What do we know about glueballs and hybrids from lattice QCD?

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Goals of lattice QCD

- brute-force black-box computations of hadronic observables
- tool to search for better ways of calculating in gauge theories
  - what dominates the path integrals? (instantons, center vortices,…)
  - construction of effective field theory of glue? (strings,…)

$L_{QCD}$

Lagrangian of QCD

hadron spectrum, structure, transitions
Current status of lattice technology

- pure gauge methods entirely satisfactory
  - improved gauge actions, anisotropic lattices
  - local pseudo-heat-bath with Creutz over-relaxation updating
  - correlation matrices, extended operators
  - significant calculations feasible using PC’s
- inclusion of quark loops \( \Rightarrow \) aye, there’s the rub
  - inversion of huge matrices by conjugate gradient required in updating and computation of quark propagators
  - especially severe as quark mass becomes light
  - all-to-all quark propagators for multi-hadron states
  - teraflop parallel computing power needed
    - SciDAC: Jlab, Fermilab, Columbia
  - significant challenge: push for few per cent accuracy in gold-plated quantities (not for glueballs!)
QCD suggests existence of states in which *gluon* field is excited
- glueballs (*excited glue*)
- hybrid mesons (*$qq\bar{q}$ + excited glue*)
- hybrid baryons (*$qqq$ + excited glue*)

such states not well understood
- quark model fails
- perturbative methods fail

lack of understanding makes identification difficult!

confront gluon field behavior
- bags, strings, …

clues to confinement
Yang-Mills SU(3) Glueball Spectrum

- pure-gauge mass spectrum well known
  - still needs some “polishing”
  - improve scalar states
- “experimental” results in simpler world (no quarks) to help build models of gluons
- glueball structure
  - constituent gluons vs flux loops?


\[ r_0^{-1} = 410(20) \text{ MeV}, \text{ states labeled by } J^{PC} \]
Glueballs (qualitative features)

- spectrum can be qualitatively understood in terms of interpolating operators of minimal dimension (Jaffe, Johnson, Ryzak, Ann. Phys. 168, 344 (1986))

  - dimension 4: \[ \text{Tr} F_{\mu\nu} F_{\alpha\beta} \Rightarrow 0^{++}, 0^{--}, 2^{++}, 2^{--} \]

  - dimension 5: \[ \text{Tr} F_{\mu\nu} D_{\rho} F_{\alpha\beta} \Rightarrow 1^{++}, 3^{++} \]

  - dimension 6: \[ \text{Tr} F_{\mu\nu} F_{\delta\sigma} F_{\alpha\beta} \Rightarrow 0^{\pm\pm}, 1^{\pm\pm}, 2^{\pm\pm}, 3^{\pm\pm} \]

\[ \text{Tr} F_{\mu\nu} \{ D_{\rho}, D_{\sigma} \} F_{\alpha\beta} \Rightarrow 1^{--}, 3^{--}, 4^{\pm\pm} \]

- of lightest 6 states, 4 have the \( J^{PC} \) of the dimension 4 operators
- absence of low-lying \( 0^{\pm\pm}, 1^{--} \) glueballs explained
Glueballs (bag model)

- qualitative agreement with bag model
  - constituent gluons are TE or TM modes in spherical cavity
  - Hartree modes with residual perturbative interactions
  - center-of-mass correction

<table>
<thead>
<tr>
<th></th>
<th>1983</th>
<th>1993</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>light baryon spectroscopy</td>
<td>static-quark potential</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>$B^{1/4}$</td>
<td>230 MeV</td>
<td>280 MeV</td>
</tr>
</tbody>
</table>

- recent calculation using another constituent gluon model shows qualitative agreement

Szczepaniak, Swanson, PLB577, 61 (2003)

Carlson, Hansson, Peterson, PRD27, 1556 (1983); J. Kuti (private communication)
Glueballs (flux tube model)

- disagreement with one particular string model
  
  Isgur, Paton, PRD31, 2910 (1985)

- future comparisons:
  - with more sophisticated string models (soliton knots)
  - AdS theories, duality
The troublesome scalar

- “curve ball” from lattice field theory
  - critical end-point in line of phase transitions in the fundamental vs adjoint coupling plane (lattice artifact)
  - $\phi^4$ continuum field theory at critical point (not QCD!)
  - affects scalar sector only
  - overcome using new action with adjoint-like term and negative coupling

Morningstar, Peardon, nucl-th/0309068
Hamiltonian glueballs

- pure-gauge glueball mass ratios from Hamiltonian approach

\[
\frac{M(1^{+-})}{M(0^{++})} = 1.91(17) \quad 1.70(5) \text{ Euclidean Monte Carlo}
\]

\[
\frac{M(0^{--})}{M(0^{++})} = 2.44(25)
\]
recent study of $0^{++}, 2^{++}, 0^{++*}$ glueballs in SU($N$), $N=2,3,4,5$
masses depend linearly on $1/N^2$
large $N \to \infty$ limits differ little from $N = 3$

Inclusion of quark loops

- scalar glueball results 2002
  - quark masses near strange
- still exploratory

The question of mixing with quarkonia

- Once quark loops included, does nature of glueballs radically change?
  - indications from lattice studies so far:  no!
  - conclusions still only tentative

- Weingarten and Lee (PRD61,014015 (2000)) examined this issue
  - glueball-quarkonium mixing energy consistent with zero (~40 MeV)
  - large variation with lattice spacing
  - quenched approximation (problematic due to ghost state contributions in scalar quarkonium propagator)
Mixing (continued)

- Recent analysis *unquenched* (Hart, Teper, PRD65, 034502 (2002))
- Wilson gauge, clover fermion action $N_f = 2$, $a \approx 0.1 \text{fm}$, $m_q \geq \frac{1}{2} m_s$
- tensor glueball mass same as pure-gauge
- scalar mass suppression: 0.85 of pure-gauge
  - not finite volume effect
  - independent of quark mass!
  - lattice artifact (another “curve ball”)
  - most likely explanation: fermion action adds “adjoint piece”

![Graph showing mass suppression](image-url)
Mixing (continued)

- mixing with quarkonia?
  - little shift of \((\pi\pi)_S\) in relevant energy region
  - glueball operator, overlap insensitive to \(m_q\)
- tentative conclusion: mixing appears \textit{weak}
- continuum limit?
  - possibly unchanged by presence of quarks loops

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Jan 14, 2004  Glueballs (C. Morningstar)
What if?

- What if unquenching changes glueball masses very little?
- Four glueball candidates identified in Bugg, Peardon, Zuo, PLB486,49 (2000)

\[ f_0(1500), \quad f_0(2105), \quad \eta(2190), \quad f_2(1980) \]

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Lattice</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ M(2^{++})/M(0^{++}) ]</td>
<td>1.39(4)</td>
<td>1.32(3)</td>
</tr>
<tr>
<td>[ M(0^{-})/M(0^{++}) ]</td>
<td>1.50(4)</td>
<td>1.46(3)</td>
</tr>
<tr>
<td>[ M(0^{*++})/M(0^{++}) ]</td>
<td>1.54(11)</td>
<td>1.40(2)</td>
</tr>
<tr>
<td>[ M(0^{-})/M(2^{++}) ]</td>
<td>1.081(12)</td>
<td>1.043(36)</td>
</tr>
</tbody>
</table>

- could bag model explain? Ted?
Prospects and future plans

- glueball structure: form factors using plaquette as probe
  - revisit the calculations of Tickle, Michael, NPB333, 593 (1990) SU2
- vacuum-glueball transition matrix elements
  - ongoing work: see progress report in hep-lat/0310013
- inclusion of quark loops
  - continued effort needed
  - better fermion actions, correlation matrices, anisotropic lattices
  - mixings with $\pi\pi$, $KK$, etc., in finite box yields information about decays
- tests of confinement using glueball spectrum
  - abelian, center projection, instantons
Heavy-quark hybrid mesons

- more amenable to theoretical treatment than light-quark hybrids
- possible treatment like diatomic molecule (Born-Oppenheimer)
  - slow heavy quarks $\leftrightarrow$ nuclei
  - fast gluon field $\leftrightarrow$ electrons
    (and light quarks)
- gluons provide adiabatic potentials $V_{Q\bar{Q}}(r)$
  - gluons fully relativistic, interacting
  - potentials computed in lattice simulations
- nonrelativistic quark motion described in leading order by solving Schrodinger equation for each $V_{Q\bar{Q}}(r)$
  \[
  \left\{ \frac{p^2}{2\mu} + V_{Q\bar{Q}}(r) \right\} \psi_{Q\bar{Q}}(r) = E \psi_{Q\bar{Q}}(r)
  \]
- conventional mesons from $\Sigma^+_g$; hybrids from $\Pi_u, \Sigma^-_u, \ldots$
Excitations of static quark potential

- Gluon field in presence of static quark-antiquark pair can be excited.
- Classification of states: (notation from molecular physics)
  - Magnitude of glue spin projected onto molecular axis
    \[ \Lambda = 0, 1, 2, \ldots \]
    \[ = \Sigma, \Pi, \Delta, \ldots \]
  - Charge conjugation + parity about midpoint
    \[ \eta = g \text{ (even)} \]
    \[ = u \text{ (odd)} \]
  - Chirality (reflections in plane containing axis)
    \[ \Sigma^+, \Sigma^- \]
    \[ \Pi, \Delta, \ldots \text{doubly degenerate} \]
    \[ (\Lambda \text{ doubling}) \]

Three scales

- small quark-antiquark separations $r$
  - excitations consistent with states from multipole OPE
- crossover region $0.5\text{fm} < r < 2\text{fm}$
  - dramatic level rearrangement
- large separations $r > 2\text{fm}$
  - excitations consistent with expectations from string models


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Possible interpretation

- small $r$
  - strong $E$ field of $q\bar{q}$-pair repels physical vacuum (dual Meissner effect) creating a *bubble*
  - important mixing of
    - gluonic modes inside bubble (low lying)
    - bubble surface modes (higher lying)

- large $r$
  - bubble stretches into thin tube of flux
  - separation of degrees of freedom
    - collective motion of tube (low lying)
    - internal gluonic modes (higher lying)
  - low-lying modes described by an effective string theory ($N\pi/r$ gaps – Goldstone modes)
Leading Born-Oppenheimer spectrum

- results obtained (in absence of light quark loops)
- good agreement with experiment below $B\bar{B}$ threshold
- plethora of hybrid states predicted (caution! quark loops)
- but is a Born-Oppenheimer treatment valid?

LBO degeneracies:

$\Sigma^+_g(S)$: $0^{--}, 1^{--}$

$\Sigma^+_g(P)$: $0^{++}, 1^{++}, 2^{++}, 1^{--}$

$\Pi_u(P)$: $0^{--}, 0^{+-}, 1^{++}, 1^{--}$,

$1^{++}, 1^{--}, 2^{--}, 2^{++}$

Charmonium LBO

- same calculation, but for charmonium
Testing LBO

- test LBO by comparison of spectrum with NRQCD simulations
  - include retardation effects, but no quark spin, no $\bar{p}^4$, no light quarks
  - allow possible mixings between adiabatic potentials
- dramatic evidence of validity of LBO
  - level splittings agree to 10% for 2 conventional mesons, 4 hybrids

\[ H_1, H'_1 = 1^{--}, 0^{--}, 1^{--}, 2^{--} \]
\[ H_2 = 1^{++}, 0^{--}, 1^{--}, 2^{--} \]
\[ H_3 = 0^{++}, 1^{--} \]

lowest hybrid 1.49(2)(5) GeV above 1S
Quark spin effects (continued)

- Burch and Toussaint, hep-lat/0305008
  - NRQCD simulations, measured mixing via $c_1 \sigma \cdot B / M$
  - mixing in bottomonium seems not to spoil BO picture
  - larger effect in charmonium

$$\langle 1H | \Upsilon \rangle \approx 0.076 - 0.11 \quad \langle 1H | J / \Psi \rangle \approx 0.18 - 0.25$$

$$\langle 1H | \eta_b \rangle \approx 0.13 - 0.19 \quad \langle 1H | \eta_c \rangle \approx 0.29 - 0.4$$
Light quark spoiler?

- spoil B.O.? → unknown
- light quarks change $V_{Q\bar{Q}}(r)$
  - small corrections at small $r$
    - fixes low-lying spectrum
  - large changes for $r > 1$ fm
    → fission into $(Q\bar{q})(\bar{Q}q)$
- states with diameters over 1 fm
  - most likely cannot exist as observable resonances
- dense spectrum of states from pure glue potentials will not be realized
  - survival of a few states conceivable given results from Bali et al.
- discrepancy with experiment above $B\bar{B}$
  - most likely due to light quark effects

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Bottomonium hybrids

- recent calculation of bottomonium hybrids confirms earlier results
  - quenched, several lattice spacings so $a \to 0$ limit taken
  - improved anisotropic gluon and fermion (clover) actions
  - good agreement with Born-Oppenheimer (but errors large)

Liao, Manke, PRD 65, 074508 (2002)
Charmonium hybrids

- recent determination of some charmonium hybrids
  - quenched, several lattice spacings for continuum limit
  - improved, anisotropic gluon and fermion (clover) actions
  - results suggest significant (but not large) corrections from LBO

Liao, Manke, hep-lat/0210030
Light-quark hybrids

- recent new determination of exotic $1^{-+}$ hybrid meson
  - improved staggered fermions (lighter quark masses)
  - quenched and unquenched, Wilson gluon action
  - $a \approx 0.09$ fm
  - lightest mass still above experiment

\[ N_f = 3, \quad m_u = m_d = m_s \] (around strange quark mass)

\[ m_u = m_d = 0.4 m_s \]

quenched continuum limit

MILC, hep-lat/0301024
Excitation of the static $3q$ system

- first excitation of the static $3q$ system recently determined
  - excitation energy about 1 GeV
  - finite spacing, finite volume errors still to be studied

Takahashi, Suganuma, hep-lat/0210024
Conclusion

- hadronic states bound by an *excited* gluon field
  - interesting new form of matter
  - promise to shed new light on confinement in QCD
- pure-gauge glueball spectrum well known, but needs polishing
- progress in including quark loops
  - *tentative* results so far ➔ little change to glueballs
- heavy-quark hybrid mesons
  - compelling physical picture from Born-Oppenheimer treatment
  - relationship to excitations of the static quark potential
  - quark spin effects do not seem to spoil BO
  - light quark loops ➔ survival issue