Finite energy sum rule determination of the scalar meson decay constants and the nature of the $a_0(980)$

Kim Maltman[a]

[a]Math & Stats, York Univ., 4700 Keele St., Toronto, ON CANADA, M3J 1P3 and CSSM, University of Adelaide, Adelaide 5005 South Australia

A form of QCD sum rules, tested and found to be very reliable in the isovector vector channel, is used to determine the scalar current decay constants of the a_0 resonances. Comparison to the corresponding decay constant of the $K_0^*(1430)$ is used to shed light on the nature of the $a_0(980)$. It is shown that the results of this analysis strongly favor the "unitarized quark model" scenario for the $a_0(980)$.

The scalar meson resonances display a spectral pattern which differs significantly from that of other channels. This anomalous pattern, together with the proximity of the $f_0(980)$ and $a_0(980)$ to $K\bar{K}$ threshold, has led to extensive debate about the nature of these resonances, and a number of distinct proposals for their structure. Among those still discussed in the literature are (1) the $K\bar{K}$ "molecule" picture, in which the $f_0(980)$ and $a_0(980)$ are viewed as weakly bound two-meson states[1]; (2) the cryptoexotic "four-quark" picture, in which the states contain both a light quark and (hidden) strange quark pair[2]; (3) the "unitarized quark model" (UQM) picture, in which the coupling of "normal" quark model meson to the nearby $K\bar{K}$ channel creates strong distortions and significant $K\bar{K}$ admixture in the meson wavefunction[3]; and (4) the "minion" picture, in which the $f_0(980)$ and $a_0(980)$ are viewed as very small scale ($\sim 0.1-0.2$ fm) excitations of the light quark vacuum condensate[4]. A major feature distinguishing the various scenarios is the spatial extent of the corresponding states. Thus, a weakly bound $K\bar{K}$ molecule should be rather large and diffuse, a cryptoexotic somewhat larger than an ordinary $q\bar{q}$ meson, a minion much smaller than an ordinary meson, while, in the UQM scenario, the states should contain a "normal" quark model core, somewhat diluted by the admixture of the longer range two meson component. Quantities proposed to test the validity of the various scenarios, $\Gamma_{f_0,a_0\rightarrow\gamma\gamma}$ and $\Gamma_{\phi\rightarrow a_0\gamma,\ f_0\gamma}$[5,6], focus precisely on this expected difference in spatial distributions. Unfortunately, the necessity of modeling the non-trivial dynamics of these processes produces systematic theoretical uncertainties, as one can readily verify from a perusal of the existing literature.

An alternate choice of short-distance-dominated observable, much less obscured by intervening dynamics, is the class of scalar meson decay constants, f_S, describing the coupling of scalar meson states, S, to that member of the octet of scalar currents, $J = (m_s - m_u)\overline{q'}q$, having the same flavor as the meson, S. (A factor of the current quark mass is included in the definition of the scalar current in order to make the decay constant physically well-defined in QCD; the common mass factor $m_s - m_u$ is chosen so that, for $\overline{q'}q = \bar{s}u$, the current corresponds to that appearing in the analysis of the strange scalar channel, studied already by a number of authors[7–9].) The relations among analogous decay constants in other channels are well-known to reflect structural and spectral information. For example, the fact that f_π/f_K differs from 1 by the $\sim 20-30\%$ expected from $SU(3)_F$ breaking confirms the assignment of π and K to the same $SU(3)_F$ multiplet. Similarly, the fact that $f_\rho^{EM}/f_{K^*}^{EM} \simeq 1.1$ confirms the assignment of the ρ and K^* to the same $SU(3)_F$ multiplet, while the observation that $f_\omega^{EM} \simeq f_\rho^{EM}/3$, rather than the value $\simeq f_\rho^{EM}/\sqrt{3}$ expected if the ω were to be the 8^{th} member of the same octet as the ρ, tells us that the vector meson sector is almost ideally mixed.

The scalar currents discussed above have been defined in such a way that the corresponding scalar decay constants should satisfy the following relations (where the approximate equality in the first case accounts for the possibility of $SU(3)_F$ breaking, and reduces to an equality in the $SU(3)_F$ limit):

(1) if the $a_0(980)$ lies in the same multiplet as the $K_0^*(1430)$, $f_{a_0(980)}$ should be $\simeq f_{K_0^*(1430)}$;

(2) if instead the $a_0(980)$ is a loosely bound molecule or cryptoexotic, $f_{K_0^*(1430)} >> f_{a_0(980)}$; and

(3) if the $a_0(980)$ is a minion, $f_{K_0^*(1430)} << f_{a_0(980)}$.

In making the statements above, we have relied on the universally-held assumption that the $K_0^*(1430)$ is an ordinary quark model state. Thus, to make progress in determining the nature of the $a_0(980)$ through the value of its decay constant, we must first determine the corresponding decay constant of this "reference" state. $f_{K_0^*(1430)}$ can, of course, be determined if the spectral function of the corresponding strange scalar current two-point function is known. Since, the $K_0^*(1430)$ is known experimentally to be essentially purely elastic[10], this means that $f_{K_0^*(1430)}$ can be determined from the $K\pi$ part of the spectral function. This can be reconstructed using unitarity, together with the experimentally determined $K\pi$ phases and the timelike scalar $K\pi$ form factor, $d_{K\pi}(s)$, measured in K_{e3}, since $d_{K\pi}(s)$ is known to satisfy an Omnes representation[8]. For a discussion of additional input assumptions required in this analysis, and some post facto justification of their validity, see Ref. [9].

To evaluate $f_{a_0(980)}$, we use a form of QCD sum rules ("pinched-weight" finite energy sum rules (FESR)[11], with weights $w(s) = (1 - s/s_0)(A - s/s_0)$, where s_0 is the radius of the circular part of the FESR contour, and A is chosen to be ~ 2 in order to suppress poorly-known instanton contributions). This class of sum rules has been tested in the isovector vector channel, where the hadronic spectral function is well-known experimentally[12], and shown to be extremely well-satisfied[11], even at scales as low as 2 GeV^2. Moreover, a fit of the decay constants in an incoherent three-Breit-Wigner spectral ansatz to the OPE side of the relevant families of sum rules allows a determination of f_ρ^{EM} accurate to within experimental error[11]. The quality of the match between the OPE-constrained spectral representation obtained in this exercise and the experimental spectral function is displayed in Figure 1, where the result of a least squares fit of the same spectral ansatz directly to the ALEPH data has also been shown for comparison.

Using this method, and assuming a resonance dominated spectral ansatz consisting, potentially, of both $a_0(980)$ and $a_0(1450)$ contributions, we have analyzed the isovector scalar correlator. We find that (1) imposing $f_{a_0(980)} \sim 0$ by hand leads to a very poor match between OPE and hadronic sides, (2) allowing $f_{a_0(980)} \neq 0$ produces an excellent match, and (3) $f_{a_0(980)} m^2_{a_0(980)} / f_{K_0^*(1430)} m^2_{K_0^*(1430)} = 0.53 \pm .10$ with a *very* conservative estimate of errors (dominated by that associated with truncating the perturbative series for the $D = 0$ term in the OPE at four-loop order). Since a very small value for $f_{a_0(980)}$, relative to $f_{K_0^*(1430)}$ would be expected in both the $K\bar{K}$ molecule scenario and the cryptoexotic scenario (in this latter case the suppression will result from a compounding of suppressions associated with the larger spatial size and the necessity of annihilating the hidden strange pair), the first observation would appear to rule out these scenarios. The fact that the $a_0(980)$ decay constant is not *much* larger than that of the $K_0^*(1430)$ similarly rules out the minion scenario. The somewhat smaller, but not greatly smaller, ratio of decay constants observed in the best fit above is precisely what one expects, qualitatively in the UQM scenario. The points above are displayed in Figure 2, where the dash-dot line is the OPE side of the sum rule employed, the solid line the best fit hadronic side with $f_{a_0(980)} = 0$ and the dotted line the best fit with both the $f_{a_0(980)}$ and $f_{a_0(1450)}$ free. The long dashed line shows the effect of replacing the $a_0(980)$ contribution by a background contribution which is five times that obtained via unitarity using the tree-level ChPT $\pi\eta$ scalar form factor of the isovector scalar current. The fact that this still produces a very poor match between the hadronic and OPE sides of the sum rule confirms that it is a narrow resonant contribution, and not a broad background contribution in the $a_0(980)$ region, which is required to satisfy the sum rule.

To conclude, our results favor an $a_0(980)$ containing a significant component from the same $SU(3)_F$ multiplet as the $K_0^*(1430)$, and hence the UQM scenario for the nature of the $a_0(980)$.

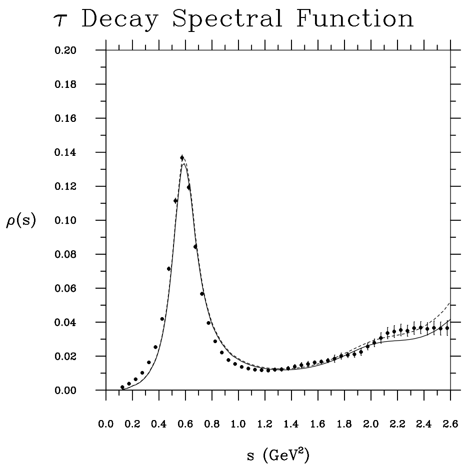

Figure 1. Isovector vector channel spectral function as reconstructed by fitting to the OPE. The data are those of the ALEPH Collaboration[12]. The solid line is the result of a least squares fit of the spectral ansatz to the ALEPH data, the dashed line the result of the fit of the same ansatz to the OPE side of a family of FESR's.

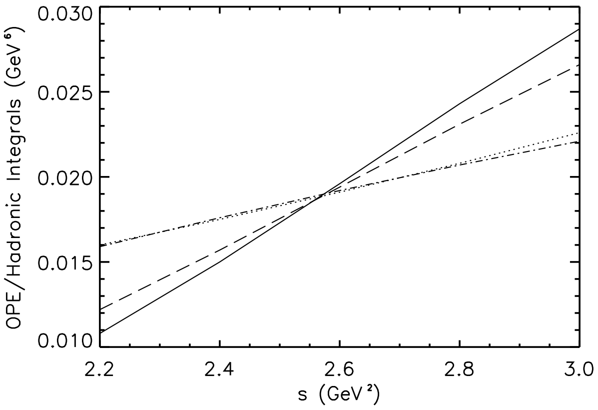

Figure 2. Isoscalar scalar sum rule fits. The dotted line is the OPE result, the dashed-dotted line the best fit with both decay constants left free, the solid line the $a_0(1450)$-only best fit, and the dashed line the best fit for the spectral ansatz consisting of an $a_0(1450)$ contribution plus 5 times the tree-level ChPT $\pi\eta$ continuum contribution below the $a_0(1450)$.

REFERENCES

1. J. Weinstein and N. Isgur, Phys. Rev. **D27**, 588 (1983).
2. R.L. Jaffe, Phys. Rev. **D15** (1977) 267, 281; *ibid.* **D17** (1978) 1444
3. N.A. Törnqvist, Phys. Rev. Lett. **49** (1982) 624 and Z. Phys. **C68** (1995) 647; N.A. Törnqvist and M. Roos, Phys. Rev. Lett. **76** (1996) 1575; E. van Beveren *et al.*, Z. Phys. **C30** (1986) 615; E. van Beveren and G. Rupp, hep-ph/9806246
4. F.E. Close, Yu. L. Dokshitzer, V.N. Gribov, V.A. Khoze and M.G. Ryskin, Phys. Lett. **B319** (1993) 291
5. T. Barnes, Phys. Lett. **165B**, 434 (1985)
6. F.E. Close, N. Isgur and S. Kumano, Nucl. Phys. **B389**, 513 (1993)
7. M. Jamin and M. Münz, Z. Phys. **C66** (1995) 633; K.G. Chetyrkin, D. Pirjol and K. Schilcher, Phys. Lett. **B404** (1997) 337; M. Jamin, Nucl. Phys. Proc. Suppl. **64** (1998) 250
8. P. Colangelo, F. De Fazio, G. Nardulli and N. Paver, Phys. Lett. **B408** (1997) 340
9. K. Maltman, hep-ph/9904370, in press Phys. Lett. **B**
10. D. Aston *et al.*, Nucl. Phys. **B296** (1988) 493; W. Dunwoodie, private communication
11. K. Maltman, Phys. Lett. **B440**, 367 (1998)
12. R. Barate *et al.* (The ALEPH Collaboration), Z. Phys. **C76** (1997) 379; Eur. Phys. J. **C4** (1998) 409