Rare Tau Decays from Extra Dimension Models

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Lepton Flavor Violation

- Because neutrinos are strictly massless in the Standard Model(SM), we have the freedom to choose neutrinos' weak eigenstates to be their mass eigenstates.
  $\Rightarrow$ No lepton flavor violation (LFV) in SM.
- Convincing evidences of non-zero neutrino masses and non-trivial mixing provided by recent results form Super-K, SNO, KamLand, and WMAP.

$$
\begin{aligned}
U_{\mathrm{MNS}} &\sim \begin{pmatrix} 0.79-0.86 & 0.50-0.61 & <0.16 \\ 0.24-0.52 & 0.44-0.69 & 0.63-0.79 \\ 0.26-0.52 & 0.47-0.71 & 0.60-0.77 \end{pmatrix} \\
\triangle M^2_{\odot} &\sim 7\times 10^{-5} eV^2,\ \triangle M^2_{\mathrm{atm}} \sim 3\times 10^{-3} eV^2
\end{aligned}
$$

$\Rightarrow$ We are now certain that LFV must happen at some level.

- In the past 5 years, we were fascinated that many long standing 4D problems can be elegantly solved by novel ideas arise from higher dimensional field theories.
  To name few: Gauge Hierarchy, Symmetry breaking, Proton Stability, Doublet-Triplet Splitting, Dark Matter Candidates....
- In this talk, I will give you few examples of models involving extra dimension which yield possible LFV signatures.

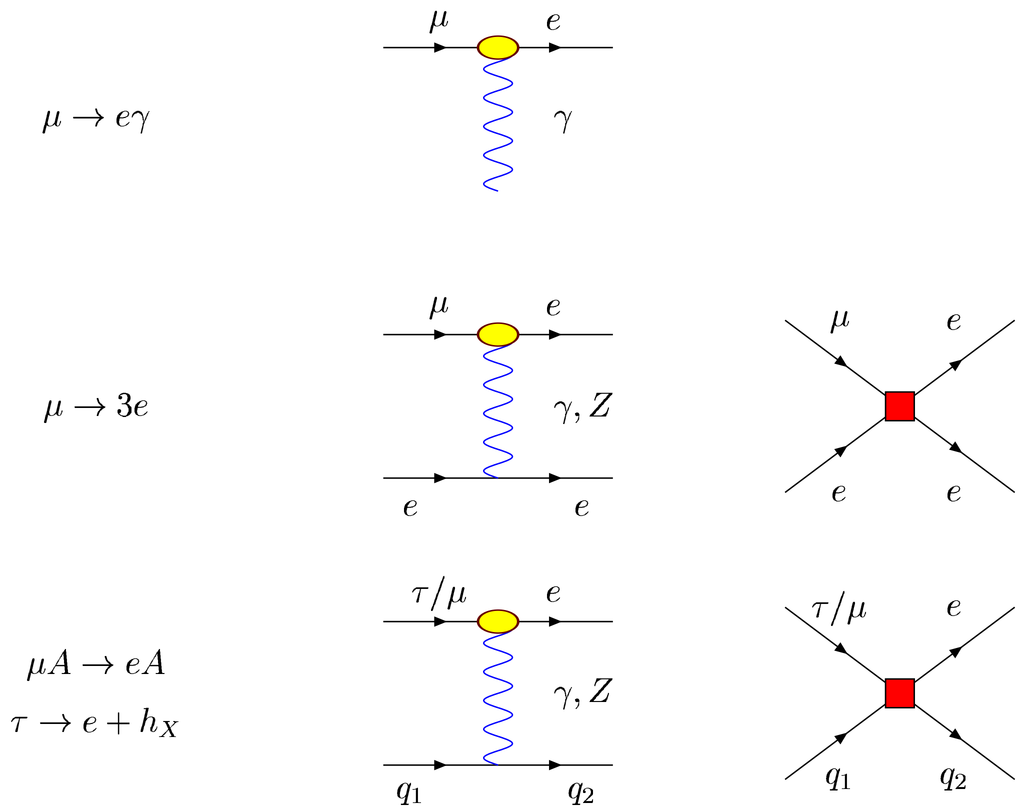
Current LFV limits

- LFV are either (1) closely related to neutrino masses or (2) directly induced.
- To probe the nature of LFV interactions it is useful to compare three processes: (1) $l \to l_1 \gamma$, (2) $l \to l_1 l_2 \bar{l}_3$ and (3) $l \to l_1 + hadrons$.

$\mu \to e\gamma$	1.2×10^{-11}	MEGA	$\tau \to \bar{\mu} e e$	1.1×10^{-7}	Babar
$\mu \to 3e$	1×10^{-12}	PSI	$\tau \to \bar{\mu} e \mu$	3.3×10^{-7}	Babar
$\mu Ti \to e Ti$	6.1×10^{-13}	PSI	$\tau \to \bar{e} e \mu$	2.7×10^{-7}	Babar
$\tau \to e\gamma$	2.7×10^{-6}	CLEO	$\tau \to \bar{e} \mu \mu$	1.3×10^{-7}	Babar
$\tau \to \mu\gamma$	3.2×10^{-7}	Belle	$\tau \to \mu K_s, e K_s$	$2.7, 2.9 \times 10^{-7}$	Belle
$\tau \to 3e$	2×10^{-7}	Babar	$\tau \to \mu h_X$	$4 - 22 \times 10^{-6}$	PDG
$\tau \to 3\mu$	1.9×10^{-7}	Babar	$\tau \to e h_X$	$2 - 35 \times 10^{-6}$	PDG

- The current experimental limits on muon LFV have already put stringent constraint on model building.
- We will focus on the neutrinoless LFV tau decay modes and discuss the possible ways to discriminate different extra dimension models and their connections to neutrino masses..

LFV Lagrangian for three processes



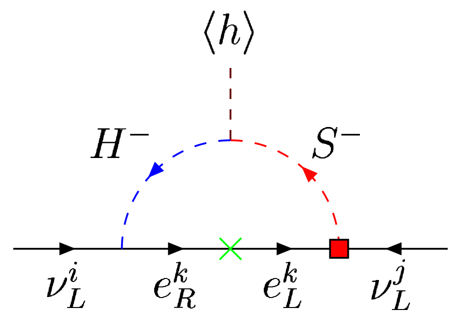
- If the dipole operator is dominated, we have simple relations

$$B(\mu \to 3e) \sim 0.006 B(\mu \to e\gamma)\,, \quad \frac{\sigma(\mu Ti \to eTi)}{\sigma(\mu Ti \to \text{capture})} \sim 0.004 B(\mu \to e\gamma)$$

- LFV in charged lepton processes is negligibly tiny for a simple seesaw or Dirac neutrino model.

Majorona Masses–Zee Model

- Other then using right-handed neutrino to give neutrinos small Dirac masses, Majorana masses can be generated through quantum corrections: A prototype of such model was first suggested by A.Zee.
- One charged singlet, S^-, and one charged doublet H^- are put in by hand.



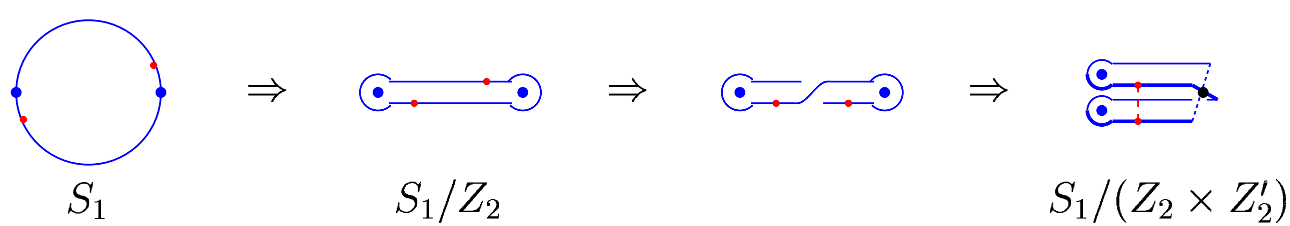
- The singlet with nontrivial hypercharge provides the necessary lepton number violation and the extra doublet to close the loop.
- The simplest Zee model gives neutrino mass matrix in the form of

$$\overline{m}_0 \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix} .$$

- Bi-maximal mixing angle at most(ruled out by the experimental data.)

Zee-Like Models in Extra Dimension

- I will discuss two Zee-like 5D GUTs models, $SU(3)_W$ and $SU(5)$, where neutrino Majorana masses are radiatively generated.
- The 5D GUTs have natural perturbations to get away from the bi-maximal mixing.
- In extra dimension models, the necessary lepton number violating scalars can be naturally imbedded into GUT multiplets.
- The symmetry breaking is achieved by assigning different orbifold parities.
- The $S_1/(Z_2 \times Z_2')$ orbifold:
  Folding the $S_1 : y \in [-\pi R, \pi R]$
  $Z_2 : y \Leftrightarrow -y$, $y \in [0, \pi R]$
  Define $y' = y - \pi R/2\ ,\ y' \in [-\pi R/2, \pi R/2]$,
  $Z_2' : y' \Leftrightarrow -y'\ ,\ y' \in [0, \pi R/2]$.



Two fixed points: $y = 0$ and $y = \pi R/2$.

- Properties of the Fourier modes on the $S_1/(Z_2 \times Z_2')$ orbifold:

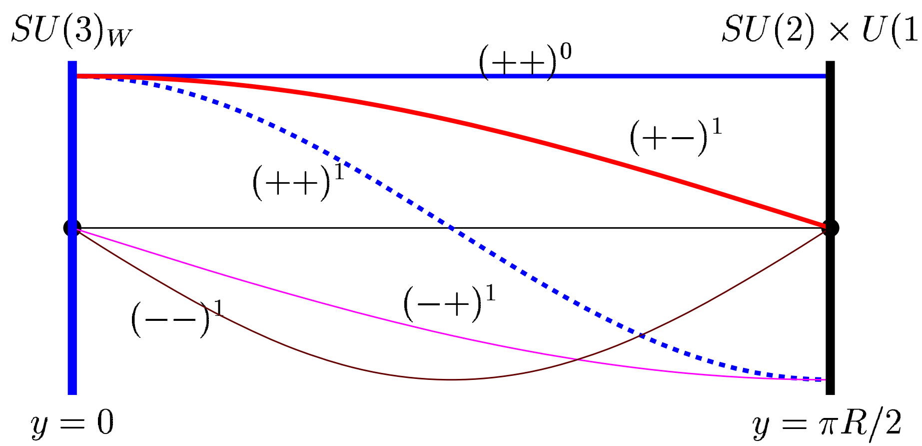
(P, P')	form	mass	$y = 0$	$y = \frac{\pi R}{2}$
$(++)$	$\frac{1}{\sqrt{\pi R}}[\frac{1}{\sqrt{2}} A_0(x) + \sum_{n=1} A_{2n}^{++}(x) \cos \frac{2ny}{R}]$	$\frac{2n}{R}$	$\surd$	$\surd$
$(+-)$	$\frac{1}{\sqrt{\pi R}}[\sum_{n=1} A_n^{+-}(x) \cos \frac{(2n-1)y}{R}]$	$\frac{(2n-1)}{R}$	$\surd$	X
$(-+)$	$\frac{1}{\sqrt{\pi R}}[\sum_{n=1} A_n^{-+}(x) \sin \frac{(2n-1)y}{R}]$	$\frac{(2n-1)}{R}$	X	$\surd$
$(--)$	$\frac{1}{\sqrt{\pi R}}[\sum_{n=1} A_n^{--}(x) \sin \frac{(2n)y}{R}]$	$\frac{2n}{R}$	X	X

- The components of a 5D field, in a specific representation, can have different orbifold parities.
- For example, in the 5D $SU(3)_W$ model, where $(e_L, \nu_L, e_R^c)^T$ form a $SU(3)$ triplet, $P = diag(+ + +)$ and $P' = diag(+ + -)$ are chosen to break bulk $SU(3)$ to $SU(2) \times U(1)$,
- The $SU(3)$ gauge matrix $\mathcal{A}_M \equiv A_M^a T^a$ is adjoint, $\mathcal{A} \to P^\dagger \mathcal{A} P$ and $\mathcal{A} \to P'^\dagger \mathcal{A} P'$. In low energy, $SU(3) \to SU(2)_L \times U(1)_Y$

$$\mathcal{A} = \frac{1}{2} \begin{pmatrix} A^3 + \frac{1}{\sqrt{3}} A^8 & \sqrt{2} T^+ & \sqrt{2} U^{++} \\ \sqrt{2} T^- & -A^3 + \frac{1}{\sqrt{3}} A^8 & \sqrt{2} V^+ \\ \sqrt{2} U^{--} & \sqrt{2} V^- & -\frac{2}{\sqrt{3}} A^8 \end{pmatrix},$$

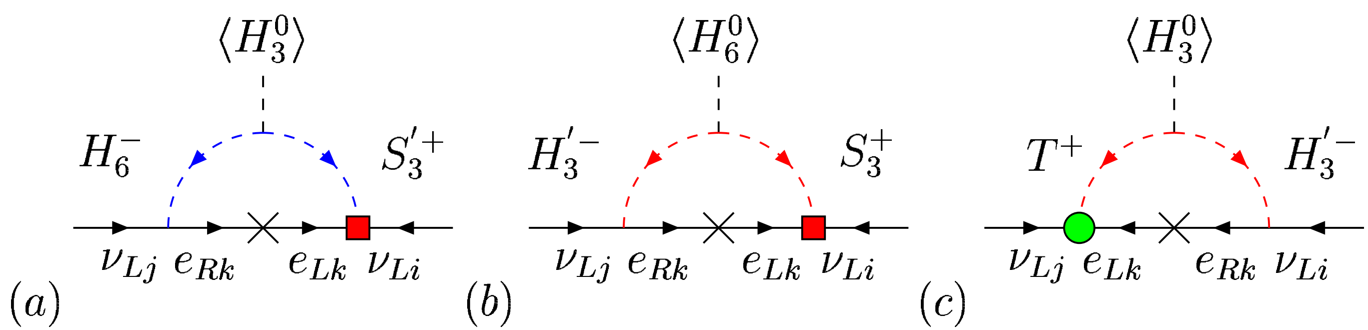
The basic idea

- The distribution in the fifth dimension of different kinds of KK modes:



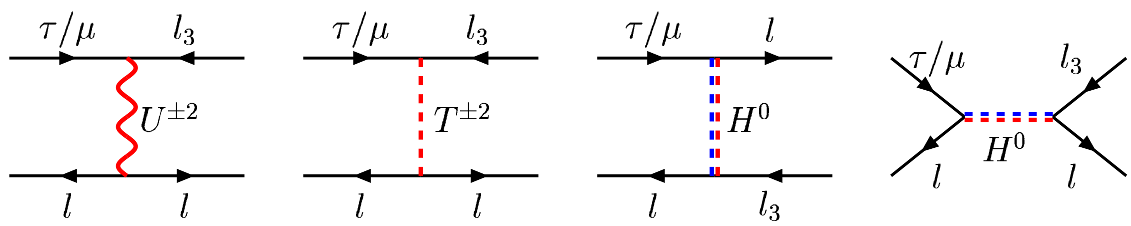
- The quarks can be located at the $SU(3)$ broken fixed point, $y = \pi R/2$.
- The leptons to be placed at the $SU(3)_W$ symmetric fixed point, $y = 0$, to avoid proton decay.
- $\sin^2\theta_W = 1/4$ at tree level.
- $1.5\text{TeV} < 1/R < 5\text{TeV}$.
- $SU(2)\times U(1)$ is broken by the usual Higgs mechanism.

- To have realistic charged lepton masses, one bulk Higgs triplet, $\phi_3(++)$, and one bulk Higgs anti-sextet, $\phi_6(+-)$, are necessary.
- Due to the requirement of $Z_2 \times Z_2'$ invariant, we also introduce another $\phi_3'(+-)$ for construction of the necessary $\mathbf{3'\bar{6}3}$ coupling.
- Branching: $\phi_3 = H_3(2, -\frac{1}{2}) + S_3(1,1)$, $\phi_3' = H_3' + S_3'$ and $\phi_6 = H_6(2, -\frac{1}{2}) + S_6(1,-2) + T(3,1)$. The $SU(2)$ singlet and triplet are naturally contained in $\mathbf{3}, \mathbf{3}'$ and $\bar{\mathbf{6}}$.



- Sub-diagram (a) yield a leading Zee-like neutrino mass matrix. Diagram-(b) and (c) give the needed perturbation to account for the data.
- The resulting mass matrix is of inverted hierarchy type and give a good fit to SuperK and SNO data.

Neutrinoless LFV Tau Decays



Tree-level LFV processes can be induced by the KK excitation of $U^{\pm 2}$ gauge boson, lepton number violating Higgs triplet and FCNC scalar(pseudoscalar).

$$
\begin{aligned}
Br(\tau \to 3\mu) &= \mathcal{F} \times \left(|\mathcal{U}_{\tau\mu}|^2 + |\mathcal{U}_{\mu\tau}|^2\right) |\mathcal{U}_{\mu\mu}|^2 + \cdots , \\
Br(\tau \to 3e) &= \mathcal{F} \times \left(|\mathcal{U}_{\tau e}|^2 + |\mathcal{U}_{e\tau}|^2\right) |\mathcal{U}_{ee}|^2 + \cdots , \\
Br(\tau \to \bar{\mu} ee) &= \mathcal{F} \times \left(|\mathcal{U}_{\tau\mu}|^2 + |\mathcal{U}_{\mu\tau}|^2\right) |\mathcal{U}_{ee}|^2 + \cdots , \\
Br(\tau \to \mu\mu\bar{e}) &= \mathcal{F} \times \left(|\mathcal{U}_{\tau e}|^2 + |\mathcal{U}_{e\tau}|^2\right) |\mathcal{U}_{\mu\mu}|^2 + \cdots , \\
Br(\tau \to \mu e\bar{e}) &= \frac{\mathcal{F}}{8} \left(|\mathcal{U}_{\tau e}|^2 + |\mathcal{U}_{e\tau}|^2\right) \left(|\mathcal{U}_{e\mu}|^2 + |\mathcal{U}_{\mu e}|^2\right) + \cdots , \\
Br(\tau \to e\mu\bar{\mu}) &= \frac{\mathcal{F}}{8} \left(|\mathcal{U}_{\tau\mu}|^2 + |\mathcal{U}_{\mu\tau}|^2\right) \left(|\mathcal{U}_{e\mu}|^2 + |\mathcal{U}_{\mu e}|^2\right) + \cdots .
\end{aligned}
$$

where $\mathcal{F} = 0.17 \times (g\pi R)^4/(512 G_F^2) = 2.65 \times 10^{-6}(2\text{TeV}/R^{-1})^4$ and the dots represent the contribution from various scalars.

Where $\mathcal{U} = U_L^\dagger U_R^*$ is the CKM-like unitary mixing matrix for the new lepton number violating charged current mediated by KK excitation of $U^{\pm 2}$:

$$\mathcal{L}_{\rm CC} = g_2 \sum_{n=1} \overline{e_{Li}} \gamma_L^\mu \mathcal{U}_{ij} e^{\mathbf{c}}_{Rj} U^{-2}_{n,\mu} + H.c$$

- Because the scalar sector gives positive contribution to these rare decays, from the unitarity of $\mathcal{U}$ the model predicts an interesting lower bond for a given $1/R$

$$Br(\tau \to 3e) > 2\mathcal{F} \times |\mathcal{U}_{ee}|^2 \left(1 - |\mathcal{U}_{ee}|^2\right)$$

- Also, because $R^{-1} < 5$ TeV,

$$Br(\tau \to 3e) > 8.0 \times 10^{-7} |\mathcal{U}_{ee}|^2 \left(1 - |\mathcal{U}_{ee}|^2\right)$$

- On the other hand, if we assume that $U^{\pm 2}$ is the dominate FCNC source,

$$Br(\tau \to 3e) < \frac{\mathcal{F}}{2}$$

Example(2): 5D $SU(5)$ GUT on $S_1/(Z_2 \times Z_2')$ orbifold

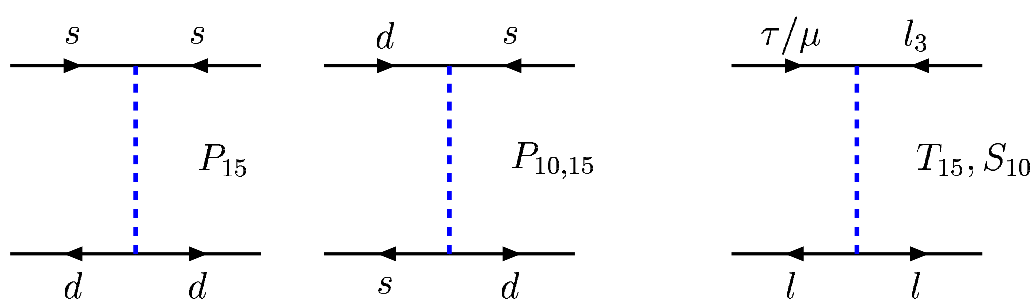
- In $SU(5)$, quarks and leptons are equal footing, $\Psi_{\bar{5}} = \{d^c, L\}, \Psi_{10} = \{Q, u^c, e^c\}$.
- Similar to $SU(3)_W$ model, but much more complicate, neutrino masses can be generated through quantum correction by using $\mathbf{10}$ or $\mathbf{15}$ bulk scalars.

$$\mathbf{15}_s(++) = P_{15}\left(6, 1, -\frac{2}{3}\right) + T_{15}(1,3,1) + C_{15}\left(3, 2, \frac{1}{6}\right)$$

$$\mathbf{10}_a(++) = P_{10}\left(\bar{3}, 1, -\frac{2}{3}\right) + S_{10}(1,1,1) + C_{10}\left(3, 2, \frac{1}{6}\right)$$

Two bulk Higgs in $\mathbf{5}, \mathbf{5}'(++)$ are introduced to break EW symmetry and form the $\overline{\mathbf{5}'}(\mathbf{10}/\mathbf{15})\bar{\mathbf{5}}$ interaction.

- $R^{-1} > 10^{14}$GeV, $\mathbf{15}$ favor the Normal Hierarchy, $\mathbf{10} \Rightarrow$ Inverted Hierarchy.
- $\mathbf{10}, \mathbf{15}$ give tree-level $L \to 3l$ decay and $K - \bar{K}$ mixing, $\Rightarrow M_{10,15} > 10^5$ GeV.



- $\triangle M_K^P$ arise from $P_{10,15}$ can be used to eliminate the ambiguity of absolute strength of Yukawa couplings.
- The ratio of Yukawa couplings can be replaced by the ratio of the corresponding elements in $\mathcal{M}_\nu$. For using **15** Higgs solely, we have

$$Br(\mu \to 3e) \sim 3.02 \times 10^{-16} \left(\frac{\triangle m_K^P}{\triangle m_K}\right)^2 \left(\frac{M_P}{M_T}\right)^4 \left(\frac{2m_{11}m_{12}}{m_{11}m_{22} + (2\frac{m_e}{m_\mu}m_{12})^2}\right)^2$$

$$Br(\mu \to 3e) : Br(\tau \to 3e) : Br(\tau \to 3\mu) : Br(\tau \to \mu ee) : Br(\tau \to e\mu\mu)$$

$$\sim \frac{m_{12}^2}{m_{22}^2} : \left(\frac{m_\mu}{m_\tau}\right)^4 \frac{m_{13}^2}{m_{22}^2} : \left(\frac{m_e}{m_\tau}\right)^4 \frac{m_{23}^2}{m_{11}^2} : \left(\frac{m_\mu}{m_\tau}\right)^4 \frac{m_{23}^2}{m_{22}^2} : \left(\frac{m_e}{m_\mu}\right)^4 \frac{m_{12}^2}{m_{11}m_{22}}.$$

- Interestingly, only the neutrino mass matrix of the form

$$\mathcal{M}_\nu \sim \overline{m}_0 \times \begin{pmatrix} \delta & 1 & 1 \\ 1 & \delta & \delta \\ 1 & \delta & \delta \end{pmatrix}, \ \delta \text{ represent some small number,}$$

have the chance to observe $\mu \to 3e$ in near future experiments.
- On the other hand, if it is **10**, we have similar expression but NO Tree-Level $\tau/\mu \to 3e, 3\mu$.

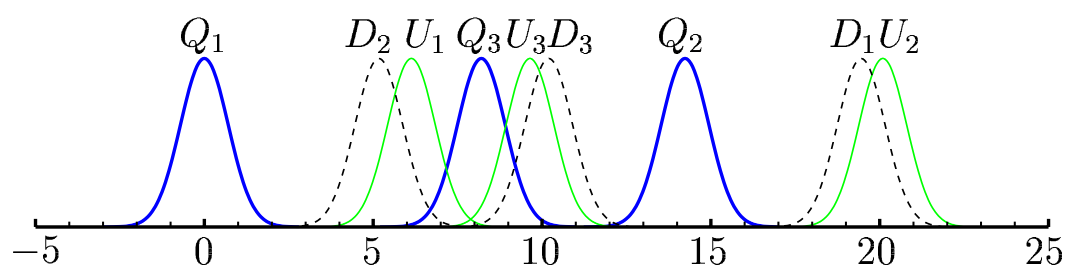
Example(3): Split Fermion Model

- 5D fermion localizes at different position, z_i, in extra dimension $y \in [-\pi R, \pi R]$, $\psi_i(x,y) = g(z_i,y)\psi(x)$,

$$g(z_i, y) = \frac{1}{(\pi\sigma^2)^{1/4}} \exp\left[-\frac{(y-z_i)^2}{2\sigma^2}\right]$$

$$g(z_1,y)g(z_2,y) = \exp\left[-\frac{(z_1-z_2)^2}{4\sigma^2}\right] g\left(\frac{z_1+z_2}{2}, y\right)$$

- Exponential Yukawa hierarchy becomes linear displacement between left-handed and right-handed fermions in the fifth dimension.
- The following map can reproduce all quarks' masses and CKM mixings



- The GEOMETRY induces genuine flavor mixing in all KK coupling!

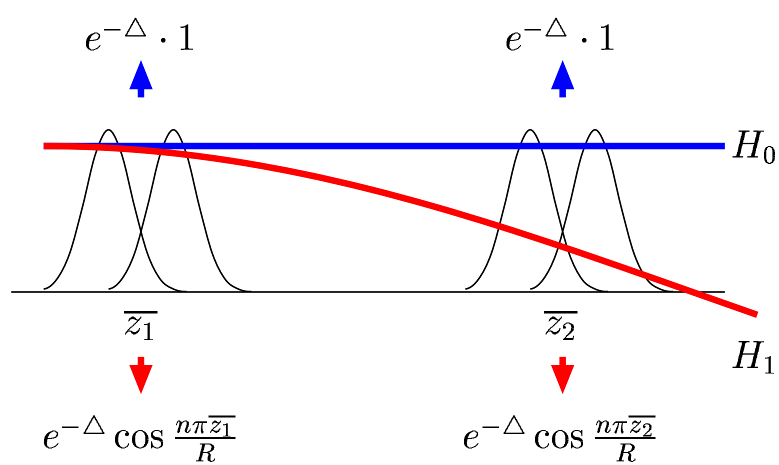
- The 4D effective mass in the gauge eigenstate is

$$\mathcal{L}_{4D}^{\mathrm{Mass}} \sim y_{eff} VEV \times \left(\overline{\psi'_{1L}}, \overline{\psi'_{2L}}\right) \begin{pmatrix} e^{-\triangle_{1\dot{1}}} & e^{-\triangle_{1\dot{2}}} \\ e^{-\triangle_{2\dot{1}}} & e^{-\triangle_{2\dot{2}}} \end{pmatrix} \begin{pmatrix} \psi'_{1R} \\ \psi'_{2R} \end{pmatrix}$$

and the mass matrix can be diagonalized by bi-unitary transformation:
$U_L^\dagger \cdot \mathcal{M} \cdot U_R = diag\,(m_1, m_2)$

- But the effective 4D Yukawa coupling to Higgs KK excitation is

$$\mathcal{L}_{4D}^{Y} \sim y_{eff} \sum_{n=1} H_n \left(\overline{\psi'_{1L}}, \overline{\psi'_{2L}}\right) \begin{pmatrix} e^{-\triangle_{1\dot{1}}} \cos\frac{n\pi\overline{z_{1\dot{1}}}}{R} & e^{-\triangle_{1\dot{2}}} \cos\frac{n\pi\overline{z_{1\dot{2}}}}{R} \\ e^{-\triangle_{2\dot{1}}} \cos\frac{n\pi\overline{z_{2\dot{1}}}}{R} & e^{-\triangle_{2\dot{2}}} \cos\frac{n\pi\overline{z_{2\dot{2}}}}{R} \end{pmatrix} \begin{pmatrix} \psi'_{1R} \\ \psi'_{2R} \end{pmatrix}$$



LFV in 5D Split Fermion Model

- In the split fermion model or any other multi-position model, LFV is generic.
- It suffers from lacking prediction power.
- However, we can conclude that (1) $\mu/\tau \to 3l$ and $\mu Ti \to eTi$(or $\tau \to lH_x$) happens at tree-level by exchanging KK scalars, photons and Z bosons; (2) $L \to l\gamma$ happen at loop level.
- We need experimental data to help us understand the lepton flavor physics.

Conclusion

- Possible detectable rare tau decays $\tau \to 3l$ can be generated by well motivated extra dimension models.
- More $\tau \to 3l$ data can be used to probe neutrino mass matrix in 5D $SU(5)$ model. Which is the example that LFV directly relates to neutrino masses.
- Totally due to geometry, the split fermion or multi-brane model give genuine LFV in all KK excitation couplings. More experimental data is needed to map out SM fermions' 5D positions.
- $SU(3)_W$ is somehow in between. The LFV data will help to better constraint Yukawa pattern.
- Opposite to other models, the 5D $SU(3)_W$ and $SU(5)$ Zee-like models genuinely predict $Br(L \to l\gamma), Br(L \to l + H_x) \ll Br(L \to 3l)$.
- On the other hand, the split fermion or general multi-brane models predict $Br(L \to l\gamma) \ll Br(L \to l + H_x) \sim Br(L \to 3l)$.

More LFV knowledge is needed!!