Gluonia, scalar and hybrid mesons in QCD

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For some experimental guidelines of the next millennium, I review the determinations of the masses, decays and mixings of the gluonia, scalar and hybrid mesons from QCD spectral sum rules and low-energy theorems, and compare them with the lattice.

1. INTRODUCTION

Since the discovery of QCD, it has been emphasized [1] that exotic mesons beyond the standard octet, exist as a consequence of the non-perturbative aspects of quantum chromodynamics (QCD). Since the understanding of the nature of the η' [2], a large amount of theoretical efforts have been furnished in the past and pursued at present for predicting the spectra of the exotics using different QCD-like models such as the flux tube [3] , the bags [4], the quark [5] and constituent gluon [6] models . In this talk, I shall review the present status of the predictions from the QCD spectral sum rules (QSSR) à la SVZ [7] (for a review, see e.g.: [8]) and from some low-energy theorems based on Ward identities, which I will compare with the lattice results.

2. QCD SPECTRAL SUM RULES (QSSR)

2.1. Description of the method

Since its discovery in 79, QSSR has proved to be a powerful method for understanding the hadronic properties in terms of the fundamental QCD parameters such as the QCD coupling α_s, the (running) quark masses and the quark and/or gluon QCD vacuum condensates. The description of the method has been often discussed in the literature, where a pedagogical introduction can be, for instance, found in the book [8]. In practice (like also the lattice), one starts the analysis from the two-point correlator:

$$\psi_H(q^2) \equiv i \int d^4x \; e^{iqx} \; \langle 0|\mathcal{T} J_H(x)\,(J_H(0))^\dagger\,|0\rangle \;, \tag{1}$$

built from the hadronic local currents $J_H(x)$, which select some specific quantum numbers. However, unlike the lattice which evaluates the correlator in the Minkowski space-time, one exploits, in the sum rule approaches, the analyticity property of the correlator which obeys the well-known Källen–Lehmann dispersion relation:

$$\psi_H(q^2) = \int_0^\infty \frac{dt}{t - q^2 - i\epsilon}\,\frac{1}{\pi}\,\mathrm{Im}\psi_H(t) \;+ ..., \tag{2}$$

where ... represent subtraction points, which are polynomials in the q^2-variable. In this way, the *sum rule* expresses in a clear way the *duality* between the integral involving the spectral function $\text{Im}\psi_H(t)$ (which can be measured experimentally), and the full correlator $\psi_H(q^2)$. The latter can be calculated directly in the QCD Euclidean space-time using perturbation theory (provided that $-q^2+m^2$ (m being the quark mass) is much greater than Λ^2), and the Wilson expansion in terms of the increasing dimensions of the quark and/or gluon condensates which simulate the non-perturbative effects of QCD.

2.2. Beyond the usual SVZ expansion

Using the Operator Product Expansion (OPE) [7], the two-point correlator reads:

$$\psi_H(q^2) \simeq \sum_{D=0,2,4,...} \frac{1}{(-q^2)^{D/2}} \sum_{dim O=D} C(q^2,\nu)\langle\mathcal{O}(\nu)\rangle \ , \tag{3}$$

where ν is an arbitrary scale that separates the long- and short-distance dynamics; C are the Wilson coefficients calculable in perturbative QCD by means of Feynman diagrams techniques; $\langle\mathcal{O}(\nu)\rangle$ are the quark and/or gluon condensates of dimension D. In this paper, we work in the massless quark limit. Then, one may expect the absence of the terms of dimension 2 due to gauge invariance. However, it has been emphasized recently [9] that the resummation of the large order terms of the perturbative series, and the effects of the higher dimension condensates due e.g. to instantons, can be mimiced by the effect of a tachyonic gluon mass, which might be understood from the short distance linear part of the QCD potential. The strength of this short distance mass has been estimated from the e^+e^- data to be [10,11]: $\frac{\alpha_s}{\pi}\lambda^2 \simeq -(0.06 \sim 0.07)$ GeV^2, which leads to the value of the square of the (short distance) string tension: $\sigma \simeq -\frac{2}{3}\alpha_s\lambda^2 \simeq [(400\pm 20)\ \text{MeV}]^2$ in an (unexpected) good agreement with the lattice result [12] of about $[(440\pm 38)\ \text{MeV}]^2$. The strengths of the vacuum condensates having dimensions $D \leq 6$ are also under good control: namely $2m\langle\bar{q}q\rangle = -m_\pi^2 f_\pi^2$ from pion PCAC, $\langle\alpha_s G^2\rangle = (0.07\pm 0.01)$ GeV^2 from $e^+e^- \to I=1$ data [10] and from the heavy quark mass-splittings [13], $\alpha_s\langle\bar{q}q\rangle^2 \simeq 5.8\times 10^{-4}$ GeV^6 [10], and $g^3\langle G^3\rangle \approx 1.2$ GeV$^2\langle\alpha_s G^2\rangle$ from dilute gaz instantons [14].

2.3. Spectral function

The spectral function is often parametrized using the “naïve” duality ansatz:

$$\frac{1}{\pi}\,\text{Im}\psi_H(t) \simeq 2M_H^{2n} f_H^2 \delta(t-M_H^2) + \text{“QCD continuum”} \times \theta(\text{t}-\text{t}_\text{c}) \ , \tag{4}$$

which has been tested [8] using e^+e^-, τ-decay data, to give a good description of the spectral integral in the sum rule analysis; f_H (analogue to f_π) is the the hadron's coupling to the current ; $2n$ is the dimension of the correlator, while t_c is the QCD continuum's threshold.

2.4. Form of the sum rules and optimization procedure

Among the different sum rules discussed in the literature [8] (Finite Energy Sum rule (FESR) [15], τ-like sum rules [16],...), we shall mainly be concerned here with:

• The exponential Laplace unsubtracted sum rule (USR) and its ratio:

$$\mathcal{L}_n(\tau) = \int_0^\infty dt\ t^n \exp(-t\tau)\ \frac{1}{\pi}\ \text{Im}\psi_H(t) \ , \qquad \mathcal{R}_n \equiv -\frac{d}{d\tau}\log\mathcal{L}_n \ , \qquad (n\geq 0)\ ; \tag{5}$$

• The subtracted sum rule (SSR):

$$\mathcal{L}_{-1}(\tau) = \int_0^\infty \frac{dt}{t}\,\exp(-t\tau)\,\frac{1}{\pi}\,\mathrm{Im}\psi_H(t) + \psi_H(0)\ . \tag{6}$$

The advantage of the Laplace sum rules with respect to the previous dispersion relation is the presence of the exponential weight factor, which enhances the contribution of the lowest resonance and low-energy region accessible experimentally. For the QCD side, this procedure has eliminated the ambiguity carried by subtraction constants, arbitrary polynomial in q^2, and has improved the convergence of the OPE by the presence of the factorial dumping factor for each condensates of given dimensions. The ratio of the sum rules is a useful quantity to work with, in the determination of the resonance mass, as it is equal to the meson mass squared, in the usual duality ansatz parametrization. As one can notice, there are "a priori" two free external parameters (τ, t_c) in the analysis. The optimized result will be (in principle) insensitive to their variations. In some cases, the t_c-stability is not reached due to the too naïve parametrization of the spectral function. One can either fixed the t_c-values by the help of FESR (local duality) or improve the parametrization of the spectral function by introducing threshold effects fixed by chiral perturbation theory, ..., in order to restore the t_c-stability of the results. The results discussed below satisfy these stability criteria.

3. UNMIXED GLUONIA CURRENTS, MASSES AND COUPLINGS

3.1. The currents

In this paper, we shall consider the lowest-dimension gluonic currents that can be built from the gluon fields $G^a_{\alpha\beta}$ and which are gauge-invariant:

$$\begin{aligned} \theta^\mu_\mu &= \beta(\alpha_s) G_{\alpha\beta}G^{\alpha\beta} + \sum_{u,d,s} m_q \bar{q}q\ , \qquad \theta_{\mu\nu} = -G^\alpha_\mu G_{\nu\alpha} + \frac{1}{4} g_{\mu\nu} G_{\alpha\beta}G^{\alpha\beta}, \\ \partial_\mu A^\mu(x) &= \left(\frac{\alpha_s}{8\pi}\right) \mathrm{tr}\ G_{\alpha\beta}\tilde{G}^{\alpha\beta} + \sum_{u,d,s} m_q \bar{q}(i\gamma_5)q\ , \qquad J_3 = g f_{abc} G^a_{\alpha\beta} G^b_{\beta,\gamma} G^c_{\gamma\alpha}\ , \end{aligned} \tag{7}$$

where the sum over colour is understood; θ^μ_μ is the trace of the energy-momentum tensor $\theta_{\mu\nu}$; $\partial_\mu A^\mu(x)$ is the $U(1)_A$ anomaly; m_q is the light quark mass and β is the β-function.

3.2. Masses and couplings

The unmixed gluonia masses from the unsubtracted QCD Spectral Sum Rules (USR) [17] are compared in Table 1 with the ones from the lattice [12,18] in the quenched approximation, where we use the conservative guessed estimate of about 15% for the different lattice systematic errors (separation of the lowest ground states from the radial excitations, which are expected to be nearby as indicated by the sum rule analysis; discretisation; quenched approximation,...). One can notice an excellent agreement between the USR and the lattice quenched results, which the mass hierarchy: $M_{0^{++}} \leq M_{0^{-+}} \approx M_{2^{++}}$, expected from some QCD inequalities [19]. However, this is not the whole story !

Table 1
Unmixed gluonia masses and couplings from QSSR [17] compared with the lattice.

J^{PC}	Name	Mass [GeV]			$\sqrt{t_c}$ [GeV]	f_H [MeV]
		Estimate	Upper Bound	Lattice [12,18]		
0^{++}	G	1.5 ± 0.2	2.16 ± 0.22	1.60 ± 0.16	2.1	390 ± 145
	$3G$	3.1	3.7		3.4	62
2^{++}	T	2.0 ± 0.1	2.7 ± 0.4	2.26 ± 0.22	2.2	80 ± 14
0^{-+}	P	2.05 ± 0.19	2.34 ± 0.42	2.19 ± 0.32	2.2	$8 \sim 17$

4. PSEUDOSCALAR GLUONIA

4.1. Testing the nature of the E/ι

We test the gluonic nature of the E/ι by determining its decay constant f_ι, from a saturation of the USR (Eq.5) and SSR (Eq.6) pseudoscalar sum rules by the η', E/ι and the gluonium P. One obtains [17] a value of f_ι consistent with zero, in agreement with the estimate from the $J/\psi \rightarrow \gamma + E/\iota$ decays ($f_\iota \approx 7$ MeV), and smaller than $f'_\eta \approx 30$ MeV [20] and $f_P \approx (8 \sim 17)$ MeV. The quarkonium-gluonium *mass mixing angle* is determined to be small ($\theta_P \approx 12°$), from the off-diagonal two-point correlator [21,8] (see also [22]):

$$\psi_{qg}(q^2) \equiv i \int d^4x \ e^{iqx} \langle 0|\mathcal{T} \left(\frac{\alpha_s}{8\pi}\right) \mathrm{tr} \ G_{\alpha\beta}\tilde{G}^{\alpha\beta}(x) \sum_{u,d,s} m_q \bar{q}(i\gamma_5)q \ (0)|0\rangle \ , \tag{8}$$

which, then, suggests a small mass shift of the physical states after mixing.
We may conclude that the E/ι is likely the radial excitation of the η or/and η'.

4.2. Radiative decays of the pseudoscalar gluonium P

Using $|\theta_P| \approx 12°$, and the OZI rule, one can predict [21,8,17]:

$$\Gamma(P \rightarrow \gamma\gamma) \approx (1.3 \pm 0.1) \ \mathrm{keV} \ , \qquad \Gamma(\mathrm{P} \rightarrow \rho\gamma) \approx (.3 \pm 0.1) \ \mathrm{keV} \ , \tag{9}$$

where the errors are probably underestimated. This result is testable at BES.

5. TENSOR GLUONIA

5.1. Spectrum

From Table 1, the lowest ground state and the radial excitations of about $\sqrt{t_c}$ are almost degenerated in masses, which suggests a rich population of the 2^{++} gluonia around 2 GeV. Though the $\zeta(2.2)$ is a good gluonium candidate [23], it may not be the lowest ground state. A complete analysis needs systematic scannings of the region above 1.9 GeV and further tests of the old BNL candidates g_T.

5.2. Quarkonium-gluonium mass mixing

A QSSR analysis of the tensor correlator [24,8], similar to the one in Eq.(8), leads to a small quarkonium-gluonium *mass mixing* angle of about $-10°$.

5.3. Tensor T decays

One starts from the universality of the vertex form factor [25]:

$$\langle\pi(p)|\theta_{\mu\nu}|\pi(p')\rangle \simeq (\text{Lorentz structure}) \times 1, \quad \text{at} \quad q^2 \equiv (p-p')^2 = 0, \tag{10}$$

and write a dispersion relation. Using the $f_2 \to \pi\pi$ data, one can deduce [8,24]:

$$\Gamma(T \to \pi\pi + KK + \eta\eta) \leq (119 \pm 36)\ \text{MeV}\ , \qquad \Gamma(T \to \pi\pi) \approx 10\ \text{MeV} \leq 70\ \text{MeV}\ , \tag{11}$$

in agreement with present data [26]. A non-relativistic relation between the 0^{++} and the 2^{++} wave functions gives the width:

$$\Gamma(T \to \gamma\gamma) \approx 0.06\ \text{keV}\ . \tag{12}$$

6. UNMIXED SCALAR GLUONIA

6.1. The need for a low mass σ_B from the sum rules

Using the mass and coupling of the scalar gluonium G in Table 1 from the USR (Eq.5), into the SSR (Eq.6) sum rules, where [14] $\psi_s(0) \simeq -16(\beta_1/\pi)\langle\alpha_s G^2\rangle$, one can notice [27,17] that one needs a low mass resonance σ_B for a consistency of the two sum rules. Using $M_{\sigma_B} \simeq 1$ GeV, one gets [27,17]: $f_{\sigma_B} \approx 1$ GeV, which is larger than $f_G \simeq .4$ GeV.

6.2. Low-energy theorems (LET) for the couplings to meson pairs

In order to estimate the couplings of the gluonium to meson pairs, we use some sets of low-energy theorems (LET) based on Ward identities for the vertex:

$$V(q^2 \equiv (p-p')^2 = 0) \equiv \langle H(p)|\theta^\mu_\mu|H(p')\rangle \simeq 2m_H^2\ , \qquad \text{and} \qquad \mathrm{V}'(0) = 1\ , \tag{13}$$

and write the vertex in a dispersive form. H can be a Goldstone boson (π, K, η_8), a η_1-$U(1)_A$-singlet , or a σ_B. Then, one obtains the sum rules for the hadronic couplings:

$$\frac{1}{4}\sum_{\sigma_B,\sigma'_B,G} g_{SHH}\sqrt{2}f_S \approx 2M_H^2\ , \qquad\qquad \frac{1}{4}\sum_{\sigma_B,\sigma'_B,G} g_{SHH}\sqrt{2}f_S/M_S^2 \simeq 1\ . \tag{14}$$

• Neglecting, to a first approximation the G-contribution, the σ_B and σ'_B widths to $\pi\pi,\ KK, ...$ (we take $M_{\sigma'} \approx 1.37$ GeV as an illustration) are [17]:

$$\Gamma(\sigma_B \to \pi\pi) \approx 0.8\ \text{GeV}\ , \qquad\qquad \Gamma(\sigma'_B \to \pi\pi) \approx 2\ \text{GeV}\ , \tag{15}$$

which suggests a huge OZI violation and seriously questions the validity of the lattice results in the quenched approximation. Similar conclusions have been reached in [28–30]. For testing the above result, one should evaluate on the lattice, the decay mixing 3-point function $V(0)$ responsible for such decays using dynamical fermions.

• The previous LET implies the *characteristic gluonium decay* (we use $M_G \approx 1.5$ GeV and assume a G-dominance) [27,8]:

$$\Gamma(G \to \eta\eta') \approx (5-10)\ \text{MeV}\ , \qquad \frac{\Gamma(G \to \eta\eta)}{\Gamma(G \to \eta\eta')} \approx 0.22 \quad : \quad g_{G\eta\eta} \simeq \sin\theta_P\ g_{G\eta\eta'}\ . \tag{16}$$

• Assuming that the G decay into $4\pi^0$ occurs through $\sigma_B\sigma_B$, and using the data for $f_0(1.37) \to (4\pi^0)_S$, one obtains [27,8]:

$$\Gamma(G \to \sigma_B\sigma_B \to 4\pi) \approx (60-140)\ \text{MeV}\ . \tag{17}$$

6.3. $\gamma\gamma$ widths and $J/\psi \to \gamma S$ radiative decays

These widths can be estimated from the quark box or anomaly diagrams [27,8]. The $\gamma\gamma$ widths of the σ, σ' and G are much smaller (factor 2 to 5) than $\Gamma(\eta' \to \gamma\gamma) \simeq 4$ keV, while $B(J/\psi \to \gamma\ \sigma, \sigma'$ and $G)$ is about 10 times smaller than $B(J/\psi \to \gamma\eta') \approx 4\ 10^{-3}$. These are typical values of gluonia widths and production rates [31].

7. UNMIXED SCALAR QUARKONIA

7.1. The $a_0(980)$

The $a_0(980)$ is the most natural meson candidate associated to the divergence of the vector current: $\partial_\mu V^\mu(x) \equiv (m_u - m_d)\bar{u}(i\gamma_5)d$. Previous different sum rule analysis of the associated two-point correlator gives [8]: $M_{a_0} \simeq 1$ GeV and the conservative range $f_{a_0} \simeq (0.5 - 1.6)$ MeV $\;(f_\pi = 93$ $MeV)$, in agreement with the value 1.8 MeV from a hadronic kaon tadpole mass difference approach plus a a_0 dominance of the $K\bar{K}$ form factor, and includes the recent sum rule determination [32]. A 3-point function sum rule analysis gives the widths [33,34,8]:

$$\Gamma(a_0 \to \eta\pi) \simeq 37 \text{ MeV} , \qquad \Gamma(a_0 \to \gamma\gamma) \simeq (0.3 - 1.5) \text{ keV} , \tag{18}$$

while from $SU(3)$ symmetry, we expect to have: $g_{a_0K^+K^0} \simeq \sqrt{\frac{3}{2}} g_{a_0\eta\pi}$. Analogous sum rule analysis in the four-quark scheme [34,8] gives similar values of the masses and hadronic couplings but implies a too small value of the $\gamma\gamma$ width due to the standard QCD loop-diagram factor suppressions. The $(\bar{u}u - \bar{d}d)$ quark assignment for the $a_0(980)$ is supported by present data and alternative approaches [28].

7.2. The isoscalar partner $S_2 \equiv \bar{u}u + \bar{d}d$ of the $a_0(980)$

Analogous analysis of the corresponding 2-point correlator gives $M_{S_2} \approx M_{a_0}$ as expected from a good $SU(2)$ symmetry, while its widths are estimated to be [17,33]:

$$\Gamma(S_2 \to \pi^+\pi^-) \simeq 120 \text{ MeV} , \qquad \Gamma(S_2 \to \gamma\gamma) \simeq \frac{25}{9}\Gamma(a_0 \to \gamma\gamma) \simeq 0.7 \text{ keV} . \tag{19}$$

7.3. The $K_0^*(1430) \equiv \bar{u}s$ and $S_3 \equiv \bar{s}s$ states

The $K_0^*(1430)$ is the natural partner of the $a_0(980)$, where their mass shift is due to $SU(3)$ breakings [8]. An analysis of the S_3 over the K_0^* 2-point functions gives [17]:

$$M_{S_3}/M_{K_0^*} \simeq 1.03 \pm 0.02 \quad \Longrightarrow \quad M_{S_3} \simeq 1474 \text{ MeV} , \qquad f_{S_3} \simeq (43 \pm 19) \text{ MeV} , \tag{20}$$

in agreement with the lattice result [35], while the 3-point function leads to [17]:

$$\Gamma(S_3 \to K^+K^-) \simeq (73 \pm 27) \text{ MeV} , \qquad \Gamma(S_3 \to \gamma\gamma) \simeq 0.4 \text{ keV} . \tag{21}$$

In the usual sum rule approach, and in the absence of large violations of the OPE at the sum rule stability points, one expects a small mixing between the S_2 and S_3 mesons before the mixing with the gluonium σ_B.

7.4. Radial excitations

The properties of the radial excitations cannot be obtained accurately from the sum rule approach, as they are part of the QCD continuum which effects are minimized in the analysis. However, as a crude approximation and using the sum rule results from the well-known channels (ρ,...), one may expect that the value of $\sqrt{t_c}$ can localize approximately the position of the first radial excitations. Using this result and some standard phenomenological arguments on the estimate of the couplings, one may expect [17]:

$$M_{S_2'} \approx 1.3 \text{ GeV}, \quad \Gamma(\mathrm{S}_2' \rightarrow \pi^+\pi^-) \approx (300 \pm 150) \text{ MeV}, \quad \Gamma(\mathrm{S}_2' \rightarrow \gamma\gamma) \approx (4 \pm 2) \text{ keV},$$
$$M_{S_3'} \approx 1.7 \text{ GeV}, \quad \Gamma(S_3' \rightarrow K^+K^-) \approx (112 \pm 50) \text{ MeV}, \quad \Gamma(S_3' \rightarrow \gamma\gamma) \approx (1 \pm .5) \text{ keV}. \quad (22)$$

7.5. We conclude that:

• Unmixed scalar quarkonia ground states are not wide, which excludes the interpretation of the low mass broad σ for being an ordinary $\bar{q}q$ state.
• There can be many states in the region around 1.3 GeV (σ', S_3, S_2'),which should mix non-trivially in order to give the observed $f_0(1.37)$ and $f_0(1.5)$ states (see next sections).
• The $f_J(1.7)$ seen to decay mainly into $\bar{K}K$ [36], if it is confirmed to be a 0^{++} state, can be *the first radial excitation of the $S_3 \equiv \bar{s}s$ state*, but *definitely not a pure gluonium.*

8. SCALAR MIXING-OLOGY

8.1. Mixing below 1 GeV and nature of the $\sigma(1000)$ and $f_0(980)$

We consider the two-component mixing scheme of the bare states (σ_B, S_2):

$$|f_0\rangle \equiv -\sin\theta_s|\sigma_B\rangle + \cos\theta_s|S_2\rangle, \qquad |\sigma\rangle \equiv \cos\theta_s|\sigma_B\rangle + \sin\theta_s|S_2\rangle \quad (23)$$

A sum rule analysis of the off-diagonal 2-point correlator [37,8,17]:

$$\psi^S_{qg}(q^2) \equiv i\int d^4x \; e^{iqx}\langle 0|\mathcal{T}\beta(\alpha_s)G^2(x)\sum_{u,d,s} m_q\bar{q}q \;(0)|0\rangle, \quad (24)$$

responsible for the mass-shift of the mixed states gives a small *mass mixing angle* of about 15°, which has been confirmed by lattice calculations using different unput for the masses [38] and from the low-energy theorems based on Ward identities of broken scale invariance [39], if one uses there the new input values [8,10] of the quark and gluon condensates. In order to have more complete discussions on the gluon content of the different states, one should also determine the *decay mixing angle*. In so doing, we use the predictions for σ_B, $S_2 \rightarrow \gamma\gamma$ obtained in the previous sections and the data $\Gamma(f_0 \rightarrow \gamma\gamma) \approx 0.3$ keV. Then, we deduce a *maximal decay mixing angle* and the widths [33,8]:

$$\begin{aligned} |\theta_s| &\simeq (40-45)^\circ, \\ \Gamma(f_0 \rightarrow \pi^+\pi^-) &\leq 134 \text{ MeV}, & g_{f_0K^+K^-}/g_{f_0\pi^+\pi^-} \approx 2, \\ \Gamma(\sigma \rightarrow \pi^+\pi^-) &\approx (300-700) \text{ MeV}, & \Gamma(\sigma \rightarrow \gamma\gamma) \approx (0.2-0.5) \text{ keV}. \end{aligned} \quad (25)$$

The huge coupling of the f_0 to $\bar{K}K$ comes from the large mixing with the σ. For this reason, the f_0 can have a large singlet component, as also suggested from independent analysis [28,29]. Extending the previous $J/\psi \rightarrow \gamma + X$ analysis into the case of the ϕ, one obtains the *new result* within this scheme [40]:

$$Br[\phi \rightarrow \gamma + f_0(980)] \approx 1.3 \times 10^{-4}, \quad (26)$$

in good agreement with the Novosibirsk data of $(1.93 \pm 0.46 \pm 0.5) \times 10^{-4}$.

8.2. Mixing above 1 GeV and nature of the $f_0(1.37)$, $f_0(1.5)$, $f_0(1.6)$ and $f_0(1.7)$

As already mentioned previously, this region is quite complicated due to the proliferation of states. Many scenarios have been proposed in the literature for trying to interpret this region [41,17,28,29,38]. However, one needs to clarify and to confirm the data [42] for selecting these different interpretations. We shall give below *some selection rules* which can already eliminate some of these different schemes:

• **The** $f_0(1.37)$ decay into $\sigma\sigma \to (4\pi^0)_S$ signals mixings with the σ, σ' and G.

• **The** $f_0(1.5)$ **of Crystal Barrel and the** $f_0(1.6)$ **of GAMS** are different objects:

– Both states like to go into $\sigma\sigma$ and $\eta'\eta$, which signals a gluon component.

– The $f_0(1.5)$ coupled to $\pi\pi$ and $\bar{K}K$ signals a $\bar{q}q$ component which may come from the S_2', S_3. Then, it can result from the $\bar{q}q$ mixings with the σ, σ' and G, like the $f_0(1.37)$.

– The $f_0(1.6)$ couples weakly to $\pi\pi$ and $\bar{K}K$, while the ratio of its $\eta\eta$ and $\eta'\eta$ is proportional to $1/\sin^2\theta_P$. These features indicate that it is an almost pure gluonium state, which can be identified with the G in Table 1 obtained in the quenched approximation (this approximation is expected to better at higher energies using $1/N_c$ arguments [27,43]).

• **The** $f_0(1.7)$ is the radial excitation S_3' of the $S_3(\bar{s}s)$ state (see section 7).

9. LIGHT 1^{-+} HYBRIDS

The experimental situation has been discussed in [44]. The sum rule analysis of the spectrum is based on the 2-point correlator $\psi(q^2)_H$ associated to the hybrid currents:

$$\mathcal{O}^V_\mu(x) \equiv: g\bar{\psi}_i\lambda_a\gamma_\nu\psi_j G^{\mu\nu}_a : \ , \qquad \mathcal{O}^A_\mu(x) \equiv: g\bar{\psi}_i\lambda_a\gamma_\nu\gamma_5\psi_j G^{\mu\nu}_a : \tag{27}$$

which shows that the lowest state is the 1^{-+}.

• An analysis of the $\tilde{\rho}(1^{-+})$ mass and coupling have been done in the past by different groups [45,46], where (unfortunately) the non-trivial QCD expressions were wrong leading to some controversial predictions [8]. The final correct QCD expression is given in [47]. The analysis has been extended recently taking into account the effect of a non-trivial $1/q^2$ term in the OPE. One obtains the preliminary results [48]:

$$M_{\tilde{\rho}} \approx 1.6 \text{ GeV} \ , \qquad f_{\tilde{\rho}} \approx (25 \sim 50) \text{ MeV} \ , \qquad (M_{\tilde{\rho}'} \approx \sqrt{t_c}) - M_{\tilde{\rho}} \approx 200 \text{ MeV} \ , \tag{28}$$

where the $\tilde{\rho}'$ is the radial excitation. One can consider this result as an improvement of the available sum rule results ranging from 1.4 to 2.1 GeV, [8], though, we cannot absolutely exclude the presence of the 1.4 GeV candidate [44]. The $\tilde{\rho}'$-$\tilde{\rho}$ mass-splitting is much smaller than $M_{\rho'} - M_\rho \simeq 700$ MeV, and can signal a rich population of 1^{-+} states above 1.6 GeV. The hadronic widths have been computed in [47,49], with the values:

$$\begin{aligned} \Gamma(\tilde{\rho} \to \rho\pi) &\approx 274 \text{ MeV} \ , & \Gamma(\tilde{\rho} \to \gamma\pi) &\approx 3 \text{ MeV} \ , \\ \Gamma(\tilde{\rho} \to \eta'\pi) &\approx 3 \text{ MeV} \ , & \Gamma(\tilde{\rho} \to \pi\pi,\ \bar{K}K,\ \eta_8\eta_8) &\approx \mathcal{O}(m_q^2) \ . \end{aligned} \tag{29}$$

• These new [48] and old [8] values of $(M_{\tilde{\rho}},\ f_{\tilde{\rho}})$, indicate that the constraint [45]:

$$\psi_H(0) \approx \frac{16\pi}{9}\alpha_s\langle\bar{\psi}\psi\rangle^2 \ , \tag{30}$$

is quite inaccurate as it underestimates the value of $\psi_H(0) \simeq 2f^2_{\tilde{\rho}}M^4_{\tilde{\rho}}$ by a factor 10. An independent measurement of this quantity, e.g. on the lattice, can be useful.

• One can measure the $SU(3)$ breakings and the mass of the $\tilde{\phi}(\bar{s}s)$ from the difference of the ratio of moments, which gives:

$$M^2_{\tilde{\phi}} - M^2_{\tilde{\rho}} \simeq \frac{20}{3}\overline{m}^2_s - \frac{160\pi^2}{9} m_s \langle \bar{s}s \rangle \tau \approx 0.3 \text{ GeV}^2 \quad \Longrightarrow \quad M_{\tilde{\phi}} \approx 1.7 \text{ GeV} \ , \tag{31}$$

a value which is in the range of the lattice results of (1.7 ∼ 2.2) GeV [50].

10. HEAVY 1^{-+} HYBRIDS

The sum rule predictions for the masses are [51,52]:

$$M_{\tilde{\psi}(\bar{c}cg)} \simeq 4.1 \text{ GeV} \ , \qquad \mathrm{M}_{\tilde{\Upsilon}(\bar{\mathrm{b}}\mathrm{bg})} \simeq 10.6 \text{ GeV} \ , \qquad \mathrm{M}_{\tilde{\mathrm{B^*}}(\bar{\mathrm{b}}\mathrm{ug})} \simeq 6.3 \text{ GeV} \ , \tag{32}$$

where the two former agree (within the 10% systematics of the two methods) with the available lattice values [53] of about 4.4 GeV [resp. 11 GeV], and are above the ψ-π [resp. Υ-π] thresholds. A check of these results is in progress [48]. Using the $1/M_b$-expansion sum rule, the $\tilde{B^*}(\bar{b}ug)$ decays are found to be [52]:

$$\Gamma(\tilde{B^*} \rightarrow B_1\pi) \approx 250 \text{ MeV} \ , \qquad \Gamma(\tilde{\mathrm{B}^*} \rightarrow \sum_{\mathrm{X}\neq\mathrm{B}_1} \mathrm{X}\pi) \approx 50 \ \text{ MeV} \ . \tag{33}$$

11. CONCLUSIONS

There are some progresses in the long run study and search for the exotics. Before some definite conclusions, one still needs improvements of the present data, and some lattice unquenched estimates of the mixing angles and widths which should complement the QCD spectral sum rule (QSSR) and low-energy theorem (LET) results. However, one can already notice that the simple $\bar{q}q$-scheme combined with (large) mixings with gluonia (the σ,...) can explain the complex 0^{++} data. It is a pleasure to thank the organizers of Hadron 99 for their invitation to present this review in this exotic country !

REFERENCES

1. M. Gell-Mann, *Acta Phys. Aust. Suppl* **9** (1972) 733; H. Fritzsch and M. Gell-Mann, *XVI Int. Conf. High-Energy Phys.*, Chicago, **Vol 2** (1972) 135.
2. E. Witten, *Nucl. Phys.* B156 (1979) 269; G. Veneziano, *Nucl. Phys.* B159 (1979) 213.
3. N. Isgur and J.E. Paton, *Phys. Rev.* **D31** (1985) 1910; T. Barnes, F.E. Close, P.R. Page and E. Swanson, *Phys. Rev.* **D55** (1997) 4157 and references therein.
4. R.L. Jaffe, *Phys. Rev.* **D15**(1977)267; M. Chanowitz and S. Sharpe, *Nucl. Phys.* B222 (1983) 211; T. Barnes, F.E. Close and F. de Viron, *Nucl. Phys.* B224 (1983) 241.
5. S. Godfrey and N. Isgur, *Phys. Rev.* **D32** (1985) 189; A. Le Yaounac et al., *Phys. Rev.* **D8** (1985) 2223 (1973); N. Tornquist, *Ann. Phys.* **123** (1979) 1.
6. E. Swanson, QCD 99 (Montpellier) and references therein.
7. M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B147** (1979) 385, 448.
8. S. Narison, Lecture Notes in Physics, **Vol. 26** (1989) ed. World Scientific.
9. V.A. Zakharov, QCD 99 (Montpellier) and references therein.
10. S. Narison, *Phys. Lett.* **B300** (1993) 293; **B361** (1995) 121.
11. K. Chetyrkin, S. Narison and V.A. Zakharov, *Nucl. Phys.* B550 (1999) 353.

12. See e.g., M. Teper, hep-ph/9711299 (1997); hep-lat 9711011 (1997).
13. S. Narison, *Phys. Lett.* **B387** (1996) 162.
14. V.A. Novikov et al, *Nucl. Phys.* **B191** (1981) 301.
15. R.A. Bertlmann, G. Launer and E. de Rafael, *Nucl. Phys.* **B250** (1985) 61; R.A. Bertlmann et al., *Z. Phys.* **C39** (1988) 231 and references therein.
16. E. Braaten, S. Narison and A. Pich, *Nucl. Phys.* **B 373** (1992) 581.
17. S. Narison, *Nucl. Phys.* **B509** (1998) 312; *Nucl. Phys. (Proc. Suppl.)* **B64** (1998) 210.
18. G. Bali et al., *Phys. Lett.* **B 309** (1994) 29; J. Sexton et al., *Nucl. Phys. (Proc. Suppl.)* **B42** (1995) 279; C.J. Morningstar and M. Peardon, hep-lat/9901004 (1999).
19. G. West, *QCD96*, Montpellier, *Nucl. Phys. (Proc. Suppl.)* **B, A54** (1997).
20. S. Narison, G. Shore and G. Veneziano, *Nucl. Phys.* **B433** (1995)209; **B546** (1999)235.
21. S. Narison, N. Pak and N. Paver, *Phys. Lett.* **B147** (1984) 162.
22. H. Lipkin, these proceedings and private communication.
23. BES collaboration: C.Z. Yuan, QCD 99 (Montpellier); Z.G. Zhao, these proceedings.
24. E. Bagan, A. Bramon and S. Narison, *Phys. Lett.* **B196** (1987) 203.
25. T.M. Aliev and M.A. Shifman, *Phys. Lett.* **B112** (1982) 401.
26. D. Bugg, these proceedings and private communication.
27. S. Narison and G. Veneziano, *Int. J. Mod. Phys* **A4, 11** (1989) 2751.
28. For a review, see e.g. L. Montanet, QCD 99 (Montpellier) and private communications.
29. P. Minkowski and W. Ochs, *Eur. Phys. J* **C9** (1999) 283 and private communications.
30. V. Anisovich et al., *Phys. Lett.* **B413** (1997) 476.
31. M.S. Chanowitz, Workshop on $\gamma\gamma$-collisions (Israel 1988).
32. K. Maltman, these proceedings.
33. A. Bramon and S. Narison, *Mod. Phys. Lett.* **A4** (1989) 1113.
34. S. Narison, *Phys. Lett.* **B175** (1986) 88.
35. W. Lee and D. Weingarten, hep-lat/9608071.
36. A. Singovski for the WA102 collaboration, QCD 99 (Montpellier); these proceedings.
37. G. Mennessier, S. Narison and N. Paver, *Phys. Lett.* **B158** (1985) 153.
38. W. Lee and D. Weingarten, hep-lat/9805029.
39. J. Ellis, H. Fujii and D. Kharzeev, hep-ph/9909322.
40. S. Narison (unpublished).
41. C.Amsler and F.E. Close, *Phys. Rev.* **D53** (1996) 296.
42. U. Gastaldi, QCD 99 (Montpellier) and private communications.
43. G. Veneziano, private communications.
44. S.U. Chung, QCD 99 (Montpellier); these proceedings; P. Eugenio, these proceedings.
45. I.I Balitsky, D.I. D'Yakonov and A.V. Yung, *Phys. Lett.* **B112** (1982) 71.
46. J. Govaerts et al., *Nucl. Phys.* **B248** (1984) 1; J.I. Latorre et al., *Phys. Lett.* **B147** (1984) 169.
47. J.I. Latorre, S. Narison and P. Pascual, *Z. Phys.* **C34** (1987) 347.
48. K. Chetyrkin, S. Narison and A. Pivovarov (to appear).
49. J. Govaerts and F. de Viron, Phys. Rev. Lett. **53** (1984) 2207.
50. C. Bernard et al., hep-lat/9809087; P. Lacock et al. hep-lat/9809022.
51. J. Govaerts et al., *Nucl. Phys.* **B258** (1985) 215; **B262** (1985) 575; **B284** (1987) 674.
52. S-L. Zhu, hep-ph/9812469; T. Huang, H. Jin and A. Zhang, hep-ph/9812430.
53. T. Manke, QCD 99 (Montpellier).