Instanton effects in hadron spectroscopy

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In this talk we will briefly review the role of instantons for the mass spectra and decay properties of light–flavour hadrons.

1. INTRODUCTION

A glance at the experimental meson spectrum reveals two distinctive global properties, each with a conspicuous exception. First of all for virtually every isovector meson there is an isoscalar meson almost degenerate in mass (and of course an additional state interpreted as the $s\bar{s}$-state). Prominent examples are: $b_1(1235)-h_1(1170)$, $a_4(2040)-f_4(2050)$, $a_6(2400)-f_6(2450)$, $\rho(770)-\omega(784)$, $\rho'(1450)-\omega'(1420)$, $\rho_3(1690)-\omega_3(1670)$, etc. This reflects the isospin independence of the quark dynamics. The exception is of course the well known splitting of the $\pi-\eta-\eta'$-system which amounts up to a few hundred MeV. Secondly, in most channels the spin-dependent splittings are rather moderate, e.g. $a_1(1230)(^3P_1)-a_2(1320)(^3P_2)$, $h_1(1170)(^1P_1)-f_1(1285)(^3P_1)-f_2(1270)(^3P_2)$, $K_2(1820)(^3D_2)-K_3^*(1780)(^3D_3)$. Exceptions here are the lowest pseudoscalar and scalar mesons in each flavour channel.

Also for light flavoured baryons there is hardly any evidence for large spin-orbit effects. An outstanding feature of the baryon spectrum is the low position of the first scalar excitations of the ground states, almost degenerate with the lowest resonances of negative parity. As an example we mention the Roper-resonance $N_{\frac{1}{2}^+}(1440)$ and its counterparts for other flavours, like e.g. $\Sigma_{\frac{1}{2}^+}(1660)$.

Any model of hadrons should address these observations. This is especially important for the identification of exotic states, like hybrids, hadronic molecules and the appreciation of the predictions on glueballs from lattice QCD. In this contribution we shall explore to what extend these features can be understood in a relativistic constituent quark model of mesons and baryons, which apart from modelling confinement through a string-like linear potential takes into account a quark interaction that is induced by instanton effects. Many results of this quark model have already been presented elsewhere [1–10]. In this contribution we will focus on some selected topics.

2. INSTANTONS

The conspicuous low masses of the π, η, K-mesons is usually attributed to the chiral invariance of the QCD-Lagrangian for vanishing (current) quark masses, which is broken spontaneously, leading to non-vanishing quark condensates, effective constituent quark masses of some 100 MeV and a nearly massless Goldstone octet of pseudoscalar states. The same assumptions also imply invariance under $\exp(i\tau\gamma_5)$-transformations, which according to the same scenario would lead to a 9th Goldstone boson. However, the η'-meson is definitely too heavy for this purpose. This feature, known as the $U_A(1)$-problem, is solved by the observation that in QCD the corresponding axial current is in fact not conserved: $\mathrm{div} j_A \propto F_{\mu\nu}\tilde{F}^{\mu\nu} \neq 0$. More quantitatively, 't Hooft [11] showed that special solutions of the QCD-Yang-Mills theory concentrated in euclidian space-time, called instantons, dominate the propagation of light quarks leading to an effective quark interaction which for three flavours takes the form

$$\Delta\mathcal{L} = g \sum_{i,k,l;i',k',l'=u,d,s} \varepsilon_{ikl}\varepsilon_{i'k'l'} \bar{\Psi}_{Ri}\bar{\Psi}_{Rk}\bar{\Psi}_{Rl} \left(2\mathcal{P}^c_{10} + 5\mathcal{P}^c_8\right) \Psi_{Li'}\Psi_{Lk'}\Psi_{Ll'} + h.c. \tag{1}$$

Normal ordering of this interaction with respect to the nontrivial QCD vacuum then leads to a one-body term contributing to the constituent quark mass, a two body interaction which we will use below and a three body term, which does not contribute for quark-antiquark and colourless three-quark states, see [2] for details. In the instanton liquid model, see e.g. [12] for an extensive review on the role of instantons in QCD, one may think of the 't Hooft interaction as a vertex that absorbs left(right)-handed quarks of each flavour and creates right(left)-handed quarks through scattering off (anti)instantons, which are randomly distributed with a density of ≈ 1 fm^{-4} and an average size of $\approx \frac{1}{3}$ fm. Mesons propagate by "hopping" of the quark-antiquark pairs between these vertices, the interaction being attractive for pseudoscalar states, repulsive for scalar states and weak for other channels. This picture is substantiated by recent analyses of QCD-lattice calculations, see e.g. [13].

In order to appreciate how this works quantitatively, we show in Fig. 1 how the instanton induced interaction explains pseudoscalar splittings: Without the instanton induced interaction the isoscalar and isovector $n\bar{n}$ ($n = u, d$) states are degenerate. The interaction for non-strange quarks with increasing coupling strength g splits the $u\bar{u} - d\bar{d}$ state (π^0) and the $u\bar{u} + d\bar{d}$ state (η_{ns}), which in turn through the non-strange-strange interaction with increasing strength g' mixes with the $s\bar{s}$ state (η_s). At the same time the kaon is lowered. In fact these results were obtained in a relativistic constituent quark model, where this interaction, which we will take to act only in the (pseudo)scalar channel, is supplemented by quark-confinement through a linear potential, which thus determines the structure in all other channels. This model we will now briefly discuss.

3. A QUARK MODEL FOR MESONS

Without doubt the nonrelativistic potential model is most successful in describing the bulk of meson and baryon resonances. However, since binding energies, especially for the pion, are too large to justify a non-relativistic treatment, the Schrödinger wave functions are really inadequate for processes involving larger momentum transfers and in particular electroweak observables can be very sensitive to relativistic components in the amplitudes, we prefer to use the Bethe-Salpeter equation as a starting point for our quark model, see [1,2] for details on the formalism. Here, we will only repeat the basic assumptions: Staying as close as possible to the non relativistic potential model we assume that the

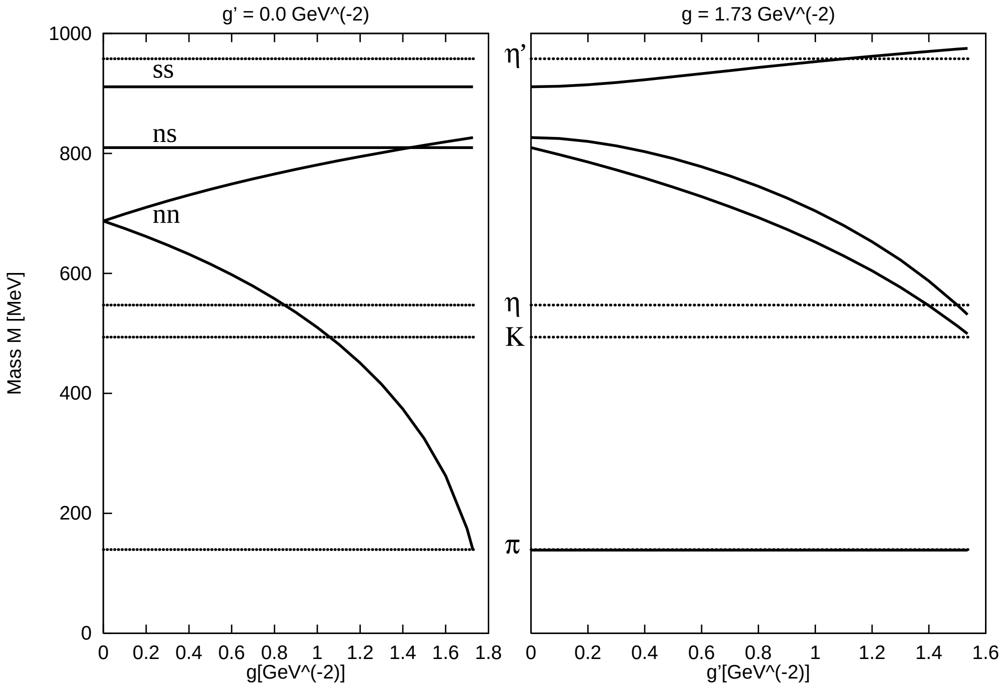

Figure 1. Instanton induced pseudoscalar meson splitting (solid lines) as a function of the coupling strength between normal quarks (g) and between normal and strange quarks (g'). Experimental values are indicated by dotted lines

quark propagators are of the free type, i.e. $S_i^F(p) = i/(\not p - m_i + i\varepsilon)$, with an effective constituent quark mass m_i, and that the interaction kernel K can be suitably approximated by an instantaneous potential, containing a confining interaction (with a spin structure that minimises spin-orbit effects) depending linearly on the quark-antiquark distance as well as 't Hoofts instanton induced interaction from Eq.(1), which acts exclusively on (pseudo)scalars. These assumptions facilitate a 3-dimensional, but covariant, reduction to the Salpeter equation, which is solved numerically by an expansion in a suitable basis, see [2,3] for details also concerning the subsequent calculation of current matrix elements in the Mandelstam-formalism. For results on various observables we refer to [2,8–10] concerning spectra, [3,5] for results on form factors and [4] for an extensive discussion of $\gamma\gamma$-decays. Here we will merely present some more recent results obtained for $\gamma\gamma^*$, see Fig. 2 and the time-like pion form factor from τ-decay, see Fig. 3.

We conclude this section by some comments on the (pseudo)scalar meson spectra, see Fig. 4: The pattern of splittings and mixings of the pseudoscalar ground state nonet discussed above is in fact repeated for the radial excitations, thus accounting for $\pi(1300)$, $\eta(1440)$, $\eta(1760)$, $K(1460)$, but leaving $\eta(1295)$ unexplained. For scalar mesons, the instanton induced force acts with opposite sign and produces an almost pure SU(3)-flavour singlet state around 1 GeV, which could be identified with the broad $f_0(400-1200)$, thus leaving $f_0(980)$ and $a_0(980)$ as non-$q\bar{q}$-states, the octet states around 1.4 GeV to

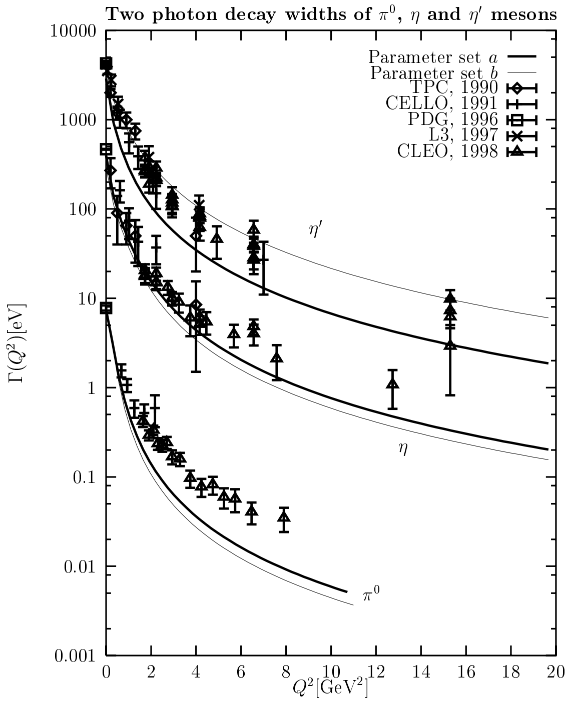

Figure 2. Comparison of calculated and experimental [15,16] two–photon decay widths of pseudoscalar mesons

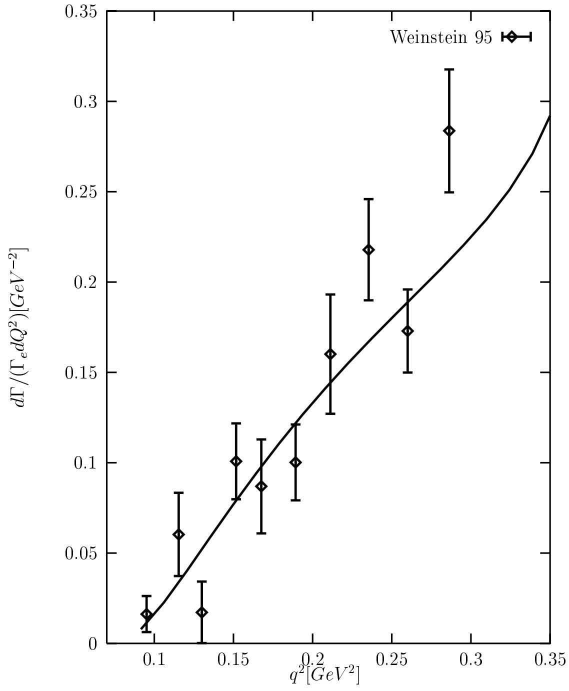

Figure 3. Comparison of calculated and experimental [14] differential decay width for $\tau \to \nu_\tau + \pi\pi$

be identified with $f_0(1500)$ or $f_0(1370)$, $a_0(1450)$ and $K_0^*(1430)$. Again, this pattern is repeated for the radial excitations. If, as reported on this conference, the $f_J(1700)$ is scalar, it would fit rather well to the radially excited flavour-singlet state at $\approx$ 1770 MeV.

4. EXTENSION TO BARYONS

The quark model discussed above can also be extended to formulate a relativistic constituent quark model of baryons. We model confinement by an instantaneous string-like three-body potential, again with a spin-dependence which minimises spin-orbit effects. As remaining quark interaction we again take the 't Hooft interaction, which for baryons has the distinctive property that it acts only if a quark pair is in an antisymmetric spin- and flavour state ("scalar diquarks"), see [9,10], which implies that flavour decuplet states are determined by the confinement potential only. That indeed a satisfactory description of e.g the Δ-resonances up to spin $\frac{15}{2}^+$ can be obtained in this manner is illustrated in Table 1. It should be remarked, that a consistent treatment of two-body interactions, as e.g. the 't Hooft interaction, in the framework of the instantaneous three-body Salpeter equation is not straightforward and still under investigation. Presently we account for these two-body interactions by a perturbative construction of an effective, instantaneous three-body interaction. In order to indicate the quality of the description of baryon spectra we compare our calculated spectra for some lower spin baryon resonances with experimental

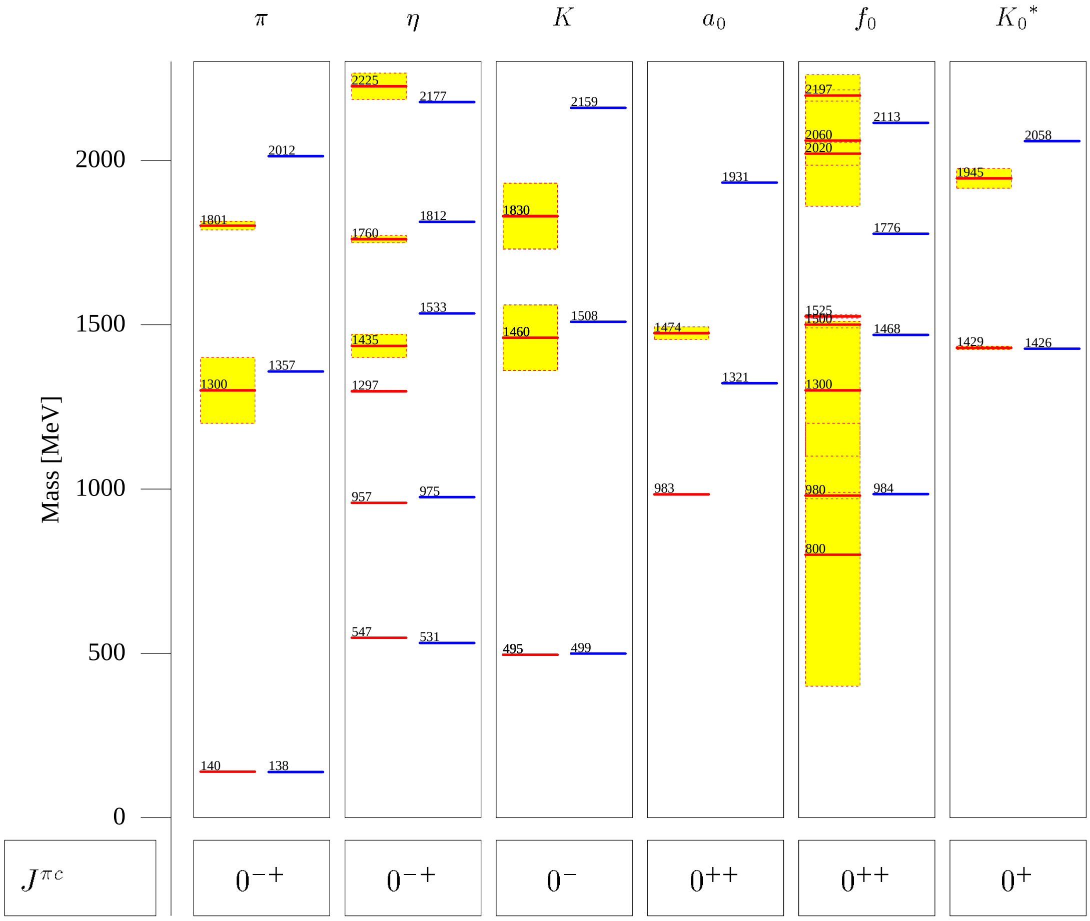

Figure 4. Comparison of experimental (left side of each column, data from [16]) and calculated (right side of each column) spectra for (pseudo)scalar mesons. The experimental uncertainty in the resonance position is indicated by a shaded box.

Table 1
Comparison of experimental [16] and calculated masses (in MeV) of Δ resonances.

J^π	exp.	calc.	J^π	exp.	calc.	J^π	exp.	calc.
$\frac{1}{2}^+$	1700-1800	1866	$\frac{5}{2}^-$	1920-1970	2180	$\frac{11}{2}^+$	2300-2500	2441
$\frac{1}{2}^-$	1615-1675	1653	$\frac{7}{2}^+$	1940-1960	1955	$\frac{11}{2}^-$		2608
$\frac{3}{2}^+$	1230-1240	1261	$\frac{7}{2}^-$	2155-2360	2180	$\frac{13}{2}^+$		2783
$\frac{3}{2}^-$	1670-1770	1628	$\frac{9}{2}^+$	2140-2550	2394	$\frac{13}{2}^-$	2550-2870	2673
$\frac{5}{2}^+$	1870-1920	1897	$\frac{9}{2}^-$	2100-2500	2276	$\frac{15}{2}^+$	2750-3190	2815

data in Table 2. It is a remarkable feature of the present relativistic treatment that, in contrast to the non-relativistic potential model, where the Roper resonance and its strange partners are in general calculated too high and are lowered only if additional interactions

Table 2
Comparison of experimental [16] and calculated baryon masses (in MeV).

	N		Λ		Σ		Ξ	
J^π	exp.	calc.	exp.	calc.	exp.	calc.	exp.	calc.
$\frac{1}{2}^+$	939	940	1116	1107	1189-1198	1189	1313-1315	1310
$\frac{1}{2}^+$	1430-1470	1518	1560-1700	1677	1630-1690	1760	1950?	1875
$\frac{1}{2}^+$	1680-1740	1729	1750-1850	1746	1738-1772	1947		2062
$\frac{1}{2}^-$	1520-1555	1434	1402-1410	1524	1600-1633	1628	1690?	1769
$\frac{1}{2}^-$	1640-1680	1659	1660-1680	1629	1730-1800	1770		1921
$\frac{3}{2}^+$	1650-1750	1688	1850-1910	1823	1375-1400	1411	1529-1533	1539
$\frac{3}{2}^-$	1515-1530	1475	1518-1522	1507	1580-1583	1669	1819-1830	1779

(usually meson (dressing) effects) are taken into account, once the parameters are fixed to obtain the correct ground state masses, the lowest positive parity excitations are already well described.

We thus see, that the main features of the light-flavoured hadron spectra can be understood on the basis of a *relativistic* constituent quark model that includes, apart from a string–like confinement, the interactions induced by instanton effects, which dominate the dynamics of light-flavoured quarks. I thank Matthias Koll, Klaus Kretzschmar, Ulrich Löring, Dirk Merten, Herbert Petry and Ralf Ricken for substantial contributions.

REFERENCES

1. J. Resag, C.R. Münz, B.C. Metsch, H.R. Petry, Nucl. Phys. A578 (1994) 397.
2. C.R. Münz, J. Resag, B.C. Metsch, H.R. Petry, Nucl. Phys. A578 (1994) 418.
3. C.R. Münz, J. Resag, B.C. Metsch, H.R. Petry, Phys. Rev. C52 (1995) 2110.
4. C.R. Münz, Nucl. Phys. A 609 (1996) 364.
5. W.I. Giersche, C.R. Münz, Phys. Rev. C 53 (1996) 2554.
6. E. Klempt, B.C. Metsch, C.R. Münz, H.R. Petry, Phys. Lett. B361 (1995) 160.
7. C. Ritter, B.C. Metsch, C.R. Münz, H.R. Petry, Phys. Lett. B380 (1996) 431.
8. B.C. Metsch, H.R. Petry, Acta Phys. Pol. B27 (1996) 3307.
9. B.C. Metsch, Proceedings of the 7th Int. Conf. on Hadron Spectrocopy, eds. S.U. Chung, H.J. Willutzki, AIP Conf. Proc. 432 (1998) 557, *ibid.* 828.
10. B.C. Metsch, Proceedings of the 8th Int. Conf. on the Structure of Baryons, eds. D.W. Menze, B.Ch. Metsch, World Scientific, Singapore, (1999) 61.
11. G. 't Hooft, Phys. Rev. D14 (1976) 3432.
12. T. Schäfer, E.V. Shuryak, Rev. Mod. Phys. 70 (1998) 323.
13. J.W. Negele, Nucl. Phys. Proc. Suppl. 73 (1999) 92.
14. A.J. Weinstein, Nucl. Phys. B Proc. Suppl. 40 (1995) 163.
15. TPC-2γ Coll., Phys. Rev. Lett. 64 (1990) 172; CELLO Coll., Z. Phys. C49 (1991) 401; L3 Coll., Phys. Lett. B148 (1998) 399; CLEO Coll., Phys. Rev. D57 (1998) 33.
16. C. Caso *et al.*, Eur. Phys. J. C3 (1998) 1.