Strong Van der Waals force in the hadron physics

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Possible strong Van der Waals interaction between hadrons is searched in the P-wave amplitude of the π-π scattering by removing the cut of the two-pion exchange spectrum as well as the unitarity cut.

1. WHY THE STRONG VAN DER WAALS FORCE APPEARS IN THE HADRON INTERACTION ?

It has been believed for a long time that the interactions between hadrons are short range, which arise from the exchanges of a set of pions or heavier particles. However in nineteen sixties our view of hadrons changed drastically, namely changed from elementary particles to composite particles. And even after the change of the view, people optimistically believed that the scattering of the elementary particles and that of the composite particles are essentially the same as long as we stay in the region of small momentum transfer and so do not explore the inside of the composite particle. However this is not true.

What is common to various composite models of hadron is that the fundamental constructive force is strong or super-strong Coulombic force. And the hadrons are regarded as the neutral bound states, for example "color charge zero state" for QCD and "magnetic charge zero state" for the dyon model[1] and so on. From the quantum fluctuation, the two step dipole-dipole interaction namely the Van der Waals interaction appears as in the case of the ordinary atoms. Therefore once we accept such a composite model, we cannot escape from the appearance of the strong Van der Waals force. So it is important to find the strong Van der Waals interaction in the hadron scatterings[2]. We shall analyse the P-wave amplitude of the π-π scattering in search for such a force.

2. THE DISPERSION RELATION AND THE KANTOR AMPLITUDE

The P-wave amplitude $a_1(\nu)$ of the π-π scattering is one of the ideal places to search for the strong Van der Waals force, because firstly we can consider $a_1(\nu)/\nu$ rather than the amplitude and secondly we can compute the spectral function of the two-pion exchange from the π-π phase shift. It is well-known that the dispersion relation of the once subtracted form is

$$\frac{K_1(\nu)}{\nu} \equiv \frac{a_1(\nu)}{\nu} - \frac{1}{\pi}\int_0^\infty d\nu' \frac{\mathrm{Im}\, a_1(\nu')}{\nu'(\nu'-\nu)} = \frac{1}{\pi}\int_{-\infty}^0 d\nu' \frac{\mathrm{Im}\, a_1(\nu')}{\nu'(\nu'-\nu)} \qquad , \tag{1}$$

*Associate member of AERI for research. e-mail address: t-sawada@fureai.or.jp where $K_1(\nu)$ is the Kantor amplitude[3], and which can be evaluated from the experimental data (phase shift and elasticity). On the other hand, the right hand side is the integration on the spectral function on the left hand cut which consists of the two-pion and four-pion exchange spectra and so on plus the extra spectrum which we are trying to find. Eq.(1) can be used to explore the missing spectral function, when the phase shift data are given.

2.1. Spectral function of the two-pion exchange

The spectral function of the two-pion exchange $\mathrm{Im}\, a_\ell^{2\pi}(\nu)$ can be evaluated from the phase shift data. We shall describe it briefly[4]. The spectral function of the s-channel $A_s(s,t)$ is the sum of the imaginary part of the partial waves:

$$A_s^{(I)}(s,t) = \sum_{L=0}^{\infty}(2L+1)\sqrt{\frac{\nu+1}{\nu}}\sin^2\delta_L^{(I)}(\nu)P_L(z_s)\ , \tag{2}$$

in which (I) is the isotopic spin and hereafter the pion mass will be used as the unit of the energy and the momentum. Since in π-π the s and t channels are essentially the same, we can obtain the t-channel spectral function $A_t(s,t)$ by simply changing the variables (ν, z_s) to (ν_t, z_t) in Eq.(2). The new variables relate to (s,t) by $\nu_t = t/4 - 1$ and by $z_t = 1 + s/2\nu_t$ respectively. $A_u(s,t)$ is obtained in the similar way. We can get the spectral function $\mathrm{Im}\, a_\ell^{2\pi}(\nu)$ by making the partial wave projection of A_t and A_u to the ℓ-state. The result is

$$\mathrm{Im}\, a_\ell^{(I)}(\nu) = \frac{1}{\nu}\int_0^{-1-\nu} d\nu_t P_\ell(1+2\frac{\nu_t+1}{\nu})\sum_{J=0}^{2}\alpha^{(I)}_{(J)}A_t^{(J)}(s,t) \tag{3}$$

where $\alpha^{(I)}_{(J)}$ is twice of the crossing matrix, or more explicitly

$$\alpha^{(I)}_{(J)} = \frac{1}{3}\begin{pmatrix} 2 & 6 & 10 \\ 2 & 3 & -5 \\ 2 & -3 & 1 \end{pmatrix}\ .$$

2.2. The missing spectrum and the discrepancy of the dispersion relation

When we compute the Kantor amplitude in Eq.(1), we shall use Wagner's fit of the P-wave phase shift[5]

$$(\frac{s_\rho}{s})(\frac{\nu}{\nu_\rho})^{3/2}\cot\delta_1^{(1)}(\nu) = \frac{s_\rho - s}{\sqrt{s_\rho}\,\Gamma_\rho}\quad . \tag{4}$$

In figure 1, $K_1(\nu)/\nu$ is plotted against ν as the full line. In the same graph the two-pion exchange contribution to the r.h.s. of Eq.(1) is also shown as the dotted curve. The dispersion relation Eq.(1) requires that these two curves must coincide up to a constant, because the four-pion exchange spectrum slowly starts at $\nu = -4$ and the main part of the spectrum is far away, and therefore gives an almost constant term in $K_1(\nu)/\nu$. However the figure indicates this is not the case, and therefore an extra force must be acting between pions in addition to the forces arising from the exchanges of pions. It is interesting to see that the shape of the cusp is close to what is expected when the Van der Waals force is acting between pions.

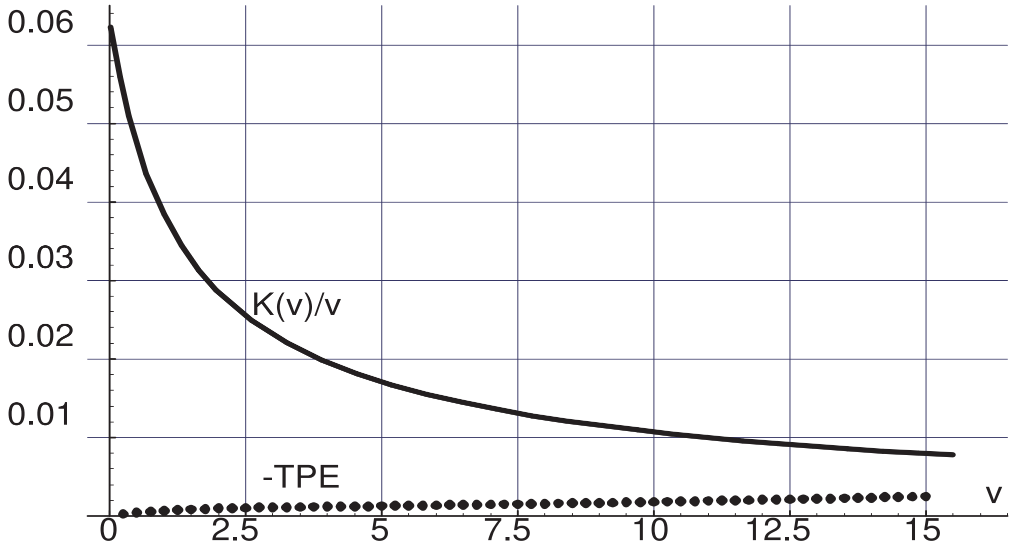

Figure 1. $K_1(\nu)/\nu$ (full line) and $-(2\pi$ exchange contribution of Eq.(1)) (dotted line).

Since the spectral function $A_t(s,t)$ relates to the potential $V(R)$, which is acting between particles with masses m_1 and m_2, by

$$V(R) = -\frac{1}{\pi m_1 m_2}\frac{1}{R}\int_0^\infty dt A_t(s,t) e^{-R\sqrt{t}} \ , \tag{5}$$

we see that the power α of the asymptotic behavior of the potential ($V(R) \sim -C/R^\alpha$) and the power γ of the threshold behavior of the spectral function ($A_t(s,t) \approx \pi C' t^\gamma + \cdots$) are related by

$$\alpha = 2\gamma + 3 \quad . \tag{6}$$

Since $\gamma = 1.5$ and $\gamma = 2.0$ for the Van der Waals potential of the London type ($\alpha = 6$) and the Casimir-Polder type ($\alpha = 7$) respectively, the singular behaviors of the Kantor amplitudes at $\nu = 0$ become

$$\frac{K_1^{(extra)}(\nu)}{\nu} = \begin{cases} c_0 - C''\sqrt{\nu} & (\text{ London type }) \\ c_0 + C''\nu\log\nu & (\text{ Casimir-Polder type }) \end{cases}$$

respectively[6]. The cusp of the figure lies something between these two cases.

2.3. Determination of the parameters of the extra spectral function

In order to determine the strength and the type of the long range force, the chi-square fit was done, in which the form of the extra spectral function is assumed to be

$$A_t^{(extra)}(s,t) = \pi C' t^\gamma e^{-\beta t} \qquad . \tag{7}$$

The exponential factor is necessary to make the integration convergent, and the parameter $\sqrt{\beta}$ has the meaning of the seize of the composite system. The result of the fit is

$$C' = 0.062, \quad \gamma = 1.92 \quad and \quad \beta = 0.16 \qquad .$$

With this extra spectrum, the equality of Eq.(1) is recovered.

3. COMMENTS

Since the Van der Waals interaction is universal[7], it must be observed also in other processes. In order to confirm the long range force unambiguously, it is recommended to use the difference of the analytic structure of the amplitudes. An extra singularity appears at $t = 0$ in the amplitude $A(s,t)$ and at $\nu = 0$ in the partial wave amplitude $a_\ell(\nu)/\nu^\ell$ respectively, although these points are regular when all the forces are short range. It is fortunate that these singular points are the end points of the physical regions, which are $-4\nu \leq t \leq 0$ for $A(s,t)$ and $\nu \geq 0$ for $a_\ell(\nu)/\nu^\ell$ respectively. Therefore we can in principle observe the extra singularity when sufficiently precise data are available. However if the data are not very precise, we must remove the known near-by spectrum and prepare the wide domain of analyticity to see the extra singularity.

If we consider the precision of the measurement, another ideal place to observe the strong Van der Waals force is the once subtracted S-wave of the proton-proton scattering $(a_0(\nu) - a_0(0))/\nu$. Here again we obtain a huge cusp at $\nu = 0$, after the one-pion exchange term is removed[8]. From the fit to the cusp, the power of the threshold behavior is $\gamma = 1.54$ and which indicates, contrary to the case of π-π, the Van der Waals force is the London type rather than the Casimir-Polder type in the p-p interaction[6].

For fixed ν, the singularity $(-t)^\gamma$ is observed as the singular behavior of the angular distribution $(1-z)^\gamma$ in the neighborhood of $z = 1$. Therefore contrary to the case of the short range force, the amplitude cannot be reproduced well by a few terms of the polynomials of z. In the low energy proton-proton scattering, we must observe a characteristic destructive interference pattern in the forward region which arises from the (electric) Coulomb potential and the tail of the strong Van der Waals potential. Since the deviation from the case of the short range force is small, the high precision measurement of $\sigma_{obs}(\theta)$ is required. In fact ratio function $R(\theta)$ defined by $(\sigma_{obs}(\theta) - \sigma_{short}(\theta))/\sigma_{obs}(\theta)$ has a dip at $\theta_{cm} = 10°$ whose depth is around 1% when the incident energy is 25 MeV.. Although it is not easy to measure the cross section $\sigma(\theta)$ of the p-p scattering in the energy range $T_{lab} = 20 \sim 30 MeV.$ with the relative accuracy 0.1% down to smaller angle θ, it is still worthwhile to do the experiment, because the fixed energy measurement is more direct in observing the strong Van der Waals force, and the confirmation of such a force serves to place the hadron physics on the sound basis. More information is available in the e-mail address written in the first page of the present paper.

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