Chiral quark model with topological ghost

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A nonlinear chiral quark model with topological ghost terms is proposed. The couplings among the light-flavor quarks, the Goldstone boson nonet and the topological ghost field are consistently incorporated in the model. Some features of the model are discussed.

1. INTRODUCTION

It is believed that the spontaneous breaking of chiral symmetry(SBCS) plays a significant role in the relatively low-energy hadronic processes. Strong evidence found in different investigations[1−3] supports such a picture that while the Goldstone particles appear due to SBCS, the bare quarks acquire their dynamical(constituent) masses. However, there is another story. It is the existence of the $U(1)_A$ anomaly. As one knows, the QCD Lagrangian indeed has an exact $U(3) \times U(3)$ chiral symmetry if light-flavor quark masses are set to be zero. Correspondingly, nine Goldstone bosons should appear in the QCD vacuum while the symmetry spontaneously broken down to the ordinary U(3). Nevertheless, the axial U(1) symmetry is in fact destroyed by the quantum effects which can be attributed to the influence of the instanton solutions of the gauge field on the physical vacuum[5,6]. As a consequence, the mass of the SU(3) singlet Goldstone boson η' is pushed up to a much heavier value than it would be. This exhibits explicitly the strong violation of the $U(1)_A$ symmetry in the real world. However, one would naturally expect that there should be obvious differences between this symmetry and the others which do not exist at all in the classical Lagrangian, and that there should exist some specific effects due to the anomaly in the relevant processes. The main aim of this work is just to study such effects in the chiral quark dynamics.

2. THE MODEL

In this paper we propose a chiral quark model in which both the mechanism of SBCS and the effect of the $U(1)_A$ anomaly are incorporated in a self-consistent way. In the description of SBCS, we take the general line of the model by Manohar and Georgi[4], and extend to U(3) by including the coupling of the flavor singlet Goldstone boson π_0. Define the Goldstone field in its nonlinear form

$$\Sigma = exp(i\pi_a\lambda_a/f) \qquad (a = 0, 1 \cdots 8) \quad , \tag{1}$$

with the normalization

$$Tr\lambda_a\lambda_b = 2\delta_{ab} \quad . \tag{2}$$

Σ transforms under $U(3)\times U(3)$ as

$$\Sigma \to L\Sigma R^\dagger \quad . \tag{3}$$

Let $\xi^2 = \Sigma$, the transformation can be rewritten as

$$\xi \to L\xi U^\dagger = U\xi R^\dagger \quad , \tag{4}$$

which defines U as a nonlinear function of the chiral transformation and the π fields. The quark fields transform as

$$\psi \to U\psi \quad . \tag{5}$$

Obeying the above transforming properties, and considering the explicit symmetry breaking effect induced by the quark masses, we can explicitly write out the various terms of the effective Lagrangian up to the order of two derivatives of Σ or one μM and one quark pair(the low energy approximation) as follows

$$\begin{aligned}\mathcal{L}_1 \;=\;& f^2\{-\frac{1}{4}tr(\partial_\mu\Sigma^\dagger\partial_\mu\Sigma)+\frac{1}{2}tr(\Sigma^\dagger+\Sigma)\mu M\} \\ & -m\bar{\psi}\psi-\frac{f'm}{\Lambda^2_{CSB}}\bar{\psi}(\xi\mu M\xi+\xi^\dagger\mu M\xi^\dagger)\psi \\ & -\bar{\psi}\gamma_\mu D_\mu\psi-ig_A\bar{\psi}\gamma_\mu A_\mu\gamma_5\psi \quad ,\end{aligned} \tag{6}$$

where we have used the same convention as in [4] for M,D_μ,V_μ and A_μ except the ξs appearing in them denoting the corresponding pseudoscalar nonet here. f,μ,m,f',Λ_{CSB} and g_A are some parameters to be fixed in the model. The gluon terms are not explicitly given in the expression. They may only provide small correction in our model, and is irrelevant to the discussion in the rest of the paper.

Furthermore it is naturally demanded that the effect of the anomaly must be properly incorporated in the chiral quark model. Through the instanton solutions of the Yang-Mills theory[5], 't Hooft[6] established the connection between the topological charge density(the ghost) and the axial anomaly. Then it was shown that the coupling between the Goldstone bosons and the topological ghost can be constructed to describe effectively the effect of the anomaly and explain the low-energy spectrum and dynamics of the pseudoscalar mesons and the structure of the vacuum[7,8]. The gauge invariant topological charge density field is expressed in term of gluon fields as

$$q(x) = \frac{g^2}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma} = \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma} \quad . \tag{7}$$

Due to the anomaly, a U(1) chiral transformation is equivalent to changing the QCD action by

$$\int d^4x\mathcal{L}_{QCD} \rightarrow \int d^4x\mathcal{L}_{QCD} + L\phi \int d^4xq(x) \quad . \tag{8}$$

L is the flavor number, ϕ the transformation parameter. Satisfying this requirement and preserving the left $SU(3) \times SU(3)$, we can further introduce various coupling terms between the ghost and the Goldstone bosons and the quarks. Such terms in the leading order of the large N_c and the low energy expansions are as follows:

$$\begin{aligned}\mathcal{L}_2 &= cq^2 + \frac{i}{2}qtr(log\Sigma - log\Sigma^{\dagger}) - \theta q \\ &+g_Q^S\bar{\psi}q\gamma_5\psi + g_Q^V\bar{\psi}\gamma_\mu\gamma_5\partial_\mu q\psi \quad . \end{aligned} \tag{9}$$

c, θ, g_Q^S and g_Q^V are the parameters.The first term is the "mass" term of the ghost. The θ term is associated with the θ-vacuum and irrelevant to our present concern.

Putting together Eqs.(6) and (9) forms the full Lagrangian in our model. To see some general features of this model, we consider the chiral limit and the tree level approximation. In this case the chiral Lagrangian is reduced as

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}(\partial_\mu\pi_a)^2 - \frac{\lambda^2}{2}(\partial_\mu q)^2 + cq^2 - \frac{\sqrt{6}}{f}\pi_0 q \\ &-\bar{\psi}(\gamma_\mu\partial\mu + m)\psi + \frac{ig_A}{2f}\bar{\psi}\gamma_\mu\partial_\mu\pi_a\lambda_a\gamma_5\psi \\ &+g_Q^S\bar{\psi}q\gamma_5\psi + g_Q^V\bar{\psi}\gamma_\mu\gamma_5\partial_\mu q\psi \quad . \end{aligned} \tag{10}$$

In the above Lagrangian we have added the kinetic energy term of the ghost , which is usually omitted in the large N_c expansion in the literature. Because we find that as shown below, this term has important influence on the resultant physics and is closely related to the mass term cq^2, hence should not be simply set as zero. If we admit that π_0 and q are the usual canonical variables (They might not be so in general. The other possibilities of quantization will be discussed elsewhere), then the Lagrangian concerning π_0 and q can be diagonalized as

$$\begin{aligned}\mathcal{L}_S &= -\frac{1}{2}(\partial_\mu\eta^{'})^2 - \frac{1}{2}(\partial_\mu q^{'})^2 - \frac{m_{\eta^{'}}^2}{2}\eta^{'2} + c^{'}_{(\lambda)}q^{'2} \\ &+\frac{ig_A}{3f}\bar{\psi}\gamma_\mu\gamma_5\partial_\mu(C_{1(\lambda)}\eta^{'} + C_{2(\lambda)}q^{'})\psi + g_Q^S\bar{\psi}\gamma_5(C_{3(\lambda)}\eta^{'} + C_{4(\lambda)}q^{'})\psi \\ &+g_Q^V\bar{\psi}\gamma_\mu\gamma_5\partial_\mu(C_{3(\lambda)}\eta^{'} + C_{4(\lambda)}q^{'})\psi \ , \end{aligned} \tag{11}$$

where $\eta^{'}$and $q^{'}$ are the physical flavor singlet and the transformed ghost field, respectively. Assuming $\lambda^2, c \geq 0$, the parameters satisfy

$$c = \frac{3}{m_{\eta^{'}}^2 f^2} - \frac{\lambda^2 m_{\eta^{'}}^2}{2} \quad , \qquad (0 \leq \lambda^2 \leq \frac{6}{m_{\eta^{'}}^4 f^2}) \quad . \tag{12}$$

It is shown that the variation of λ or c may lead to very different values of the coefficients c',C_1,C_2,C_3 and C_4 and therefore results in different physics. The values of λ and c should be defined by further theoretical study and/or experimental information rather than are fixed *a priori*.

3. SUMMARY

We propose a chiral quark model which properly incorporates the effects of the axial U(1) anomaly by introducing the couplings of the topological ghost field with both the quark fields and the pseudoscalar nonet fields. It provides a new picture of the chiral quark dynamics, which indicates that not only the quarks are coupled with the whole pseudoscalar nonet, but also they interact with an additional but virtual degree of freedom, the ghost field, due to the anomaly effect from the QCD vacuum. Further discussion of this model and some specific results will be given elsewhere. The application of the model to the relevant physical processes are under investigation.

This work is partly supported by the National Natural Science Foundation of China. We are grateful to Chao-tsi Chang for his useful discussions and comments.

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