UGNT and PPGEN

Discribing Phase Space and Angular
Distribution in J/ψ and ψ' Multiple Cascade Decay Processes

ZHANG Dahua

Institute of High Energy Physics, Beijing 100039, China

On the 9th BES Annual Meeting.
June 6-8, 2000, Dalian

1. **Introduction**
   (a) More Accurate Event Selection Efficiency
   (b) Partial Wave Analysis (PWA)
   (c) Other Usage
2. **Helicity Amplitude**
3. **Some comments and discussion**
   (a) The full amplitude quantum correlation
   (b) The Lorentz boost

1 Introduction

Generator UGNT has been resumed in BES software library after near ten years interruption in order to meet the challenge of fifty million J/ψ data analysis. From this generator, the events with correct phase space and angular distribution in any cascade decay final states can be generated depending only on the assignments of particle's spins, parities as well as helicity amplitudes involving in the decay chain without any further calculations. Making such a generator is necessary and useful to get more accurate measurement results in J/ψ physics analysis.

- **More accurate event selection efficiency**

  Sometimes, the difference of efficiencies with and without correct angular distribution is rather large. For example, in

$$\begin{aligned} \psi' \to \pi^0 \quad & J/\psi \\ & \hookrightarrow \quad \mu^+\mu^-(\gamma) \end{aligned} \qquad (1)$$

  with generator HOWL

$$\epsilon_{howl} = 5.92\%$$

  (no radiation correction),
  with UGNT

$$\epsilon_{ugnt} = 5.89\%, \qquad (2)$$

  but with PPGNT

$$\epsilon_{ppgen} = 4.14\%,$$

  shifted near 30%, with the same cut condation. The last number, the minimal one, 4.14%, is partly caused by some mis-sampling procedure in PPGEN.

- **In Partial Wave Analysis (PWA)**

  The Monte Carlo integration of parameter and detector efficience dependent normalization constant is an important quantity:

$$\begin{aligned} N(A^{(j)'s}, \epsilon) &= \int W(\phi_j, \theta_j, A^{(j)'s}) \times \epsilon(r, \theta, \phi,)d\Phi \\ &\equiv \frac{\Phi}{N_{MC}} \sum_{event=1}^{N_{MC}} W(\phi_j, \theta_j, A^{(j)'s}) \end{aligned} \qquad (3)$$

  where
  ϕ_j, θ_j — the polar and azimuthal angles of particles flying out from the jth vertex, in the vertex itself rest system;
  ϵ — the detector efficiency which is usually a function of detector configuration;
  dΦ — Phase space integration element;
  Φ— Phase space total volume, which is a constant depending on the final state particle number and their masses;
  N_{MC} — the total number of Monte Carlo events generated in use of the integration.

  A large source of the calculating error of N(a(j),ϵ) comes from the large rises and falls of the distribution function W, in phase space. A common way to reduce this kind of errors is to adopt the importance sampling methods(ISM), i.e. to make a special MC sample with an approximate angular distribution

$$W_0 = \mathrm{W}(\phi_j,\, \theta_j,\, A_0(\mathrm{j}))$$

closing to the expectation distribution to carry on the integration weighted by the ratio

$$W(\phi_j,\theta_j,\ A(j))/W_0;$$

i.e.

$$N(A^{(j)'s},\epsilon) = \frac{\Phi}{N_{MC}} \sum_{W0\ sampling\ event=1}^{N_{MC}} \times W(\phi_j,\theta_j,A^{(j)\prime s})/W_0. \quad (4)$$

With the **iteration method**, e.g. using the first fitted function as next approximate distribution W_0 to fit again and then repeat if necessary, one could get better analysis results. With the help of generator UGNT, the ISM can be used conveniently in PWA.

- **Other Usage**

  UGNT can also act as a tool to help people to understand decay angular distributions, spin-parity angular correlations in study and to help people to check his/her calculations of the angular distribution in various decay process to carry on PWA fitting or other works.

2 Helicity Amplitute

In the spin-parity analysis, there are several ways, such as the partial wave analysis, the covariant tensor forms, the multipole expansion methods and the helicity amplitude formulas. Because of the clear writing rules of the helicity amplitude formula for fortran language, we use it in calculating routine of UGNT.

The key step in the simulation is to calculate the angular distribution function in the routine:

$$W(\phi_j,\quad \theta_j\quad ,A^{(j)\prime S}) = \sum_{M'=\pm1,m_i|ive(i)=0} |\sum_{m_k|ive(k)\neq 0} \prod_j A^{(j)}_{m_{i_{(j,1)}},m_{i_{(j,2)}}} D^{J_j}_{m_{in}(j),m_{i(j,1)}-m_{i(j,2)}}(\phi_j,\theta_j,0)|^2, \quad (5)$$

where

M' – the ψ' magnetic quantum number,

j – the vertex number,

i, k, i(j,1), i(j,2), in(j)– the track's number,

i(j,1), i(j,2) – the two tracks number off jth vertex,

in(j) – the track number into jth vertex,

ive(i) – the vertex number of the ith track goes in, if 0, means ith track is the final particle.

The remained question is the sum range of m_i's . Of course, firstly,

$$-J_i \leq m_i \leq J_i,$$
$$-J_i \leq |m_{i(j,1)} - m_{i(j,2)}| \leq J_i;$$

and from parity conservation for any decay vertex.

$$A^{(j)}_{-m_1 -m_2} = (-1)^{J_{(j)} - J_1 - J_2} \times \eta_1 \eta_2 \eta_j A^{(j)*}_{m_1 m_2}.$$

From Bose-Einstein Statistic, if particle 1 and particle 2 are the identical particles, then we have

$$A^{(j)}_{m_1 m_2} = A^{(j)}_{m_2 m_1}.$$

Sampling with the phase space generating package SAGE in BES library together, we can generate all the events with required phase space and angular distributions just by inputting all particle's masses, spins, parities and the decay chain's topology defined by SOBCARDs.

3 Some comments and discussion

- **The full amplitude quantum coorelation**

  As we see, in Eq. 5) the amplitude norm square,used in generator UGNT is taken sum over all mediate states, i.e. all the coherent effects are taken into account. In any cases the generator can give correct correlations of particle's spin-parities in the angular distribution. For example, in

$$\begin{array}{ll} \psi' \to \pi^+\pi^- & J/\psi \\ & \hookrightarrow \quad l^+ l^- (\gamma) \end{array} \tag{6}$$

  which is mainly an E1 E1 dipole transition between two $c\bar{c}$ states, in the first vertex

$$\psi' \to S^0(\pi^+\pi^-) \quad J/\psi \quad ,$$

  the helicity amplitudes will be

$$\begin{aligned} A(1,0) &= \sqrt{1/3}, \\ A(0,0) &= -\sqrt{1/3}, \\ A(-1,0) &= \sqrt{1/3} \end{aligned}$$

  as well known from Clebsc-Gordan coefficients. This assignment of polarization A's allows the initial polarizition of ψ' transfer completely to J/ψ system. So the lepton's angular distribution with respect to the beam direction should be almost the same as that in the case ψ' decay to lepton pair directly, as

$$1 + \cos^2(\theta)$$

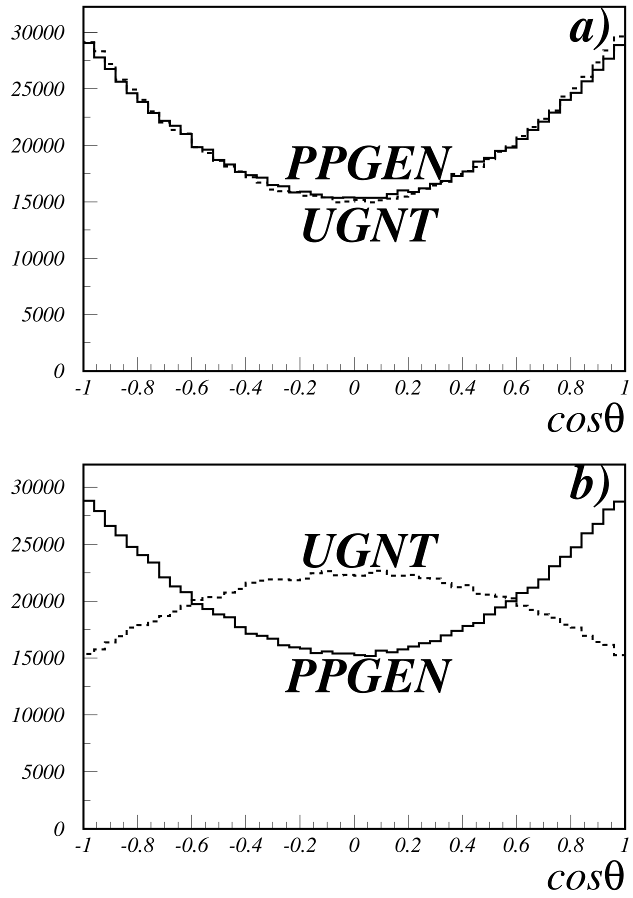

Figure 1: The μ^+ angular distributions in J/ψ rest frame to beam dirrection. a) for $\psi' \to \pi^+\pi^- J/\psi \to \pi^+\pi^-\mu^+\mu^-(\gamma)$ and b) for $\psi' \to \pi^0 J/\psi \to \pi^0\mu^+\mu^-(\gamma)$, solid line PPGEN and dashed line UGNT.

shown in Fig. 1 a).

But, in the M1 M1 dipole transition Eq. 1), for J/ψ there is no longitudinal polarization component, the initial polarization of ψ' would undergo great distortion and the l^+l^- direction distribution to beam axis will be greatly different from the 1+cos$^2(\theta)$, as show in Fig. 1 b) dashed line. But in PPGEN, because there is no any correlation between J/ψ and emission particles (π^+ π^- sys. or π^0) the l^+l^- distribution in the π^0 emission is still the same as that of π^+ π^- emission, see Fig. 1 b) solid line. This is why the efficience of ψ' to π^0 J/ψ obtained from PPGEN in Eq. 2) lost so much.

- **The Lorentz boost**

  As we know, All the helicity angles, θ's and ϕ's in the D functions in Eq. 5) are Lorentz invariant, i.e. they can all be represented by Lorentz invariants such as masses, 4 dot-products of momenta etc. All momenta concerned in our system are produced from those angles and Lorentz invariant momenta in CM system. So after we rotate all those momenta to beam coordinate of lab system, we have to boost them from their own CM system to lab. system carefully. Because the 3-vector parameter space of Lorentz transformation are not the common 3-dimension Euclidean space, two successive Lorentz boost generally are not identical to a certain(pure) Lorentz boost. Additional rotation would be caused by this operation and from the same initial system to the same final system, different operation routines would lead to differrent rotations, which will certainly affect our final momentum configuration. To check if our procedures are correct or not we can calculate the θ's using the momenta in lab. system and masses of relative particles and compare them with the original θ's used in generator. For example in cascade process

$$\begin{array}{lll} & \psi' \to \pi^0 & J/\psi \\ & & J/\psi \;\; \to \rho^0 \pi 0 \\ \rho^0 \to \pi^+\pi^- & & \end{array}$$

the helicity angle θ of $\pi^+\pi^-$ in ρ^0 rest frame has

$$cos\theta = \frac{[(m_{J/\psi}^2 - m_\rho^2 - m_{\pi^0}^2)(m_\rho^2 + m_{\pi^-}^2 - m_{\pi^+}^2) - 4m_\rho^2(p_- \cdot p_0)]}{\lambda(m_\rho, m_{\pi^+}, m_{\pi^-})\lambda(m_{J/\psi}, m_\rho, m_{\pi^0})} \tag{7}$$

where

$$\lambda(m_1, m_2, m_3) = \sqrt{m_1^4 + m_2^4 + m_3^4 - 2m_1^2m_2^2 - 2m_2^2m_3^2 - 2m_3^2m_1^2}$$

Fig.2 shows the diffrences between $cos(\theta)_{calcul}$ and $cos(\theta)_{original}$ in both generators UGNT and PPGEN. We can see, the Lorentz boosts in PPGEN might not be correct. In UGNT, the boost way is step by step boosted back from daughter system to her parents system then to grandpa system, finally up to lab. system. Only this way can guarantee those θ's and ϕ's unchange and so guarantee the Lorentz invariant of the whole treatment. This way is quite different from the conventional treatment in the HEP experimets and also that in PPGEN. The conventional way is to boost the oldest generation to lab. system firstly, then youngers can be boosted directly back to lab. system once after her parents back to lab. Those differences of Lorentz boost may not greatly affect the determination of detector efficiences but the erroneous information about the helicity angles will certainly impact the fitting works in physics analysis.

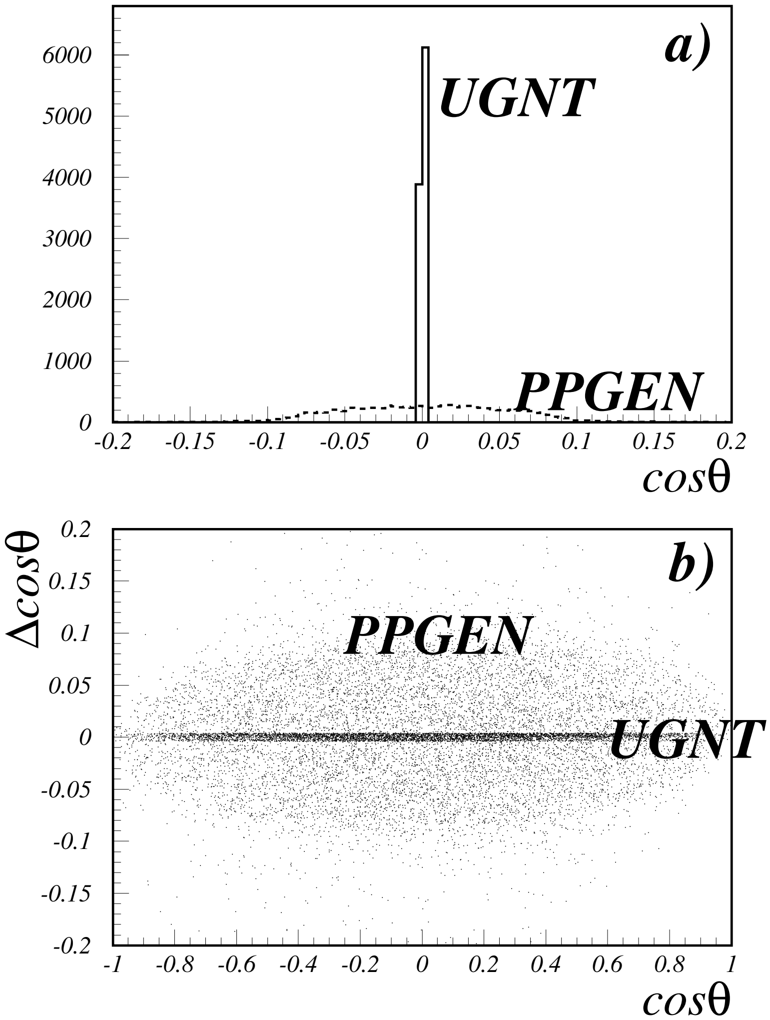

Figure 2: The differences of helicity angles θ's of $\pi^+\pi^-$ in ρ rest frame between original generator used and calculated from prodused momenta by the two generators for $\psi' \to \pi^0 J/\psi \to \pi^0\pi^0\rho^0 \to \pi^0\pi^0\pi^+\pi^-$, in a) solid line UGNT and dashed line PPGEN.