Review on α_s at LEP

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To measure the strong coupling α_s from event shape observables two ingredients are necessary: a perturbative prediction containing the dependence of observables on α_s and a description of the hadronisation process to match the perturbative prediction with the hadronic data.

As perturbative prediction $\mathcal{O}(\alpha_s^2)$, NLLA and combined calculations are available. Beside the well known Monte-Carlo based models also analytical predictions, so called power corrections, exist to describe the hadronisation. Advantages and disadvantages of the different resulting methods for determining the strong coupling and its energy dependence will be discussed, the newest DELPHI results will be presented, and an overview of the LEP results will be included.

1. INTRODUCTION

The theoretical description of hadron production in e^+e^--annihilation consists of three parts. The first part is based on the fundamentals of the standard model: Feynman diagrams are used to (perturbatively) calculate the electroweak process of e^+e^--annihilation and the evolution of partons under the strong interaction. At some stage the partons must be combined to become hadrons. This hadronisation process cannot be described by perturbation theory and thus builds a second (non-perturbative) part. Finally the decay of unstable hadrons, which can be described by kinematics using experimentally measured decay rates, need to be included before the prediction can be confronted with data.

Several different predictions exist for the two first parts. Calculations of the evolution of quarks and gluons are available in fixed order $\mathcal{O}(\alpha_s^2)$, recently some observables became available in $\mathcal{O}(\alpha_s^3)$, and in the next to leading log approximation (NLLA), which resums large logarithms to all orders of α_s. When combining $\mathcal{O}(\alpha_s^2)$ with NLLA matching ambiguities occur leading to even more competing predictions.

To describe the second part (the hadronisation) usually generator based models are used. The most reliable of these models are the Lund string fragmentation, implemented in JETSET, and the Cluster fragmentation, implemented in HERWIG. Recently analytical predictions, so called power corrections, became available to describe the influence of hadronisation on event shape observables.

Given the many different possibilities to determine α_s it was necessary to restrict this review due to time and space limitation. I've chosen to review the analyses using event shape observables to determine α_s, as this field currently is very active at LEP.

In the following I will first discuss the standard method for measuring α_s from event shapes, which uses $\mathcal{O}(\alpha_s^2)$, NLLA or the combined $\mathcal{O}(\alpha_s^2)$+NLLA prediction in combination with generator based hadronisation models. In the second part the energy dependence of α_s will be discussed introducing the alternative analytic description of hadronisation by power corrections. Finally first results obtained using the new $\mathcal{O}(\alpha_s^3)$ predictions are presented. In all three sections DELPHI-analyses shall serve as showcases.

2. CONSISTENT DETERMINATION OF THE STRONG COUPLING

The original motivation of repeating an α_s-measurement with LEP1 data in DELPHI was to include the event orientation, given e.g. by the polar angle of the Thrust axis θ_T, into the fit. Second order coefficients including such an event orientation became available in $\mathcal{O}(\alpha_s^2)$ through the EVENT2 program by Catani and Seymour [1]. Using the fully reprocessed LEP1 data, event shapes were measured very precisely in eight bins of θ_T. The small systematic errors result not only from the good quality of the final data reprocessing, but also from the fact that all detector corrections in this measurement were naturally calculated for each θ_T-bin separately.

The second order prediction for such distributions now contains second order coefficients A and B which depend on the observables value itself and on θ_T:

$$\frac{1}{N}\frac{\mathrm{d}^2 N}{\mathrm{d}y\,\mathrm{d}\theta_T} = A(y,\theta_T)\frac{\alpha_s(\mu)}{2\pi} + \left(A(y,\theta_T)\cdot 2\pi b_0 \ln\left(\frac{\mu^2}{E_{\mathrm{cm}}^2}\right) + B(y,\theta_T)\right)\left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 \tag{1}$$

μ being the renormalisation scale and $b_0 = (33-2N_f)/12\pi$. Beside α_s this formula contains renormalisation scale x_μ parameterized as $x_\mu = \mu^2/E_{\mathrm{cm}}^2$ as a free parameter.

As a first step the dependence of the resulting α_s-values on this parameter were investigated. It is found that different observables acquire largely different dependencies on x_μ. Also the scales with the optimal χ^2/ndf vary widely (see Fig. 1 left and middle). In spite of this large variation of the optimal scale, the α_s results corresponding to the experimentally optimised scales show a smaller spread for a large number of different event shapes, than the results obtained with $x_\mu = 1$. In contrast to the results with scale 1, optimised scales yield consistent values of α_s without assuming any renormalisation scale error (see Fig. 1 right). In addition the scale dependence of α_s near the optimised scale is smaller than for $x_\mu = 1$, so that the scale variation between half and twice the chosen value of x_μ leads to a smaller scale uncertainty for optimised scales.

Averaging over all 18 investigated observables the final result [2] is

$$\alpha_s(M_{\mathrm{Z}}) = 0.1228 \pm 0.0119 \quad (x_\mu = 1) \tag{2}$$

$$\alpha_s(M_{\mathrm{Z}}) = 0.1173 \pm 0.0026 \quad (\text{opt. scales}), \tag{3}$$

where for the optimised scales also the b-quark mass corrections are included. The overall fit quality of these results is far better for the optimised scales than for $x_\mu = 1$, as can be seen from the $\Delta\chi^2$ in Fig. 1. Thus the larger spread of the results with $x_\mu = 1$ can be attributed to a less stable fit procedure which is caused by the bad agreement between data and the prediction.

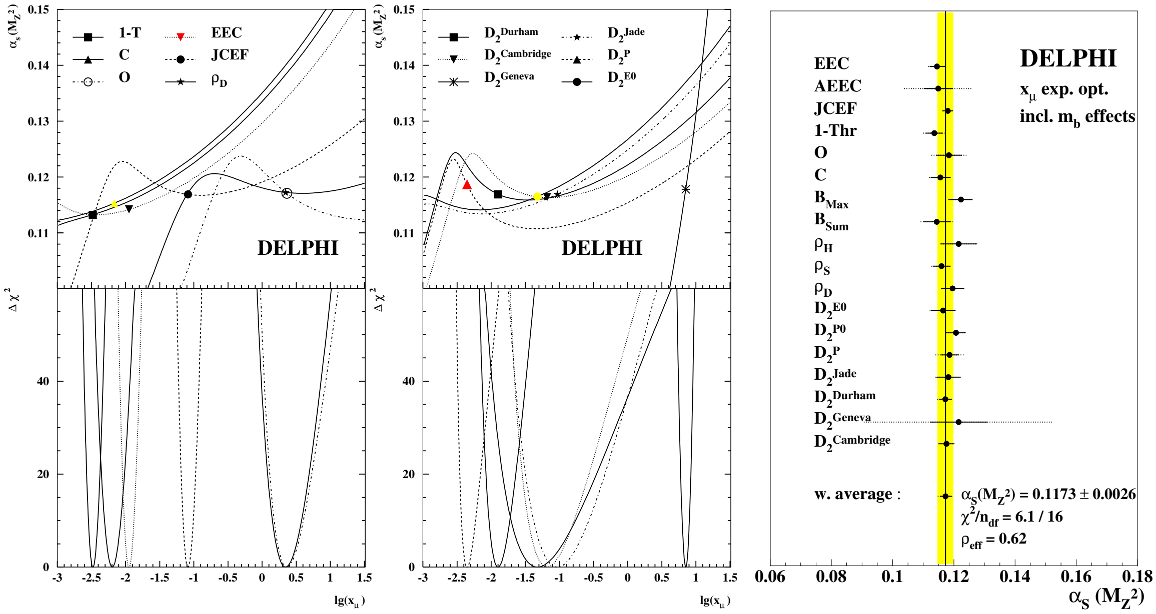


Figure 1. Dependence of the resulting α_s on the renormalisation scale for different event shapes (upper parts in left and middle plot) and the corresponding $\Delta\chi^2$-values (lower parts). Resulting α_s values for 18 different event shape observables using experimentally optimised scale and including corrections for b-quark mass (right plot).

To crosscheck the results the scale dependence was also investigated for NLLA and combined $\mathcal{O}(\alpha_s^2)$+NLLA predictions. In contrast to the $\mathcal{O}(\alpha_s^2)$ results the required relative renormalisation scales are close to one and thus there is no significant change compared to the results obtained with $x_\mu = 1$. Moreover, because of the limited fit range available for NLLA and because of the matching ambiguities for combined $\mathcal{O}(\alpha_s^2)$+NLLA predictions, the total error for these methods is larger than for the $\mathcal{O}(\alpha_s^2)$ result:

$$\alpha_s(M_Z) = 0.116 \pm 0.006 \qquad \text{NLLA} \tag{4}$$

$$\alpha_s(M_Z) = 0.119 \pm 0.005 \qquad \mathcal{O}(\alpha_s^2)\text{+NLLA} \tag{5}$$

Both results are in good agreement with each other and with the $\mathcal{O}(\alpha_s^2)$ results [2].

There are two other publication in which optimised scales are used to determine α_s at $E_{\rm cm} = M_Z$: Already in 1992 Opal [3] stated to have found a clear improvement in the fit quality with optimised scales using 14 observables. They quoted

$$\alpha_s(M_Z) = 0.118^{+0.007}_{-0.003} \quad \text{(opt. scales)}, \tag{6}$$

where the error includes a scale variation from the optimised scale upto a scale of 1.

In 1996 Burrows et. al. [4] investigated 15 Observables using SLD-data. In contrast to Opal and Delphi they found no significant reduction of the spread of α_s-values, though the shift to lower values of α_s when using optimised scales was reproduced:

$$\alpha_s(M_Z) = 0.1265 \pm 0.0076 \quad (x_\mu = 1) \tag{7}$$

$$\alpha_s(M_Z) = 0.1173 \pm 0.0071 \quad \text{(opt. scales)}. \tag{8}$$

In spite of the extra theoretical uncertainties due to matching ambiguities three of four LEP experiments today use the combined $\mathcal{O}(\alpha_s^2)$+NLLA calculation to determine their central α_s-value (second error at L3 is the theoretical component):

$$\begin{array}{lcll} \alpha_s(M_{\rm Z}) & = & 0.1216 \pm 0.0039 & \text{Aleph [5]} \\ \alpha_s(M_{\rm Z}) & = & 0.1220 \pm 0.0015 \pm 0.0060 & \text{L3 [6]} \\ \alpha_s(M_{\rm Z}) & = & 0.120 \quad \pm 0.006 & \text{Opal [3].} \end{array} \tag{9}$$

3. ENERGY DEPENDENCE

The increase of beam energy accomplished during the LEP2 programme gives access to the energy dependence of event shapes and thereby to the energy dependence of α_s.

3.1. Power Corrections

Using data at different energies allows also to replace the generator based hadronisation models by an analytical ansatz. For mean values this ansatz (which was developed by Dokshitzer and Webber [7]) describes the hadronisation by an additive term:

$$\langle f\rangle = \frac{1}{\sigma_{\rm tot}}\int f\frac{{\rm d}f}{{\rm d}\sigma}{\rm d}\sigma = \langle f_{\rm pert}\rangle + \langle f_{\rm pow}\rangle \quad . \tag{10}$$

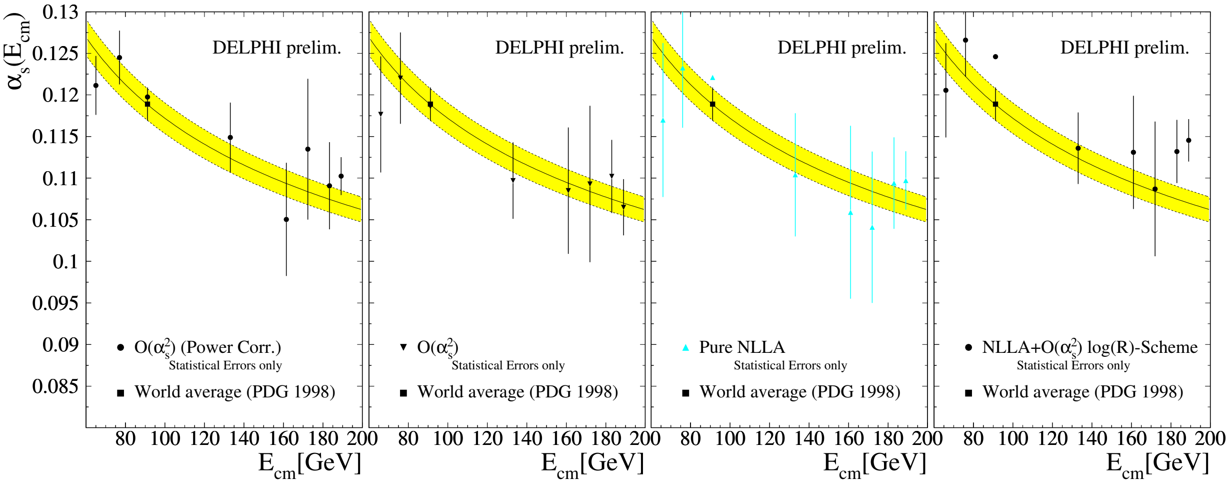
The 2nd order perturbative prediction is given by Eq. (1) with coefficients A and B integrated over y and θ_T. The power correction term is falling off like the inverse centre-of-mass energy and is given by

$$\langle f_{\rm pow}\rangle = c_f\frac{4C_F}{\pi^2}\mathcal{M}\frac{\mu_I}{E_{\rm cm}}\left[\alpha_0(\mu_I) - \alpha_s(\mu) - \left(b_0\cdot\log\frac{\mu^2}{\mu_I^2} + \frac{K}{2\pi} + 2b_0\right)\alpha_s^2(\mu)\right] \tag{11}$$

where α_0 is a non-perturbative parameter accounting for the contributions to the event shape below an infrared matching scale μ_I, $K = (67/18 - \pi^2/6)C_A - 5N_f/9$. The Milan factor $\mathcal{M}$ is set to 1.8, which corresponds to three active flavours in the non-perturbative region [8]. The observable-dependent coefficient c_f is 2 and 1 for $f = \langle 1-T\rangle$ and $f = \langle M_{\rm h}^2/E_{\rm vis}^2\rangle$, respectively. For $\langle B_{\rm max}\rangle$ the coefficient is itself energy dependent: $c_{\langle B_{\rm max}\rangle} \sim 1/\sqrt{\alpha_s(E_{\rm cm})}$ [9]. The infrared matching scale is set to 2 GeV as suggested by the authors [7], the renormalisation scale μ is set to be equal to $E_{\rm cm}$. Beside α_s these

Observable	$\alpha_0(2\,\text{GeV})$	$\alpha_s(M_{\rm Z})$	χ^2/ndf
$\langle B_{\rm max}\rangle$	$0.407 \pm 0.022 \pm 0.055$	$0.117 \pm 0.0012 \pm 0.0015$	8.28/21
$\langle 1-T\rangle$	$0.493 \pm 0.009 \pm 0.006$	$0.119 \pm 0.0014 \pm 0.0067$	64.0/32
$\langle M_{\rm h}^2/E_{\rm vis}^2\rangle$	$0.545 \pm 0.023 \pm 0.017$	$0.120 \pm 0.0020 \pm 0.0050$	8.28/21

Table 1
Determination of α_0 in a combined fit of α_0 and α_s using a large number of measurements [10] at different energies [12]. The first error is calculated from the fit, the second contains the renormalisation scale dependence.



	Power Corr.	Monte Carlo based hadronisation models		
	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^2)$	NLLA	$\mathcal{O}(\alpha_s^2)$+NLLA
$\frac{\mathrm{d}\alpha_s^{-1}}{\mathrm{d}\log E_{\mathrm{cm}}}$	$1.07{\pm}0.19 \pm 0.20$	$1.23{\pm}0.31 \pm 0.19$	$1.41{\pm}0.31 \pm 0.27$	$1.12{\pm}0.22 \pm 0.18$

Figure 2. Comparison of α_s-results obtained by DELPHI at different centre-of-mass energies using Monte Carlo generator based hadronisation models as well as power correction. Below each plot the result for the logarithmic energy slope of the inverse coupling is given.

formulae contain α_0 as the only free parameter. In order to measure α_s from individual high energy data this parameter has to be known.

To infer α_0, a combined fit of α_s and α_0 to a large set of measurements at different energies [10] is performed [11,12]. For $E_{\mathrm{cm}} \geq M_{\mathrm{Z}}$ only DELPHI measurements are included in the fit. The resulting values of α_0 for $\langle B_{\max}\rangle$, $\langle 1-T\rangle$ and $\langle M_{\mathrm{h}}^2/E_{\mathrm{vis}}^2\rangle$ are summarised in Tab. 1. Even though the found α_0 values are experimentally inconsistent, the universality of α_0 is not violated because the expected theoretical precision allows deviations of upto 20%. The α_0 values are consistent with the corresponding analyses of L3 [6] and OPAL/JADE [13].

After fixing α_0 to the values found, α_s can be calculated individually for each energy using Eqs. (10–11). The results for energies between 65 GeV and 189 GeV of this method and of the traditional methods described in the previous section are compared in Fig. 2. All methods give consistent results.

3.2. Energy dependence of α

To measure the energy dependence of α_s DELPHI uses the logarithmic energy slope of the inverse coupling. This quantity is directly proportional to the Callan-Symanzik β-function and is independent of α_s and of E_{cm} to first order:

$$\frac{\mathrm{d}\alpha_s^{-1}}{\mathrm{d}\log E_{\mathrm{cm}}} = -\frac{1}{\alpha_s^2}\beta_{\alpha_s} = \frac{\beta_0}{2\pi} + \frac{\beta_1}{4\pi^2}\alpha_s + \ldots \simeq 1.27 \qquad (12)$$

The numerical value represents the QCD prediction calculated in second order for energies between 91 GeV and 200 GeV using the PDGs world average of α_s. The energy dependence of this derivative in the given range and the uncertainty of α_s influence this value by about one unit in the last digit.

The result with the smallest systematic error is obtained from $\mathcal{O}(\alpha_s^2)$+NLLA fits:

$$\frac{\mathrm{d}\alpha_s^{-1}}{\mathrm{d}\log E_{\mathrm{cm}}} \quad = \quad 1.12 \pm 0.22 \pm 0.18 \tag{13}$$

in good agreement with the QCD expectation.

Instead of determining a value for the energy dependence explicitly, L3 checks the running of α_s by applying a combined fit to all energies assuming the standard model running to $\mathcal{O}(\alpha_s^3)$. This yields a $\chi^2/\mathrm{ndf} = 15.4/12$ also indicating a good agreement [6].

4. $\mathcal{O}(\alpha_s^3)$

Third order calculations for four-parton final states have become available recently [14, 15]. These calculations can be used to measure α_s in next to leading order from event shapes that acquire non-trivial values only for four and more partons, like the four-jet-rate R_4:

$$R_4 = B\left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 + \left(B\cdot 2\pi b_0 \log\frac{\mu^2}{E_{\mathrm{cm}}^2} + C\right)\left(\frac{\alpha_s(\mu)}{2\pi}\right)^3 \tag{14}$$

Observables of this kind are uncorrelated to the observables discussed so far which are based on three-jet-like configurations. The reduced number of relevant events is partly compensated by the (due to quadratic α_s dependence) increased sensitivity in Eq. (14).

Delphi investigated four-jets-rates $R_4(y_{\mathrm{cut}})$ at a given y_{cut} for the Durham cluster algorithm [16]. It shows good agreement with the prediction for $y_{\mathrm{cut}} > 0.002$. Using $y_{\mathrm{cut}} = 0.0025$ the resulting α_s values are consistent with the results shown in the previous section, but they have larger statistical errors. Also the running obtained from this analysis is in good agreement:

$$\frac{\mathrm{d}\alpha_s^{-1}}{\mathrm{d}\log E_{\mathrm{cm}}} \quad = \quad 1.16 \pm 0.46 \quad . \tag{15}$$

This new calculation has also been investigated by Aleph, but was not yet used to determine the strong coupling.

5. SUMMARY

In this talk it was tried to give an overview over the current state of the art in measuring α_s from event shape observables at LEP. Beside different perturbative calculations several hadronisation models (generator based and analytical ones) exist. The previous standard choice for the perturbative calculation $\mathcal{O}(\alpha_s^2)$+NLLA is questioned by Delphi due to poor data description and unsatisfactory consistency.

In that sense measurements of α_s from LEP data are still in full progress. In addition to the method used, special attention is paid to tests of the running, also new theoretical developments like power corrections and $\mathcal{O}(\alpha_s^3)$ gain due recognition.

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