Moments Analysis on $f_J(1710)$ in incomplete Phase Space for Radiative Decay J/$\Psi \to \gamma K^+K^-$

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Abstract

We discuss the moment analysis method for the 0^{++} and 2^{++} coupling mode of $f_J(1710)$ for incomplete phase space in the radiative decay J/$\psi \to \gamma K^+K^-$, and give a new formulation of analyzing the $f_J(1710)$ structure for the BES detector at the Beijing Electron-Positron Collider.

The lattice QCD calculations predicts [1] that the lightest scalar glueball might be about 1.5~1.7 GeV. So we have special interests in the meson of this energy range. $f_J(1710)$ is one of such resonances. In this paper, we discuss decay mode J/$\psi \to \gamma K^+K^-$ in the e^+e^- via $J/\psi \to \gamma + f_J(1710)$. In the channel $J/\psi \to \gamma + f_J(1710) \to \gamma + K\overline{K}$, the state $f_J(1710)$ was identified as 0^{++} by MARK II [2], but later the data of BES [3] indicate that the $J = 2$ component cannot be excluded, therefore very possibly, $f_J(1710)$ is a mixture of $J = 0$ and $J = 2$ components. In this scenario, by helicity amplitude method, one can easily write down the angular distribution of $\gamma K\overline{K}$.

In the center of mass frame of J/ψ which is the laboratory frame, $\vec{p}_+$ of positron is defined as the z-direction; $\vec{p}_\gamma$ of photon is on the x-z plane; y direction is parallel to the direction of $\vec{p}_+ \times \vec{p}_\gamma$. In this frame, the helicity of J/ψ can only be ± 1, i.e. $\lambda_J = \pm 1$ [4]; no $\lambda_J = 0$ contribution. The angular distribution is [5]

$$\begin{aligned}
W(\Omega_\gamma, \Omega) &= \sum_{\lambda_J,\lambda_\gamma,\lambda_1,\lambda_{1'}} \frac{5}{4\pi} A_{\lambda_\gamma,\lambda_1} A_{\lambda_\gamma,\lambda_{1'}} \cdot D^{1*}_{\lambda_J,\lambda_\gamma-\lambda_1}(\Omega_\gamma) \cdot D^{1}_{\lambda_J,\lambda_\gamma-\lambda_{1'}}(\Omega_\gamma) \cdot D^{2*}_{\lambda_1,0}(\Omega) \cdot D^{2}_{\lambda_{1'},0}(\Omega) \\
&+ \frac{1}{4\pi} B^2_{\lambda_\gamma,0} \cdot D^{1*}_{\lambda_J,\lambda_\gamma}(\Omega_\gamma) \cdot D^{1}_{\lambda_J,\lambda_\gamma}(\Omega_\gamma) \cdot D^{0*}_{0,0}(\Omega) \cdot D^{0}_{0,0}(\Omega) \\
&+ \frac{\sqrt{5}}{2\pi} Re[(\gamma A_{\lambda_\gamma,\lambda_1} B_{\lambda_\gamma,0}) \cdot D^{1*}_{\lambda_J,\lambda_\gamma-\lambda_1}(\Omega_\gamma) \cdot D^{1}_{\lambda_J,\lambda_\gamma}(\Omega_\gamma) \cdot D^{2*}_{\lambda_1,0}(\Omega) \cdot D^{0}_{0,0}(\Omega)] \qquad (1)
\end{aligned}$$

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where B_{10} is the helicity amplitude for 0^{++} component, A_{10}, A_{11} and A_{12} are those for 2^{++}, and $\gamma \equiv 2cos\alpha$ represents the interference between 0^{++} and 2^{++}. In the expression, Ω_γ is the solid angle of the 3-momentum of the emitted photon in the laboratory frame and Ω is the solid angle of $K\overline{K}$ in the c-m frame of $f_J(1710)$.

If there is an endcap detector, 4π solid angle is measured, then one can integrate out Ω_γ and Ω independently, in other words, Ω_γ and Ω are independent integration variables. In this case, it is very easy to extract A_{ij} and B_{ij} from $W(\Omega_\gamma, \Omega)$. For this aim, the standard procedure is to calculate the moments.

The moments for radiative decay $J/\psi \to \gamma X(X \to M\overline{M})$ where M are mesons, $Tr \equiv T(j,l,m)$ with 0^{++} and 2^{++} components can be written as[3]:

$$Tr \;=\; \int W(\theta_v, \Omega) Re[D^j_{0,-m}(0,\theta_v,0) {D^l_{m,0}}^*(\Omega)] dcos\theta_v d\Omega. \tag{2}$$

Thus, we get ten non-zero moments as follows[3]:

$$\begin{aligned}
T(000) &\sim 8[(A_{10}^2 + A_{11}^2 + A_{12}^2) + B_{10}^2], \\
T(200) &\sim \frac{4}{5}[(A_{10}^2 - 2A_{11}^2 + A_{12}^2) + B_{10}^2], \\
T(020) &\sim 8[\frac{1}{7}(2A_{10}^2 + A_{11}^2 - 2A_{12}^2) + \frac{1}{\sqrt{5}}A_{10}B_{10}\gamma], \\
T(220) &\sim \frac{8}{5}[\frac{1}{7}(A_{10}^2 - A_{11}^2 - 2A_{12}^2) + \frac{1}{2\sqrt{5}}A_{10}B_{10}\gamma], \\
T(221) &\sim \frac{4}{5}[\frac{1}{7}(\sqrt{3}A_{10}A_{11} - 3\sqrt{2}A_{11}A_{12} + \frac{\sqrt{3}}{2\sqrt{5}}A_{11}B_{10}\gamma], \\
T(222) &\sim \frac{4\sqrt{6}}{5}[-\frac{2}{7}A_{10}A_{12} + \frac{1}{2\sqrt{5}}A_{12}B_{10}\gamma], \\
T(040) &\sim \frac{8}{21}(6A_{10}^2 - 4A_{11}^2 + A_{12}^2), \\
T(240) &\sim \frac{4}{105}(6A_{10}^2 - 8A_{11}^2 + A_{12}^2), \\
T(241) &\sim \frac{4}{105}(3\sqrt{10}A_{10}A_{11} + \sqrt{15}A_{11}A_{12}), \\
T(242) &\sim \frac{4\sqrt{10}}{35}A_{10}A_{12}.
\end{aligned} \tag{3}$$

In fact, to determine the fractions of 0^{++} and 2^{++} in $f_J(1710)$ resonance, A_{ij} and B_{ij} give the direct information. These coefficients can be calculated in theoretical models. Comparing these values with data, one can determine what particles they are. Therefore the key point is how to correctly extract A_{ij}^{exp} and B_{ij}^{exp} from the measured angular distribution data $W(\Omega_\gamma, \Omega)$.

As aforementioned, as long as complete 4π solid angles are measured, integrations with respect to Ω_γ and Ω can be independently carried out. However, when the solid angles are not complete, or say, the phase space for the final states is not complete, namely as in BES there is no endcap detector, so that a certain solid angles are not measurable. Thus Ω_γ and Ω are not independent and the orthogonal and normalizable conditions of the D-functions no longer hold. So we need to evaluate all the moments corresponding to the incomplete solid angles and obtain a new equation group.

In order to study the effect of incomplete phase space, we have employed the Monte-Carlo method to simulate the radiative process. At first we deal with the complete space integration, and get the theoretical results of Eq.(3). Then we add the cut condition which is the same as in BES[3] into the computer program and get results for the incomplete phase space .

With a straightforward calculation, the new equations are obtained:

$$
\begin{aligned}
T(000) \sim\ & 8.02A_{10}^2 + 9.14A_{11}^2 + 7.24A_{12}^2 + .43A_{10}A_{11} + 1.13A_{10}A_{12} - \\
& .44A_{11}A_{12} + 7.58B_{10}^2 + .55A_{10}B_{10}\gamma + \\
& .33A_{11}B_{10}\gamma - 1.10A_{12}B_{10}\gamma \\
T(200) \sim\ & -.415A_{10}^2 - 1.005A_{11}^2 + .007A_{12}^2 + .001A_{10}A_{11} - \\
& .105A_{10}A_{12} - .079A_{11}A_{12} - .150B_{10}^2 - .271A_{10}B_{10}\gamma + \\
& .043A_{11}B_{10}\gamma + .132A_{12}B_{10}\gamma \\
T(020) \sim\ & 1.27A_{10}^2 + .75A_{11}^2 - .96A_{12}^2 + .15A_{10}A_{11} - .32A_{10}A_{12} - \\
& .02A_{11}A_{12} + .10B_{10}^2 + 1.68A_{10}B_{10}\gamma + .05A_{11}B_{10}\gamma + \\
& .12A_{12}B_{10}\gamma \\
T(220) \sim\ & -.340A_{10}^2 - .183A_{11}^2 - .021A_{12}^2 + .013A_{10}A_{11} + .074A_{10}A_{12} \\
& -.116B_{10}^2 - .177A_{10}B_{10}\gamma - .022A_{12}B_{10}\gamma \\
T(221) \sim\ & .176A_{10}^2 + .174A_{11}^2 + .107A_{12}^2 + .115A_{10}A_{11} + .016A_{10}A_{12} - \\
& .223A_{11}A_{12} + .187B_{10}^2 + .105A_{10}B_{10}\gamma + .148A_{11}B_{10}\gamma +
\end{aligned}
$$

$$
\begin{aligned}
& .016A_{12}B_{10}\gamma \\
T(222) \sim\ & -.224A_{10}^2-.392A_{11}^2-.433A_{12}^2+.011A_{10}A_{11}-.267A_{10}A_{12}- \\
& .016A_{11}A_{12}-.322B_{10}^2+.171A_{10}B_{10}\gamma+.011A_{11}B_{10}\gamma+ \\
& .216A_{12}B_{10}\gamma \\
T(040) \sim\ & 1.141A_{10}^2-.737A_{11}^2+.135A_{12}^2+.011A_{10}A_{11}+.141A_{10}A_{12}+ \\
& .046A_{11}A_{12}+.018B_{10}^2+.093A_{10}B_{10}\gamma-.023A_{11}B_{10}\gamma \\
T(240) \sim\ & -.442A_{10}^2+.334A_{11}^2+.028A_{12}^2-.014A_{10}A_{11}-.073A_{10}A_{12}+ \\
& .037A_{11}A_{12}-.073B_{10}^2-.218A_{10}B_{10}\gamma-.023A_{11}B_{10}\gamma- \\
& .010A_{12}B_{10}\gamma \\
T(241) \sim\ & .155A_{10}^2+.074A_{11}^2-.036A_{12}^2+.166A_{10}A_{11}+.017A_{10}A_{12}+ \\
& .051A_{11}A_{12}+.041B_{10}^2+.130A_{10}B_{10}\gamma \\
T(242) \sim\ & -.314A_{11}^2+.076A_{12}^2+.031A_{10}A_{11}+.187A_{10}A_{12}- \\
& .019A_{11}A_{12}-.047B_{10}^2-.154A_{10}B_{10}\gamma+.017A_{11}B_{10}\gamma. \qquad (4)
\end{aligned}
$$

One can compare this new formulation (4) with eq.(3) and notice that the change is not negligible. Thus we can expect that considering the effects of incomplete phase space, the analysis based on the old formulation which assumed a complete phase space might lead to a conclusion about the fractions of 0^{++} and 2^{++} in $f_J(1710)$ notably deviating from real values. The BES moment data[3] should be re-analyzed based on the new formulation.

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