$K^*(892)$ Electromagnetic Mass Anomaly[1]

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Abstract

Comparing the experimental data of $K^{*0}(892)-$mass splitting with the theoretical prediction to the splitting caused by $u-d$ quark mass difference, we found out that the EM-mass of neutral K^* is larger than one of charged K^*. This is unusal, and was called as EM-mass anomaly of K^*. Employing the $U(3)_L \times U(3)_R$ chiral theory of mesons, we derived this anomaly effect explicitly. The studies are hadron dynamics dependent. It is meaningful and helpful to improve the experimental measurment of the mass splitting of $K^*(892)$ for solving this EM-mass anomaly puzzle.

1 Introduction

In the Review of Particle Data Group[1], two measurements of the mass differences between neutral $K^{*0}(892)$ ($\bar{s}\gamma_\mu d$ or $\bar{d}\gamma_\mu s$) and charged $K^{*\pm}(892)$ ($\bar{s}\gamma_\mu u$ or $\bar{u}\gamma_\mu s$) have been reported:

1. The masses of neutral and charged $K^*(892)$ are measured individually:

$$m_{K^{*0}} = 896.10 \pm 0.28; \quad m_{K^{*\pm}} = 891.66 \pm 0.26.$$

Then, the mass splitting of K^* is

$$(m_{K^{*0}} - m_{K^{*\pm}})_{\text{expt}(1)} = 4.54 \pm 0.54 MeV. \tag{1}$$

2. Another datum on this mass splitting is due to derict measurement of it. The result is

$$(m_{K^{*0}} - m_{K^{*\pm}})_{\text{expt}(2)} = 6.7 \pm 1.2 MeV. \tag{2}$$

Note that both of results are larger than $(m(K^0) - m(K^\pm)) \simeq 4MeV$.

The mass splitting of K^* are caused both by the $u-$ and $d-$quark mass difference and by the electromagnetic (EM-) mass difference between neutral and charged $K^*(892)$. Namely, we have

$$(m_{K^{*0}} - m_{K^{*\pm}})_{\text{expt}} = (m_{K^{*0}} - m_{K^{*\pm}})_{\text{Q}} + (m_{K^{*0}} - m_{K^{*\pm}})_{\text{EM}} \tag{3}$$

where $(m_{K^{*0}} - m_{K^{*\pm}})_{\text{Q}}$ denotes the contribution due to $u-$ and $d-$quark mass difference, and $(m_{K^{*0}} - m_{K^{*\pm}})_{\text{EM}}$, the EM-mass difference between K^{*0} and $K^{*\pm}$. In ref.[2], the mass splitting caused by $u-$ and $d-$quark mass difference has been derived in the formalism of the chiral quark model[3][4], which is

$$(m_{K^{*0}} - m_{K^{*\pm}})_{\text{Q}} = 3.07 MeV. \tag{4}$$

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From eqs.(1)−(4), we can see that

$$(m_{K^{*0}} - m_{K^{*\pm}})_{\mathrm{EM}} \simeq 0.93MeV \text{ to } 4.83MeV, \tag{5}$$

and therefore

$$(m_{K^{*0}})_{\mathrm{EM}} > (m_{K^{*\pm}})_{\mathrm{EM}}. \tag{6}$$

This result is surprising and unusual because it is contrary to the common knowledge of EM-masses for hadrons, that the neutral particle's self EM-energe should be smaller than the charged one, such as $(m_{\mathrm{neutron}})_{\mathrm{EM}} < (m_{\mathrm{proton}})_{\mathrm{EM}}$[5], $(m_{\pi^0})_{\mathrm{EM}} < (m_{\pi^\pm})_{\mathrm{EM}}$[6][7], $(m_{K^0})_{\mathrm{EM}} < (m_{K^\pm})_{\mathrm{EM}}$[7][8] and $(m_{a_1^0})_{\mathrm{EM}} < (m_{a_1^\pm})_{\mathrm{EM}}$[7] and so on. In ref[9], therefore, the eq.(6) has been called as the EM-mass anomaly of $K^*(892)$. In this talk, I try to present a brief review on this subject in order to urge further experimental studies on the mass splitting of K^*. Actully, since 1978[10], there is no significant effort paid to measure this splitting derictly in experiments. Lacking of corresponding theoretical discussions may be one of the resons why the experimenst did not pay enough attention to this problem. I shall address that the physics hidden in the mass splitting of $K^*(892)$ is interesting, and to improve the measurement of the splitting is worthy and meaningful. In particular, it is practical to do so by studying the processes of $J/\Psi \longrightarrow \bar{K}K^*$ and $K^* \longrightarrow K\pi$ in BES on BEPC.

2 Chiral quark model and U(3)$_L$×U(3)$_R$ chiral theory of mesons

The chiral quark model, originated by Weinberg, and then developed by Manohar and Georgi[3], provides a QCD-inspired description on the simple constituent quark model. In the view of Manohar-Georgi model, between the chiral symmetry breaking scale($\Lambda_{\chi SB} \sim 1-2$GeV) and the confinement scale ($\Lambda_{QCD} \sim 0.1-0.3$GeV), the dynamical field freedom are constituent quarks(quasi-particle of quarks), gluons and Goldstone bosons associated with chiral symmetry spontaneously breaking. In this quasiparticle description, the effective gluon coupling is small and interactions between quarks and Goldstone bosons is important. The external gauge fields(e.g., photon field) can be introduced by localizing the global chiral symmetry. On the other hand, it is well known that in the electromagnetic interaction of mesons, the vector mesons play an essential role through VMD(Vector Meson Dominate)[11]. Therefore, it is quite natural to extend chiral quark model to include spin-1 meson resonances via VMD and via minimal coupling principle.

The U(3)$_L$×U(3)$_R$ chiral theory of mesons has been studied extensively[4]. The basic inputs for it are the pseudoscalar decay constants f_P, vector meson mass m_V and a universal coupling constant g. Predictions of this model are in good agreement with data[4]. In particular, in Ref.[12] it has been shown that the low energy limit of this theory is equivalent to the chiral perturbation theory, and the QCD constraints in Ref.[13] are satisfied by this model. Therefore, as an effective model of QCD, the U(3)$_L$×U(3)$_R$ chiral theory of mesons is reliable.

We begin at the simplest parametrization of chiral quark model is[3]

$$\mathcal{L}_\chi = \bar{\psi}(x)(i\gamma \cdot \partial - mu(x))\psi(x) \tag{7}$$

where

$$
\begin{aligned}
\psi &= (u,d,s)^T, \\
u(x) &= \frac{1}{2}(1-\gamma_5)U(x)+\frac{1}{2}(1+\gamma_5)U^\dagger, \\
U(x) &= \exp(2i\lambda_a\Phi_a/f_0),
\end{aligned}
$$

and λ_a are Gell-Mann matrices of SU(3), Φ_a are fields of pseudoscalar meson octet and $f_0 \simeq f_\pi = 186MeV$, m is a parameter related to the quark condensate.

This Lagrangian is invariant under global chiral symmetry transformation. The vector($\mathcal{V}_\mu$) and axial-vector($\mathcal{A}_\mu$) external field are introduced into $\mathcal{L}_\chi$ due to the requirment of local chiral symmetry, i.e., we can replace the derivative operator in Eq.(1) by covariant derivative operator with affine connection(or gauge potential) $\mathcal{V}_\mu + \mathcal{A}_\mu\gamma_5$ as follows:

$$
\partial_\mu \to \tilde{\nabla}_\mu = \partial_\mu - i(\mathcal{V}_\mu + \mathcal{A}_\mu\gamma_5). \tag{8}
$$

Then we obtain a theory describing strong and elecro-weak interactions of mesons.

Spin-1 meson resonances can be included via VMD, i.e., via substitution of a new affine connection $((\mathcal{V}+v)_\mu + (\mathcal{A}+a)_\mu\gamma_5)$ for the former one $\mathcal{V}_\mu + \mathcal{A}_\mu\gamma_5$ in Eq.(2),

$$
\tilde{\nabla}_\mu \to \tilde{\tilde{\nabla}}_\mu \equiv \partial_\mu - i((\mathcal{V}+v)_\mu + (\mathcal{A}+a)_\mu\gamma_5), \tag{9}
$$

with

$$
\begin{aligned}
a_\mu &= \tau_i a^i_\mu + \lambda_\alpha K^\alpha_{1\mu} + (\frac{2}{3}+\frac{1}{\sqrt{3}}\lambda_8)f_\mu + (\frac{1}{3}-\frac{1}{\sqrt{3}}\lambda_8)f_{s\mu}, \\
v_\mu &= \tau_i \rho^i_\mu + \lambda_\alpha K^{*\alpha}_\mu + (\frac{2}{3}+\frac{1}{\sqrt{3}}\lambda_8)\omega_\mu + (\frac{1}{3}-\frac{1}{\sqrt{3}}\lambda_8)\phi_\mu,
\end{aligned} \tag{10}
$$

where i=1, 2, 3 and α=4, 5, 6, 7.

Now, the author of Ref.[4] came to this extention and proposed a Lagrangian,

$$
\mathcal{L} = \bar{\psi}(x)(i\gamma\cdot(\partial - i((e_0QA+v)+a\gamma_5)) - mu(x))\psi(x) + \frac{1}{2}m_V^2(v_{\mu a}v^\mu_a + a_{\mu a}a^\mu_a), \tag{11}
$$

where A_μ is photon field, Q is the electric charge. The mass term of v and a is chiral gauge invariant because the v and a transform homogeneously under local $U(3)_L \times U(3)_R$ symmetry. Note that there are no kinetic terms in Eq.(5) for all meson fields, since they are treated as composited fields of quark fields instead of the fundamental fields. The kinetic terms for these fields will be generated via loop effects of quarks.

Following Ref. [4], the effective Lagrangian of mesons (indicated by $\mathcal{M}$) are obtained through integrating over the quark fields,

$$
\exp\{i\int d^4x\mathcal{L}^{\mathcal{M}}\} = \int[d\psi][d\bar{\psi}]\exp\{i\int d^4x\mathcal{L}\}. \tag{12}
$$

Using the dimensional regularization, and in the chiral limit, the effective Lagrangian $\mathcal{L}_{RE}$ (normal parity part) and $\mathcal{L}_{IM}$ (abnormal parity part) have been evaluated in Refs.[4]. The Lagrangian describing normal parity processes reads

$$\begin{aligned}
\mathcal{L}_{RE} &= \frac{F^2}{16}TrD_\mu UD^\mu U^\dagger - \frac{g^2}{16}Tr(L_{\mu\nu}L^{\mu\nu} + R_{\mu\nu}R^{\mu\nu}) \\
&+i\frac{N_C}{2(4\pi)^2}Tr(D_\mu UD_\nu U^\dagger L_{\nu\mu} + D_\mu^\dagger UD_\nu UR_{\nu\mu}) + \frac{N_C}{6(4\pi)^2}TrD_\mu D_\nu UD^\mu D^\nu U^\dagger \\
&-\frac{N_C}{12(4\pi)^2}Tr(D_\mu UD^\mu U^\dagger D_\nu UD^\nu U^\dagger + D_\mu U^\dagger D^\mu UD_\nu U^\dagger D^\nu U - D_\mu UD^\nu U^\dagger D_\mu UD^\nu U^\dagger) \\
&+\frac{1}{8}m_0^2 Tr(L_\mu L^\mu + R_\mu R^\mu), \qquad (13)
\end{aligned}$$

where

$$\begin{aligned}
L_\mu &= v_\mu - a_\mu \ , \quad R_\mu = v_\mu + a_\mu \ , \\
D_\mu U &= \bigtriangledown_\mu U - iL_\mu U + iUR_\mu, \\
D_\mu U^\dagger &= \bigtriangledown_\mu U^\dagger - iR_\mu U^\dagger + iU^\dagger L_\mu, \\
L_{\mu\nu} &= \bigtriangledown_\mu L_\nu - \bigtriangledown_\nu L_\mu - i[L_\mu, L_\nu] + e_0 QF_{\mu\nu}^L \ , \\
R_{\mu\nu} &= \bigtriangledown_\mu R_\nu - \bigtriangledown_\nu R_\mu - i[R_\mu, R_\nu] + e_0 QF_{\mu\nu}^R \ , \\
\bigtriangledown_\mu \Psi &= \partial_\mu \Psi + [-ie_0 QA_\mu, \Psi] \qquad \Psi = U, U^\dagger, L, R, \\
F_{\mu\nu}^L &= F_{\mu\nu}^R = \partial_\mu A_\nu - \partial_\nu A_\mu \ . \qquad (14)
\end{aligned}$$

Here an universal coupling constant g has been introduced to absorb the logarithmic divergence due to the integral of quark loop:

$$\frac{F^2}{16} = \frac{N_c}{(4\pi)^2}m^2\frac{D}{2}\Gamma(2-\frac{D}{2}), \qquad (15)$$

$$g^2 = \frac{8}{3}\frac{N_c}{(4\pi)^2}\frac{D}{4}\Gamma(2-\frac{D}{2}) = \frac{1}{6}\frac{F^2}{m^2}. \qquad (16)$$

In Lagrangian (13) the field $a_\mu(x)$ mixes with $\partial_\mu\Phi(x)$, which should be diagonalized conveniently via field redefinition,

$$\begin{aligned}
&L_\mu \to L_\mu + i\frac{c}{g}U \bigtriangledown_\mu U^\dagger \ , \\
&R_\mu \to R_\mu + i\frac{c}{g}U^\dagger \bigtriangledown_\mu U \ . \qquad (17)
\end{aligned}$$

Then the following equations are derived,

$$\frac{F^2}{f_\pi^2}(1-\frac{2c}{g}) = 1, \quad c = \frac{f_\pi^2}{2gm_\rho^2}, \quad \text{for two-flavor.} \qquad (18)$$

$$\frac{F^2}{f_K^2}(1-\frac{2c'}{g}) = 1, \quad c' = \frac{f_K^2}{2gm_{K^*}^2}, \quad \text{for three-flavor} \qquad (19)$$

It should be stressed that this field redefinition is different from the one in Ref.[4], $a_\mu \to a_\mu - \frac{c}{g}\partial_\mu\Phi$. Eq.(17) keeps the chiral symmetry explicitly. It plays an important role when we check the electromagnetic gauge invariance of the decay amplitude in following sections. Eq.(7)-Eq.(11) provide the formalism employed in this paper and all the calculations are performed in the chiral limit.

The quark mass term can be inserted into the chiral Lagrangian $\mathcal{L}$(eq.(11)) in standard way, i.e.,

$$\mathcal{L} \longrightarrow \mathcal{L} + \bar{\psi}(-M)\psi \tag{20}$$

where $M = diag(m_u, m_d, m_s)$.

3 u- and d-quark mass difference and $(m_{K^{*0}} - m_{K^{*\pm}})_{\rm Q}$

Starting from eqs. (11)(20), in which the quark mass term are included, and using the similar procedure presented in Ref. [4], we can get the effective lagrangian beyond the chiral limit. The nonzero quark masses will yield other terms in addition to the effective lagrangian in the chiral limit. At the leading order in quark mass expansion, the masses of the pseudoscalar mesons will be no longer zero,

$$(m^2_{\pi^+})_Q = (m^2_{\pi^0})_Q = -\frac{2}{f^2_\pi}(m_u + m_d)\langle 0|\bar{\psi}\psi|0\rangle, \tag{21}$$

$$(m^2_{K^+})_Q = -\frac{2}{f^2_\pi}(m_u + m_s)\langle 0|\bar{\psi}\psi|0\rangle, \tag{22}$$

$$(m^2_{K^0})_Q = -\frac{2}{f^2_\pi}(m_d + m_s)\langle 0|\bar{\psi}\psi|0\rangle, \tag{23}$$

$$(m^2_{\eta})_Q = -\frac{2}{3f^2_\pi}(m_u + m_d + 4m_s)\langle 0|\bar{\psi}\psi|0\rangle, \tag{24}$$

where $\langle 0|\bar{\psi}\psi|0\rangle$ is the quark condensate of the light flavors [4, 7]. Note that

$$(m^2_{K^\pm} - m^2_{K^0})_{\rm expt} = (m^2_{K^\pm} - m^2_{K^0})_{\rm Q} + (m^2_{K^\pm} - m^2_{K^0})_{\rm EM}.$$

and $(m^2_{K^\pm} - m^2_{K^0})_{\rm EM}$ has been calculated in ref[7] in this formalism. The result is

$$(m^2_{K^\pm} - m^2_{K^0})_{\rm EM} = 2m_K \times 2.5MeV.$$

Then, using the experimental value of mass difference between K^+ and K^0 [1] together with eqs. (21)-(23), we can get

$$\frac{m_u + m_d}{m_u - m_d} = \frac{m^2_\pi}{(m^2_{K^+} - m^2_{K^0})_{\rm expt} - (m^2_{K^+} - m^2_{K^0})_{\rm EM}} = 2.83. \tag{25}$$

Now let us to derive $m_u - m_d$. For this purpose, we consider $\rho^0 - \omega$ mixing in the $U(3)_L \times U(3)_R$ chiral theory of mesons.

For the vector mesons, at the leading order in quark mass expansion, there are no the explicit contributions of their masses. Consequently, there is no such $\rho^0 - \omega$ mixing which is independent of the momentum (see below). However, an additional kinetic term of the vector mesons will appear in the effective lagrangian. Therefore, to the order of m_q, all the kinetic terms of the vector mesons induced from eqs.(11),(20) and (12) are as follows,

$$\mathcal{L}^V_{kin} = -\frac{1}{8}g^2 Tr v_{\mu\nu}v^{\mu\nu} + \frac{2N_c}{3(4\pi)^2 m} Tr\, M\, v_{\mu\nu}v^{\mu\nu} + \text{higher order terms.} \tag{26}$$

The second term in eq.(26) contains the $\rho^0 - \omega$ mixing due to isospin symmetry breaking, which can be easily derived

$$\mathcal{L}_{\rho\omega} = \frac{1}{4\pi^2}\frac{m_u - m_d}{m}\rho^0_{\mu\nu}\omega^{\mu\nu}. \tag{27}$$

However, the vector meson fields in eq.(26) is not physical fields, therefore we should make the kinetic terms of the vector mesons fields in the standard form by redefining these fields. For ρ-mesons, from eq.(26), we have

$$\mathcal{L}^\rho_{kin} = -\frac{1}{4}g^2(1 - \frac{1}{2\pi^2 g^2}\frac{m_u + m_d}{m})\rho^i_{\mu\nu}\rho^{i\mu\nu}. \tag{28}$$

The physical ρ-mesons field should be defined as

$$\rho_\mu \longrightarrow \frac{1}{g\sqrt{1 - \frac{1}{2\pi^2 g^2}\frac{m_u+m_d}{m}}}\rho_\mu.$$

Then, the physical mass of ρ mesons is

$$m^2_\rho = \frac{m^2_1}{g^2(1 - \frac{1}{2\pi^2 g^2}\frac{m_u+m_d}{m})} = \frac{m^2_V}{1 - \frac{1}{2\pi^2 g^2}\frac{m_u+m_d}{m}}. \tag{29}$$

Expanding eq.(29) to the first order of m_q, we obtain

$$m_\rho = m_V(1 + \frac{1}{4\pi^2 g^2}\frac{m_u + m_d}{m}), \tag{30}$$

here $m_V = \frac{m_1}{g}$ is the vector meson masses in the chiral limit. Likewise, for ω, K^*, and ϕ mesons, we can get

$$m_\omega = m_V(1 + \frac{1}{4\pi^2 g^2}\frac{m_u + m_d}{m}), \tag{31}$$

$$m_{K^{*\pm}} = m_V(1 + \frac{1}{4\pi^2 g^2}\frac{m_u + m_s}{m}), \tag{32}$$

$$m_{K^{*0}} = m_V(1 + \frac{1}{4\pi^2 g^2}\frac{m_d + m_s}{m}), \tag{33}$$

$$m_\phi = m_V(1 + \frac{1}{2\pi^2 g^2}\frac{m_s}{m}). \tag{34}$$

where m_V is the vector mesons masses in the chiral limit. The mass splittings of the vector mesons from the quark mass effect are reduced [eqs. (30)-(34)] although no explicit vector meson mass terms generated from the leading order quark mass expansion appear in the effective lagrangian. Due to the definition of the physical vector mesons, the $\rho^0 - \omega$ mixing of eq.(27) should be translated into

$$\mathcal{L}_{\rho^0\omega} = \frac{1}{4\pi^2 g^2}\frac{m_u - m_d}{m}\rho^0_{\mu\nu}\omega^{\mu\nu}. \tag{35}$$

The higher order terms in quark mass expansion have been ignored in the above equation.

Eq.(35) represents the $\rho^0-\omega$ mixing induced by the isospin symmetry breaking caused by $u-d$ quark mass difference. Now, we try to get the contribution of $\rho^0-\omega$ mixing from electromagnetic interactions. The photon field has been introduced in eq. (11). Following Refs. [4], the direct couplings of neutral vector meson fields (ρ^0, ω, and ϕ) and the photon fields read

$$\mathcal{L}_{\rho\gamma} = -\frac{1}{2}\frac{e}{f_\rho}\rho^0_{\mu\nu}(\partial^\mu A^\nu - \partial^\nu A^\mu), \tag{36}$$

$$\mathcal{L}_{\omega\gamma} = -\frac{1}{2}\frac{e}{f_\omega}\omega_{\mu\nu}(\partial^\mu A^\nu - \partial^\nu A^\mu), \tag{37}$$

$$\mathcal{L}_{\phi\gamma} = -\frac{1}{2}\frac{e}{f_\phi}\phi_{\mu\nu}(\partial^\mu A^\nu - \partial^\nu A^\mu), \tag{38}$$

where

$$\frac{1}{f_\rho} = \frac{1}{2}g, \quad \frac{1}{f_\omega} = \frac{1}{6}g, \quad \frac{1}{f_\phi} = -\frac{1}{3\sqrt{2}}g. \tag{39}$$

Eqs. (36) and (37) will lead to $\rho^0-\omega$ mixing at the order of $\alpha_{\rm EM}$ through the transition process $\rho \to \gamma \to \omega$, which is

$$\mathcal{L}_{\rho\omega} = \frac{1}{24}e^2 g^2 \rho^0_{\mu\nu}\omega^{\mu\nu}. \tag{40}$$

From eqs.(35) and (40), the total $\rho^0-\omega$ mixing is

$$\mathcal{L}_{\rho\omega} = \frac{1}{4\pi^2 g^2}\frac{m_u - m_d}{m}\rho^0_{\mu\nu}\omega^{\mu\nu} + \frac{1}{24}e^2 g^2 \rho^0_{\mu\nu}\omega^{\mu\nu}. \tag{41}$$

Note that the vector mesons mass terms do not lead to any $\rho^0-\omega$ mixing, therefore, the mixing $\mathcal{L}_{\rho\omega}$ in the present theory comes entirely from the quark loop. In the standard way, the two-point Green function associated with $\rho^0-\omega$ mixing is

$$\begin{aligned}\Pi^{\rho\omega}_{\mu\nu}(q^2) &= i\int d^4x\, e^{iqx}\langle 0|T\rho_\mu(x)\omega_\nu(0)\exp\{i\int d^4y\,\mathcal{L}_{\rho\omega}(y)\}|0\rangle, \\ &= (g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})\frac{\theta^{\rho\omega}(q^2)}{(q^2-m_\rho^2)(q^2-m_\omega^2)},\end{aligned} \tag{42}$$

where

$$\theta^{\rho\omega}(q^2) = 2q^2(\frac{m_u-m_d}{4\pi^2 g^2 m} + \frac{1}{24}e^2 g^2). \tag{43}$$

$\Pi^{\rho\omega}_{\mu\nu}(q^2)$ is of transversal structure, which is due to the the conservation of the vector currents. From eq.(43), the off-shell $\rho^0-\omega$ mixing amplitude $\theta^{\rho\omega}(q^2)$ is obviously momentum dependent, and vanishes at $q^2=0$. This is consistent with the argument by O'Connell *et al.* in Ref. [14] that this mixing amplitude must vanish at the transition from time-like to space-like four-momentum within a broad class of models.

Now, we consider the on-shell $\rho^0-\omega$ mixing amplitude. From eq. (43), we have

$$\theta^{\rho\omega}(m_\rho^2) = 2m_\rho^2(\frac{m_u-m_d}{4\pi^2 g^2 m} + \frac{1}{24}e^2 g^2). \tag{44}$$

The mass difference of ρ and ω mesons has been ignored here. Note that the universal coupling constant g has been fixed in this theory [4, 7], and m can be determined from eqs.(16) and (18). Therefore, the electromagnetic contribution of the on-shell mixing amplitude [the second term in eq.(44)] is calculable. By taking g=0.39, f_π=186 MeV, and m_ρ=768.5 MeV, $\theta^{\rho\omega}(m_\rho^2)$ from the transition $\rho \to \gamma \to \omega$ is about 0.686×10^{-3} GeV^2. The $\rho^0 - \omega$ mixing leads to the G-parity forbidden decay of the ω meson, $\omega \to \rho^0 \to \pi^+\pi^-$. By using the experimental value of the decay width $\Gamma(\omega \to \pi^+\pi^-)$, one can get the total on-shell $\rho^0 - \omega$ mixing amplitude [17]. Therefore, by subtracting the electromagnetic contribution from the value extracted from this decay, the full contribution of the mixing amplitude due to isospin symmetry breaking will be obtained. This could provide an important constraint on the u-d quark mass difference.

Following Refs. [18, 17, 19], the decay width of the process $\omega \to \rho^0 \to \pi^+\pi^-$ is expressed as

$$\Gamma(\omega \to \pi^+\pi^-) = \left|\frac{\theta^{\rho\omega}(m_\rho^2)}{m_\omega^2 - m_\rho^2 - i(m_\omega\Gamma_\omega - m_\rho\Gamma_\rho)}\right|^2 \Gamma(\rho \to \pi^+\pi^-), \tag{45}$$

here Γ_ρ and Γ_ω are the widths of ρ and ω mesons respectively, and the decay width of $\rho \to \pi^+\pi^-$ has been calculated in Ref. [4]

$$\begin{aligned}\Gamma(\rho \to \pi^+\pi^-) &= \frac{f_{\rho\pi\pi}^2}{48\pi} m_\rho (1 - \frac{4m_\pi^2}{m_\rho^2})^{\frac{3}{2}}, \\ f_{\rho\pi\pi} &= \frac{2}{g}\{1 + \frac{m_\rho^2}{2\pi^2 f_\pi^2}[(1 - \frac{2c}{g})^2 - 4\pi^2 c^2]\}.\end{aligned} \tag{46}$$

Using the experimental data $B(\omega \to \pi^+\pi^-) = 2.21 \pm 0.30\%$ together with eqs.(45) and (46), we get

$$\theta^{\rho\omega}(m_\rho^2) = -(4.21 \pm 0.28) \times 10^{-3}\ \mathrm{GeV}^2, \tag{47}$$

which agrees well with the value $-(4.52 \pm 0.60) \times 10^{-3}$ GeV^2 obtained by Coon and Barrett [17]. The error bar in eq.(47)is from the uncertainty in the branch ratio of the process $\omega \to \pi^+\pi^-$, and we have neglected the mass difference between m_ρ and m_ω and the width Γ_ω in the denominator of eq.(45). The sign of the on-shell mixing amplitude has been discussed in Ref. [20], and it is determined from the relative phase of the ω to the ρ amplitude in the reaction $e^+e^- \to \pi^+\pi^-$ near m_ρ and m_ω.

More recently, the $\rho^0 - \omega$ mixing is investigated based on the analysis of $e^+e^- \to \pi^+\pi^-$ in Refs. [15, 16]. Particularly, in Ref. [16], the magnitude, phase and the s dependence of the mixing have been determined from the pion form-factor in the timelike region, and the smaller absolute value of the on-shell mixing amplitude (compared with the value by Coon and Barrett [17]) has been obtained. As pointed out by O'Connell *et al.* in Ref. [21], the value of the mixing amplitude given in Ref. [17] is not the value which provides the optimal fit to the pion electromagnetic form-factor.

Combining eq.(47) and eq.(44), we can obtain the u-d quark mass difference

$$m_d - m_u = 6.14 \pm 0.36\ \mathrm{MeV}. \tag{48}$$

Using eqs.(25) and (48), we have

$$m_u + m_d = 17.40 \pm 1.02 \text{ MeV},$$
$$m_u = 5.64 \pm 0.32 \text{ MeV}, \quad m_d = 11.76 \pm 0.70 \text{ MeV}.$$

Then, from eq.(30), we get the vector meson mass in the chiral limit m_V is 759.5$\pm$1.0 MeV. In fact, the value of $2\pi^2 g^2 m$ is about 745 MeV, which is not far from m_V. So it is a good approximation to set $m_V = 2\pi^2 g^2 m$. Thus, eqs. (30)-(34) are simplified as follows

$$\begin{aligned}
m_\rho &= m_\omega = m_V + \frac{m_u + m_d}{2}, \\
m_{K^{*\pm}} &= m_V + \frac{m_u + m_s}{2}, \\
m_{K^{*0}} &= m_V + \frac{m_d + m_s}{2}, \\
m_\phi &= m_V + m_s.
\end{aligned}$$

Finally, we get the desired result

$$(m_{K^{*0}} - m_{K^{*\pm}})_{\mathrm{Q}} = \frac{1}{2}(m_d - m_u) = 3.07 \pm 0.18 MeV. \tag{49}$$

where eq (48) has been used. This is just eq.(4), and indicates the unusual result of eq.(6) holds.

4 Electromagnetic mass difference between $K^{*0}(892)$ and $K^{*\pm}(892)$

In the previous section, the deriving of the significant result of $(m_{k^{*0}})_{EM} > (m_{k^{*\pm}})_{EM}$ is based on the comparing the theoretical estimate of $(m_{K^{*0}} - m_{K^{*\pm}})_{\mathrm{Q}}$ to experimental data of K^*-mass splitting. It is not a direct dynamical calculation to the EM-mass splitting of K^*. Such a calculation is necessary and meaningfull because it will reveal the dynamical reasons why eq.(6) holds. In this section, we calculate EM-mass splitting of K^* in the $U(3)_L \times U(3)_R$ chiral theory of mesons[7, 9].

It is nontrivial to evaluate the energies of virtual photons around the low-lying mesons (or EM masses of mesons) in effective theories of low energy QCD. During the past three decades, many studies had been done on the EM masses of pseudoscalar mesons π and K[6, 7, 8], which are useful for determining the light quark mass parameters in QCD and remarkable to understand the dynamics of π and K-mesons. It is natural and necessary to extend the EM-mass studies to vector mesons. In refs.[7, 9], the EM-masses of vector-mesons have been calculated in the $U(3)_L \times U(3)_R$ chiral theory of mesons.

The vector and axial-vector mesons have long been thought as Massive Yang-Mills (MYM) particles[22]. In the $U(3)_L \times U(3)_R$ chiral theory of mesons[4], the dynamics guiding vector and axial-vector mesons is derived from eqs.(11) and (12). It is found that this dynamics of vector mesons is of the MYM field theory with Gauge Wess-Zumino-Witten anomaly (GWZW)[23] (or Bardeen anomaly[24]) and of gauging nonlinear σ model. One of the attractive features of this theory is that the EM masses of all lowlying mesons (including vector's) can be calculated and the results are finite. The point is that the universal coupling constant g (see eq(16)) is related to a cutoff Λ in the theory. Therefore, it is legitimate to use g (or Λ) to factorize the logarithm divergence in the loop calculations (including photon loops) of this truncated field theory.

Generally, in order to get the virtual photon contributions to the masses of mesons, we use $\mathcal{L}_i(\Phi, \gamma, ...)|_{\Phi=\pi,K,K^*,...}$ to calculate the following S-matrix

$$S_\Phi = \langle\Phi|T\exp[i\int dx^4\mathcal{L}_i(\Phi,\gamma,...)] - 1|\Phi\rangle|_{\Phi=\pi,K,K^*,...}. \tag{50}$$

On the other hand S_Φ can also be expressed in terms of the effective lagrangian of Φ as

$$S_\Phi = \langle\Phi|i\int d^4x\mathcal{L}_{\text{eff}}(\Phi)|\Phi\rangle.$$

Noting $\mathcal{L} = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}m_\Phi^2\Phi^2$, then the electromagnetic interaction correction to the mass of Φ reads

$$\delta m_\Phi^2 = \frac{2iS_\Phi}{\langle\Phi|\Phi^2|\Phi\rangle}. \tag{51}$$

where $\langle\Phi|\Phi^2|\Phi\rangle = \langle\Phi|\int d^4x\Phi^2(x)|\Phi\rangle$.

We adopt dimensional regularization to do loop-calculations, and take the most generic linear gauge condition for electromagnetic fields to all diagram calculations in this letter. Namely, the A_μ-propagator with an arbitrary gauge constant a is taken to be

$$\begin{aligned}
\Delta_{F\ \mu\nu}^{(\gamma)}(x-y) &= \int\frac{d^4k}{(2\pi)^4}\Delta_{F\mu\nu}^{(\gamma)}(k)e^{-ik(x-y)},\\
\Delta_{F\ \mu\nu}^{(\gamma)}(k) &= \frac{-i}{k^2}[g_{\mu\nu} - (1-a)\frac{k_\mu k_\nu}{k^2}].
\end{aligned} \tag{52}$$

The calculations in this letter are of $O(\alpha_{em})$ and one-loop. The interactions between K^* and other mesons consist three parts: gauging non-linear σ model, MYM lagrangian and GWZW lagrangian[4]. There are two sorts of vertices which contribute to $(m_{K^{*0}}^2)_{EM} - (m_{K^{*+}}^2)_{EM}$: three points vertices and four points vertices. Due to VMD (vector meson dominance), the former has to be the coupling of K^* to a neutral vector meson plus other field, and the latter is the interaction of two K^* with two neutral vector-mesons. Here, the neutral vector mesons are ρ^0, ω and ϕ, and using following VMD substitutions

$$\rho_\mu^0 \longrightarrow \frac{e}{f_\rho}A_\mu, \quad \omega_\mu \longrightarrow \frac{e}{f_\omega}A_\mu, \quad \phi_\mu \longrightarrow \frac{e}{f_\phi}A_\mu \tag{53}$$

we get $\mathcal{L}(K^*, \gamma)$ which are usfull for calculations of EM masses of K^* (where f_ρ, f_ω, f_ϕ, to see eq.(39)). It is easy to be sure that all contributions to $(m_{K^{*0}}^2)_{EM} - (m_{K^{*+}}^2)_{EM}$ come from MYM- and GWZW- parts of K^*-lagrangian, and no contributions are from gauging nonlinear $\sigma-$model. This is a special feature for K^*-dynamics (Note that to π, K and a_1, the contributions to their EM masses coming from the gauging nonlinear $\sigma-$model do exist[7]). In the following, we derive $\mathcal{L}(K^*)_{MYM}$ and $\mathcal{L}(K^*)_{GWZW}$, and calculate their contributions to the EM masses of K^* respectively.

From eqs.(11)(12), MYM lagrangian related to K^* reads

$$\mathcal{L}_{MYM} = -\frac{g^2}{8}Tr(\partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu] - i[a_\mu, a_\nu])^2 + \text{mass terms of } v, \tag{54}$$

where $v_\mu = \tau_i\rho^i_\mu + \lambda_a K^{*a}_\mu + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)\omega_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)\phi_\mu$, and a_μ is axial vector field (notations of [4] are used). The three-point MYM vertex contrbuting to K^*'s EM masses and four-point one read from eq.(54)

$$\mathcal{L}^{(3)}_{MYM}(K^{*\pm}) = -\frac{i}{g}(\rho^0_{\mu\nu} + \omega_{\mu\nu} - \sqrt{2}\phi_{\mu\nu})K^{*+\mu}K^{*-\nu}$$
$$+\frac{i}{g}(\rho^0_\mu + \omega_\mu - \sqrt{2}\phi_\mu)(K^{*+\mu\nu}K^{*-}_\nu - K^{*-\mu\nu}K^{*+}_\nu), \tag{55}$$
$$\mathcal{L}^{(3)}_{MYM}(K^{*0}) = -\frac{i}{g}(-\rho^0_{\mu\nu} + \omega_{\mu\nu} - \sqrt{2}\phi_{\mu\nu})K^{*0\mu}\bar{K}^{*0\nu}$$
$$+\frac{i}{g}(-\rho^0_\mu + \omega_\mu - \sqrt{2}\phi_\mu)(K^{*0\mu\nu}\bar{K}^{*0}_\nu - \bar{K}^{*0\mu\nu}K^{*0}_\nu), \tag{56}$$
$$\mathcal{L}^{(4)}_{MYM}(K^{*\pm}) = -\frac{1}{g^2}(\omega^\mu - \sqrt{2}\phi^\mu)\{2\rho^0_\mu K^{*+\nu}K^{*-}_\nu$$
$$-\rho^{0\nu}(K^{*+}_\mu K^{*-}_\nu + K^{*+}_\nu K^{*-}_\mu)\}, \tag{57}$$
$$\mathcal{L}^{(4)}_{MYM}(K^{*0}) = \frac{1}{g^2}(\omega^\mu - \sqrt{2}\phi^\mu)\{2\rho^0_\mu K^{*0\nu}\bar{K}^{*0}_\nu$$
$$-\rho^{0\nu}(K^{*0}_\mu \bar{K}^{*0}_\nu + K^{*0}_\nu \bar{K}^{*0}_\mu)\}, \tag{58}$$

where $v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu$, $(v = \rho,\ \omega,\ \phi,\ K^*)$. The corresponding direct couplings of K^* to photon, $\mathcal{L}^{(3)}_{MYM}(K^{*\pm},\gamma), \mathcal{L}^{(4)}_{MYM}(K^{*\pm},\gamma), \mathcal{L}^{(4)}_{MYM}(K^*v\gamma)|_{v=\rho,\omega,\phi}$ and $\mathcal{L}^{(3)}_{MYM}(K^{*0},\gamma) = \mathcal{L}^{(4)}_{MYM}(K^{*0},\gamma) = 0$, can be obtained through the VMD substitutions eq.(53) respectively. Thus the S-matrix is

$$S = \langle K^*|T\{\exp i\int d^4x[\mathcal{L}_{MYM}(K^*) + \mathcal{L}_{MYM}(K^*\gamma) + \mathcal{L}_{\rho\gamma} + \mathcal{L}_{\omega\gamma} + \mathcal{L}_{\phi\gamma}] - 1\}|K^*\rangle \tag{59}$$

which can be calculated in the standard way.Then the EM-mass is given by formula of (51).

Firstly, let's check the gauge-independence of the S-matrix. The gauge-dependent part of the S-matrix derived from eqs.(52),(59) and $\mathcal{L}^{(3)}_{MYM}(K^{*\pm}\gamma)$ is

$$S^{(3)G}_{MYM} = \frac{3a}{4}e^2\langle K^{*+}K^{*-}\rangle\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}, \tag{60}$$

where $\langle K^{*+}K^{*-}\rangle = \int d^4x\langle K^{*+}|\frac{1}{2}(K^{*2}_1 + K^{*2}_2)|K^{*-}\rangle$. Similarly, the gauge-dependent part corresponding to $\mathcal{L}^{(4)}_{MYM}(K^{*\pm}\gamma)$ is

$$S^{(4)G}_{MYM} = \frac{-3a}{4}e^2\langle K^{*+}K^{*-}\rangle\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}, \tag{61}$$

From eqs.(60),(61), we have $S^{(3)G}_{MYM} + S^{(4)G}_{MYM} = 0$. Therefore the EM-mass calculations of MYM part is gauge-independent. Furthermore, when one takes the dimensional regularization, $S^{(3)G}_{MYM} = S^{(4)G}_{MYM} = 0$, then $S^{(3)}_{MYM}$ and $S^{(4)}_{MYM}$ become gauge-independent individually. To the GWZW anomaly part, since there is an antisymmetric tensor $\epsilon^{\mu\nu\lambda\rho}$ in the lagrangian (see below), the gauge-independency is obvious. So we conclude that our EM-mass calculations in this letter is gauge-independent.

The EM-mass square difference corresponding to the contributions of three-point MYM vertices theory can be calculated directly. The result is

$$\begin{aligned}(m^2_{EM}(K^{*0}) &- m^2_{EM}(K^{*\pm}))^{(3)}_{MYM} = \frac{-ie^2}{3}\int\frac{d^4k}{(2\pi)^4}m^2_\rho \\ &\times \frac{1}{-k^2(-k^2+2p\cdot k)(-k^2+m^2_\rho)} \\ &\times (\frac{m^2_\omega}{-k^2+m^2_\omega}+\frac{2m^2_\phi}{-k^2+m^2_\phi}) \\ &\times \{3k^2+4m^2_{K^*}-\frac{k^4}{m^2_{K^*}}-4\frac{(k\cdot p)^2}{k^2}+2k\cdot p \\ &+\frac{\langle(k\cdot K^{*+})(k\cdot K^{*-})\rangle}{\langle K^{*+}K^{*-}\rangle}(9+\frac{k^2}{m^2_{K^*}}-\frac{2k\cdot p}{k^2})\},\end{aligned} \tag{62}$$

where p is 4-momentum of K^*, i.e., $p^2 = m^2_{K^*}$. It is standard to carry the Feynman integration in eq.(62) through to the end. The logarithmic divergences in it are factorized by eq.(16). Then we have

$$(m^2_{EM}(K^{*0}) - m^2_{EM}(K^{*\pm}))^{(3)}_{MYM} = \Delta_\omega + \Delta_\phi \tag{63}$$

where

$$\begin{aligned}\Delta_\omega &= \frac{e^2m^2_\rho m^2_\omega}{12\pi^2}\{\frac{45m^2_{K^*}}{16m^2_\rho m^2_\omega}-\frac{49m^4_{K^*}(m^2_\rho+m^2_\omega)}{180m^4_\rho m^4_\omega} \\ &+ \int^1_0 dx\frac{1}{m^2_\omega-m^2_\rho}[\frac{m^2_{K^*}}{m^2_\omega}(Y_\omega(\frac{5}{2}x-\frac{7}{2})-1-2x^2+x^3)logY_\omega \\ &-\frac{m^2_{K^*}}{m^2_\rho}(Y_\rho(\frac{5}{2}x-\frac{7}{2})-1-2x^2+x^3)logY_\rho \\ &+\frac{m^4_{K^*}}{m^4_\omega}(\frac{1}{8}Y^2_\omega+\frac{1+x^2}{2}Y_\omega+x^2)logY_\omega \\ &-\frac{m^4_{K^*}}{m^4_\rho}(\frac{1}{8}Y^2_\rho+\frac{1+x^2}{2}Y_\rho+x^2)logY_\rho]\} \\ &- \frac{e^2m^2_\rho m^2_\omega}{m^2_{K^*}}(\frac{1}{32}g^2+\frac{1}{64\pi^2}log\frac{f^2_\pi}{6(g^2m^2_\rho-f^2_\pi)} \\ &+\frac{1}{48\pi^2}-\frac{1}{64\pi^2}\frac{m^2_\omega}{m^2_\omega-m^2_\rho}log\frac{m^2_\omega}{m^2_\rho}) \\ &+ \frac{e^2}{64\pi^2}\frac{m^2_\rho m^2_\omega}{m^2_\omega-m^2_\rho}log\frac{m^2_\omega}{m^2_\rho},\end{aligned} \tag{64}$$

$$\Delta_\phi = 2\Delta_\omega(m^2_\omega\to m^2_\phi), \tag{65}$$

where

$$Y_i = x^2+\frac{m^2_i}{m^2_{K^*}}(1-x),\ \ i=\rho,\ \omega,\ \phi.$$

Substituting the experimental values of $f_\pi(=0.186GeV)$, m_ρ, m_ω, m_ϕ and m_{K^*} into eqs.(63) (64) and (65), we get the numerical result

$$(m^2_{EM}(K^{*0}) - m^2_{EM}(K^{*\pm}))^{(3)}_{MYM} = 1.57\pm0.02\times10^{-3}GeV^2. \tag{66}$$

Similarly, the four-point MYM-vertex contributions to EM-mass difference of K^* can be calculated by using $\mathcal{L}^{(4)}_{MYM}(K^*), \mathcal{L}^{(4)}_{MYM}(K^*\gamma)$, $\mathcal{L}^{(4)}_{MYM}(K^*v\gamma)|_{v=\rho,\omega,\phi}, \mathcal{L}_{\rho\gamma}, \mathcal{L}_{\omega\gamma}$ and $\mathcal{L}_{\phi\gamma}$. It is straightforward to get the result as follows

$$\begin{aligned}&(m^2_{EM}(K^{*0}) - m^2_{EM}(K^{*\pm}))^{(4)}_{MYM} \\ &= \frac{3e^2}{64\pi^2}\{\frac{m^2_\rho m^2_\omega}{m^2_\omega - m^2_\rho} log\frac{m^2_\omega}{m^2_\rho} + 2\frac{m^2_\rho m^2_\phi}{m^2_\phi - m^2_\rho} log\frac{m^2_\phi}{m^2_\rho}\}.\end{aligned} \tag{67}$$

Numerically, we have

$$(m^2_{EM}(K^{*0}) - m^2_{EM}(K^{*\pm}))^{(4)}_{MYM} = 9.38 \times 10^{-4} GeV^2. \tag{68}$$

In the GWZW anomaly part of the $U(3)_L \times U(3)_R$ chiral theory of mesons there are vertices of $K^* - (\rho, \omega, \phi) - K$. So this part makes contributions to the EM-masses of K^*, which should be taken into account. The anomaly lagrangians for $K^{*\pm}$ and K^{*0} read respectively

$$\begin{aligned}\mathcal{L}_{GWZW}(K^{*\pm}) &= -\frac{N_c}{2\pi^2 g^2 f_K}\epsilon^{\mu\nu\alpha\beta}(K^{*+}_\mu \partial_\beta K^- + K^{*-}_\mu \partial_\beta K^+) \\ &\quad \times(\frac{1}{2}\partial_\nu \rho^0_\alpha + \frac{1}{2}\partial_\nu \omega_\alpha + \frac{\sqrt{2}}{2}\partial_\nu \phi_\alpha), \qquad (69) \\ \mathcal{L}_{GWZW}(K^{*0}) &= -\frac{N_c}{2\pi^2 g^2 f_K}\epsilon^{\mu\nu\alpha\beta}(K^{*0}_\mu \partial_\beta \bar{K}^0 + \bar{K}^{*0}_\mu \partial_\beta K^0) \\ &\quad \times(-\frac{1}{2}\partial_\nu \rho^0_\alpha + \frac{1}{2}\partial_\nu \omega_\alpha + \frac{\sqrt{2}}{2}\partial_\nu \phi_\alpha), \qquad (70)\end{aligned}$$

where f_K is determined by the following equation,

$$\frac{f^2_K}{1 - \frac{f^2_K}{g^2 m^2_{K^*}}} = \frac{f^2_\pi}{1 - \frac{f^2_\pi}{g^2 m^2_\rho}}$$

The corresponding direct couplings of K^* to photon, $\mathcal{L}_{GWZW}(K^{*\pm}, \gamma)$ and $\mathcal{L}_{GWZW}(K^{*0}, \gamma)$, can be obtained through the VMD substitutions of eq.(53). Then, the GWZW's contribution to the EM-mass difference of K^* can be computed directly. The result is

$$\begin{aligned}(m^2_{EM}(K^{*0}) - m^2_{EM}(K^{*\pm}))_{GWZW} &= \frac{-9e^2}{2\pi^4 g^2 f^2_K}\int_0^1 dx_1 \int_0^{x_1} dx_2 \int_0^{x_2} dx_3 \\ &\times m^2_{K^*}\frac{m^2_\rho}{48\pi^2}(\frac{m^2_\omega}{M^2_a} - \frac{2m^2_\phi}{M^2_b}),\end{aligned} \tag{71}$$

where

$$\begin{aligned}M^2_a &= m^2_\omega x_3 + m^2_\rho(x_2 - x_3) + m^2_K(1 - x_1) - m^2_{K^*}x_1(1 - x_1), \\ M^2_b &= m^2_\phi x_3 + m^2_\rho(x_2 - x_3) + m^2_K(1 - x_1) - m^2_{K^*}x_1(1 - x_1).\end{aligned}$$

Numerically, we have

$$(m^2_{EM}(K^{*0}) - m^2_{EM}(K^{*\pm}))_{GWZW} = 6.95 \pm 0.33 \times 10^{-4} GeV^2. \tag{72}$$

The full EM-mass difference between K^{*0} and $K^{*\pm}$ is the sum of eqs.(66),(68) and (72), which reads

$$(m_{K^{*0}} - m_{K^{*\pm}})_{EM} = 1.79 \pm 0.03 MeV. \tag{73}$$

In above computations, there are no any adjustable parameters. In other words, when one takes the experimental value of $m_\rho, m_a, m_{K^*}, m_K, m_\omega, m_\phi$ and f_π as inputs, then $(m_{K^{*0}} - m_{K^{*\pm}})_{EM}$ is fixed. The fact that this value is positive is just the claim of eq.(6). We can see also that the contribution of MYM part is significantly larger than one of GWZW part,

$$(m_{K^{*0}} - m_{K^{*\pm}})_{EM}^{MYM} : (m_{K^{*0}} - m_{K^{*\pm}})_{EM}^{GWZW} = 3.75 : 1. \tag{74}$$

Therefore, under VMD, this EM-mass difference is dominated by the MYM lagrangian of K^*.

The total mass difference between K^{*0} and $K^{*\pm}$ should be expressed as

$$\begin{aligned} &(m_{K^{*0}} - m_{K^{*\pm}})_{\rm total} \\ &= \quad (m_{K^{*0}} - m_{K^{*\pm}})_{\rm Q} + (m_{K^{*0}} - m_{K^{*\pm}})_{EM}, \end{aligned} \tag{75}$$

where $(m_{K^{*0}} - m_{K^{*\pm}})_{\rm Q}$ is due to $u - d$ quark mass difference, which has been calculated in the previous section (see eq.(49)). Then the prediction is

$$(m_{K^{*0}} - m_{K^{*\pm}})_{total} = (3.07 \pm 0.18 MeV) + (1.79 \pm 0.03 MeV) = 4.86 \pm 0.21 MeV. \tag{76}$$

which is in good agreement with the experimental data of eqs.(1) and (2). This fact means that the experiment favors to support the theoretical prediction of eq.(73) even though the experimental error is still large so far.

To conclude: In terms of the $U(3)_L \times U(3)_R$ chiral fields theory of mesons, all one-loop diagrams contributing to the electromagnetic mass difference of K^* have been calculated. The calculations are gauge independent. The gauging non-σ model part of the theory has no contribution to EM-masses of K^* while MYM- and GWZW- parts make contributions. MYM- and GWZW-parts make the EM-mass of K^{*0} larger than one of $K^{*\pm}$ respectively, but MYM-part is dominant. Then we conclude that in this reliable theory of mesons the non-abelian gauge fields structure of Yang-Mills theory causes the EM-mass of neutral K^* larger than one of charged K^*. This effect of YM theory is remarkable. It is interesting that the experiment supports it.

Similar studies can be done to other vector mesons and such an effect will also emerge. However, the things will not be so clear as in the K^* case becouse the non-linear σ model part of the meson's dynamics will be involved in the evaluations[9].

5 Summary

Comparing the experimental data of $K^{*0}(892)-$mass splitting with the theoretical prediction to the splitting caused by $u - d$ quark mass difference, we found out that the EM-mass of neutral K^* is larger than one of charged K^*. This is unusal, and was called as EM-mass anomaly of K^*. Employing the $U(3)_L \times U(3)_R$ chiral theory of mesons, we derived this anomaly effect explicitly.

The studies are hadron dynamics dependent. We conclude that it is meaningful and helpful to improve the experimental measurment of the mass splitting of $K^*(892)$ for solving this EM-mass anomaly puzzle.

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