On the mixing amplitude of J/ψ and vector glueball O

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We study the mixing angle $\theta_{O\psi}$ and mixing amplitude $f_{O\psi}$ of J/ψ and vector glueball O based on the framework of potential models of heavy quarks and constituent gluons. While the state vectors of J/ψ and O are constructed from the wave functions of few-body Schroedinger equations, the mixing dynamics is governed by perturbative QCD. We obtain a value of the mixing angle of $|\tan\theta_{O\psi}| \approx 0.015$ and the mixing amplitude of $|f_{O\psi}(m^2_{O\psi})| \approx 0.008$ GeV^2. Within our approximation scheme, this result is comparable with phenomenological analysis.

1. INTRODUCTION

Among various solutions attempting to resolve the "$\rho - \pi$ anomaly" in the charmonium decays [1] [2], the resonance enhancement model, as proposed by Hou and Soni [3] and later generalized by Brodsky, Lepage, and Tuan [4], requires the existence of a vector glueball O [5]. Since both the rest energy $m_O - i\frac{\Gamma_O}{2}$ and the mixing amplitude $f_{O\psi}, f_{O\psi'}$ with charmonium J/ψ and ψ' are needed in the analysis of "$\rho - \pi$ anomaly "[6], we feel that it is necessary to study these issues within a hadronic model and henceforth provide some quantitative information. A naive approach [7] based on a constituent model of gluons [8] was used earlier to study the glueball spectrum in the pure Yang-Mills gauge theory. In the present study [9], we extend this framework by including the quark-gluon interaction and calculate the mixing angle $\theta_{O\psi}$ within a nonrelativistic approximation.

2. SETUP OF THIS PROBLEM

In the non-relativistic framework, we can calculate the mixing angle $\tan\theta_{O\psi}(\vec{Q})$ between the physical composites J/ψ, O, and pre-mixed hadrons $c\bar{c}$ and ggg via the evolution operator $U(T,-T) = e^{-2iHT}$,

$$\begin{pmatrix} |J/\psi(\vec{Q})\rangle \\ |O(\vec{Q})\rangle \end{pmatrix}_{NR} = \begin{pmatrix} \cos\theta_{O\psi}(\vec{Q}) & \sin\theta_{O\psi}(\vec{Q}) \\ -\sin\theta_{O\psi}(\vec{Q}) & \cos\theta_{O\psi}(\vec{Q}) \end{pmatrix} \begin{pmatrix} |c\bar{c}(\vec{Q})\rangle \\ |ggg(\vec{Q})\rangle \end{pmatrix}_{NR}, \tag{1}$$

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$$-\tan\theta_{O\psi}(\vec{Q}) = \lim_{T\to\infty\cdot e^{i\epsilon}} \frac{\langle ggg(\vec{Q})|U(T,-T)|c\bar{c}(\vec{Q})\rangle_{NR}}{\langle c\bar{c}(\vec{Q})|U(T,-T)|c\bar{c}(\vec{Q})\rangle_{NR}} \approx \langle ggg(\vec{Q})|U(T,-T)|c\bar{c}(\vec{Q})\rangle_{NR}. \quad (2)$$

The mixing angle $\theta_{O\psi}$ is then related to a relativistically normalized mixing amplitude $f_{O\psi}$, neglecting the decay widths of both J/ψ and O, by

$$\begin{aligned} f_{O\psi}(q^2) &= \left[(\gamma\delta - \alpha\beta)q^2 + (\alpha\beta m_{ggg}^2 - \gamma\delta m_{c\bar{c}}^2)\right]\sin\theta_{O\psi}\cos\theta_{O\psi}\ , & (3)\\ &\approx \left[(\delta-\beta)q^2 + (\beta m_{ggg}^2 - \delta m_{c\bar{c}}^2)\right]\theta_{O\psi}\ , & (4)\end{aligned}$$

$$\begin{aligned} \alpha(\vec{Q}) &\equiv \sqrt{\tfrac{\omega_{J/\psi}(\vec{Q})}{\omega_{c\bar{c}}(\vec{Q})}} = \left(\tfrac{\vec{Q}^2+m_{J/\psi}^2}{\vec{Q}^2+m_{c\bar{c}}^2}\right)^{1/4}, \quad & \beta(\vec{Q}) &\equiv \sqrt{\tfrac{\omega_{J/\psi}(\vec{Q})}{\omega_{ggg}(\vec{Q})}} = \left(\tfrac{\vec{Q}^2+m_{J/\psi}^2}{\vec{Q}^2+m_{ggg}^2}\right)^{1/4}, \\ \gamma(\vec{Q}) &\equiv \sqrt{\tfrac{\omega_{O}(\vec{Q})}{\omega_{ggg}(\vec{Q})}} = \left(\tfrac{\vec{Q}^2+m_{O}^2}{\vec{Q}^2+m_{ggg}^2}\right)^{1/4}, \quad & \delta(\vec{Q}) &\equiv \sqrt{\tfrac{\omega_{O}(\vec{Q})}{\omega_{c\bar{c}}(\vec{Q})}} = \left(\tfrac{\vec{Q}^2+m_{O}^2}{\vec{Q}^2+m_{c\bar{c}}^2}\right)^{1/4}, \end{aligned} \quad (5)$$

where in the second line we assume a small mixing angle and keep terms only to first order in $\theta_{O\psi}$.

With these definitions at hand, our task in this problem is to calculate the transition amplitude in the framework of constituent models of charm quark and gluons. Specifically, we need to solve for the state vectors of $c\bar{c}$ and ggg and use Feynman rules to calculate annihilation amplitude between constituents. The transition amplitude between $c\bar{c}$ and ggg composites can then be expressed as a convolution of the bound state wave functions and the annihilation amplitude among quark/gluon constituents.

3. DETAILS OF THE CALCULATIONS

In this section, we outline the basic ingredients in our calculations of the transition amplitude. Our normalizations and conventions follow that of [10].

1. The state vector of the $c\bar{c}$ particle:

$$\begin{aligned} |c\bar{c}(\vec{P},\vec{\lambda})\rangle &= \frac{1}{\sqrt{V}}\int\frac{d^3\vec{p}_1}{(2\pi)^3\sqrt{2\omega_c(\vec{p}_1)}}\int\frac{d^3\vec{p}_2}{(2\pi)^3\sqrt{2\omega_c(\vec{p}_2)}}(2\pi)^3\delta^{(3)}(\vec{P}-\vec{p}_1-\vec{p}_2) \\ &\times\ \frac{1}{\sqrt{6}}\ \tilde{\Psi}_{c\bar{c}}(\frac{\vec{p}_1-\vec{p}_2}{2})\ b_r^{\alpha\dagger}(\vec{p}_1)\ \vec{\lambda}\cdot(\vec{\sigma}\ \epsilon)_{rs}\ d_s^{\alpha\dagger}(\vec{p}_2)|0\rangle, \end{aligned} \quad (6)$$

where $\tilde{\Psi}_{c\bar{c}}(\vec{p})$ is the momentum space wave function of $c\bar{c}$. The spin and color wave function are given by $S_{rs}(\vec{\lambda}) \equiv \frac{1}{\sqrt{2}}\vec{\lambda}\cdot(\vec{\sigma}\ \epsilon)_{rs}$, $U^{\alpha\beta} \equiv \frac{1}{\sqrt{3}}\delta^{\alpha\beta}$, respectively.

2. The state vector of the ggg particle:

$$\begin{aligned} |ggg(\vec{K},\vec{\zeta})\rangle &\sim \frac{1}{\sqrt{V}}\left[\Pi_{i=1}^3\int\frac{d^3\vec{k}_i}{(2\pi)^3\sqrt{2\omega_g(\vec{k}_i)}}\right]\ (2\pi)^3\delta^{(3)}(\vec{K}-\sum_{i=1}^3\vec{k}_i) \\ &\times\ \tilde{\Phi}_{ggg}(\vec{k}_i;\vec{K})\ T_{mpq}(\vec{\zeta})\ V^{abc}\ \frac{1}{\sqrt{6}}\ G_m^{a\dagger}(\vec{k}_1)\ G_p^{b\dagger}(\vec{k}_2)\ G_q^{c\dagger}(\vec{k}_3)|0\rangle\ . \end{aligned} \quad (7)$$

$$\tilde{\Phi}_{ggg}(\vec{k}_1,\vec{k}_2,\vec{k}_3;\vec{K}) \equiv \left(\frac{6\pi}{a^2 m_g^2}\right)^{3/2} \exp\left[-\frac{1}{2a^2m_g^2}\left(-\vec{k}_1\vec{k}_2-\vec{k}_2\vec{k}_3-\vec{k}_3\vec{k}_1+\frac{\vec{K}^2}{3}\right)\right] \quad (8)$$

is a trial wave function of ggg, and the variational parameter a is fixed at 0.64 to obtain a stable glueball spectrum. The totally symmetric color wave function is given by $V^{abc} \equiv \sqrt{\frac{3}{40}}d^{abc}$. Please refer to [9] for the definitions of creation operators of the constituent gluons $G_m^{a\dagger}(\vec{k})$ and the spin wave function $T_{mpq}(\vec{\zeta})$.

3. The annihilation amplitude:

Using the Feynman rules for charm quarks and constituent gluons, we can write down the annihilation amplitude corresponding to the process $c+\bar{c} \leftrightarrow ggg$,

$$\mathcal{A}_{c\bar{c}\leftrightarrow ggg} = \bar{v}_n(\vec{p}_2,r)u_m(\vec{p}_1,s)S_{rs}(\vec{\lambda})T_{ijl}(\vec{\zeta})G_{\mu i}(\vec{k}_1)G_{\nu j}(\vec{k}_2)G_{\rho l}(\vec{k}_3)U^{\alpha\beta}V^{abc}A^{abc,\mu\nu\rho}_{nm,\alpha\beta}, \quad (9)$$

where

$$\begin{aligned} A^{abc,\mu\nu\rho}_{nm,\alpha\beta} &\equiv (-ig_s)^3\left[\gamma^\mu\frac{i}{\not{p}_1-\not{k}_1-m_c}\gamma^\nu\frac{i}{\not{p}_1-\not{k}_1-\not{k}_2-m_c}\gamma^\rho\right]_{nm}\left[\frac{\lambda^a}{2}\frac{\lambda^b}{2}\frac{\lambda^c}{2}\right]_{\alpha\beta} \\ &+ \ 6 \text{ permutations}, \end{aligned} \quad (10)$$

and the external legs for massive gluons are defined as $G^{\mu i}(\vec{k}_1) \equiv -g^{\mu i} + \frac{k_1^\mu k_1^i}{m_g^2}$.

To simplify the algebra, we perform a momentum expansion,

$$\begin{aligned} \mathcal{A}_{c\bar{c}\leftrightarrow ggg}(\vec{p}_i;\vec{k}_j) &= \mathcal{A}_{c\bar{c}\leftrightarrow ggg}(\vec{p}_i=0;\vec{k}_j=0) + \left(\vec{p}_i\frac{\partial}{\partial\vec{p}_i}+\vec{k}_j\frac{\partial}{\partial\vec{k}_j}\right)\mathcal{A}_{c\bar{c}\leftrightarrow ggg}|_{\vec{p}_i=0;\vec{k}_j=0} \\ &+ \text{ higher order terms in } \frac{p_i}{m_c},\frac{k_j}{m_g} \approx \frac{1.06\ g_s^3}{\sqrt{3}m_g(m_g-2m_c)}. \end{aligned} \quad (11)$$

4. Mixing angle as a convolution:

A generalized Fermi's Golden Rules No.2 can be applied to the transition amplitude and the mixing angle formula Eq.(2). With appropriate normalizations, we have

$$\begin{aligned} -\tan\theta_{O\psi}(\vec{P}) &= \int\frac{d^3\vec{p}_1}{(2\pi)^3\sqrt{2\omega_c(\vec{p}_1)}}\int\frac{d^3\vec{k}_1}{(2\pi)^3\sqrt{2\omega_g(\vec{k}_1)}}\int\frac{d^3\vec{k}_2}{(2\pi)^3\sqrt{2\omega_g(\vec{k}_2)}} \\ &\times(2\pi)\ \delta(\sum_{i=1}^{2}\omega_c(\vec{p}_i)-\sum_{j=1}^{3}\omega_g(\vec{k}_j))\ \frac{\tilde{\Psi}_{c\bar{c}}(\vec{p}_1)\ \mathcal{A}(\vec{p}_1;\vec{k}_1,\vec{k}_2)\ \tilde{\Phi}^*_{ggg}(\vec{k}_1,\vec{k}_2)}{\sqrt{6}\sqrt{2\omega_c(\vec{P}-\vec{p}_1)}\sqrt{2\omega_g(\vec{P}-\vec{k}_1-\vec{k}_2)}}. \end{aligned} \quad (12)$$

Since our model is based on a nonrelativistic approximation, we can perform a momentum expansion on a convoluted wave function of the vector glueball ggg. The momentum integration of the wave function of the $c\bar{c}$ then generates a factor of $c\bar{c}$ spatial wave function at origin $g_s^3\Psi_{c\bar{c}}(0)$, which can be extracted from the total hadronic deacy rate of J/ψ [10]. Using the mass inputs $m_c = 1.5$ GeV and $m_g = 0.7 \sim 1.0$ GeV, and the variational parameter $a = 0.64$, our calculation give a value of mixing angle with $\tan\theta_{O\psi}(\vec{Q}=0) \approx 0.015$.

5. Systematics of our approximations:

   In this mixing angle calculations, we have made several approximations:

   (a) Only nearest states, namely, $c\bar{c}$ and ggg, are considered.
   (b) Mixing dynamics between quarkonium and glueball is treated perturbatively in the strong coupling constant α_s.
   (c) Assuming a nonrelativistic picture, we expand the annihilation amplitude $\mathcal{A}$ in powers of constituent momenta and keep leading term only.
   (d) A variational solution of the vector glueball wave function is used for the convolution formula.

   For a discussion of possible improvement, see our paper [9] for details.

4. SUMMARY AND CONCLUSIONS

In this paper, we study the mixing angle $\tan\theta_{O\psi}$ and the mixing amplitude $f_{O\psi}$ between $c\bar{c}$ and vector glueball ggg based on the potential models of heavy quarks and constituent gluons and the perturbative dynamics of QCD. From this simple model calculation, we get the mixing angle at $|\tan\theta_{O\psi}| \approx 0.015$.

If we take the J/ψ mass at $m_{c\bar{c}} \approx m_{J/\psi} \approx 3096$ MeV and glueball mass at $m_{ggg} \approx m_O \approx 3168$ MeV, the mixing amplitude, as converted from the mixing angle, is equal to 0.008 GeV^2. In comparison with the phenomenological analysis [6], our results are off by a factor of two, which lies in the ballpark within our approximations.

It is unlikely to be a fortuitous coincidence that a naive picture of constituent gluons can give reasonable estimates for both glueball mass spectrum and mixing with quarkonium state, as the later quantity is more sensitive to the actual shape of the glueball wave function. At this stage our result seems to be encouraging, and indicates that a nonrelativistic approximation and more important, the constituent gluon picture, does capture some grains of truth behind this phenomenological puzzle.

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