Large angle QED processes at e^+e^- colliders at energies below 3 GeV

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Abstract

QED processes at electron–positron colliders are considerd. We present differential cross–sections for large–angle Bhabha scattering, annihilation into muons and photons. Radiative corrections in the first order are taken into account exactly. Leading logarithmic contributions are calculated in all orders by means of the structure–function method. An accuracy of the calculation can be estimated about 0.2%.

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1 Introduction

At the existing VEPP–2M e^+e^- collider and planned meson factories QED processes of the lowest order of the perturbation theory (PT) play an important role. These processes are to be considered as an essential background when extracting subtle mesons properties from experimental data. QED processes are used also for calibration and monitoring purposes. For instance, large–angle Bhabha scattering is used for a precise determination of luminosity at e^+e^- colliders. That is the reason, why radiative corrections (RC) to QED processes are to be considered in detail.

A lot of attention was paid to the problem of radiative correction calculations to various QED processes, starting more than 50 years ago [1], concerning mainly the lowest order of

PT calculations. The accuracy requirements of modern experiments, however, exceed the ones provided by the first order RC. Unfortunately, calculations of radiative corrections in higher orders encounter tremendous technical difficulties. Nevertheless, powerful methods developed in quantum chromodynamics (see paper [2] and references therein) provide a possibility to improve essentially the results obtained earlier. At first we mean the methods based on the renormalization group ideas and on the factorization theorem. Thay allow us to get a differential cross–section of a certain process similar to the Drell–Yan process cross–section, and to consider leading logarithmic RC of higher orders. Meanwhile, nonleading contributions are to be taken from the lowest order PT calculations.

The aim of our paper is to provide relevant theoretical formulae for QED processes, which are required for experiments with CMD–2 and SND detectors at VEPP–2M (Novosibirsk) [3] collider, at DAΦNE (Frascati) [4] and BEPC/BES (Beijing) [5] machines. Formulae cited below may be applied also for higher energies (see the Conclusions), if one will take into account weak interaction and higher hadronic resonance contributions.

A cross–section calculated with an account of radiative corrections (RC) in the n-th order of perturbation theory (PT) contains enhanced contributions of the form $(\alpha/\pi)^n L^n$, where $L = \ln(s/m_e^2)$ is the *large* logarithm (for $s \sim 1$ GeV^2, $L \approx 15$). We call these contributions the leading ones. Nonleading contributions have the order $(\alpha/\pi)^n L^m$, $m < n$. The terms, proportional to $(\alpha/\pi)^n L^n$, can be calculated by means of the structure function method [6]. The structure function formalism, based on the renormalization group approach, permits one to keep all leading terms of order $(\alpha L/\pi)^n$, $n = 0, 1, 2\ldots$ explicitly.

In the first order of PT the nonleading terms, proportional to (α/π), can be accounted by means of so–called $\mathcal{K}$–factor, $\mathcal{K} = 1 + (\alpha/\pi)K$. As for nonleading terms of the order $(\alpha/\pi)^2 L$, they can be correctly calculated in the two–loop approximation. We will consider them in further publications [7]. Fortunately, this second order nonleading RC are small: $(\alpha/\pi)^2 L \sim 10^{-4}$. So, the presented formulae guarantee theoretical precision of the order 0.2%. Considering hard photon emission in the first order of PT we distinguish the contributions due to the initial state radiation, the final state radiation, and their interference. The first one always contains large logarithms. The ones due to the final state radiation as well as the initial–final state interference do not enhance the corrections considerably, except the case of Bhabha scattering. In the case of hard photon radiation large logarithms come from kinematical regions, where the photona are emitted along electron and positron beams. We will call this kinematics as the collinear one.

Our work consist in explicit calculations of the Born and the first order RC contributions to differential cross–sections. We apply the structure function method to increase the accuracy.

The paper is organized as follows. In Section 2 we consider $\mu^+\mu^-$ production. Both charge–even and charge–odd contributions to the differential cross–section are evaluated. In Sect. 3 we consider the electron–positron scattering process at large angles. In Sect. 4 we investigate electron–positron annihilation into photons. In the Conclusions we discuss the results obtained and estimate the provided precision. Numerical illustrations are given for a realistic experimental set up.

2 Muon Pair Production

Consider the process

$$e^+(p_+) + e^-(p_-) \to \mu^+(q_+) + \mu^-(q_-).$$

Taking into account the photon and Z–boson intermediate states, the differential cross–section in the Born approximation (in the framework of the Standard Model) has the form:

$$\begin{aligned} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\{1+K_W\}, \qquad \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega} = \frac{\alpha^2\beta}{4s}\Big(2-\beta^2(1-c^2)\Big), \\ \beta &= \sqrt{1-4m_\mu^2/s}, \qquad c = \cos\widehat{\boldsymbol{p}_-\boldsymbol{q}}_-, \qquad s=(p_++p_-)^2 = 4\varepsilon^2, \end{aligned} \tag{1}$$

where the centre–of–mass reference system of the initial beams is applied.

Here K_W (we put the explicit expression for it in the Conclusions) represents contributions due to Z–boson intermediate states (see [8] for example), we will neglect them within the accepted precision: $K_W \sim s/M_Z^2 \lesssim 10^{-3}$. We will drop also all other contributions due to weak interactions in higher orders.

Consider first the even, with respect to the $c \leftrightarrow -c$ permutation, part of the one–loop virtual and soft radiative corrections. Using the known in the literature Dirac and Pauli muon form factors and the known soft photon contributions, we obtain

$$\begin{aligned} \frac{\mathrm{d}\sigma^{B+S+V}_{\mathrm{even}}}{\mathrm{d}\Omega} &= \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\frac{1}{|1-\Pi(s)|^2}\biggl\{1+\frac{2\alpha}{\pi}\biggl[\biggl[L-2+\frac{1+\beta^2}{2\beta}l_\beta\biggr]\ln\frac{\Delta\varepsilon}{\varepsilon} \\ &+ \frac{3}{4}(L-1)+K_{\mathrm{even}}\biggr]\biggr\}, \\ K_{\mathrm{even}} &= \frac{\pi^2}{6}-\frac{5}{4}+\rho\Big(\frac{1+\beta^2}{2\beta}-\frac{1}{2}+\frac{1}{4\beta}\Big)+\ln\frac{1+\beta}{2}\left(\frac{1}{2\beta}+\frac{1+\beta^2}{\beta}\right) \\ &- \frac{1-\beta^2}{2\beta}\frac{l_\beta}{2-\beta^2(1-c^2)}+\frac{1+\beta^2}{2\beta}\biggl[\frac{\pi^2}{6}+2\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right)+\rho\ln\frac{1+\beta}{2\beta^2} \\ &+ 2\ln\frac{1+\beta}{2}\ln\frac{1+\beta}{2\beta^2}\biggr], \\ l_\beta &= \ln\frac{1+\beta}{1-\beta}, \quad \rho=\ln\frac{s}{m_\mu^2} \quad L=\ln\frac{s}{m_e^2}, \quad \mathrm{Li}_2(x) \equiv -\int\limits_0^x\frac{\mathrm{d}t}{t}\ln(1-t). \end{aligned} \tag{2}$$

where $\Delta\varepsilon \ll \varepsilon$ is the maximum energy of soft photon in the centre–of–mass system. $\Pi(s)$ is the vacuum polarization operator including electron, muon, tau–meson and hadron contributions [9] (see Appendix 1).

The odd part of the one–loop correction comes from the interference of Born and box Feynman diagrams and from the interference part of the soft photon emission contribution. It causes the charge asymmetry of the process:

$$\eta = [\mathrm{d}\sigma(c)-\mathrm{d}\sigma(-c)]/[\mathrm{d}\sigma(c)+\mathrm{d}\sigma(-c)] \neq 0. \tag{3}$$

The odd part of the differential cross–section has the following form:

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^{S+V}_{\mathrm{odd}}}{\mathrm{d}\Omega} &= \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\,\frac{2\alpha}{\pi}\Big[2\ln\frac{\Delta\varepsilon}{\varepsilon}\ln\frac{1-\beta c}{1+\beta c}+K_{\mathrm{odd}}\Big], \qquad (4)\\
K_{\mathrm{odd}} &= \frac{1}{2}l_-^2 - L_-(\rho+l_-) + \mathrm{Li}_2\left(\frac{1-\beta^2}{2(1-\beta c)}\right) + \mathrm{Li}_2\left(\frac{\beta^2(1-c^2)}{1+\beta^2-2\beta c}\right)\\
&- \int\limits_0^{1-\beta^2}\frac{\mathrm{d}x}{x}f(x)\Big(1-\frac{x(1+\beta^2-2\beta c)}{(1-\beta c)^2}\Big)^{-\frac{1}{2}} + \frac{1}{2-\beta^2(1-c^2)}\\
&\times \Big\{-\frac{1-2\beta^2+\beta^2c^2}{1+\beta^2-2\beta c}(\rho+l_-) - \frac{1}{4}(1-\beta^2)\Big[l_-^2-2L_-(l_-+\rho)\\
&+ 2\mathrm{Li}_2\left(\frac{1-\beta^2}{2(1-\beta c)}\right)\Big] + \beta c\Big[-\frac{\rho}{2\beta^2} + \left(\frac{\pi^2}{12}+\frac{1}{4}\rho^2\right)\left(1-\frac{1}{\beta}-\frac{\beta}{2}+\frac{1}{2\beta^3}\right)\\
&+ \frac{1}{\beta}(-1-\frac{\beta^2}{2}+\frac{1}{2\beta^2})\Big(\rho\ln\frac{1+\beta}{2} - 2\mathrm{Li}_2\left(\frac{1-\beta}{2}\right) - \mathrm{Li}_2\left(-\frac{1-\beta}{1+\beta}\right)\Big)\\
&- \frac{1}{2}l_-^2 + L_-(\rho+l_-) - \mathrm{Li}_2\left(\frac{1-\beta^2}{2(1-\beta c)}\right)\Big]\Big\} - (c\to -c),\\
f(x) &= \Big(\frac{1}{\sqrt{1-x}}-1\Big)\ln\frac{\sqrt{x}}{2} - \frac{1}{\sqrt{1-x}}\ln\frac{1+\sqrt{1-x}}{2}\,,\\
l_- &= \ln\frac{1-\beta c}{2}\,, \qquad L_- = \ln\Big(1-\frac{1-\beta^2}{2(1-\beta c)}\Big).
\end{aligned}
$$

In the ultra–relativistic limit ($\beta\to 1$) for the even part we obtain:

$$
\begin{aligned}
\left(\frac{\mathrm{d}\sigma^{B+S+V}_{\mathrm{even}}}{\mathrm{d}\Omega}\right)_{\beta\to 1} &= \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\,\frac{1}{|1-\Pi(s)|^2}\Big\{1+\frac{2\alpha}{\pi}\Big[(-2+L+\rho)\ln\frac{\Delta\varepsilon}{\varepsilon}\\
&- 2+\frac{3}{4}L+\frac{3}{4}\rho+\frac{\pi^2}{3}\Big]\Big\}. \qquad (5)
\end{aligned}
$$

For the odd part in this limit we obtained the same as Khriplovich [10]:

$$
\begin{aligned}
\left(\frac{\mathrm{d}\sigma^{S+V}_{\mathrm{odd}}}{\mathrm{d}\Omega}\right)_{\beta\to 1} &= \frac{\alpha^3}{s\pi}\Big\{2(1+c^2)\Big[\ln(\mathrm{ctg}\frac{\theta}{2})\ln\frac{\varepsilon}{\Delta\varepsilon}+\frac{1}{2}\ln^2(\sin\frac{\theta}{2})-\frac{1}{2}\ln^2(\cos\frac{\theta}{2})\\
&- \frac{1}{4}\mathrm{Li}_2(\sin^2\frac{\theta}{2})+\frac{1}{4}\mathrm{Li}_2(\cos^2\frac{\theta}{2})\Big]+\cos^2\frac{\theta}{2}\ln(\sin\frac{\theta}{2})-\sin^2\frac{\theta}{2}\ln(\cos\frac{\theta}{2})\\
&- \cos\theta\Big[\ln^2(\cos\frac{\theta}{2})+\ln^2(\sin\frac{\theta}{2})\Big]\Big\}. \qquad (6)
\end{aligned}
$$

Consider now the process of hard photon emission

$$
e^+(p_+)+e^-(p_-)\to\mu^+(q_+)+\mu^-(q_-)+\gamma(k). \tag{7}
$$

It was studied in detail [13]. The photon energy is assumed to be larger than $\Delta\varepsilon$. The differential cross–section has the form

$$
\mathrm{d}\sigma = \frac{\alpha^3}{2\pi^2s^2}R\,\mathrm{d}\Gamma, \qquad \mathrm{d}\Gamma = \frac{\mathrm{d}^3q_-\mathrm{d}^3q_+\mathrm{d}^3k}{q_-^0q_+^0k^0}\delta^{(4)}(p_++p_--q_--q_+-k), \tag{8}
$$

$$R = \frac{s}{16(4\pi\alpha)^3} \sum_{spins} |M|^2 = R_e + R_\mu + R_{e\mu}\,.$$

The contribution due to the initial state radiation reads

$$\begin{aligned}
R_e &= A_e + B_e, \qquad (9)\\
A_e &= \frac{s}{s_1^2}\biggl\{\frac{s}{\chi_+\chi_-}\left(\frac{1}{2}tt_1 + \frac{1}{2}uu_1 + sm_\mu^2\right) + \frac{1}{\chi_-}(2m_\mu^2\chi_+ - u_1\chi'_+ - t_1\chi'_-)\\
&+ \frac{1}{\chi_+}(2m_\mu^2\chi_- - u\chi'_- - t\chi'_+) - \frac{1}{2}\Bigl(\frac{u_1}{\chi_+} - \frac{t}{\chi_-}\Bigr)(u - t_1) - \frac{1}{2}\Bigl(\frac{t_1}{\chi_+} - \frac{u}{\chi_-}\Bigr)(t - u_1)\\
&- \frac{s}{2\chi_+}\left(2m_\mu^2 + \frac{1}{\chi_-}(2m_\mu^2\chi_+ - u_1\chi'_+ - t_1\chi'_-)\right)\\
&- \frac{s}{2\chi_-}\left(2m_\mu^2 + \frac{1}{\chi_+}(2m_\mu^2\chi_- - u\chi'_- - t\chi'_+)\right)\biggr\},\\
B_e &= -\frac{s}{s_1^2}\biggl\{\left[\frac{m_e^2}{\chi_+^2} + \frac{m_e^2}{\chi_-^2}\right]\left(\frac{1}{2}tt_1 + \frac{1}{2}uu_1 + sm_\mu^2\right) - \frac{m_e^2}{\chi_+^2}(2m_\mu^2\chi_- - u\chi'_- - t\chi'_+)\\
&- \frac{m_e^2}{\chi_-^2}(2m_\mu^2\chi_+ - u_1\chi'_+ - t_1\chi'_-)\biggr\},\\
s &= 2p_+p_-, \quad s_1 = (q_+ + q_-)^2, \quad t = -2p_-q_-, \quad t_1 = -2p_+q_+,\\
u &= -2p_-q_+, \quad u_1 = -2p_+q_-, \quad \chi_\pm = p_\pm k, \quad \chi'_\pm = q_\pm k.
\end{aligned}$$

The final state radiation and the interference of the initial and final state radiation contributions are

$$\begin{aligned}
R_\mu &= A_\mu + B_\mu, \qquad (10)\\
A_\mu &= \frac{1}{s}\biggl\{(tt_1 + uu_1 + 2sm_\mu^2)\frac{q_-q_+}{\chi'_+\chi'_-} - \frac{4m_\mu^2\chi_+\chi_-}{\chi'_+\chi'_-} - \frac{t_1\chi_- + u\chi_+}{\chi'_-} - \frac{u_1\chi_- + t\chi_+}{\chi'_+}\\
&+ \left(\frac{t_1}{2\chi'_+} - \frac{u_1}{2\chi'_-}\right)(t-u) + \left(\frac{u}{2\chi'_+} - \frac{t}{2\chi'_-}\right)(u_1 - t_1)\\
&- \frac{q_+q_-}{\chi'_-\chi'_+}\left[(u_1 + t_1)\chi_- + (u+t)\chi_+\right]\biggr\},\\
B_\mu &= -\frac{tt_1 + uu_1 + 2sm_\mu^2}{2s}\left(\frac{m_\mu^2}{(\chi'_+)^2} + \frac{m_\mu^2}{(\chi'_-)^2}\right) + \frac{1}{s}\Bigl(\frac{m_\mu^2}{(\chi'_-)^2}(t_1\chi_- + u\chi_+)\\
&+ \frac{m_\mu^2}{(\chi'_+)^2}(u_1\chi_- + t\chi_+)\Bigr),
\end{aligned}$$

$$\begin{aligned}
R_{e\mu} &= -\frac{tt_1 + uu_1 + 2sm_\mu^2}{2s_1}\left(\frac{t}{\chi_-\chi'_-} + \frac{t_1}{\chi_+\chi'_+} - \frac{u}{\chi_-\chi'_+} - \frac{u_1}{\chi_+\chi'_-}\right)\\
&- \frac{2}{s_1}\Bigl\{-t - t_1 + u + u_1 - \frac{1}{2}u_1\Bigl(\frac{p_-}{\chi_-} - \frac{q_+}{\chi'_+}\Bigr)(Q\chi'_+ + P\chi_-)\\
&- \frac{1}{2}t_1\Bigl(\frac{p_-}{\chi_-} - \frac{q_-}{\chi'_-}\Bigr)(Q\chi'_- - P\chi_-) - m_\mu^2(\chi_+ + \chi_-)(QP)
\end{aligned}$$

$$
\begin{aligned}
&+ && \left(\frac{m_\mu^2}{\chi'_+} - \frac{m_\mu^2}{\chi'_-}\right)(\chi_- - \chi_+) - \frac{1}{2}t\Big(\frac{q_+}{\chi'_+} - \frac{p_+}{\chi_+}\Big)(Q\chi'_+ - P\chi_+) \\
&- && \frac{1}{2}u\Big(\frac{q_-}{\chi'_-} - \frac{p_+}{\chi_+}\Big)(Q\chi'_- + P\chi_+)\Big\}, \qquad (11) \\
P &= && \frac{p_+}{\chi_+} - \frac{p_-}{\chi_-}, \qquad Q = \frac{q_-}{\chi'_-} - \frac{q_+}{\chi'_+}.
\end{aligned}
$$

Quantity R_e contains collinear and infrared singularities. A_μ and $A_{e\mu}$ have only infrared singularities. B_μ and $B_{e\mu}$ are free from singularities. Quantity R in the ultra–relativistic case is given in Appendix 3.

After algebraic transformations we get

$$
\begin{aligned}
R_e &= \frac{s}{\chi_-\chi_+}B - \frac{m_e^2}{2\chi_-^2}\,\frac{(t_1^2 + u_1^2 + 2m_\mu^2 s_1)}{s_1^2} - \frac{m_e^2}{2\chi_+^2}\,\frac{(t^2 + u^2 + 2m_\mu^2 s_1)}{s_1^2} + \frac{m_\mu^2}{s_1^2}\Delta_{s_1 s_1}\,, \\
R_{e\mu} &= B\Big(\frac{u}{\chi_-\chi'_+} + \frac{u_1}{\chi_+\chi'_-} - \frac{t}{\chi_-\chi'_-} - \frac{t_1}{\chi_+\chi'_+}\Big) + \frac{m_\mu^2}{ss_1}\Delta_{ss_1}\,, \qquad (12) \\
R_\mu &= \frac{s_1}{\chi'_-\chi'_+}B + \frac{m_\mu^2}{s^2}\Delta_{ss}\,, \qquad B = \frac{u^2 + u_1^2 + t^2 + t_1^2}{4ss_1}, \\
\Delta_{s_1 s_1} &= -\frac{(t+u)^2 + (t_1 + u_1)^2}{2\chi_-\chi_+}, \\
\Delta_{ss} &= -\frac{u^2 + t_1^2 + 2sm_\mu^2}{2(\chi'_-)^2} - \frac{u_1^2 + t^2 + 2sm_\mu^2}{2(\chi'_+)^2} + \frac{1}{\chi'_-\chi'_+}(ss_1 - s^2 + tu + t_1u_1 - 2sm_\mu^2), \\
\Delta_{ss_1} &= \frac{s+s_1}{2}\Big(\frac{u}{\chi_-\chi'_+} + \frac{u_1}{\chi_+\chi'_-} - \frac{t}{\chi_-\chi'_-} - \frac{t_1}{\chi_+\chi'_+}\Big) + \frac{2(u-t_1)}{\chi'_-} + \frac{2(u_1 - t)}{\chi'_+}.
\end{aligned}
$$

Note that from these expressions one may in a moment obtain the corresponding matrix element of the cross symmetrical process $e^-\mu^+ \to e^-\mu^+\gamma$. To be rigorous, we have note that in the cross symmetrical channel one has to take into account some additional terms proportional to m_e^2. In our channel they can be shown as follows:

$$
\begin{aligned}
&R_\mu \to R_\mu + \frac{m_e^2}{s^2}\Delta'_{ss}, \\
&\Delta'_{ss} = \frac{2m_\mu^2 s_1}{\chi'_-\chi'_+} + \frac{t+u_1}{\chi'_-} + \frac{t_1+u}{\chi'_+} + \frac{4m_\mu^2}{\chi'_-} + \frac{4m_\mu^2}{\chi'_+}. \qquad (13)
\end{aligned}
$$

We checked the matrix element by a comparison with the one used in `FORTRAN` program [14], describing electron–muon scattering.

The sum of the hard photon contribution, integrated over the photon phase volume with the condition $k^0 > \Delta\varepsilon$, and the contribution due to the soft and virtual photon emission does not depend on the auxiliary parameter $\Delta = \Delta\varepsilon/\varepsilon \ll 1$. The main contribution, proportional to the large logarithm, comes from the integration of R_e in the case of collinear kinematics of photon emission. For definiteness let us consider the case when the photon moves close to the initial electron direction:

$$
\widehat{\boldsymbol{p}_-\boldsymbol{k}} = \theta \le \theta_0 \ll 1, \quad \theta_0 \gg \frac{m_e}{\varepsilon}. \qquad (14)
$$

Here we can use

$$R_e\Big|_{\boldsymbol{k}\|\boldsymbol{p}_-} = \frac{s^2}{s_1^2}\left\{\frac{1+(1-x)^2}{x\chi_-} - \frac{m_e^2}{\chi_-^2}(1-x)\right\}\frac{tt_1+uu_1+2sm_\mu^2}{2}, \tag{15}$$

where x is the energy fraction carried away by the emitted photon, $x = k^0/\varepsilon = 1 - s_1/s$. Performing the integration over the photon emission angles, we can present the corresponding part of the cross–section (a similar contribution of the hard photon emission along the positron is included below also) in the form

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_-}\right)_{\mathrm{coll}} = C + D, \tag{16}$$

$$C = \frac{\alpha}{2\pi}\left(\ln\frac{s}{m_e^2} - 1\right)\int\limits_{\Delta}^{1}\mathrm{d}x\frac{1+(1-x)^2}{x}\left[\frac{\mathrm{d}\tilde{\sigma}_0(1-x,1)}{\mathrm{d}\Omega_-} + \frac{\mathrm{d}\tilde{\sigma}_0(1,1-x)}{\mathrm{d}\Omega_-}\right],$$

$$D = \frac{\alpha}{2\pi}\int\limits_{\Delta}^{1}\mathrm{d}x\left\{x + \frac{1+(1-x)^2}{x}\ln\frac{\theta_0^2}{4}\right\}\left[\frac{\mathrm{d}\tilde{\sigma}_0(1-x,1)}{\mathrm{d}\Omega_-} + \frac{\mathrm{d}\tilde{\sigma}_0(1,1-x)}{\mathrm{d}\Omega_-}\right],$$

where $\mathrm{d}\tilde{\sigma}_0(1-x_1,1-x_2)/\mathrm{d}\Omega_-$ is the so–called *shifted* Born differential cross–section. It describes the process $e^+(p_+(1-x_2)) + e^-(p_-(1-x_1)) \to \mu^+(q_+) + \mu^-(q_-)$,

$$\frac{\mathrm{d}\tilde{\sigma}_0(z_1,z_2)}{\mathrm{d}\Omega_-} = \frac{\alpha^2}{4s}\,\frac{y_1[z_1^2(Y_1-y_1c)^2 + z_2^2(Y_1+y_1c)^2 + 8z_1z_2m_\mu^2/s]}{z_1^3z_2^3[z_1+z_2-(z_1-z_2)cY_1/y_1]}, \tag{17}$$

$$y_{1,2}^2 = Y_{1,2}^2 - \frac{4m_\mu^2}{s}, \quad Y_{1,2} = \frac{q^0_{-,+}}{\varepsilon}, \quad z_{1,2} = 1 - x_{1,2}\,.$$

Using the conservation laws

$$\begin{aligned} &z_1 + z_2 = Y_1 + Y_2, \qquad z_1 - z_2 = y_1c_- + y_2c_+, \\ &y_1\sqrt{1-c_-^2} = y_2\sqrt{1-c_+^2}, \quad c_- \equiv c, \quad c_+ = \cos\widehat{\boldsymbol{p}_-\boldsymbol{q}_+}\,, \end{aligned} \tag{18}$$

we obtain the energy fraction of the created muon

$$\begin{aligned} Y_1 \;&=\; \frac{4m_\mu^2}{s}\,\frac{(z_2-z_1)c}{2z_1z_2 + [4z_1^2z_2^2 - 4(m_\mu^2/s)((z_1+z_2)^2 - (z_1-z_2)^2c^2)]^{1/2}} \\ &+\; \frac{2z_1z_2}{z_1+z_2-c(z_1-z_2)}\,. \end{aligned} \tag{19}$$

Quantity C, after adding the corrections due to soft and virtual photons, turns out to be the lowest order perturbative expansion of the convolution of the structure function $\mathcal{D}$ with the shifted Born differential cross–section. Quantity D plays role of a compensating term. Namely, in the sum with the contribution of the cross–section due to hard ($k^0 > \Delta\varepsilon$) photon emission at angles (with respect to the electron and positron) larger than θ_0 the dependence on the auxiliary parameters will cancel.

Here we remind about experimental conditions of the final particles detection mentioned above. They are to be imposed explicitly by introducing the restriction of the following kind:

$$\Theta(z_1,z_2) = \Theta(Y_1-y_{\mathrm{th}})\Theta(Y_2-y_{\mathrm{th}})\Theta(\cos^2\Psi_0 - c_+^2)\Theta(\cos^2\Psi_0 - c_-^2), \quad c_+ = \widehat{\boldsymbol{p}_+\boldsymbol{q}_+}\,, \tag{20}$$

where $y_{\rm th}\varepsilon = \varepsilon_{\rm th}$ is the threshold of the detectors, angle Ψ_0 determines the *dead* cones, surrounding beam axes, unattainable for detection. More detailed cuts can be implemented in a Monte Carlo program, using the formulae given above.

The leading contributions to the cross–section, containing large logarithm L, as may be recognized, combine to the kernel of Altarelli–Parisi–Lipatov evolution equation:

$$\begin{aligned}
\mathrm{d}\sigma &= \int \mathrm{d}z_1\mathrm{d}z_2\mathcal{D}(z_1)\mathcal{D}(z_2)\frac{\mathrm{d}\tilde\sigma_0(z_1,z_2)}{|1-\Pi(sz_1z_2)|^2}, \qquad (21)\\
\mathcal{D}(z) &= \delta(1-z)+\frac{\alpha}{2\pi}(L-1)P^{(1)}(z)+\left(\frac{\alpha}{2\pi}\right)^2\frac{(L-1)^2}{2!}P^{(2)}(z)+\dots\ ,\\
P^{(1)}(z) &= \lim_{\Delta\to 0}\left\{\delta(1-z)(2\ln\Delta+\frac{3}{2})+\Theta(1-z-\Delta)\frac{1+z^2}{1-z}\right\},\\
P^{(2)}(z) &= \int\limits_z^1\frac{\mathrm{d}t}{t}P^{(1)}(t)P^{(1)}\left(\frac{z}{t}\right), \qquad \int\limits_0^1 \mathrm{d}zP^{(1,2)}(z)=0.
\end{aligned}$$

This formula is valid in the leading logarithmical approximation. We will modify it by including nonleading contributions and using the smoothed representation for structure functions [6]:

$$\begin{aligned}
\mathcal{D}(z,s) &= \mathcal{D}^{\gamma}(z,s)+\mathcal{D}^{e^+e^-}(z,s), \qquad (22)\\
\mathcal{D}^{\gamma}(z,s) &= \frac{1}{2}b\Big(1-z\Big)^{\frac{b}{2}-1}\Big[1+\frac{3}{8}b+\frac{b^2}{16}\Big(\frac{9}{8}-\frac{\pi^2}{3}\Big)\Big]\\
&- \frac{1}{4}b(1+z)+\frac{1}{32}b^2\Big(4(1+z)\ln\frac{1}{1-z}+\frac{1+3z^2}{1-z}\ln\frac{1}{z}-5-z\Big),\\
\mathcal{D}^{e^+e^-}(z,s) &= \frac{1}{2}b\Big(1-z\Big)^{\frac{b}{2}-1}\Big[-\frac{b^2}{288}(2L-15)\Big]\\
&+ \Big(\frac{\alpha}{\pi}\Big)^2\Big[\frac{1}{12(1-z)}\Big(1-z-\frac{2m_e}{\varepsilon}\Big)^{\frac{b}{2}}\Big(\ln\frac{s(1-z)^2}{m_e^2}-\frac{5}{3}\Big)^2\\
&\times \Big(1+z^2+\frac{b}{6}\Big(\ln\frac{s(1-z)^2}{m_e^2}-\frac{5}{3}\Big)\Big)+\frac{1}{4}L^2\Big(\frac{2}{3}\,\frac{1-z^3}{z}+\frac{1}{2}(1-z)\\
&+ (1+z)\ln z\Big)\Big]\Theta(1-z-\frac{2m_e}{\varepsilon}), \qquad b=\frac{2\alpha}{\pi}(L-1).
\end{aligned}$$

In comparison with the corresponding formula in ref. [6] we shifted the terms, arising due to virtual e^+e^- pair production corrections, from $\mathcal{D}^{\gamma}$ into $\mathcal{D}^{e^+e^-}$.

Finally, the differential cross–section can be presented in the form

$$\begin{aligned}
\frac{\mathrm{d}\sigma^{e^+e^-\to\mu^+\mu^-(\gamma)}}{\mathrm{d}\Omega_-} &= \int\limits_{z_{\rm min}}^1\int\limits_{z_{\rm min}}^1 \mathrm{d}z_1\mathrm{d}z_2\,\frac{\mathcal{D}(z_1,s)\mathcal{D}(z_2,s)}{|1-\Pi(sz_1z_2)|^2}\,\frac{\mathrm{d}\tilde\sigma_0(z_1,z_2)}{\mathrm{d}\Omega_-}\Big(1+\frac{\alpha}{\pi}K\Big)\\
&+\left\{\frac{\alpha^3}{2\pi^2s^2}\int\limits_{\substack{k^0>\Delta\varepsilon\\ \widehat{kp_\pm}>\theta_0}}\frac{R_e|_{m_e=0}}{|1-\Pi(s_1)|^2}\,\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega_-}\ +\ \frac{D}{|1-\Pi(s_1)|^2}\right\}
\end{aligned}$$

$$
\begin{aligned}
&+\Bigg\{\frac{\alpha^3}{2\pi^2 s^2}\int\limits_{k^0>\Delta\varepsilon}\Bigg(\mathrm{Re}\,\frac{R_{e\mu}}{(1-\Pi(s_1))(1-\Pi(s))^*}+\frac{R_\mu}{|1-\Pi(s)|^2}\Bigg)\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega_-} \\
&+\mathrm{Re}\,\frac{C_{e\mu}}{(1-\Pi(s_1))(1-\Pi(s))^*}+\frac{C_\mu}{|1-\Pi(s)|^2}\Bigg\}, \qquad (23)\\
&C_\mu=\frac{2\alpha}{\pi}\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega_-}\ln\frac{\Delta\varepsilon}{\varepsilon}\Big(\frac{1+\beta^2}{2\beta}\ln\frac{1+\beta}{1-\beta}-1\Big), \qquad z_{\min}=\frac{2m_\mu}{2\varepsilon-m_\mu}, \\
&C_{e\mu}=\frac{4\alpha}{\pi}\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega_-}\ln\frac{\Delta\varepsilon}{\varepsilon}\,\ln\frac{1-\beta c}{1+\beta c}, \qquad K=K_{\mathrm{odd}}+K_{\mathrm{even}},
\end{aligned}
$$

where D, $C_{e\mu}$ and C_μ are compensating terms, which provide cancellation of auxiliary parameters Δ and θ_0 inside figure brackets. In the first term, containing $\mathcal{D}$ functions, we gather all leading terms. A part of nonleading terms proportional to the Born cross–section is written as the $\mathcal{K}$–factor. The rest nonleading terms are written as two additional terms. The compensating term D (see Eq. (16)) comes from the integration in the collinear region of hard photon emission. Quantities C_μ and $C_{e\mu}$ come from the even and odd parts of the differential cross–section (arising due to soft and virtual corrections), respectively. Here we consider the phase volumes of two ($\mathrm{d}\Omega_-$) and three ($\mathrm{d}\Gamma$) final particles as the ones, which already include all required experimental cuts. Using the conservation laws Eq. (18) and concrete experimental conditions one can define the lower limits of the integration over z_1 and z_2.

There is a peculiar feature in the spectrum of hard photons. Namely, in the end of the spectrum the differential cross–section is proportional to the factor

$$
I(s_1)=\frac{2m_\mu^2+s_1}{s_1^2}\sqrt{1-\frac{4m_\mu^2}{s_1}}\,, \qquad (24)
$$

which defines a peak at $s_1\approx 5.6m_\mu^2$. It comes from the Feynman diagrams describing the emission by the initial particles [15].

3 Large–Angle Bhabha Scattering

The cross–section of Bhabha scattering (corrected by the vacuum polarization factor), which enter into the Drell–Yan form of corrected cross–section, has a bit more complicated form, as far as the scattering and annihilation amplitudes and their interference are to be taken into account. We remind here the form of the Lorentz–invariant matrix element module squared in the Born approximation:

$$
\begin{aligned}
R_0(s,t,u) &= \frac{1}{16(4\pi\alpha)^4}\sum_{\mathrm{spins}}\Big|\mathcal{M}(e^-(p_-)+e^+(p_+)\to e^-(p'_-)+e^+(p'_+))\Big|^2 \\
&= \frac{s^2+u^2}{2t^2}+\frac{u^2+t^2}{2s^2}+\frac{u^2}{st}\,, \qquad (25)\\
s &= (p_-+p_+)^2, \quad t=(p_--p'_-)^2, \quad u=(p_--p'_+)^2, \quad s+t+u=\mathcal{O}(m_e^2).
\end{aligned}
$$

The first term in the right hand side describes the scattering–type Feynman diagram square. The second one corresponds to the square of the annihilation–type diagram. And the third one deals with the interference of the two diagrams. A more compact representation of R_0 is also useful, $R_0 = (1\ +\ s/t\ +\ t/s)^2$. The differential cross–section in the Born approximation has the form

$$\frac{\mathrm{d}\sigma_0^{\mathrm{Born}}}{\mathrm{d}\Omega_-} = \frac{\alpha^2}{4s}\left(\frac{3+c^2}{1-c}\right)^2 . \tag{26}$$

We will need also quantity R for arbitrary energies of initial particles. Suppose that the initial electron and positron lost a certain energy fraction. The corresponding kinematics is defined as follows:

$$\begin{aligned}
& e^-(z_1p_-)\ +\ e^+(z_2p_+)\ \longrightarrow\ e^-(\tilde{p}_-)\ +\ e^+(\tilde{p}_+), \\
& \tilde{s} = sz_1z_2, \qquad \tilde{t} = -\frac{1}{2}sz_1Y_1(1-c), \qquad \tilde{u} = -\frac{1}{2}sz_2Y_1(1+c), \\
& Y_1 = \frac{\tilde{p}_-^0}{\varepsilon} = \frac{2z_1z_2}{a}, \qquad a = z_1+z_2-(z_1-z_2)c.
\end{aligned}$$

Here the *shifted* Born cross–section corrected by vacuum polarization insertions into virtual photon propagators reads

$$\begin{aligned}
\mathrm{d}\tilde{\sigma}_0(z_1,z_2) &= \frac{4\alpha^2}{sa^2}\biggl\{\frac{1}{|1-\Pi(\tilde{t})|^2}\,\frac{a^2+z_2^2(1+c)^2}{2z_1^2(1-c)^2} + \frac{1}{|1-\Pi(\tilde{s})|^2}\,\frac{z_1^2(1-c)^2+z_2^2(1+c)^2}{2a^2} \\
&- \mathrm{Re}\,\frac{1}{(1-\Pi(\tilde{t}))(1-\Pi(\tilde{s}))^*}\,\frac{z_2^2(1+c)^2}{az_1(1-c)}\biggr\}\mathrm{d}\Omega_- \, .
\end{aligned} \tag{27}$$

Rewriting the known results [11, 12] for the cross–section in the Born approximation with one–loop virtual corrections to it and with the other ones arising due to soft photon emission, we obtain

$$\begin{aligned}
\frac{\mathrm{d}\sigma_{B+S+V}}{\mathrm{d}\Omega_-} &= \frac{\mathrm{d}\tilde{\sigma}_0(1,1)}{\mathrm{d}\Omega_-}\biggl\{1+\frac{2\alpha}{\pi}(L-1)\left[2\ln\frac{\Delta\varepsilon}{\varepsilon}+\frac{3}{2}\right] \\
&- \frac{8\alpha}{\pi}\ln(\mathrm{ctg}\frac{\theta}{2})\ln\frac{\Delta\varepsilon}{\varepsilon}+\frac{\alpha}{\pi}K_{SV}\biggr\},
\end{aligned} \tag{28}$$

where

$$\begin{aligned}
K_{SV} &= -1-2\mathrm{Li}_2(\sin^2\frac{\theta}{2})+2\mathrm{Li}_2(\cos^2\frac{\theta}{2})+\frac{1}{(3+c^2)^2}\biggl[\frac{\pi^2}{3}(2c^4-3c^3-15c) \\
&+ 2(2c^4-3c^3+9c^2+3c+21)\ln^2(\sin\frac{\theta}{2})-4(c^4+c^2-2c)\ln^2(\cos\frac{\theta}{2}) \\
&- 4(c^3+4c^2+5c+6)\ln^2(\mathrm{tg}\frac{\theta}{2})+2(c^3-3c^2+7c-5)\ln(\cos\frac{\theta}{2}) \\
&+ (\frac{10}{3}c^3+10c^2+2c+38)\ln(\sin\frac{\theta}{2})\biggr]
\end{aligned} \tag{29}$$

is the part of the $\mathcal{K}$–factor coming from soft and virtual photon corrections,

$$\frac{\mathrm{d}\tilde{\sigma}_0(1,1)}{\mathrm{d}\Omega_-} = \frac{\alpha^2}{s}\biggl\{\frac{5+2c+c^2}{2(1-c)^2|1-\Pi(t)|^2}+\frac{1+c^2}{4|1-\Pi(s)|^2}$$

$$
\begin{aligned}
&- \quad \mathrm{Re}\,\frac{(1+c)^2}{2(1-c)(1-\Pi(t))(1-\Pi(s))^*}\Bigg\}, \qquad\qquad (30)\\
s &= 4\varepsilon^2, \quad t = -s\,\frac{1-c}{2}, \quad u = -s\,\frac{1+c}{2}, \quad c = \cos\theta, \quad \theta = \widehat{\boldsymbol{p}_-\boldsymbol{p}'_-}\,.
\end{aligned}
$$

Quantity $\Delta\varepsilon$ in Eq. (28) is the maximum energy of emitted soft photons. $\Pi(s)$ and $\Pi(t)$ are the vacuum polarization operators in the s and t channels. In the Conclusions we will estimate the contribution of weak interactions.

Consider now the process of hard photon (with the energy $\omega = k^0 > \Delta\varepsilon$) emission

$$
e^+(p_+) \;+\; e^-(p_-) \;\to\; e^+(p'_+) \;+\; e^-(p'_-) \;+\; \gamma(k).
$$

We start with the differential cross–section in the form suggested by F.A. Berends et al. [11] (which is valid for scattering angles being large compared with m_e/ε):

$$
\begin{aligned}
\mathrm{d}\sigma_{\mathrm{hard}} &= \frac{\alpha^3}{2\pi^2 s}\,R_{e\bar{e}\gamma}\,\mathrm{d}\Gamma, \qquad \mathrm{d}\Gamma = \frac{\mathrm{d}^3p'_+\mathrm{d}^3p'_-\mathrm{d}^3k}{\varepsilon'_+\varepsilon'_-k^0}\delta^{(4)}(p_+ + p_- - p'_+ - p'_- - k), \qquad (31)\\
R_{e\bar{e}\gamma} &= \frac{WT}{4} - \frac{m_e^2}{(\chi'_+)^2}\left(\frac{s}{t}+\frac{t}{s}+1\right)^2 - \frac{m_e^2}{(\chi'_-)^2}\left(\frac{s}{t_1}+\frac{t_1}{s}+1\right)^2\\
&- \frac{m_e^2}{\chi_+^2}\left(\frac{s_1}{t}+\frac{t}{s_1}+1\right)^2 - \frac{m_e^2}{\chi_-^2}\left(\frac{s_1}{t_1}+\frac{t_1}{s_1}+1\right)^2,
\end{aligned}
$$

where

$$
\begin{aligned}
W &= \frac{s}{\chi_+\chi_-} + \frac{s_1}{\chi'_+\chi'_-} - \frac{t_1}{\chi'_+\chi_+} - \frac{t}{\chi'_-\chi_-} + \frac{u}{\chi'_+\chi_-} + \frac{u_1}{\chi'_-\chi_+},\\
T &= \frac{ss_1(s^2+s_1^2) + tt_1(t^2+t_1^2) + uu_1(u^2+u_1^2)}{ss_1tt_1},
\end{aligned}
$$

and the invariants are defined as

$$
\begin{aligned}
s &= 2p_-p_+, \quad s_1 = 2p'_-p'_+, \quad t = -2p_-p'_-, \quad t_1 = -2p_+p'_+,\\
u &= -2p_-p'_+, \quad u_1 = -2p_+p'_-, \quad \chi_\pm = kp_\pm, \quad \chi'_\pm = kp'_\pm.
\end{aligned}
$$

It is convenient to extract the contribution of the collinear kinematics. We do that for the following reasons. First, it is natural to separate the region with very a sharp behaviour of the cross–section and to consider it carefully. Second, we keep in mind the idea of the leading logarithm factorization, which is valid in all orders of the perturbation theory. We will evaluate the collinear kinematical regions in two different ways. The first one (the quasireal electron approximation) is suitable for a generalization in order to account higher order leading corrections by means of the structure function method. In this way we will obtain below the leading logarithmic contributions and the compensating terms, which will provide the cancellation of auxiliary parameters. The second one (the direct calculation) is more rigorous, it can be used as a check of the first one. We discuss it in detail in Appendix 2.

To obtain explicit formulae for compensators it is needed to consider four kinematical regions corresponding to hard photon emission inside narrow cones, surrounding the initial and final charged particle momenta. The vertices of the cones are taken in the interaction point. We introduce a small auxiliary parameter θ_0, it should obey the restriction

$$m_e/\sqrt{s} \ll \theta_0 \ll 1. \tag{32}$$

So, we define a collinear kinematical region, as the part of the whole phase space, in which the hard photon is emitted within the cone of θ_0 polar angle with respect to the direction of motion of one of the charged particles.

Using the method of quasireal electrons [16], the matrix element $\mathcal{M}$ (squared and summed up over polarization states) of the process of hard photon emission can be expressed through a *shifted* matrix element of the process without photon emission (see Eq. (4) in [16]):

$$\begin{aligned}
\sum|\mathcal{M}(p_1,k,p_1',\mathcal{X})|^2 &= 4\pi\alpha\left[\frac{1+(1-x)^2}{x(1-x)}\,\frac{1}{kp_1}-\frac{m^2}{(kp_1)^2}\right]\sum|\mathcal{M}_0(p_1-k,p_1',\mathcal{X})|^2, \\
\sum|\mathcal{M}(p_1,p_1',k,\mathcal{X})|^2 &= 4\pi\alpha\left[\frac{y^2+Y^2}{\omega Y}\,\frac{\varepsilon}{kp_1'}-\frac{m^2}{(kp_1')^2}\right]\sum|\mathcal{M}_0(p_1,p_1'+k,\mathcal{X})|^2, \qquad (33)\\
x &= \frac{\omega}{\varepsilon}, \quad p_1^0=\varepsilon, \quad y=\frac{{p_1'}^0}{\varepsilon}, \quad Y=x+y,
\end{aligned}$$

where $\mathcal{X}$ denotes the momenta of non–radiating incoming and outgoing particles in a concrete process. The integration over the phase volume of the emitted photon inside the narrow cone, surrounding its parent charged particle momentum, gives the following factors:

$$\begin{aligned}
&\frac{4\alpha}{16\pi^2}\int\frac{\mathrm{d}^3k}{\omega}\left[\frac{1+(1-x)^2}{x(1-x)}\,\frac{1}{kp_1}-\frac{m^2}{(kp_1)^2}\right] = \frac{\alpha}{2\pi}\,\frac{\mathrm{d}z_1}{z_1}\Big[P_\Theta^{(1)}(z_1)\Big(L-1+\ln\frac{\theta_0^2}{4}\Big) \\
&\qquad +1-z_1\Big], \quad z_1=1-x, \qquad (34)\\
&\frac{4\alpha}{16\pi^2}\int\frac{\mathrm{d}^3k}{\omega}\left[\frac{y^2+Y^2}{xY}\,\frac{1}{kp_1'}-\frac{m^2}{(kp_1')^2}\right] = \frac{\alpha}{2\pi}\,\frac{\mathrm{d}z_3}{z_3}\Big[P_\Theta^{(1)}(z_3)\Big(L-1+\ln\frac{\theta_0^2}{4} \\
&\qquad +2\ln z_3\Big)+1-z_3\Big], \qquad z_3=1-\frac{\omega}{{p_1'}^0+\omega}=1-\frac{x}{Y}\,.
\end{aligned}$$

Note that the terms proportional to $(L-1)$ contain the kernel $P^{(1)}$ (see Eq. (21)) of Altarelli–Parisi–Lipatov evolution equations (more precisely, they contain Θ–part of the nonsinglet kernel):

$$P_\Theta^{(1)}(z) = \frac{1+z^2}{1-z}\Theta(1-z-\Delta).$$

Collecting the contributions of the four collinear regions, we obtain

$$\begin{aligned}
\frac{\mathrm{d}\sigma_{\mathrm{coll}}}{\mathrm{d}\Omega_-} &= \frac{\alpha}{\pi}\int\limits_\Delta^1\frac{\mathrm{d}x}{x}\left\{\left[\left(1-x+\frac{x^2}{2}\right)\left(L-1+\ln\frac{\theta_0^2}{4}+2\ln(1-x)\right)+\frac{x^2}{2}\right]\right.\\
&\times\; 2\,\frac{\mathrm{d}\tilde\sigma_0(1,1)}{\mathrm{d}\Omega_-}+\left[\left(1-x+\frac{x^2}{2}\right)\left(L-1+\ln\frac{\theta_0^2}{4}\right)+\frac{x^2}{2}\right]\\
&\times\;\left.\left[\frac{\mathrm{d}\tilde\sigma_0(1-x,1)}{\mathrm{d}\Omega_-}+\frac{\mathrm{d}\tilde\sigma_0(1,1-x)}{\mathrm{d}\Omega_-}\right]\right\}, \qquad (35)
\end{aligned}$$

where the shifted Born cross–section is defined in Eq. (27).

Adding the contributions of virtual and soft photon emission, we restore the complete kernel. Generalizing the procedure for the case of photon emission by all charged particles, we come to the representation of the cross–section in the leading logarithmic approximation. The final expression for the cross–section therefore has the form

$$\frac{\mathrm{d}\sigma^{e^+e^-\to e^+e^-(\gamma)}}{\mathrm{d}\Omega_-} = \int\limits_{\bar{z}_1}^{1} \mathrm{d}z_1 \int\limits_{\bar{z}_2}^{1} \mathrm{d}z_2\, \mathcal{D}(z_1)\mathcal{D}(z_2) \frac{\mathrm{d}\tilde{\sigma}_0(z_1,z_2)}{\mathrm{d}\Omega_-} \left(1+\frac{\alpha}{\pi}K_{SV}\right)\Theta$$

$$\times \int\limits_{y_{\mathrm{th}}}^{Y_1} \frac{\mathrm{d}y_1}{Y_1} \int\limits_{y_{\mathrm{th}}}^{Y_2} \frac{\mathrm{d}y_2}{Y_2}\, \mathcal{D}(\frac{y_1}{Y_1})\mathcal{D}(\frac{y_2}{Y_2})$$

$$+\frac{\alpha}{\pi}\int\limits_{\Delta}^{1} \frac{\mathrm{d}x}{x} \left\{ \left[\left(1-x+\frac{x^2}{2}\right) \ln\frac{\theta_0^2(1-x)^2}{4} + \frac{x^2}{2}\right] 2\, \frac{\mathrm{d}\sigma_0^{\mathrm{Born}}}{\phantom{\mathrm{d}\Omega}} \right.$$