TAU PHYSICS

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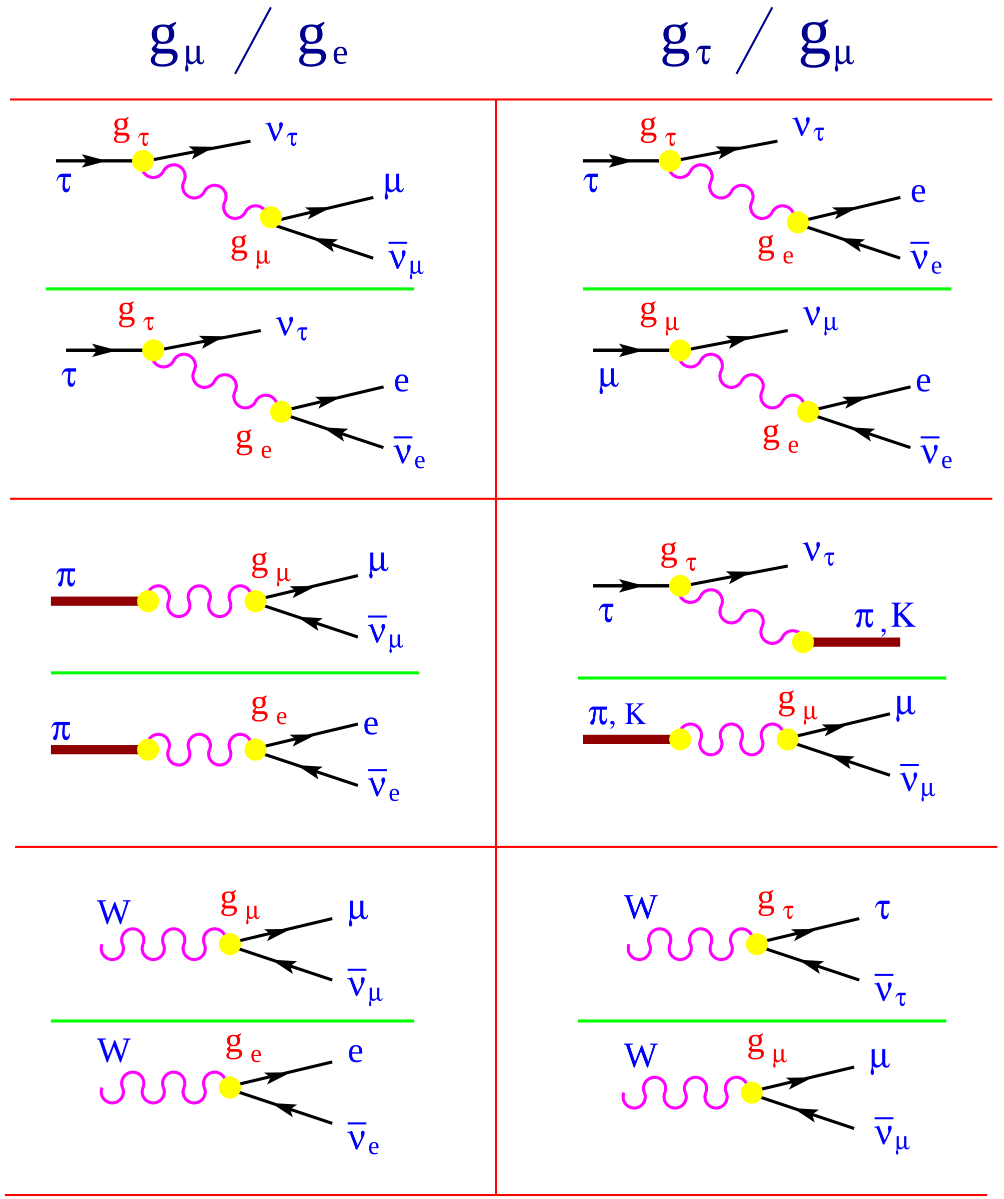
- Lepton Universality. Lorentz Structure
- Hadronic Decays: QCD Tests, m_s, $|V_{us}|$
- New Physics: Lepton Number Violation, $\not{CP}$, ...

BESIII / CLEO-c Workshop, Beijing, 13–15 January 2004

CHARGED–CURRENT UNIVERSALITY



	$\|g_\mu/g_e\|$
$B_{\tau\to\mu}/B_{\tau\to e}$	0.9999 ± 0.0020
$B_{\pi\to e}/B_{\pi\to\mu}$	1.0017 ± 0.0015
$B_{W\to\mu/e}$	0.998 ± 0.011

	$\|g_\tau/g_\mu\|$
$B_{\tau\to e}\ \tau_\mu/\tau_\tau$	1.0004 ± 0.0023
$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	0.9999 ± 0.0036
$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$	0.979 ± 0.017
$B_{W\to\tau/\mu}$	1.030 ± 0.013

	$\|g_\tau/g_e\|$
$B_{\tau\to\mu}\ \tau_\mu/\tau_\tau$	1.0002 ± 0.0022
$B_{W\to\tau/e}$	1.028 ± 0.013

Present Accuracy

$B_{\tau\to\mu} = (17.33 \pm 0.06)\%$	$3.5 \cdot 10^{-3}$
$B_{\tau\to e} = (17.81 \pm 0.06)\%$	$3.4 \cdot 10^{-3}$
$\tau_\tau = (290.6 \pm 0.9)$ fs	$3.1 \cdot 10^{-3}$
$m_\tau = (1776.99\,^{+\,0.29}_{-\,0.26})$ MeV	$1.6 \cdot 10^{-4}$
m_τ^5	$0.8 \cdot 10^{-3}$
$\tau_\mu = (2.19703 \pm 0.00004) \cdot 10^{-6}$ s	$1.8 \cdot 10^{-5}$

New Experiments: BELLE, BABAR, CLEO-C, BESIII, TWIST, PSI

Improved Theoretical Calculations:

- $e^+e^- \to \tau^+\tau^-$ (threshold, NNLO) Ruiz-Femenía–A.P., Voloshin
- μ decay (lifetime /spectrum) van Ritbergen-Stuart-Malde, Steinhauser-Seidensticker, Arbuzov-Czarnecki-Gaponenko-Melnikov, Freitas et al, Awramik-Czakon, Davydychev-Schilcher-Spiesberger, Fisher et al, . . .

LORENTZ STRUCTURE $(l^- \to \nu_l \, l'^- \bar{\nu}_{l'})$

$$\mathcal{H} = 4 \, \frac{G_{l'l}}{\sqrt{2}} \sum_{n,\epsilon,\omega} g^n_{\epsilon\omega} \left[\overline{l'_\epsilon} \, \Gamma^n \, (\nu_{l'})_\sigma \right] \left[\overline{(\nu_l)_\lambda} \, \Gamma_n \, l_\omega \right]$$

$$\Gamma^S = I \; ; \; \Gamma^V = \gamma^\mu \; ; \; \Gamma^T = \frac{\sigma^{\mu\nu}}{\sqrt{2}}$$

$$\epsilon, \omega, \sigma, \lambda = L, R$$

$$\Gamma \propto Q_{LL} + Q_{LR} + Q_{RL} + Q_{RR} \equiv 1 \equiv \sum_{n,\epsilon,\omega} |\tilde{g}^n_{\epsilon\omega}|^2$$

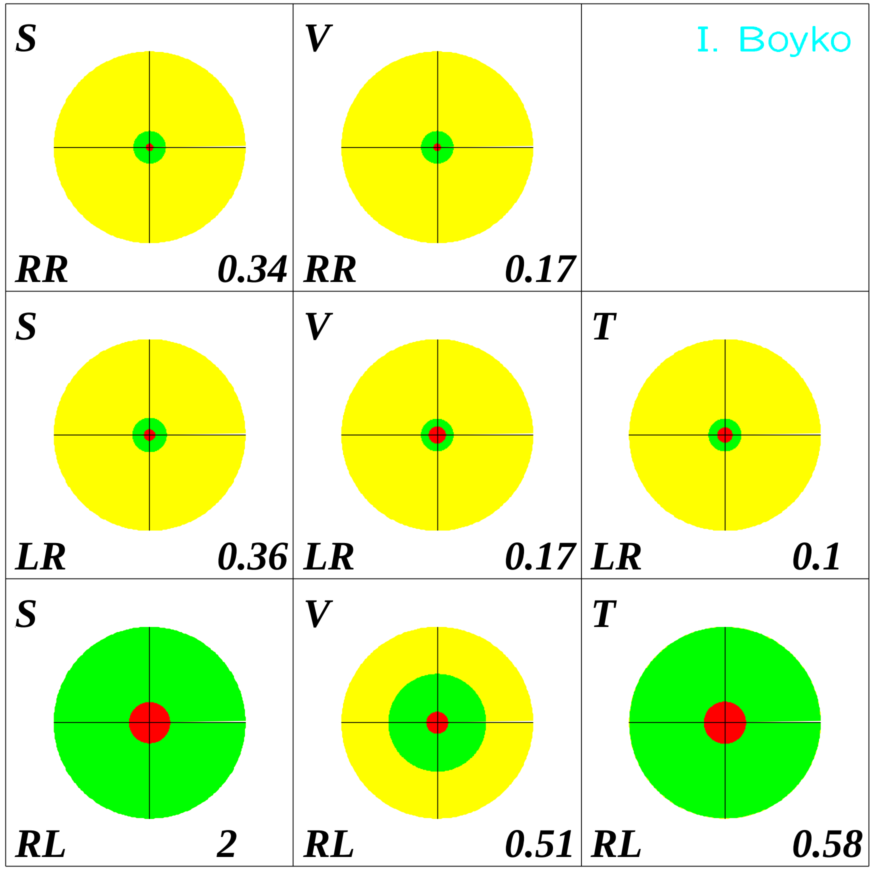
$$= \frac{1}{4} \left(|g^S_{RR}|^2 + |g^S_{RL}|^2 + |g^S_{LR}|^2 + |g^S_{LL}|^2 \right)$$
$$+ \left(|g^V_{RR}|^2 + |g^V_{RL}|^2 + |g^V_{LR}|^2 + |g^V_{LL}|^2 \right)$$
$$+ 3 \left(|g^T_{RL}|^2 + |g^T_{LR}|^2 \right)$$

3×10 complex couplings $g^n_{\epsilon\omega}$

$\mu \to e, \; \tau \to \mu, \; \tau \to e$

Standard Model: $G_{l'l} = G_F \; ; \; g^V_{LL} = 1$

90% CL, e/μ universality assumed



HADRONIC TAU DECAYS

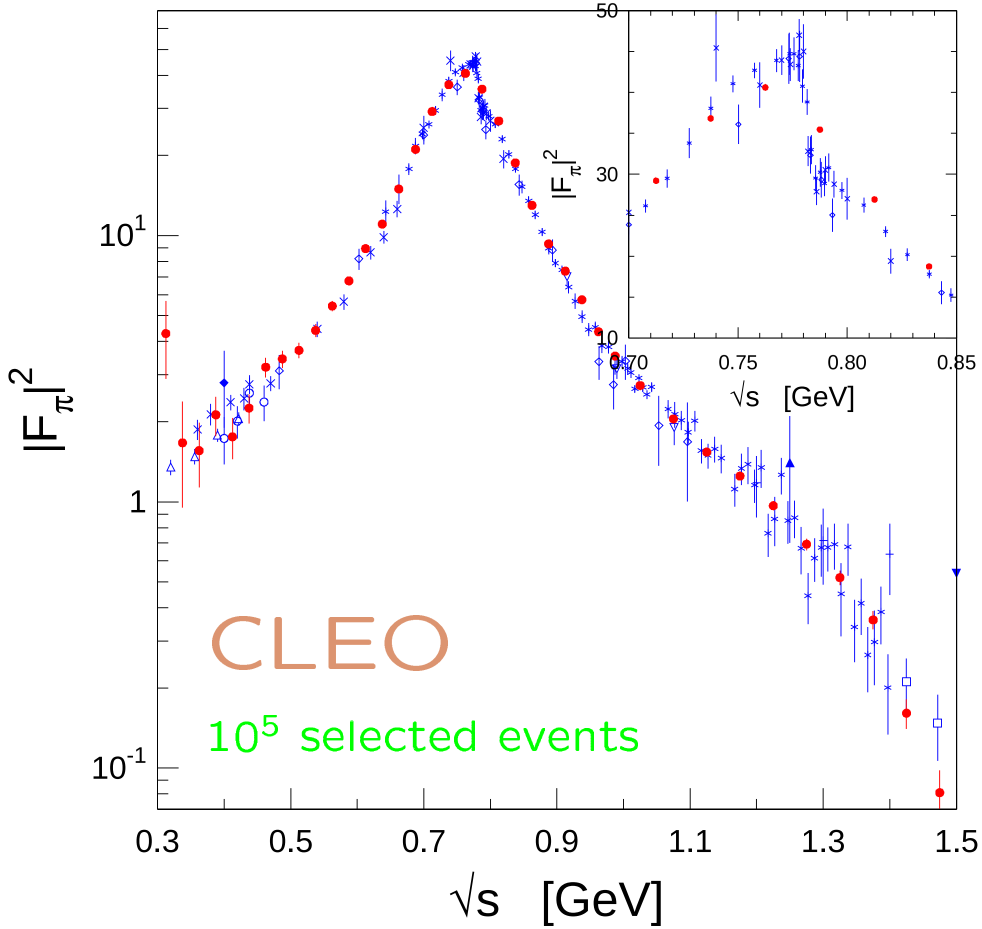
$\tau^- \to \nu_\tau H^-$ probes the hadronic $V-A$ current $\langle H^- | \bar{d}_\theta \gamma^\mu (1-\gamma_5) u | 0 \rangle$

$$\tau^- \to \nu_\tau \pi^- \pi^0$$

$$\langle \pi^- \pi^0 | \bar{d} \gamma^\mu u | 0 \rangle \equiv \sqrt{2}\, F_\pi(s)\, (p_{\pi^-} - p_{\pi^0})^\mu$$

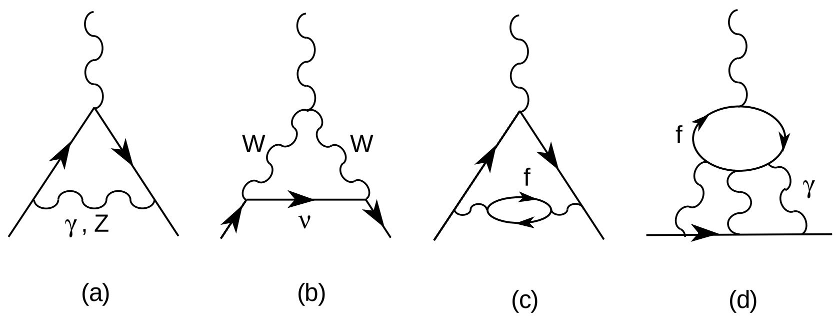
Pion Form Factor

τ data slightly above e^+e^- data



Br($\tau^- \to \nu_\tau \pi^- \pi^0$) higher than CVC prediction by $2.9\,\sigma$ [Davier]

Muon Anomalous Magnetic Moment



$$a_l \equiv \frac{1}{2}\,(g_l - 2) \approx \frac{\alpha}{2\pi}$$

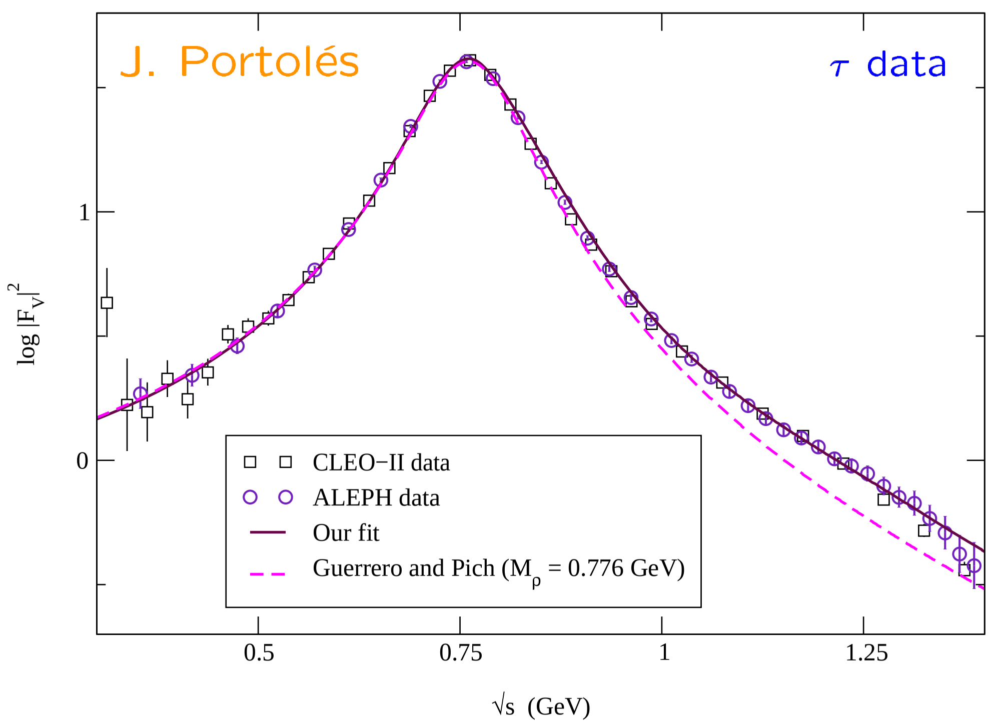
$$a_e = (115\,965\,218.69 \pm 0.41) \times 10^{-11} \quad \longrightarrow \quad \alpha^{-1} = 137.035\,998\,75\,(52)$$

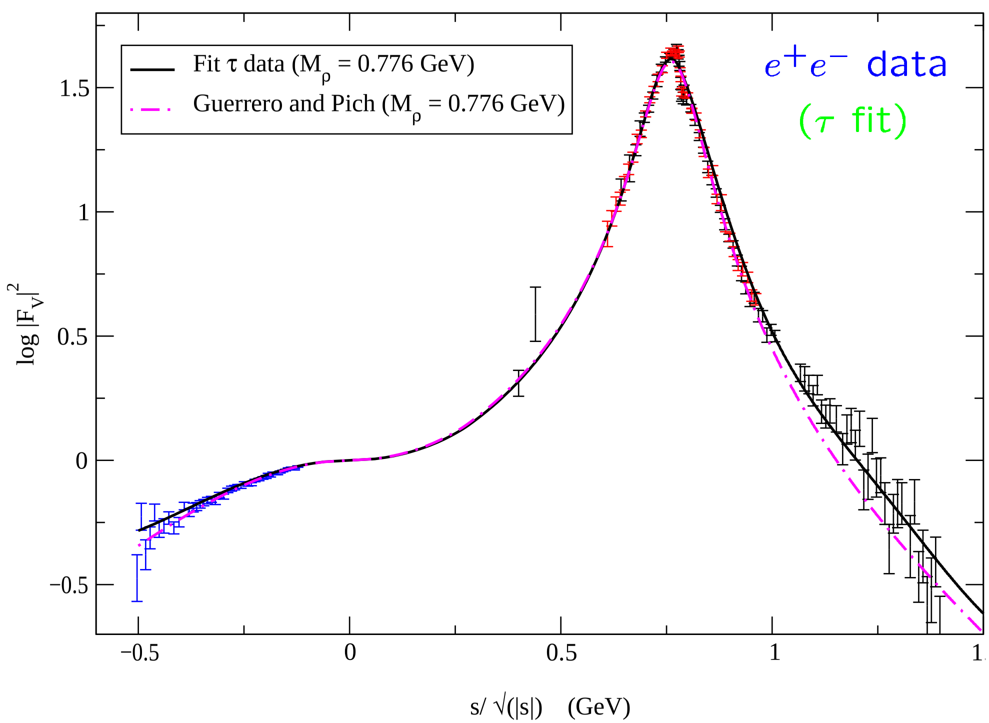
$$a_\mu = (11\,659\,203 \pm 8) \times 10^{-10}$$

BNL-E821

$10^{10} \times a_\mu^{\text{th}}$	$= 11\,658\,470.4 \pm 1.5$	QED	Kinoshita–Nio
	$+ 15.4 \pm 0.2$	EW	Czarnecki–Marciano–Vainshtein
	$+ 705.2 \pm 7.2$	hvp $(711.0 \pm 5.8)_\tau$, $(696.3 \pm 7.2)_{e^+e^-}$	Davier
	$- 10.0 \pm 0.6$	hvp NLO	Krause
	$+ 13.6 \pm 3.5$	light – by – light	Melnikov–Vainshtein, Knecht et al
	$= 11\,659\,194.6 \pm 8.2$	$(11\,659\,200.4 \pm 7.0)_\tau$, $(11\,659\,185.7 \pm 8.2)_{e^+e^-}$	
$a_\mu - a_\mu^{\text{th}}$	$= 0.73\,\sigma$	$0.24\,\sigma$	$1.51\,\sigma$

PION FORM FACTOR





Th: Analyticity, Unitarity, Chiral Symmetry (χPT, RχT), QCD, $N_C \to \infty$

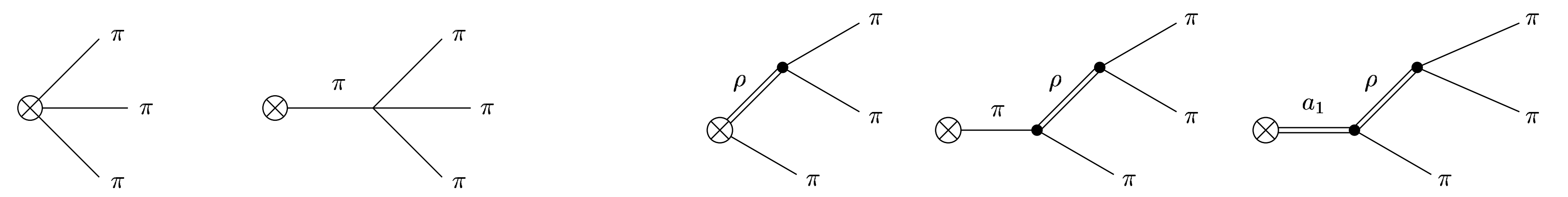
$$F_\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s - i M_\rho \Gamma_\rho(s)} \exp\left\{ -\frac{s}{96\pi^2 f_\pi^2} \mathrm{Re}\left[A\left(\frac{M_\pi^2}{s}, \frac{M_\pi^2}{M_\rho^2} \right) \right] \right\}$$

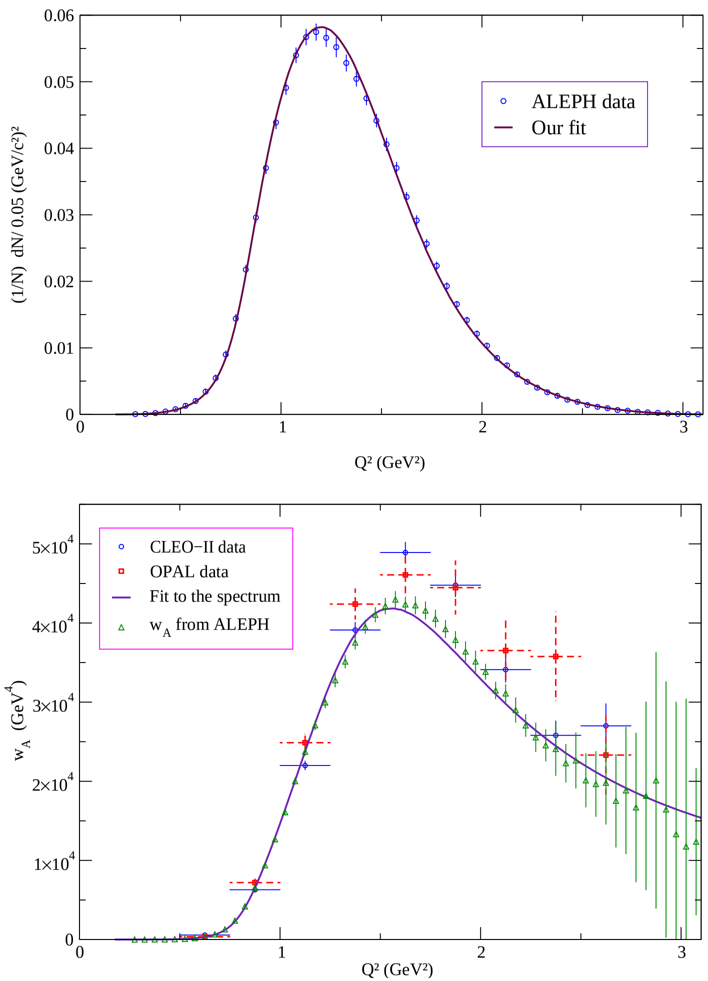
Guerrero–Pich '97

$$A(x,y) \equiv \log y + 8x - \frac{5}{3} + \sigma_x^3 \log\left(\frac{\sigma_x+1}{\sigma_x-1}\right) \quad ; \quad \sigma_x \equiv \sqrt{1-4x} \quad ; \quad \Gamma_\rho(s) = \theta(s - 4M_\pi^2)\, \sigma_\pi^3 \, \frac{M_\rho s}{96\pi f_\pi^2}$$

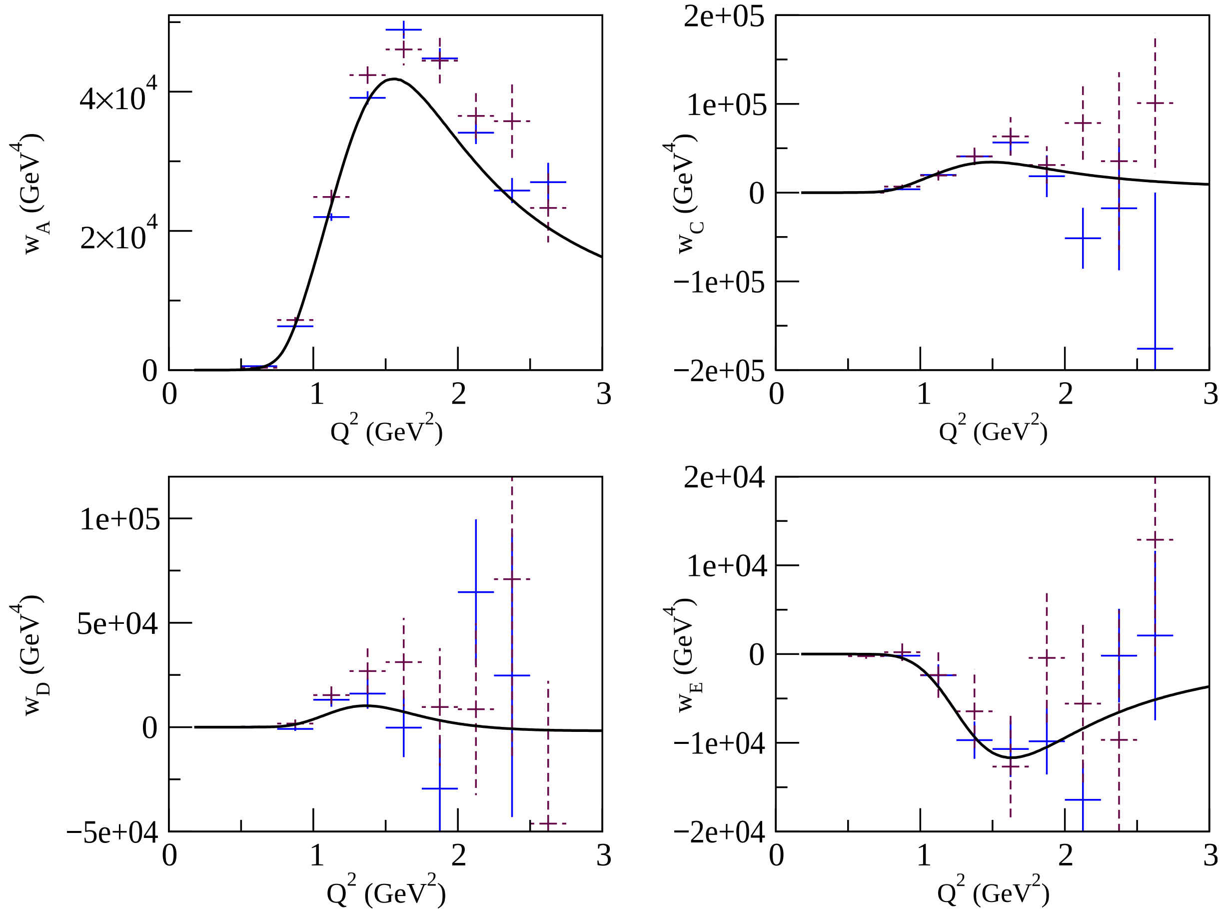
$\tau \to 3\pi\,\nu_\tau$

Dumm-Pich-Portolés



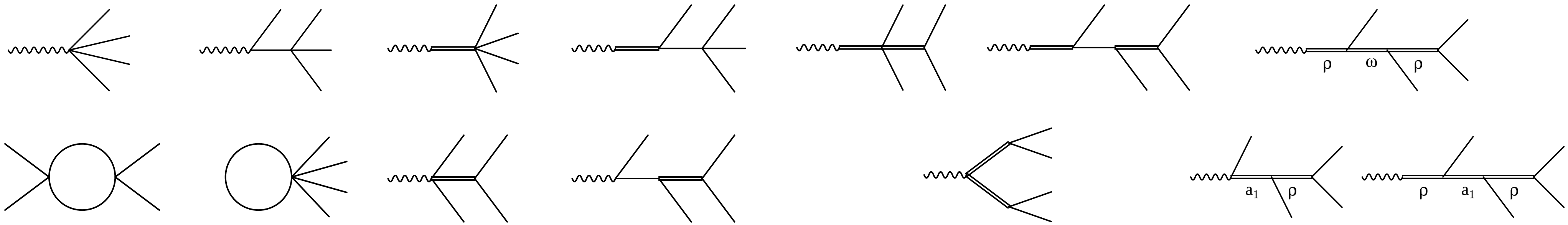


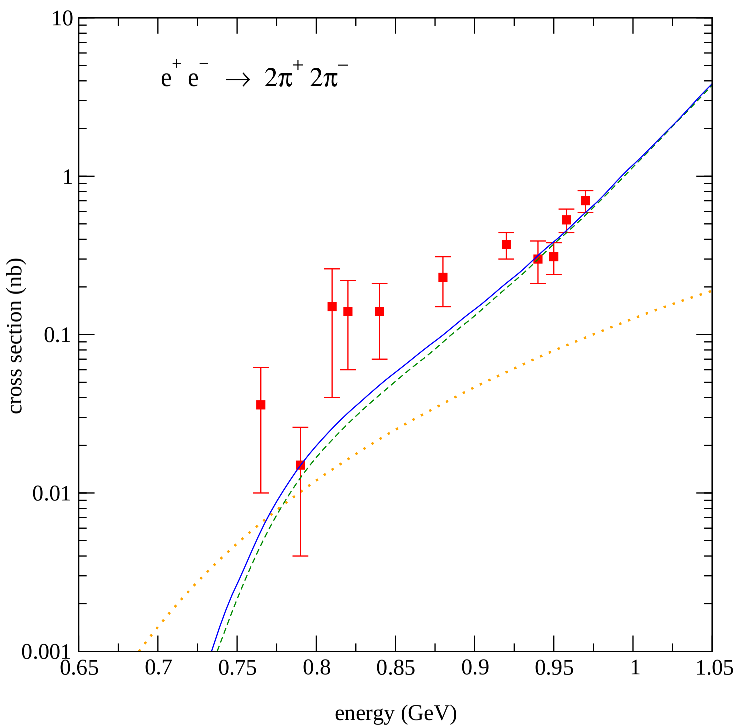
CLEO-II, OPAL

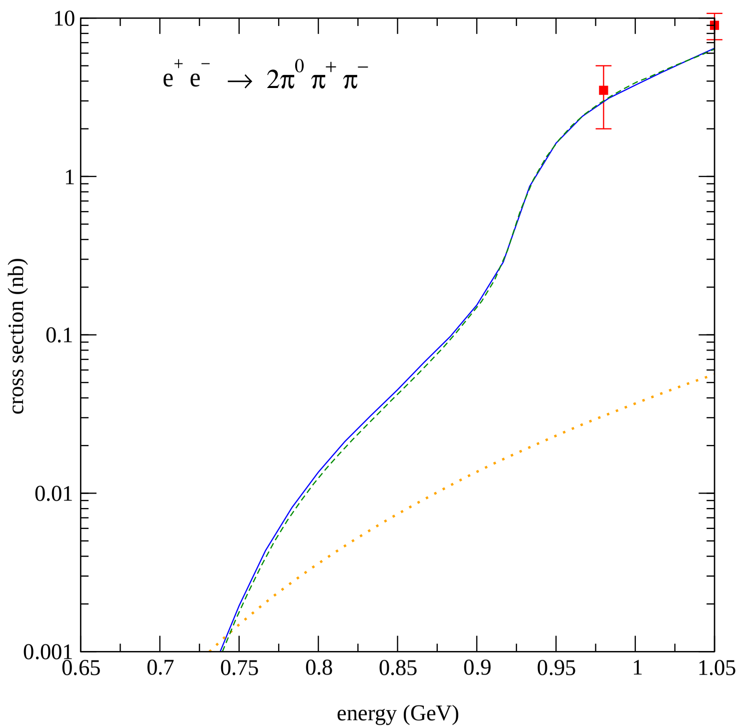


$e^+e^- \to 4\,\pi$ / $\tau \to \nu_\tau\, 4\,\pi$

Ecker–Unterdorfer







Global factor 2/27 missing in previous analyses (TAUOLA)

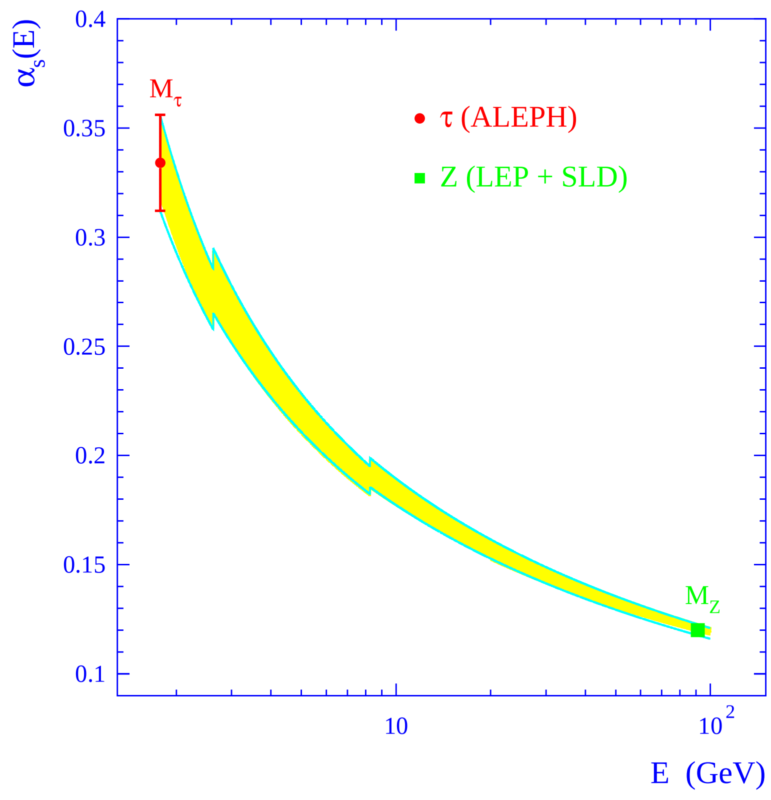
QCD TESTS

Braaten–Narison–Pich

$$R_\tau \equiv \frac{\Gamma(\tau \to \nu_\tau + \text{had})}{\Gamma(\tau \to \nu_\tau e \bar{\nu}_e)} = R_\tau^V + R_\tau^A + R_\tau^S = N_C \, S_{EW} \left\{1 + \delta'_{EW} + \delta_P + \delta_{NP}\right\}$$

$$\delta_P = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + \cdots \approx 20\% \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

$$S_{EW} = 1.0194 \quad ; \quad \delta'_{EW} = 0.0010 \qquad ; \qquad \delta_{NP} = \sum_{n\geq 2} \frac{C_{2n}}{m_\tau^{2n}} \sim \frac{C_6}{m_\tau^6} < 1\%$$



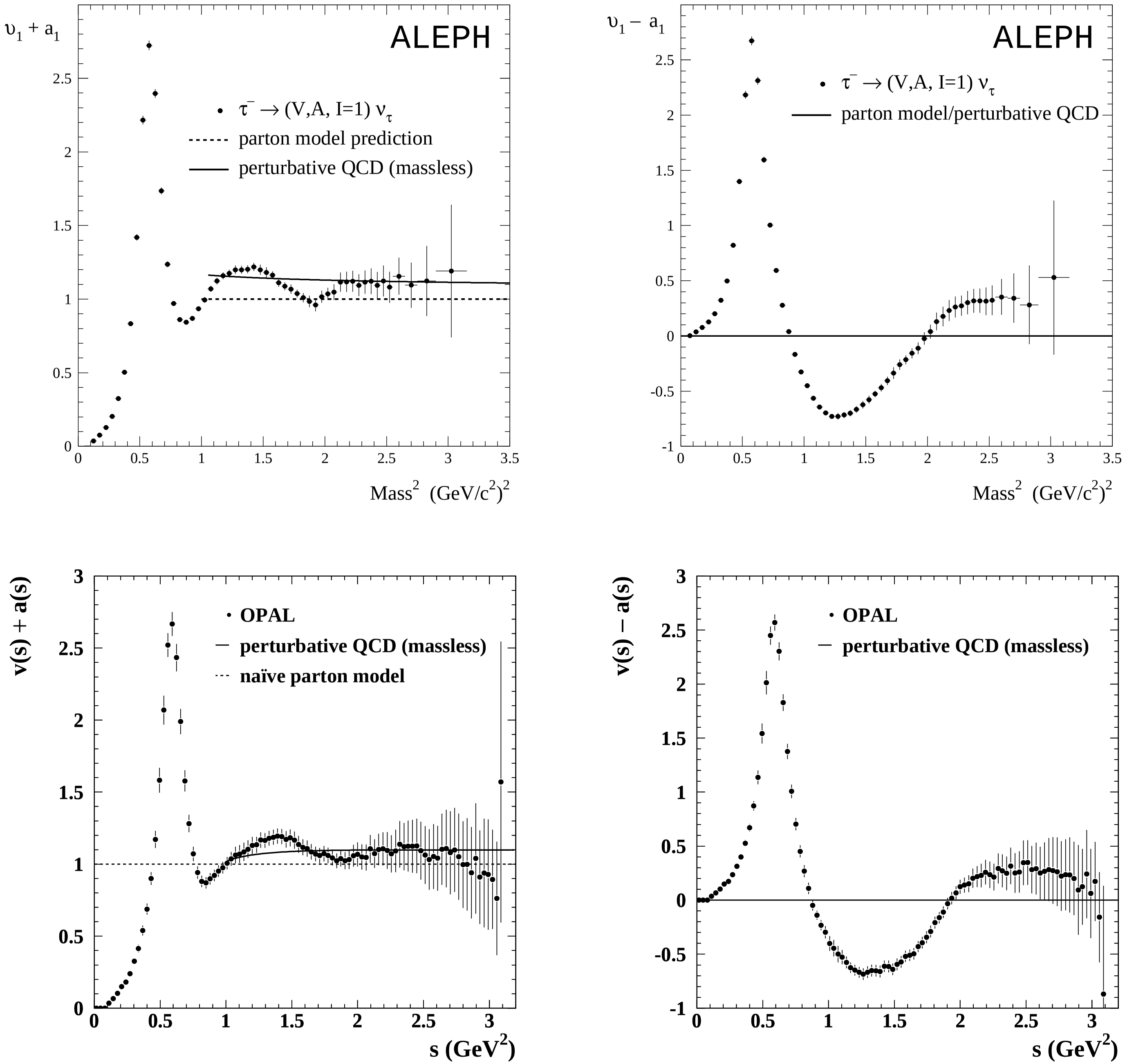
$$\Longrightarrow \quad \alpha_s(M_Z^2) = 0.120 \pm 0.003$$

Similar predictions for $R_{\tau,V}$, $R_{\tau,A}$, $R_{\tau,S}$

$$R_\tau^{kl}(s_0) \equiv \int_0^{s_0} ds \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_\tau}{ds}$$

$\delta_{NP} = -0.003 \pm 0.004$ (fitted from data)

SPECTRAL FUNCTIONS



Important Information on QCD

CVC

a_μ^{had} , $\alpha(M_Z^2)$

Chiral Sum Rules

(SCSB)

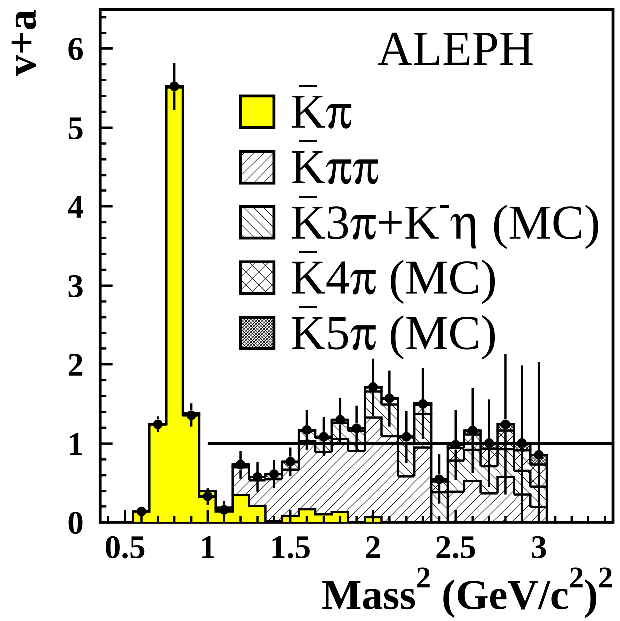
f_π , $m_{\pi^\pm}^2 - m_{\pi^0}^2$

$F_A/\langle r_\pi^2 \rangle$, . . .

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} \approx 24 \frac{m_s^2(m_\tau)}{m_\tau^2} \Delta_{kl}(a_\tau)$$

Prades–Pich

Strange Spectral Function



(k,l)	δR_τ^{kl} (ALEPH)	$m_s(m_\tau)$ (MeV)
$(0,0)$	0.374 ± 0.133	$132 \pm 29_{exp} \pm 14_{th}$
$(1,0)$	0.398 ± 0.078	$120 \pm 16_{exp} \pm 16_{th}$
$(2,0)$	0.399 ± 0.054	$117 \pm 12_{exp} \pm 21_{th}$

Chen et al

$$m_s(m_\tau) = (120 \pm 11_{\text{exp}} \pm 8_{V_{us}} \pm 19_{\text{th}})\ \text{MeV}$$

$$R_{\tau,S}^{kl} \equiv \int ds\, w_{kl}(s)\, \frac{dR_{\tau,S}}{ds}$$

$$w_{kl}(s) = \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l$$

$$m_s(1\ \text{GeV}) = \left(160\,^{+28}_{-35}\right)\ \text{MeV}$$

$$m_s(2\ \text{GeV}) = \left(116\,^{+20}_{-25}\right)\ \text{MeV}$$

Gámiz-Jamin-Pich-Prades-Schwab 2003

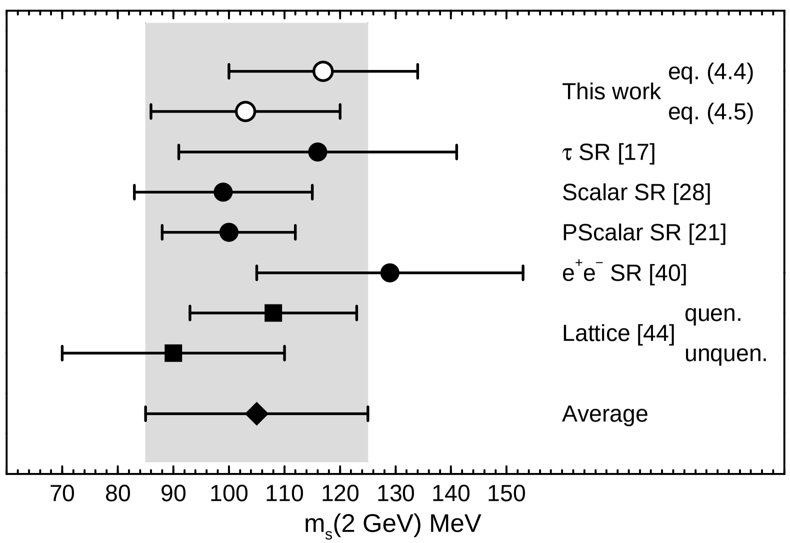
- Bad perturbative convergence of $\Delta_{kl}^{L}(a_\tau)$

→ $J = 0$ spectral function from non-τ data / theory

- Strong sensitivity to $|V_{us}|$

Unitarity CKM fit: $m_s(2\ \text{GeV}) = (117 \pm 17)$ MeV

PDG average: $m_s(2\ \text{GeV}) = (103 \pm 17)$ MeV



Precise measurements of δR_τ^{kl}

↓

$\mathbf{m_s}$ **and** $|\mathbf{V_{us}}|$ determination

$m_\nu \neq 0 \longrightarrow$ **NEW PHYSICS**

Lepton Number Violation. Lepton Mixing. Leptonic $\not{C}\not{P}$

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_d \frac{c_d}{\Lambda^{d-4}} O_d$$

- **1** $SU(2)_L \otimes U(1)_Y$ invariant operator with $d = 5$

$$\frac{c_{ij}}{\Lambda} \bar{L}_i \tilde{\phi} \tilde{\phi}^t L_j^c \xrightarrow{\text{SSB}} -\frac{1}{2} \bar{\nu}_{iL} M_{ij} \nu_{jL}^c \quad , \quad M_{ij} = -\frac{c_{ij}}{\Lambda} v^2$$

Small Majorana Mass: $m_\nu \gtrsim 0.05$ eV $\longrightarrow$ $\Lambda / c_{ij} \lesssim 10^{15}$ GeV

- Present Limits on $\tau \to l_1 l_2 l_3$ decays $\sim 2 \times 10^{-7}$ BABAR, BELLE

$$\Lambda / \sqrt{c_6} > 6 \text{ TeV}$$

SUMMARY

- Lepton Universality tested to rather good accuracy
- $V-A$ Structure verified in $\mu \to e\,\bar{\nu}_e\,\nu_\mu$, but not yet in $\tau \to l\,\bar{\nu}_l\,\nu_\tau$
- The τ is a wonderful QCD Laboratory
  - Exclusive: Resonances, χPT, . . .
  - Inclusive: α_s , m_s , V_{us} , $\langle 0|G^2|0\rangle$, . . .
- Open questions in $(g-2)_\mu$
- First hints of **New Physics**: ν **Masses and Mixings**

There is an exciting future ahead