Effect of $\sigma(600)$-production in $p\bar{p} \rightarrow 3\pi^0$ at rest

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The $\pi^0\pi^0$ mass spectra and angular distributions around $K\bar{K}$-threshold and at 1.5 GeV in $p\bar{p}$(at rest) $\rightarrow 3\pi^0$ in the Crystal Barrel experiment are reanalyzed by applying the new method, which is consistent with unitarity of S-matrix and expressed directly by resonance parameters. The effects of light σ-meson production are clearly seen to improve the fit with σ, in comparing with the fit without σ.

1. INTRODUCTION

The light iso-singlet scalar σ meson plays an important role in the mechanism of spontaneous breaking of chiral symmetry, and to confirm its real existence is one of the most important topics in hadron physics. Recently in various $\pi\pi$-production experiments[2,5] a broad peak is observed in mass spectra below 1 GeV. Conventionally this peak was regarded as a mere non-resonant background, basing on the "universality argument,"[4] since no σ was seen in $\pi\pi$ scattering at that time. However, at present the $\pi\pi$-scattering phase shift $\delta_S^{I=0}$ is reanalyzed by many authors[3,1,2] and the existence of σ meson is strongly suggested. A reason of missing σ in the conventional analysis is pointed out to be due to overlooking the cancellation mechanism,[2] which is guaranteed by chiral symmetry, between the effects of σ and of repulsive $\pi\pi$-interaction. Moreover, the conventional treatment, based on the universality argument, of the low mass broad peak was shown to be not correct, and a new effective (VMW) method is proposed to analyze resonance productions. In this method the production amplitude $\mathcal{F}$ is directly represented by the sum of Breit-Wigner amplitudes with production couplings and phase factors (, including initial strong phases) of relevant resonances. The consistency of this method with the unitarity is seen from the following field theoretical viewpoint[2].

Presently, after knowing the quark physics, the strong interaction $\mathcal{L}_{\rm str}$ among hadrons (in our example, mesons) is regarded as a residual interaction of QCD among color singlet $q\bar{q}$-bound states, the "bare states," denoted as $\pi = \bar{\pi}, \bar{\sigma}, \bar{f}_0$ and $\bar{f}_2$. In switching off $\mathcal{L}_{\rm str}$, the bare states appear as stable particles with zero widths. In switching on $\mathcal{L}_{\rm str}$, they change into the physical states with finite widths. The unitarity of S-matrix is guaranteed automatically by the hermiticity of $\mathcal{L}_{\rm str}$. The VMW method is obtained directly as the representation of production amplitude by physical state bases, with a diagonal mass and width.

The VMW method is already applied[2] to the analyses of pp-central collision $pp \to pp\pi^0\pi^0$ and $J/\psi \to \omega\pi\pi$ decay, leading to a strong evidence of existence of the light σ meson. In this talk we apply this method to the analysis of $p\bar{p} \to 3\pi^0$ at rest.

2. AMPLITUDE DESCRIBING $p\bar{p} \to 3\pi^0$ AT REST BY VMW METHOD

2.1. $\mathcal{L}_{\rm str}$ for $p\bar{p} \to 3\pi^0$ at rest

We apply the iso-bar model, describing the process in following two steps: In the first step the $p\bar{p}$ annihilates into the resonance f(f_0 and f_2) and π^0, and the f decays into $2\pi^0$ in the second step. Since both p and $\bar{p}$ are at rest, the relative momentum $p_\mu = p_{p\mu} - p_{\bar{p}\mu}$ (and the relative angular momentum $L_{p\bar{p}}$) between p and $\bar{p}$ is 0. Charge conjugation parity P_C of $f\pi^0$-system is +1. Thus the three types of $(\bar{p}, p)$ bi-linear forms with P_C=+1 are possible: $\bar{p}i\gamma_5 p, \bar{p}i\gamma_5\gamma_\mu p$ and $\bar{p}p$. The second type reduces to the first type by using the equation $-\partial_\mu(\bar{p}i\gamma_5 p) = 2m_p\bar{p}i\gamma_5\gamma_\mu p + \bar{p}i\gamma_5 i\sigma_{\mu\nu} \overleftrightarrow{\partial_\nu}\ p = 2m_p\bar{p}i\gamma_5\gamma_\mu p$ (, derived from Dirac equation and the above "rest" condition), and the third type is forbidden by parity. Thus only the first type remains.[1] The most simple form of $\mathcal{L}_{\rm str}^{1,2}$ describing the 1st step $p\bar{p} \to f\pi^0$ and the 2nd step $f \to 2\pi^0$ is given, respectively, by

$$\mathcal{L}_{\rm str}^1 = \sum_{\bar{f}_0,\bar{f}_2} (\bar{\xi}_{\bar{f}_0}\bar{p}i\gamma_5 p\bar{f}_0\pi^0 + \bar{\xi}_{\bar{f}_2}\bar{p}i\gamma_5 p\bar{f}_{2\mu\nu}\partial_\mu\partial_\nu\pi^0),\ \mathcal{L}_{\rm str}^2 = \sum_{\bar{f}_0,\bar{f}_2} (\bar{g}_{\bar{f}_0}\bar{f}_0\pi^2 + \bar{g}_{\bar{f}_2}\bar{f}_{2\mu\nu}(\pi\ \overleftrightarrow{\partial_\mu}\overleftrightarrow{\partial_\nu}\ \pi)).$$

2.2. Amplitude by VMW method

First denoting the three π^0 as π_1, π_2 and π_3 with momenta p_1, p_2 and p_3, respectively, we consider the π_1 and π_2 forming the resonance f with squared mass s_{12} (where $s_{ij} \equiv -(p_i + p_j)^2$) and with momentum $|\mathbf{p}|$ in z-direction. The π_1 has momentum $|\mathbf{q}|$ and polar angle θ in the f rest frame. In the lowest order in bare state representation the amplitude is $2im_p f^{s_p s_{\bar{p}}}(\sum_{\bar{f}_0} \frac{\bar{\xi}_{\bar{f}_0}\bar{g}_{\bar{f}_0}}{\bar{m}^2_{\bar{f}_0} - s_{12}} + \sum_{\bar{f}_2} \frac{\bar{\xi}_{\bar{f}_2}\bar{g}_{\bar{f}_2}N(s_{12},{\rm cos}\theta_{12})}{\bar{m}^2_{\bar{f}_2} - s_{12}})$, where $N(s_{12}, {\rm cos}\theta_{12}) = -\frac{(s_{23}-s_{31})^2}{4} + \frac{16m_p^2\mathbf{p}^2\mathbf{q}^2}{3s_{12}}$,[2] and $2im_p f^{s_p s_{\bar{p}}} \equiv \bar{p}(\mathbf{0}, s_{\bar{p}})i\gamma_5 p(\mathbf{0}, s_p)$($s_{\bar{p}}$ and s_p being spin of $\bar{p}$ and p, respectively, and $f^{++} = f^{--} = 0, f^{+-} = -f^{-+} = -1$). Owing to the effect of final (and initial) state interaction, the "full order" of the amplitude in physical state representation is given by

$$A_{s_p s_{\bar{p}}}(s_{12}, {\rm cos}\theta_{12}) = 2im_p f^{s_p s_{\bar{p}}} \left(\sum_{f_0} \frac{r_{f_0} e^{i\theta_{f_0}}}{m^2_{f_0} - s_{12} - i\sqrt{s_{12}}\Gamma_{f_0}(s_{12})} + \sum_{f_2} \frac{r_{f_2} e^{i\theta_{f_2}} N(s_{12}, {\rm cos}\theta_{12})}{m^2_{f_2} - s_{12} - i\sqrt{s_{12}}\Gamma_{f_2}} \right).$$

The symmetric amplitude $\mathcal{F}_{s_p s_{\bar{p}}}$ is obtained simply by its cyclic sum as
$\mathcal{F}_{s_p s_{\bar{p}}}(s_{12}, s_{23}, s_{31}) = A_{s_p s_{\bar{p}}}(s_{12}, {\rm cos}\theta_{12}) + A_{s_p s_{\bar{p}}}(s_{23}, {\rm cos}\theta_{23}) + A_{s_p s_{\bar{p}}}(s_{31}, {\rm cos}\theta_{31})$.
Cross section is given by $d\sigma \sim \int_{2m_\pi}^{2m_p - m_\pi} d\sqrt{s_{12}} \frac{\sqrt{s_{12}}}{\pi} \frac{|\mathbf{p}|}{8\pi m_p} \frac{|\mathbf{q}|}{8\pi\sqrt{s_{12}}} \int_{-1}^1 d{\rm cos}\theta \overline{|\mathcal{F}(s_{12}, s_{23}, s_{31})|^2}$, where $\overline{|\mathcal{F}(s_{12}, s_{23}, s_{31})|^2} \equiv (1/4)\sum_{s_p, s_{\bar{p}}} |\mathcal{F}_{s_p s_{\bar{p}}}(s_{12}, s_{23}, s_{31})|^2 = (1/2)|\mathcal{F}_{+-}(s_{12}, s_{23}, s_{31})|^2$, and $s_{12} + s_{23} + s_{13} = 4m_p^2 + 3m_\pi^2$, $s_{23} = -4m_p(|\mathbf{p}||\mathbf{q}|/\sqrt{s_{12}}){\rm cos}\theta + m_\pi^2 + 2m_p E_3$; $E_3 = \sqrt{m_\pi^2 + \mathbf{p}^2} = (4m_p^2 - s_{12} + m_\pi^2)/4m_p$.

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[1] This implies initial $p\bar{p}$ is in ${}^1S_0$ state. In the original analysis[5], the phenomenological parameters related with ${}^3P_1$ and ${}^3P_2$ states, considered not to contribute since of the above rest condition, are included.

[2] N is obtained by the calculation of $N = -p_{3\mu}p_{3\nu}\mathcal{P}_{\mu\nu;\lambda\kappa}(p_1 - p_2)_\lambda(p_1 - p_2)_\kappa$, where we use the tensor projection operator $\mathcal{P}_{\mu\nu;\lambda\kappa}$ with mass squared s_{12} instead of $m^2_{f_2}$.

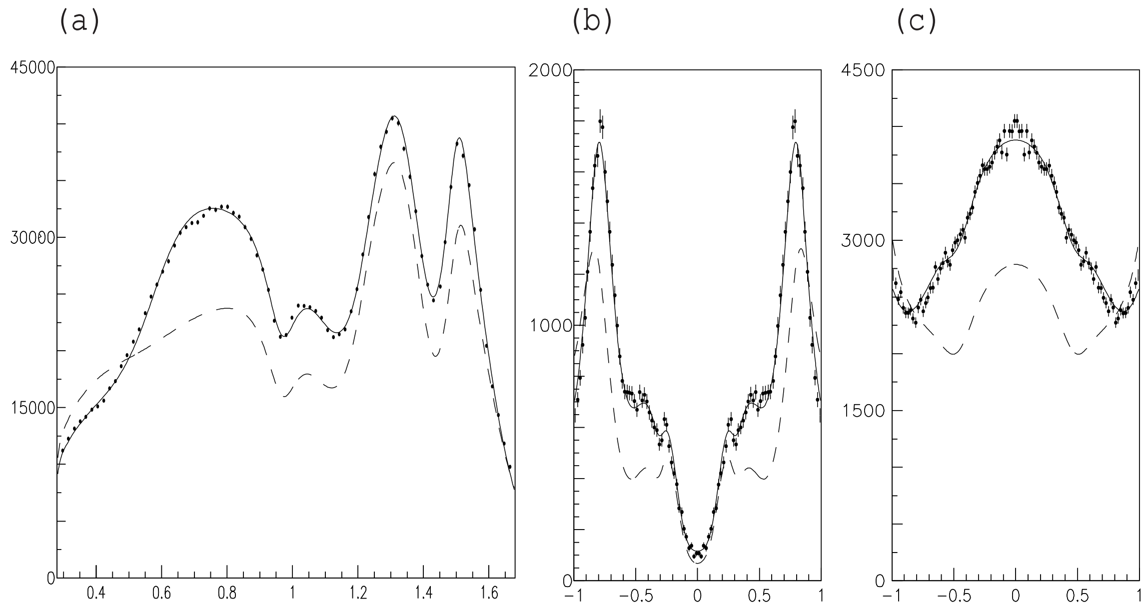

Figure 1. Result of the fit: (a) $\pi^0\pi^0$ mass distribution, and cosθ distributions (b) around $K\bar{K}$-threshold and (c) at 1.5 GeV. Dashed lines represent the spectra obtained by setting $r_\sigma = 0$.

3. RESULTS

We use as experimental data the $\pi^0\pi^0$ mass spectra and angular distributions around $K\bar{K}$-threshold and at 1.5 GeV, which are published in the paper[5] by Crystal Barrel collaboration. We take into consideration as the physical particles $f_0 = \sigma$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_2 = f_2(1275)$,$f_2(1565)$. Preliminary result of our fit is shown in Fig. 1. The mass and width of σ obtained are $m_\sigma = 560 \pm 25$MeV and $\Gamma_\sigma = 350 \pm 50$MeV (error corresponding to the 5σ-deviation), and the χ^2 is given by $858/(282-25) = 3.3$. The spectra given by setting $r_\sigma = 0$ are also given by dashed lines. Effect of σ-production is seen to be crucially important in reproducing the structure of mass spectra below 1 GeV.

We have also tried to fit without introducing the σ Breit-Wigner amplitude. The corresponding χ^2 is 3465/(282-21)=13.3, which is much worse than our best fit with σ meson. This gives a strong evidence for σ-existence.

4. COMPARISON OF THE METHODS OF ANALYSES BETWEEN CB's AND OURS

In the original analysis by Crystal Barrel collaboration the $\pi\pi$ scattering and production amplitudes are given, respectively, as in the $\mathcal{K}$ matrix representation: $\mathcal{T} = \mathcal{K}/(1-i\rho\mathcal{K})$, $\mathcal{F} = \mathcal{P}/(1-i\rho\mathcal{K})$, where $\mathcal{K}$-matrix and $\mathcal{P}$-matrix are taken in pole dominative form; $\mathcal{K} = \sum_\alpha g_\alpha^2/(m_\alpha^2 - s) + c_{\rm BG}$, $\mathcal{P} = \sum_\alpha e^{i\theta_\alpha}\xi_\alpha g_\alpha/(m_\alpha^2 - s)$. Here the summation is taken for the $\mathcal{K}$-matrix states α, which are related to the physical states corresponding to the poles of $\mathcal{T}$ (or $\mathcal{F}$). In the case with no production phases, $\theta_\alpha = 0$, the $\mathcal{F}$ and $\mathcal{T}$ have the same phase, and the unitarity(final state interaction theorem) is satisfied. In their analysis this phase was set to be the experimental scattering phase shift $\delta_S^{I=0}$. However, in the above $\mathcal{K}$-matrix parametrization, when s is close to m_α^2, $\mathcal{K}$ diverges and the phase must take the value $90^\circ(+n\times 180^\circ)$. This gives a very strong constraint for the value of m_α. The experimental $\delta_S^{I=0}$ passes through 90° at about $\sqrt{s}\simeq 900$ MeV, and so the m_α becomes $m_\alpha \simeq 900$ MeV , which is much larger than m_σ(=600MeV). Thus, no existence of light σ is implicitly assumed from the beginning.

In our method, the $\delta_S^{I=0}$ is analyzed by introducing the repulsive background phase shift $\delta_{\rm BG}$, which is required from chiral symmetry.[2] [3] Correspondingly, the $\mathcal{F}$ is represented by $\mathcal{F}=\frac{\mathcal{P}^{\rm Res}}{1-i\rho\mathcal{K}^{\rm Res}}e^{i\delta_{\rm BG}}$,[4] which satisfies the final state interaction theorem. This $\mathcal{F}$ is rewritten into the form, applied in VMW method, in the physical state representation. In our approach, whether σ meson exists or not is determined directly from the experimental data themselves, as was done in §3.

5. CONCLUSION

Through the above results the effects of production of light $\sigma(600)$ meson are clearly shown, while in the original analysis by Crystal Barrel collaboration, no existence of light σ is implicitly assumed from the beginning. Mass and width of σ is obtained as $m_\sigma = 560 \pm 25$MeV, $\Gamma_\sigma = 350 \pm 50$MeV, which are consistent with those obtained in our previous phase shift analysis[1] $((m_\sigma,\Gamma_\sigma)$=$(535\sim 675, 385\pm 70)$MeV). However, the effect of $\sigma(600)$ in this process is difficult to be discriminated from that of $f_0(1370)$ due to the statistics property of $3\pi^0$ system. In order to avoid this, it is desirable to analyze also the process, $\bar{p}n\rightarrow \pi^0\pi^0\pi^-$, through the similar method.

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[3] In our method the S-matrix is parametrized by $S=S^{\rm Res}S^{\rm BG}$ and the $\mathcal{K}$-matrix by $\mathcal{K}=\frac{\mathcal{K}^{\rm Res}+\mathcal{K}_{\rm BG}}{1-\rho^2\mathcal{K}^{\rm Res}\mathcal{K}_{\rm BG}}$. The denominator removes the poles of $\mathcal{K}^{\rm Res}=\sum_\alpha g_\alpha^2/(m_\alpha^2-s)$ in the total $\mathcal{K}$-matrix and we can take the small $m_\alpha\simeq 600$MeV. On the other hand, the background matrix, c_{BG}, in the conventional $\mathcal{K}$-matrix cannot describe the global phase motion corresponding to $\delta_{\rm BG}$ in our method, and the small value of m_α is not permissible.

[4] Here we neglect the possible effect of non-resonant $3\pi^0$ production.