Fig. 2. The gluon-scalar vertex $\Gamma_2^\beta$ has to be proportional to $(k_1^\beta + k_2^\beta)$. When contracted with the gluon propagator and the $\psi(2S)J/\psi$-gluon vertex $\Gamma_2^{\mu\nu\alpha}$, gauge invariance will ensure the contribution vanish. For spin-$n(n \geq 2)$ meson, the gluon-meson vertex contracted with the spin-$n$ meson propagator will contain a factor $(k_1 + k_2)^2 - m^2$, which gives a zero on the mass shell instead of a pole. So we come to the conclusion:*there is no $\pi$-$\pi$ resonance other than those with spin-1.*

Because the isospins of $\psi(2S)$ and $J/\psi$ are all zero, isospin symmetry will require the two pions to form an isospin singlet, which is symmetric under the interchange of the two pions' four-momenta. Since the amplitude for an odd spin meson's decay into two pions will be anti-symmetric when interchanging the pions' momenta, we see that any spin-1 resonance here is suppressed by isospin symmetry.

Nor can we find any strong resonance in $J/\psi\pi^\pm$ channels. $\psi(2S)$ decays mainly through contact interactions. Therefor, $M_{\psi'}$(or heavy quark mass) can be taken as the typical energy scale of the process.

Not all terms in Eq. (16) should be kept when fitting data. CP violating $\mathcal{O}(k^2)$ terms, $f_5$, $f_7$ and $f_{11}$, are strongly suppressed and we can not to see them when we have not enough data. $f_9$ and $f_{12}$ terms should also be dropped because they are $\mathcal{O}(k^2)$ and break isospin symmetry. Eight terms are kept:

$$\begin{aligned}\Gamma^{\mu\nu} = &\left\{ f_0 + \frac{[p\cdot(k_1-k_2)]}{M_{\psi'}^2} f_1 + \frac{(k_1\cdot k_2)}{M_{\psi'}^2} f_2 + \frac{[p\cdot(k_1-k_2)]^2}{M_{\psi'}^4} f_3 \right\} g^{\mu\nu} \\ &+ \frac{(k_1^\mu k_1^\nu + k_2^\mu k_2^\nu)}{M_{\psi'}^2} f_4 + \frac{(k_1^\mu k_2^\nu + k_2^\mu k_1^\nu)}{M_{\psi'}^2} f_6 + i\frac{(A_1^{\mu\nu} + A_2^{\mu\nu})}{M_{\psi'}^2} f_8 + i\frac{(A_1^{\mu\nu} - A_2^{\mu\nu})}{M_{\psi'}^2} f_{10}. \qquad (23)\end{aligned}$$

The $f_1$ and $f_8$ term in the amplitude are CP-violating.

TABLE I. Symmetry properties of the amplitude. The symbol "×" under a parameter means that the corresponding symmetry is violated when the parameter has a non-zero value.

| | $f_0$ | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8$ | $f_9$ | $f_{10}$ | $f_{11}$ | $f_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Order of $k$ | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 2 | 1 | 2 | 2 |
| Isospin Symmetry | √ | × | √ | √ | √ | × | √ | × | √ | × | × | √ | × |
| C Parity | √ | × | √ | √ | √ | × | √ | × | √ | × | × | √ | × |
| P Parity | √ | √ | √ | √ | √ | √ | √ | √ | × | × | × | × | × |
| CP Parity | √ | × | √ | √ | √ | × | √ | × | × | √ | √ | × | √ |

For the process $\psi(2S) \to J/\psi\pi^0\pi^0$, boson symmetry of the two pions demands the amplitude invariant when the four-momenta $k_1$ and $k_2$ are interchanged. Similar analysis leads to an effective vertex of the form

$$\Gamma'^{\mu\nu} = \left\{ g_0 + \frac{(k_1 \cdot k_2)}{M_{\psi'}^2} g_1 + \frac{[p \cdot (k_1 - k_2)]^2}{M_{\psi'}^4} g_2 \right\} g^{\mu\nu} + \frac{(k_1^\mu k_1^\nu + k_2^\mu k_2^\nu)}{M_{\psi'}^2} g_3 \\ + \frac{(k_1^\mu k_2^\nu + k_2^\mu k_1^\nu)}{M_{\psi'}^2} g_4 + i\frac{(A_1^{\mu\nu} + A_2^{\mu\nu})}{M_{\psi'}^2} g_5 + i\frac{(A_1^{\mu\nu} - A_2^{\mu\nu})[p \cdot (k_1 - k_2)]}{M_{\psi'}^4} g_6, \tag{24}$$

and the Feynman amplitude is

$$\mathcal{M}'_{\lambda,\sigma}(p, q, k_1, k_2) = e_{\psi'}^\mu(\vec{p}, \lambda) e_{J/\psi}^{*\nu}(\vec{q}, \sigma) \Gamma'_{\mu\nu}. \tag{25}$$

The $g_0$, $g_1$, $g_2$, $g_3$ and $g_4$ terms are CP conserving, while $g_5$, $g_6$ terms are CP violating. Argument for why we take $M_{\psi'}$ as the energy scale is similar to the above, and we only need to point out the fact that *any odd spin particle's decay into two identical spin-0 particles is strictly forbidden by boson symmetry.* Relative phases between $g_0$, $g_1$, $g_2$, $g_3$ and $g_4$ are small. The CP breaking $\mathcal{O}(k^2)$ term $g_6$ can be set to zero when one fits data since it is strongly suppressed.

The cross section of such a three-body decay depends on five variables: $E_1$ and $E_2$, the energies of the two pions; and $(\alpha, \beta, \gamma)$, the Euler angles that specify the orientation of the final system relative to the initial particle [3]. The cross section of $\psi(2S) \to J/\psi\pi\pi$ does not depend on the variable $\alpha$ provided that $\psi(2S)$ is produced by $e^+e^-$ collision, and the $\alpha$ represents the rotation angle around the beam line. We have

$$k_1 \cdot k_2 = (E_1 + E_2) M_{\psi'} - \tfrac{1}{2}(M_{\psi'}^2 - M_{J/\psi}^2 + 2m_\pi^2), \tag{26}$$

$$p \cdot (k_1 - k_2) = (E_1 - E_2) M_{\psi'} \tag{27}$$

and the nine independent terms in Eq. (14) represent different angular distributions. Those terms proportional to $|f_1|^2$, $|f_8|^2$, $f_1 f_8^*$ and $f_1^* f_8$ in the cross section can be ignored since it is relatively small. The coherent part that proportional $f_8$ (or $f_8^*$) has a significantly different distribution comparing with backgrounds. The part proportional to $f_1$ (or $f_1^*$) is an odd distribution when we exchange $E_1$ with $E_2$ in the Dalitz plot, while the backgrounds are even distributions. These two facts will help us to distinguish the CP-violating parts from background contributions, and will remarkably improve the precision of the measurement of the CP-violating parameters $f_1$ and $f_8$. Unlike the case in the decays of $K^0 - \bar{K}^0$ complex, CP violating terms here are not mixed with backgrounds. Fitting experimental data with such differential cross sections can give more information on CP violation than the partial width, *e.g.*, if we integrate out the variable $p \cdot (k_1 - k_2)$, the part proportional to $f_1$(or $f_1^*$) will vanish.

We would like to point out that to sum over the spins of $J/\psi$ or averaging over the spins of $\psi(2S)$ in the cross section when comparing with data(as some references) is incorrect. $\psi(2S)$ are polarized. The density matrix of $\psi(2S)$ is determined by a vector coupling with $e^+e^-$ at $10^{-4}$ precision. And because $J/\psi$ is re-constructed through $e^+e^-$ and $\mu^+\mu^-$ channels, $J/\psi$ with helicity-0 has a much less chance to be selected than those with helicity-$\pm1$.

One might doubt if the CP-violating effects in the processes can be measured, given that they are mediated by the strong interactions. The Standard Model prediction of the CP-violating effects in the processes are difficult to evaluate and it is out of the scope of our present paper. However, we can give a rough estimation. The ratio of the weak interactions and the strong interactions at $c\bar{c}$-meson energy scale should be taken as $G_F m_{J/\psi}^2 \sim 10^{-4}$. There is a unique source for CP violation in the Standard Model. Any CP-violating effects in the Standard Model is proportional to the rephasing-invariant $Im\Delta^{(4)}$ [11]. The magnitude of CP violation comparing with the weak interactions can be represented by the $\eta$-parameter measured in $K^0$ decays, $\eta \sim 2\times10^{-3}$ [3]. So we come to the conclusion that CP-violating effect in the Standard Model, when compared with the strong interactions, is of the order $10^{-7}$.

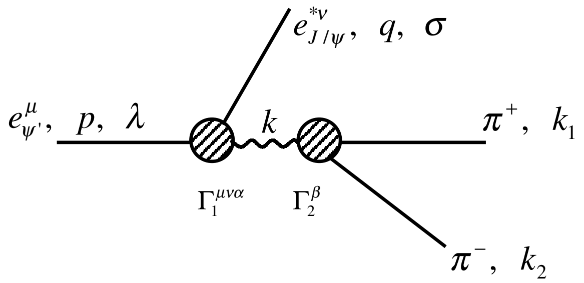


FIG. 3. Single gluon transition violates isospin symmetry.

But it is not the case when a process is flavor disconnected and the strong interactions in it are much smaller. As we have discussed above, there is no resonance in $\psi' \to J/\psi\pi\pi$ processes. Now suppose the decay is through single gluon exchange, as shown in Fig. 3. Gauge invariance demands that

$$k^\alpha \Gamma_{1\mu\nu\alpha} e^{\mu}_{\psi'} e^{*\nu}_{J/\psi} = 0, \tag{28}$$

so the only possible form of the gluon-$\pi$-$\pi$ vertex is

$$\Gamma_2^\beta = c(k_1 - k_2)^\beta, \tag{29}$$

with $c$ a constant. The two pions must form a isospin singlet provided the isospin symmetry conserved, and this require $\Gamma_2^\beta$ symmetric when exchanging $k_1$ and $k_2$. So the single gluon transition in this decay, comparing with flavor connected processes, is suppressed by $\alpha_S$ and isospin symmetry. Two-gluon transitions keeping isospin symmetry are suppressed by $\alpha_S^2$. Be advised that $\alpha_S$ is not the coupling constant in QCD which is very large at low energy. Here it is the effective coupling of gluon and should be very small. The precise value of such transitions are very difficult to calculate. However, we can evaluate such a suppression by comparing the decay $\phi \to \pi^+\pi^-$ with $\omega \to \pi^+\pi^-$. The difference between two processes is that the former one is flavor disconnected(both processes violate isospin symmetry). The amplitude of the two processes are all of the form

$$A_\lambda = c_{\phi,\omega} e^\mu(p,\lambda)(p_{\pi^+} - p_{\pi^-})_\mu, \tag{30}$$

here $c_\phi$ and $c_\omega$ are coupling constants. The ratio of the partial width of the two processes is

$$\frac{(1-4m_\pi^2/m_\phi^2)^{3/2}|c_\phi|^2 m_\phi}{(1-4m_\pi^2/m_\omega^2)^{3/2}|c_\omega|^2 m_\omega} \approx 1.4\left|\frac{c_\phi}{c_\omega}\right|^2 \tag{31}$$

The experimental value of the partial widths for the decay $\phi \to \pi^+\pi^-$ is $4.43 \times (8 \times 10^{-5}) \approx 3.5 \times 10^{-4}$ MeV, and $8.41 \times 2.21\% \approx 0.19$ MeV for $\omega \to \pi^+\pi^-$ [3]. One can find that the coupling constant of a flavor disconnected vertex is suppressed by a factor of $10^{-2}$. Now we can conclude that the CP-violating effect predicted by the Standard Model in the process $\psi(2S) \to J/\psi\pi\pi$ is of the order $10^{-5}$, comparing with the background amplitudes. For the process $\Upsilon(2S) \to \Upsilon(1S)$, it is estimated to be of $10^{-4}$ order.

It is believed that the Standard Model does not provide the complete description of CP violation in nature. Almost any extension of the Standard Model has additional sources of CP violating effects. In addition, theories that explain the observed baryon asymmetry of the universe must include new sources of CP violation [12]. The Standard Model can not generate a large enough matter-antimatter imbalance to produce the baryon number to entropy ratio observed in the universe today [13–15]. If the CP violating effect beyond the Standard Model is about ten times larger, it will be of the order $10^{-4}$ comparing with backgrounds.

BES has accumulated $3.8 \times 10^6$ $\psi(2S)$ events [16]. These events even make it possible to see the $\pi^+\pi^-$ D-wave ($\mathcal{O}(k^2)$) in the $J/\psi\pi^+\pi^-$ decay mode [5,10], although the D-wave is highly suppressed [17]. Noticing that the leading CP violating terms ($f_1$,$f_8$ and $g_5$) are $\mathcal{O}(k)$, it is significative to determine whether they are zero using current data. With BEPC has the ability to accumulate $10^8$ events within a few months, and it is scheduled for a upgrade. The upgraded BEPC will be able to accumulate $10^9 - 10^{10}$ events and can measure the CP violating parameters at a high accuracy. Provided that the CP violating effects beyond the Standard Model are of $10^{-4}$ order, they can be detected in the near future.

We have given the model independent amplitudes for the decay $\psi(2S) \to J/\psi\pi\pi$ and suggest to search for CP violation in these processes. It is practical to measure the CP violation parameters. And it has the advantage that CP violation observables can be directly measured, so that one does not need to track two or more CP-conjugate processes. The extension of our results to the case of $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$ and $\Upsilon(3S) \to \Upsilon(2S)\pi\pi$ is straightforward. $\Upsilon(2S)$ and $\Upsilon(3S)$ can be produced at B-factories.

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