What do we know about glueballs and hybrids from lattice QCD?

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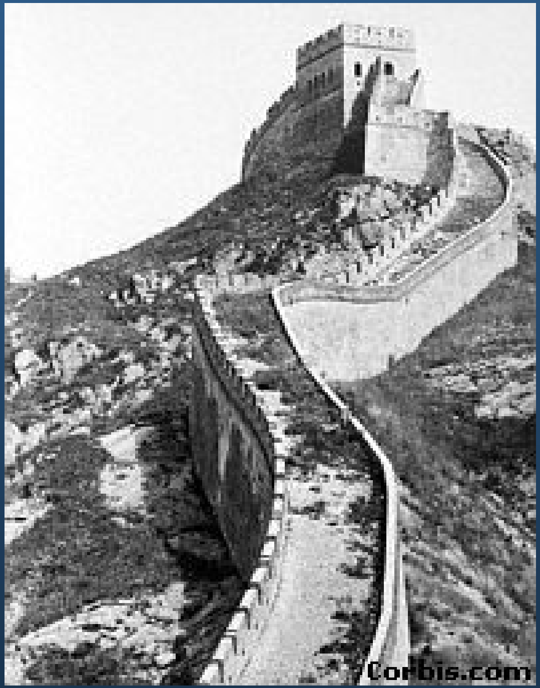
Goals of lattice QCD

- brute-force black-box computations of hadronic observables
- tool to search for better ways of calculating in gauge theories
  - ❑ what dominates the path integrals? (instantons, center vortices,…)
  - ❑ construction of effective field theory of glue? (strings,…)

L_{QCD}

Lagrangian of QCD



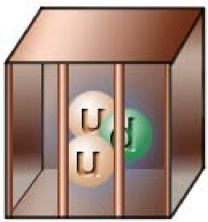
hadron spectrum,
structure,
transitions

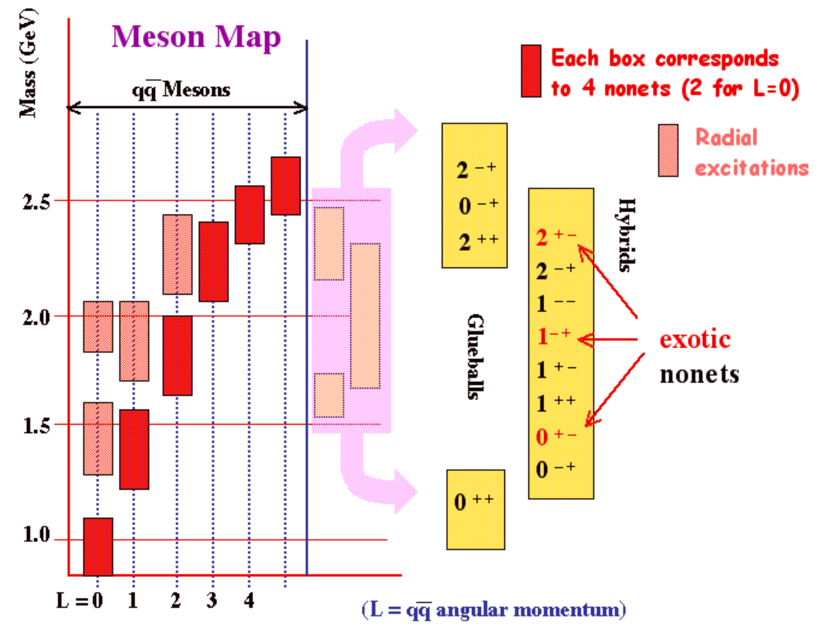
Current status of lattice technology

- pure gauge methods entirely satisfactory
  - improved gauge actions, anisotropic lattices
  - local pseudo-heat-bath with Creutz over-relaxation updating
  - correlation matrices, extended operators
  - significant calculations feasible using PC's
- inclusion of quark loops → *aye, there's the rub*
  - inversion of huge matrices by conjugate gradient required in updating and computation of quark propagators
  - especially severe as quark mass becomes light
  - all-to-all quark propagators for multi-hadron states
  - teraflop parallel computing power needed
    - SciDAC: Jlab, Fermilab, Columbia
  - significant challenge: push for *few per cent accuracy* in *gold-plated quantities* (not for glueballs!)

Gluonic excitations (new form of matter)

- QCD suggests existence of states in which *gluon* field is excited
  - glueballs (*excited glue*)
  - hybrid mesons ($q\bar{q}$ + *excited glue*)
  - hybrid baryons (qqq + *excited glue*)
- such states not well understood
  - quark model fails
  - perturbative methods fail
- lack of understanding makes identification difficult!
- confront gluon field behavior
  - bags, strings, …
- clues to confinement

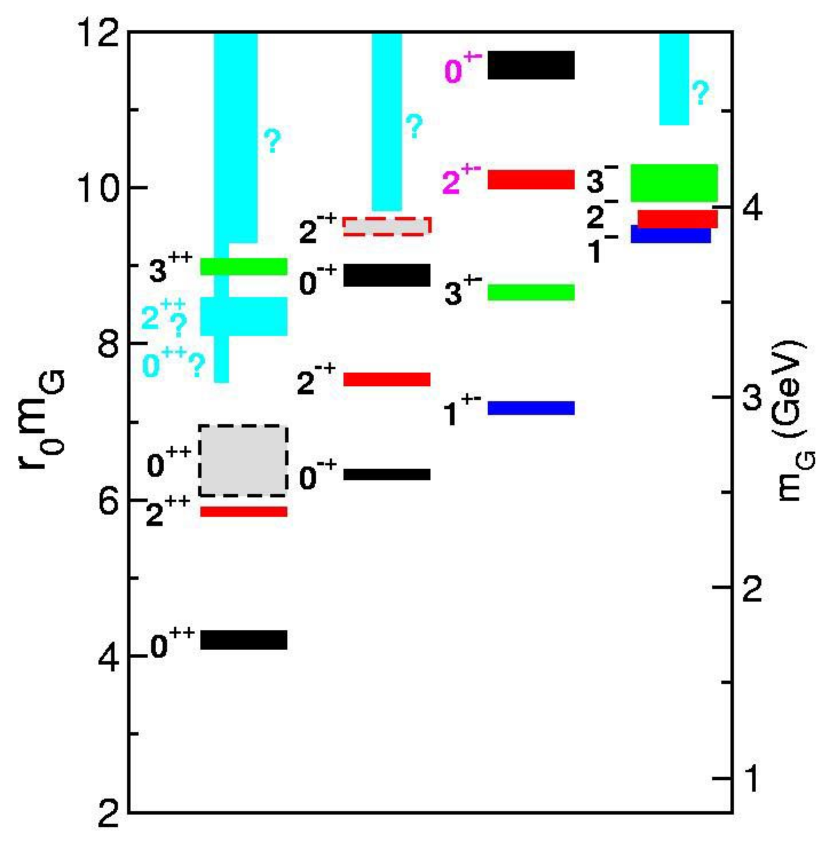




Yang-Mills SU(3) Glueball Spectrum

- pure-gauge mass spectrum well known
  - still needs some “polishing”
  - improve scalar states
- “experimental” results in simpler world (no quarks) to help build models of gluons
- glueball structure
  - constituent gluons vs flux loops?

C. Morningstar and M. Peardon, Phys. Rev. D 60, 034509 (1999)



$r_0^{-1} = 410(20)$ MeV, states labeled by J^{PC}

Glueballs (qualitative features)

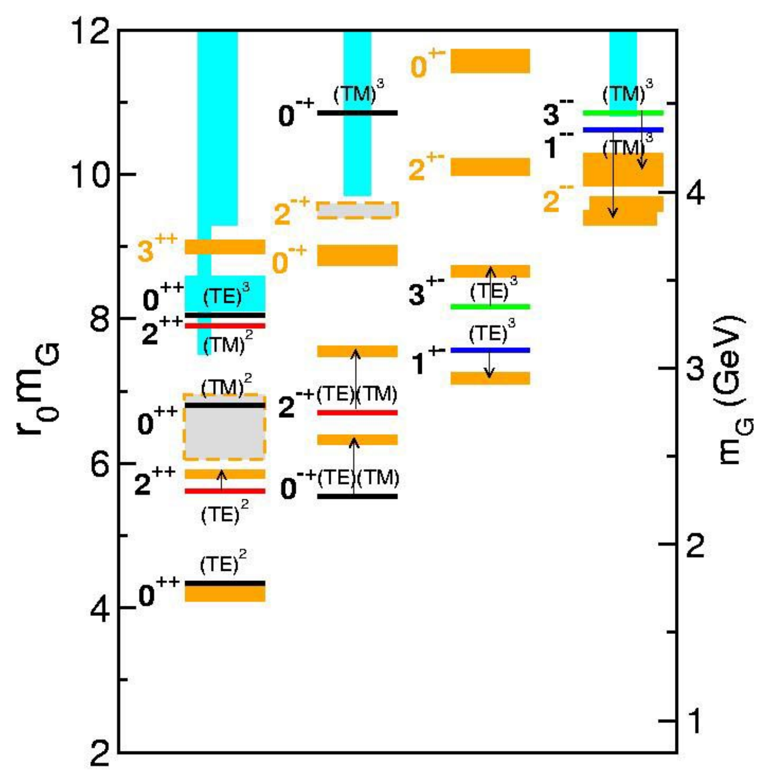
- spectrum can be qualitatively understood in terms of interpolating operators of minimal dimension (Jaffe,Johnson,Ryzak, Ann. Phys. **168**, 344 (1986))
  - dimension 4: $\mathrm{Tr}\, F_{\mu\nu} F_{\alpha\beta} \Rightarrow 0^{++}, 0^{-+}, 2^{++}, 2^{-+}$
  - dimension 5: $\mathrm{Tr}\, F_{\mu\nu} D_\rho F_{\alpha\beta} \Rightarrow 1^{++}, 3^{++}$
  - dimension 6: $\mathrm{Tr}\, F_{\mu\nu} F_{\delta\sigma} F_{\alpha\beta} \Rightarrow 0^{\pm+}, 1^{\pm\pm}, 2^{\pm\pm}, 3^{\pm-}$
    $\mathrm{Tr}\, F_{\mu\nu} \{D_\rho, D_\sigma\} F_{\alpha\beta} \Rightarrow 1^{-+}, 3^{-+}, 4^{\pm+}$
- of lightest 6 states, 4 have the J^{PC} of the dimension 4 operators
- absence of low-lying $0^{\pm-}, 1^{-+}$ glueballs explained

Glueballs (bag model)

- qualitative agreement with bag model
    - constituent gluons are TE or TM modes in spherical cavity
    - Hartree modes with residual perturbative interactions
    - center-of-mass correction

	1983	1993
	light baryon spectroscopy	static-quark potential
α_s	1.0	0.5
$B^{1/4}$	230 MeV	280 MeV

Carlson, Hansson, Peterson, PRD27, 1556 (1983);
J. Kuti (private communication)



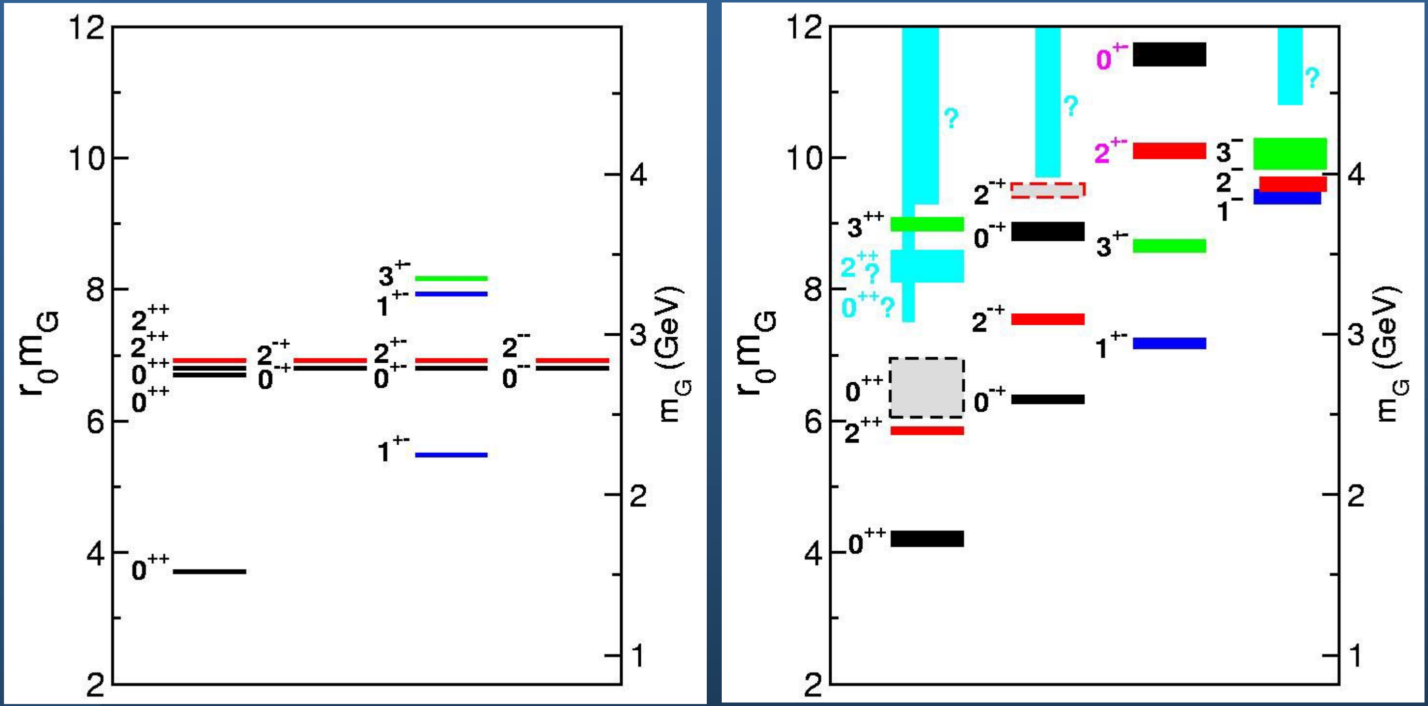
- recent calculation using another constituent gluon model shows qualitative agreement

Szczepaniak, Swanson, PLB577, 61 (2003)

Glueballs (flux tube model)

- disagreement with one particular string model

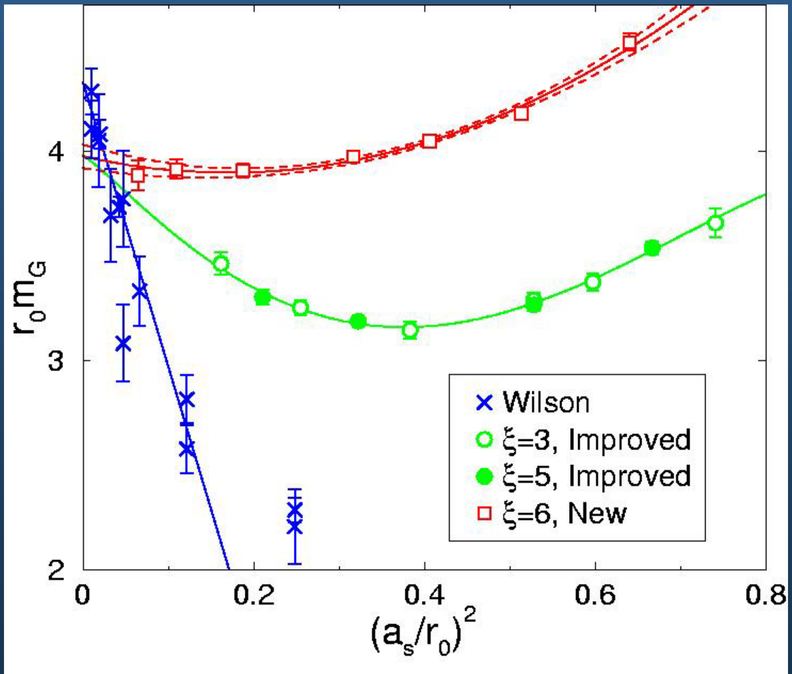
Isgur, Paton, PRD**31**, 2910 (1985)



- future comparisons:
  - with more sophisticated string models (soliton knots)
  - AdS theories, duality

The troublesome scalar

- "curve ball" from lattice field theory
  - critical end-point in line of phase transitions in the fundamental vs adjoint coupling plane (lattice artifact)
  - ϕ^4 continuum field theory at critical point (not QCD!)
  - affects scalar sector *only*
  - overcome using new action with adjoint-like term and negative coupling



Morningstar, Peardon, nucl-th/0309068

Hamiltonian glueballs

- pure-gauge glueball mass ratios from Hamiltonian approach
  - Chinese effort: Hu, Luo, Chen, Fang, Guo, Commun.Theor.Phys. **28**, 327 (1997)

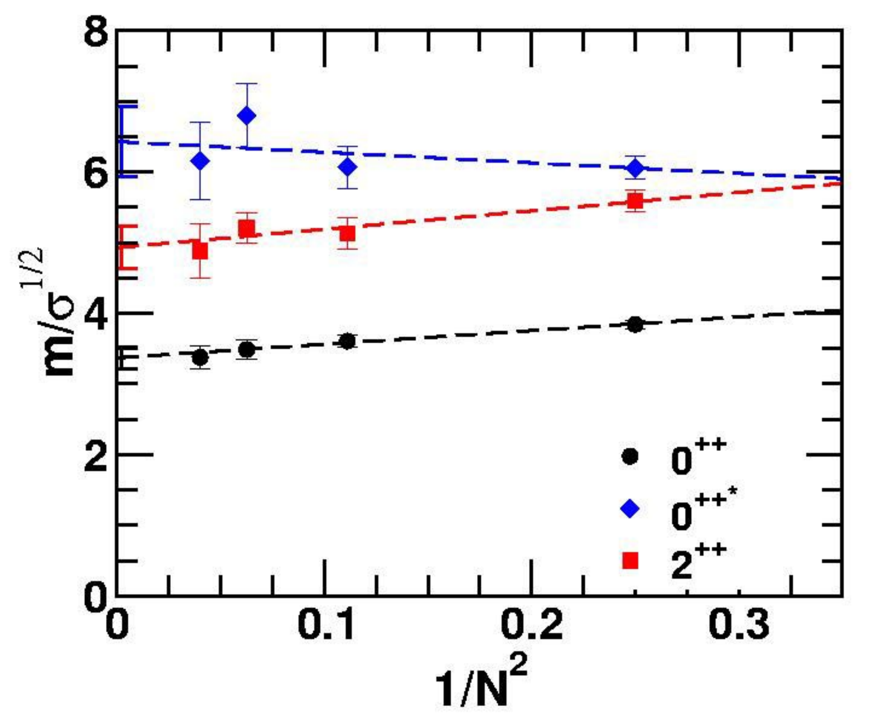
$$\frac{M\left(1^{+-}\right)}{M\left(0^{++}\right)} = 1.91(17) \qquad 1.70(5)\ \text{Euclidean Monte Carlo}$$

$$\frac{M\left(0^{--}\right)}{M\left(0^{++}\right)} = 2.44(25)$$

SU(N) Glueballs

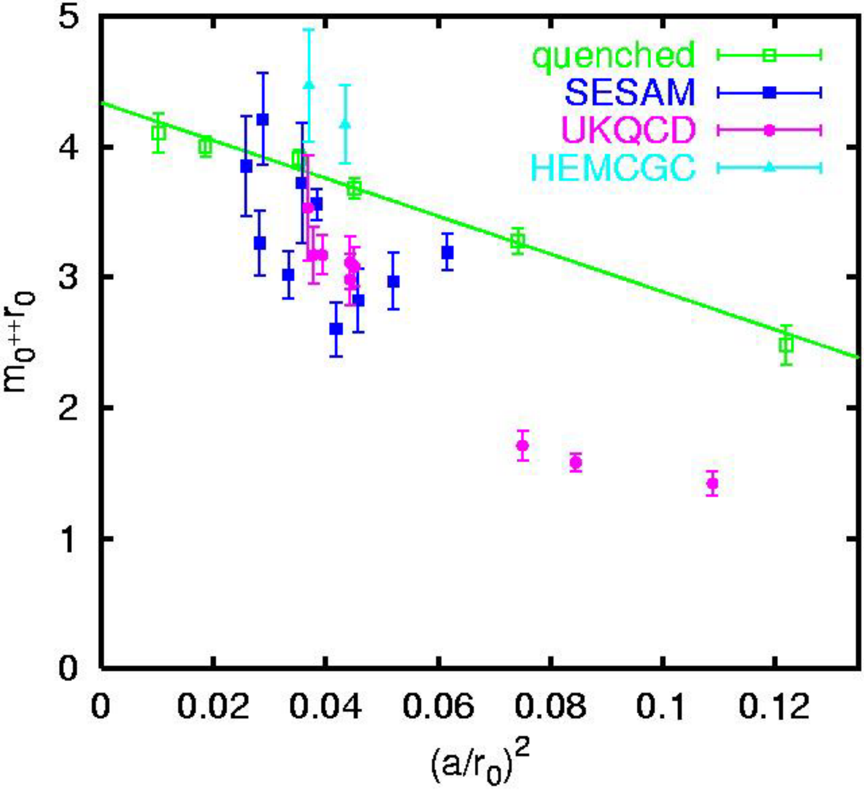
- recent study of $0^{++}, 2^{++}, 0^{++*}$ glueballs in SU(*N*), *N*=2,3,4,5
- masses depend linearly on $1/N^2$
- large $N \rightarrow \infty$ limits differ little from $N = 3$

Lucini, Teper, JHEP **06**, 050 (2001).



Inclusion of quark loops

- scalar glueball results 2002
  - quark masses near strange
- still exploratory



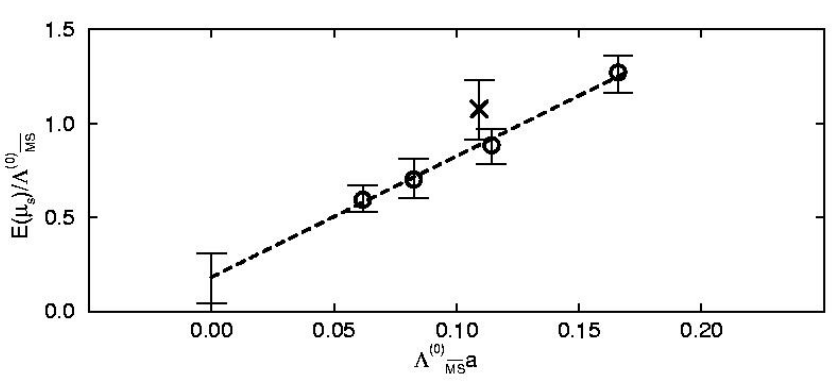
SESAM: PRD**62**, 054503 (2000)

UKQCD: PRD**65**, 014508 (2002)

HEMCGC: PRD**44**, 2090 (1991)

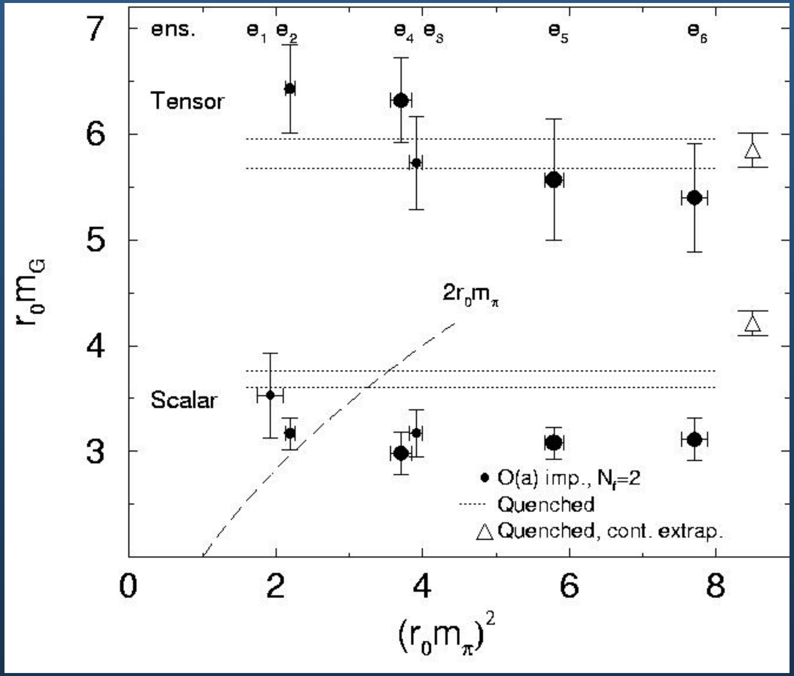
The question of mixing with quarkonia

- Once quark loops included, does nature of glueballs radically change?
  - indications from lattice studies so far: *no!*
  - conclusions still only *tentative*
- Weingarten and Lee (PRD**61**,014015 (2000)) examined this issue
  - glueball-quarkonium mixing energy consistent with zero (~40 MeV)
  - large variation with lattice spacing
  - *quenched* approximation (problematic due to ghost state contributions in scalar quarkonium propagator)



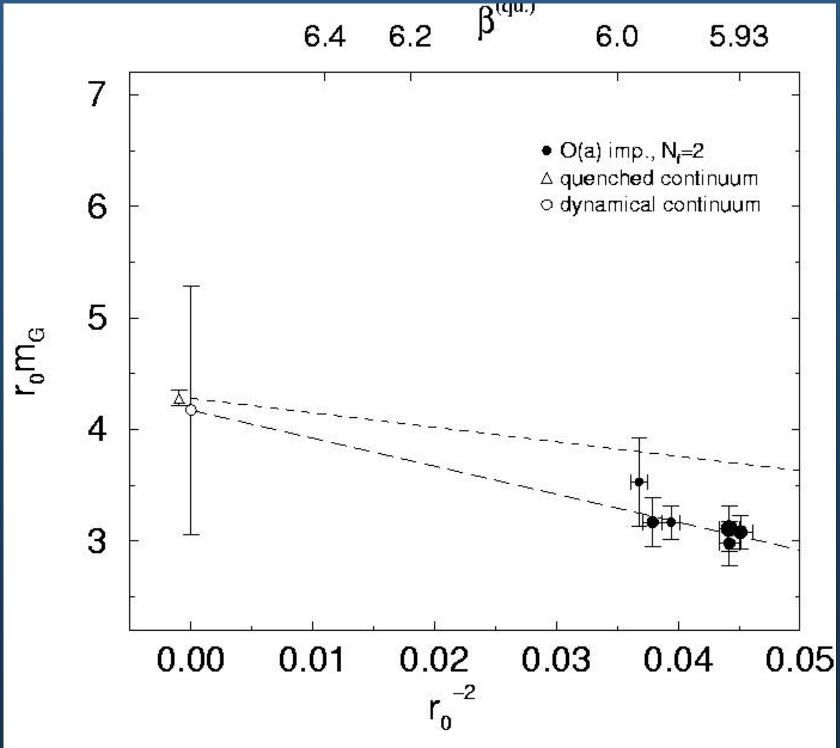
Mixing (continued)

- Recent analysis *un*quenched (Hart, Teper, PRD65, 034502 (2002))
- Wilson gauge, clover fermion action $N_f = 2, \quad a \approx 0.1\,\mathrm{fm}, \quad m_q \geq \frac{1}{2} m_s$
- tensor glueball mass same as pure-gauge
- scalar mass suppression: 0.85 of pure-gauge
  - not finite volume effect
  - independent of quark mass! → lattice artifact (another "curve ball")
  - most likely explanation: fermion action adds "adjoint piece"



Mixing (continued)

- mixing with quarkonia?
  - little shift of $(\pi\pi)_S$ in relevant energy region
  - glueball operator, overlap insensitive to m_q
- *tentative* conclusion: mixing appears *weak*
- continuum limit?
  - possibly unchanged by presence of quarks loops



What if?

- What if unquenching changes glueball masses very little?
- Four glueball candidates identified in Bugg, Peardon, Zuo, PLB486,49 (2000)

$$f_0(1500),\quad f_0(2105),\quad \eta(2190),\quad f_2(1980)$$

Ratio	Lattice	Experiment
$M(2^{++})/M(0^{++})$	1.39(4)	1.32(3)
$M(0^{-+})/M(0^{++})$	1.50(4)	1.46(3)
$M(0^{*++})/M(0^{++})$	1.54(11)	1.40(2)
$M(0^{-+})/M(2^{++})$	1.081(12)	1.043(36)

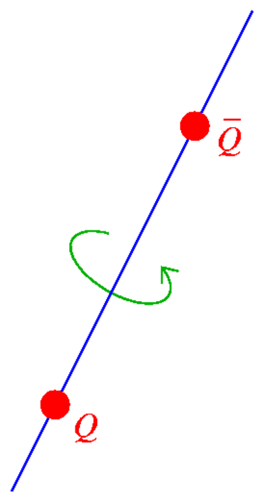
- could bag model explain? Ted?

Prospects and future plans

- glueball structure: form factors using plaquette as probe
  - ☐ revisit the calculations of Tickle, Michael, NPB333, 593 (1990) SU2
- vacuum-glueball transition matrix elements
  - ☐ ongoing work: see progress report in hep-lat/0310013
- inclusion of quark loops
  - ☐ continued effort needed
  - ☐ better fermion actions, correlation matrices, anisotropic lattices
  - ☐ mixings with $\pi\pi$, KK, *etc.*, in finite box yields information about decays
- tests of confinement using glueball spectrum
  - ☐ abelian, center projection, instantons

Heavy-quark hybrid mesons

- more amenable to theoretical treatment than light-quark hybrids
- early work: Hasenfratz, Horgan, Kuti, Richard (1980), Michael, Griffiths, Rakow (1983)
- possible treatment like diatomic molecule (Born-Oppenheimer)
  - slow heavy quarks ←→ nuclei
  - fast gluon field ←→ electrons (and light quarks)



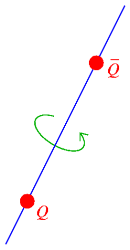
- gluons provide adiabatic potentials $V_{Q\bar{Q}}(r)$
  - gluons fully relativistic, interacting
  - potentials computed in lattice simulations
- nonrelativistic quark motion described in *leading order* by solving Schrodinger equation for each $V_{Q\bar{Q}}(r)$

$$\left\{\frac{p^2}{2\mu}+V_{Q\bar{Q}}(r)\right\}\psi_{Q\bar{Q}}(r)=E\,\psi_{Q\bar{Q}}(r)$$

- conventional mesons from Σ_g^+ ; hybrids from $\Pi_u, \Sigma_u^-, \ldots$

Excitations of static quark potential

- gluon field in presence of static quark-antiquark pair can be *excited*
- classification of states: (notation from molecular physics)
  - ☐ magnitude of glue spin projected onto molecular axis



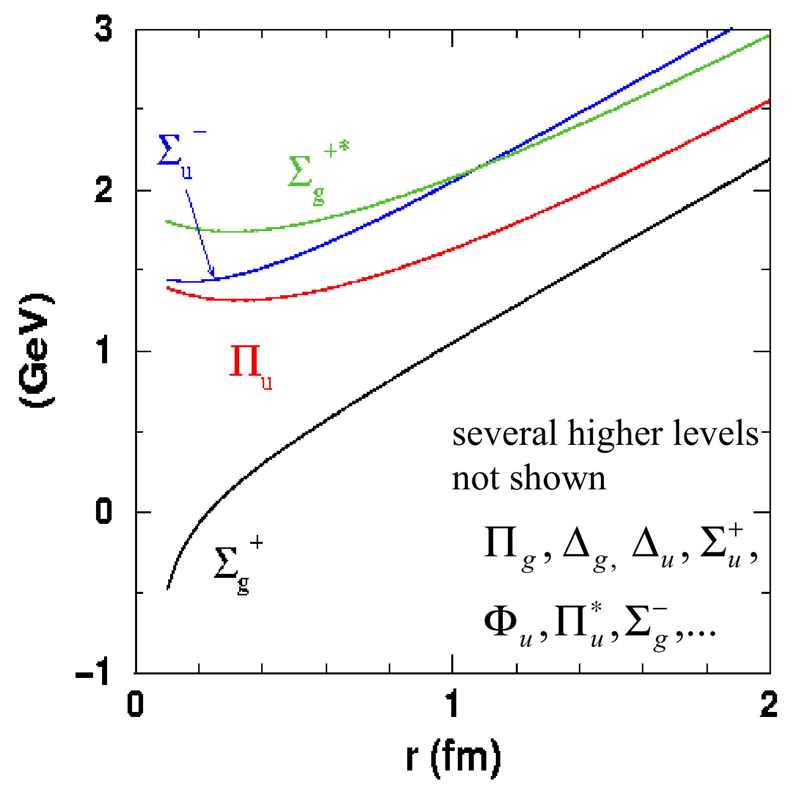
$$\Lambda = 0,1,2,\ldots$$
$$= \Sigma, \Pi, \Delta, \ldots$$

  - ☐ charge conjugation + parity about midpoint

$$\eta = g \text{ (even)}$$
$$= u \text{ (odd)}$$

  - ☐ chirality (reflections in plane containing axis) Σ^+, Σ^-

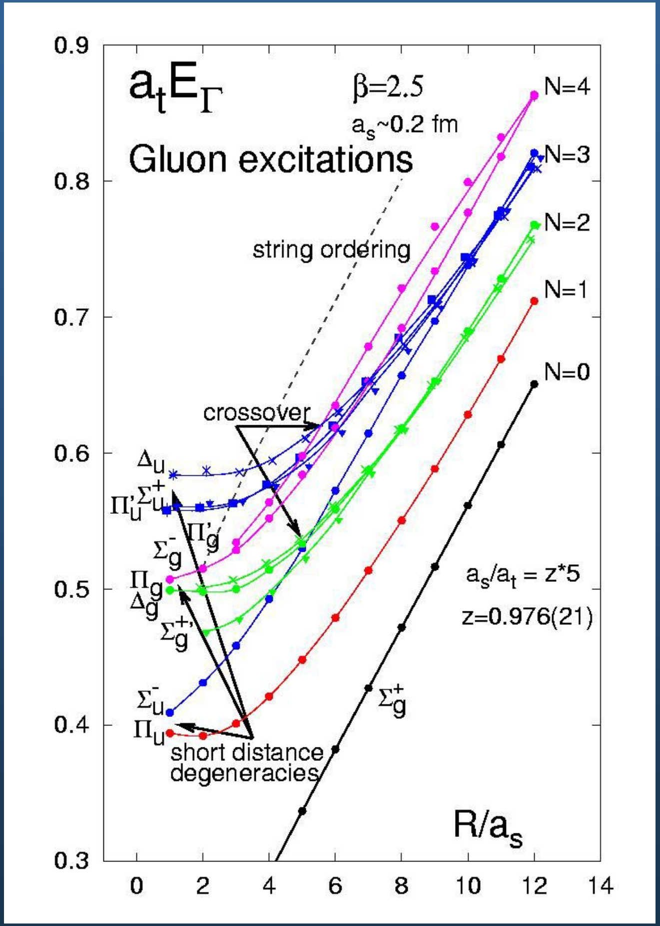
$\Pi, \Delta, \ldots$ doubly degenerate (Λ doubling)



Juge, Kuti, Morningstar, PRL 90, 161601 (2003)

Three scales

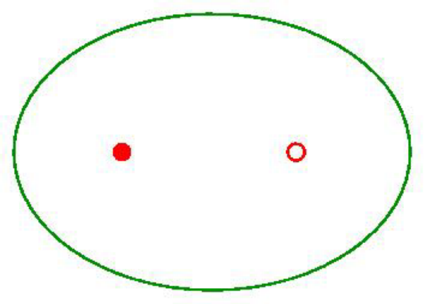
- small quark-antiquark separations r
  - excitations consistent with states from multipole OPE
- crossover region $0.5\mathrm{fm} < r < 2\mathrm{fm}$
  - dramatic level rearrangement
- large separations $r > 2\mathrm{fm}$
  - excitations consistent with expectations from string models

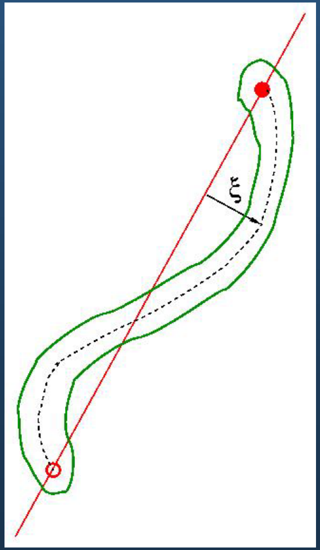


Juge, Kuti, Morningstar, PRL 90, 161601 (2003)

Possible interpretation

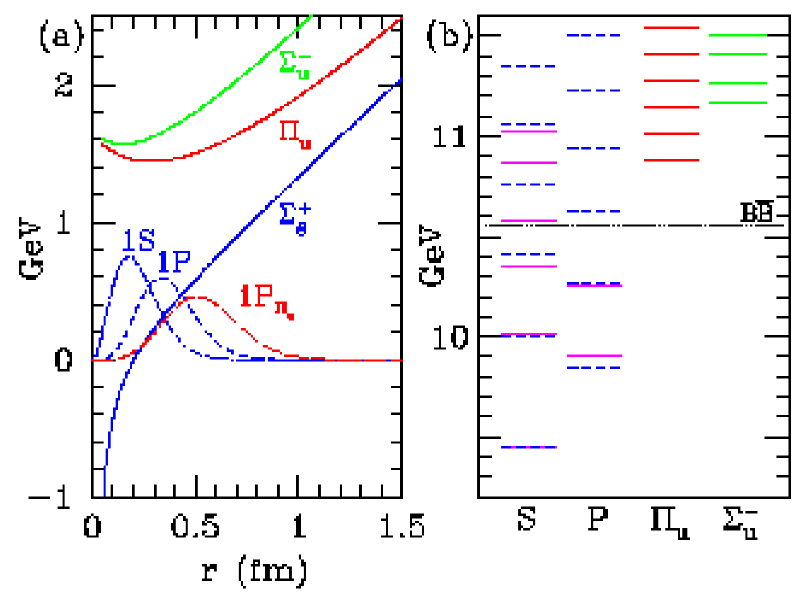
- small r
  - strong E field of $q\bar{q}$-pair repels physical vacuum (dual Meissner effect) creating a *bubble*
  - important mixing of
    - gluonic modes inside bubble (low lying)
    - bubble surface modes (higher lying)
- large r
  - bubble stretches into thin tube of flux
  - separation of degrees of freedom
    - collective motion of tube (low lying)
    - internal gluonic modes (higher lying)
  - low-lying modes described by an effective string theory ($N\pi/r$ gaps – Goldstone modes)





Leading Born-Oppenheimer spectrum

- results obtained (in absence of light quark loops)
- good agreement with experiment below $B\overline{B}$ threshold
- plethora of hybrid states predicted (caution! quark loops)
- but is a Born-Oppenheimer treatment valid?



LBO degeneracies:

$\Sigma_g^+(S): \quad 0^{-+}, 1^{--}$

$\Sigma_g^+(P): \quad 0^{++}, 1^{++}, 2^{++}, 1^{+-}$

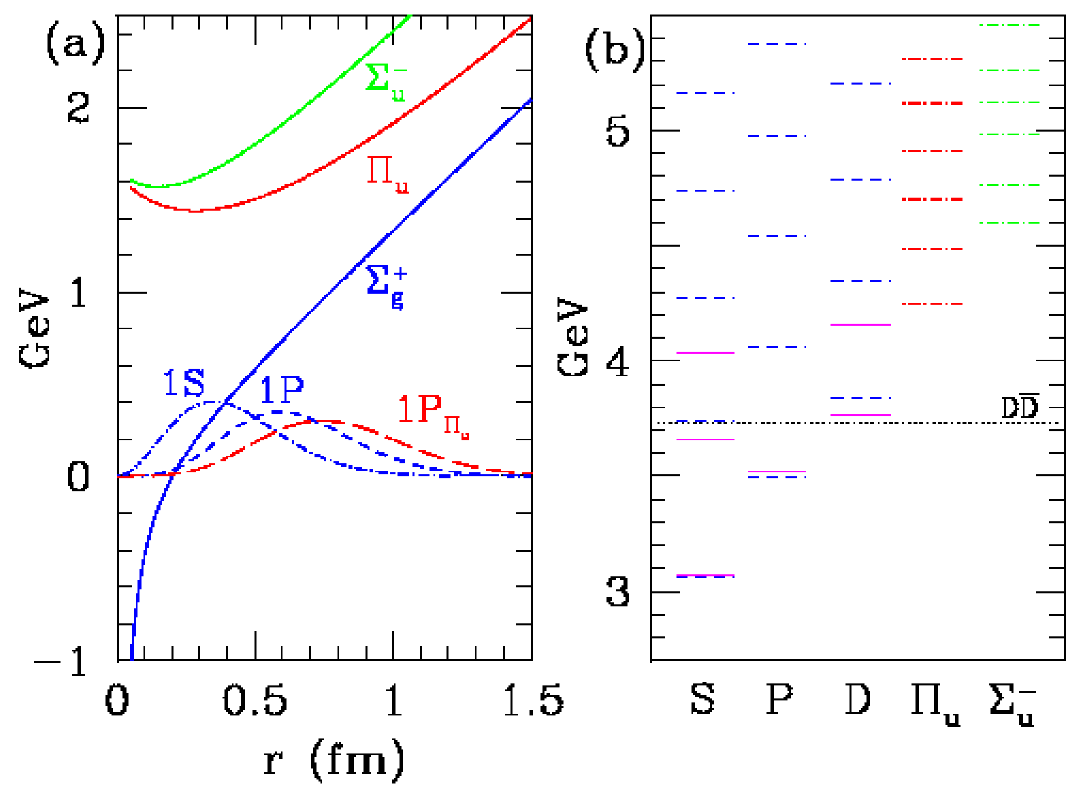
$\Pi_u(P): \quad 0^{-+}, 0^{+-}, 1^{++}, 1^{--},$

$1^{+-}, 1^{-+}, 2^{+-}, 2^{-+}$

Juge, Kuti, Morningstar, Phys Rev Lett **82**, 4400 (1999)

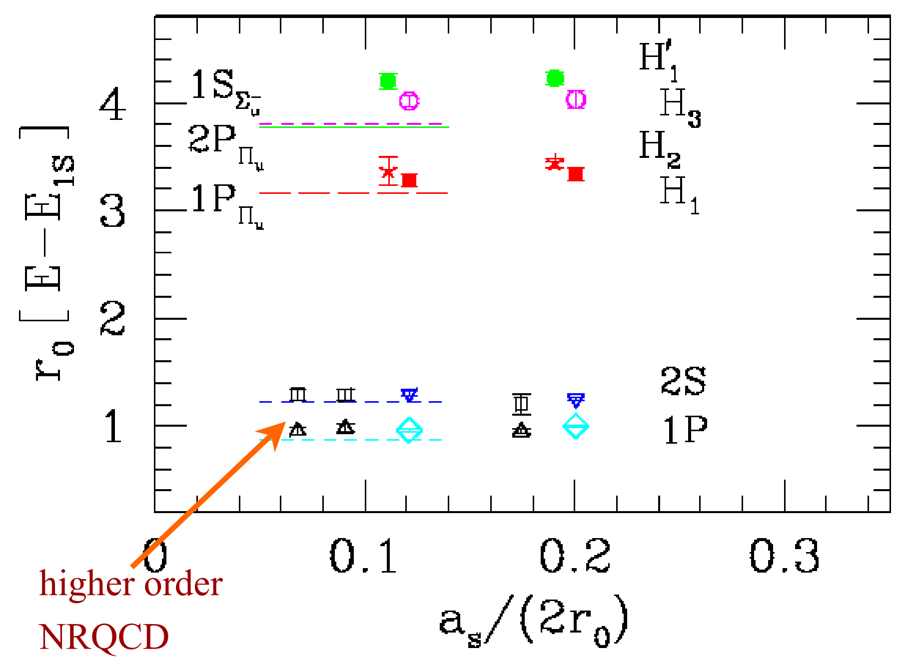
Charmonium LBO

- same calculation, but for charmonium



Testing LBO

- test LBO by comparison of spectrum with NRQCD simulations
  - include retardation effects, but no quark spin, no $\vec{p}^4$, no light quarks
  - allow possible mixings between adiabatic potentials
- dramatic evidence of validity of LBO
  - level splittings agree to 10% for 2 conventional mesons, 4 hybrids

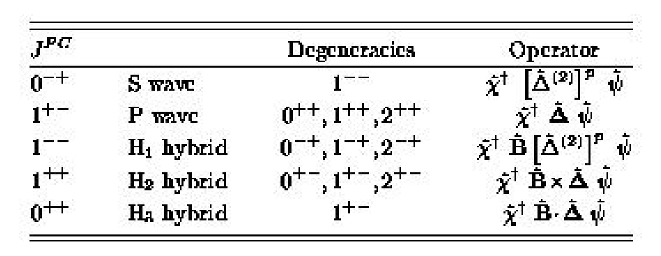


higher order NRQCD

$$H_1, H_1' = 1^{--}, 0^{-+}, 1^{-+}, 2^{-+}$$

$$H_2 = 1^{++}, 0^{+-}, 1^{+-}, 2^{+-}$$

$$H_3 = 0^{++}, 1^{+-}$$



J^{PC}		Degeneracies	Operator
0^{-+}	S wave	1^{--}	$\hat{\chi}^\dagger\ [\hat{\mathbf{\Delta}}^{(2)}]^p\ \hat{\psi}$
1^{+-}	P wave	$0^{++}, 1^{++}, 2^{++}$	$\hat{\chi}^\dagger\ \hat{\mathbf{\Delta}}\ \hat{\psi}$
1^{--}	H_1 hybrid	$0^{-+}, 1^{-+}, 2^{-+}$	$\hat{\chi}^\dagger\ \hat{\mathbf{B}}\,[\hat{\mathbf{\Delta}}^{(2)}]^p\ \hat{\psi}$
1^{++}	H_2 hybrid	$0^{+-}, 1^{+-}, 2^{+-}$	$\hat{\chi}^\dagger\ \hat{\mathbf{B}}\times\hat{\mathbf{\Delta}}\ \hat{\psi}$
0^{++}	H_3 hybrid	1^{+-}	$\hat{\chi}^\dagger\ \hat{\mathbf{B}}\cdot\hat{\mathbf{\Delta}}\ \hat{\psi}$

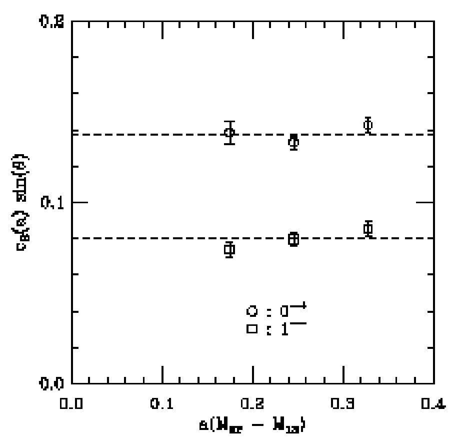
lowest hybrid 1.49(2)(5) GeV above 1S

Quark spin effects (continued)

- Burch and Toussaint, hep-lat/0305008
  - NRQCD simulations, measured mixing via $c_1\,\sigma\cdot B/M$
  - mixing in bottomonium seems not to spoil BO picture
  - larger effect in charmonium

$$\langle 1H \,|\, \Upsilon\rangle \approx 0.076-0.11 \qquad \langle 1H \,|\, J/\Psi\rangle \approx 0.18-0.25$$

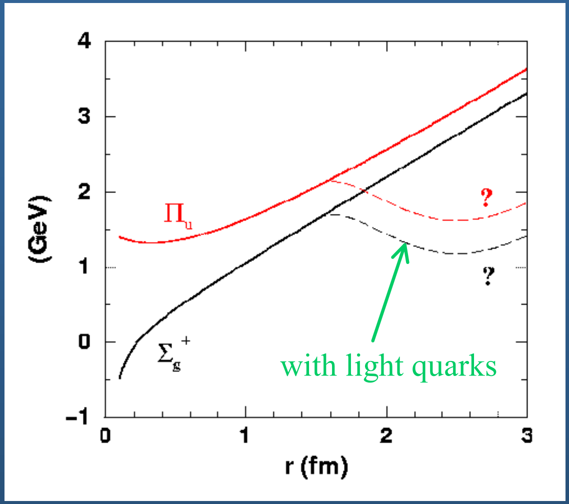
$$\langle 1H \,|\, \eta_b\rangle \approx 0.13-0.19 \qquad \langle 1H \,|\, \eta_c\rangle \approx 0.29-0.4$$



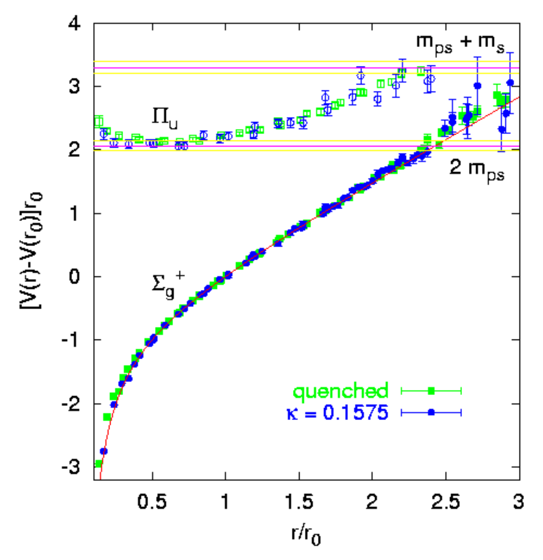
Light quark spoiler?

- spoil B.O.? → unknown
- light quarks change $V_{Q\bar{Q}}(r)$
  - small corrections at small r
    - fixes low-lying spectrum
  - large changes for $r>1$ fm

    → fission into $(Q\bar{q})(\bar{Q}q)$
- states with diameters over 1 fm
  - most likely *cannot exist* as observable resonances
- dense spectrum of states from pure glue potentials will not be realized
  - survival of a few states conceivable given results from Bali *et al.*
- discrepancy with experiment above $B\bar{B}$
  - most likely due to light quark effects



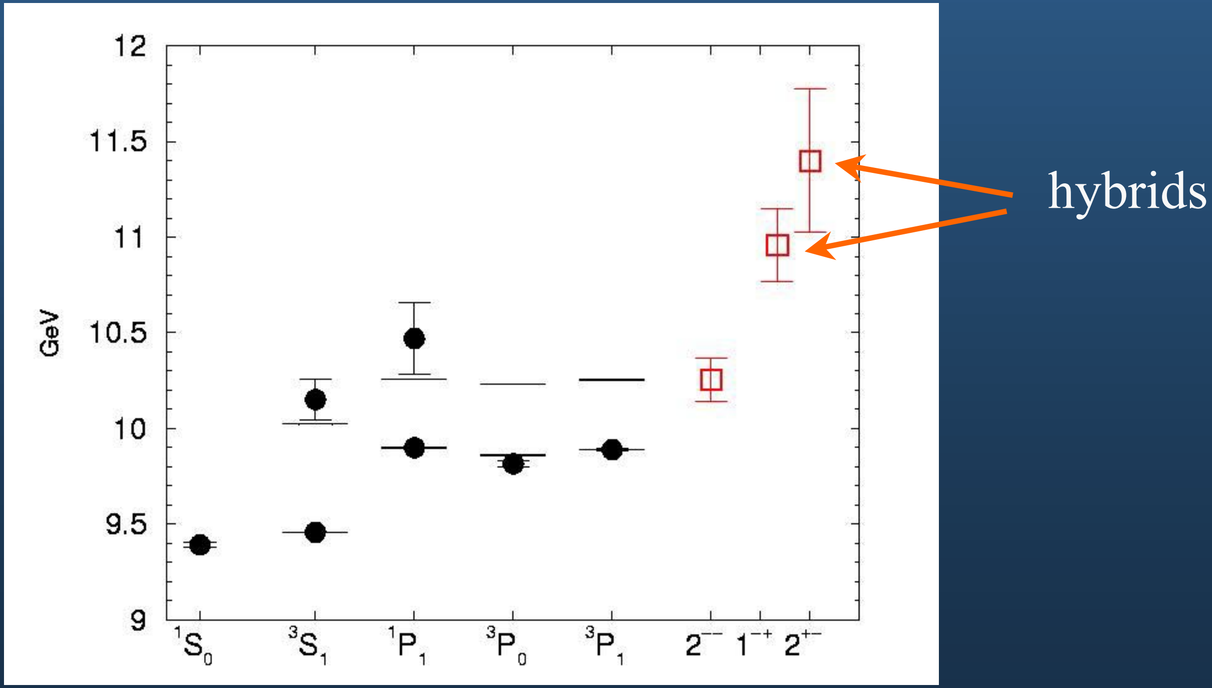
with light quarks



Bottomonium hybrids

- recent calculation of bottomonium hybrids confirms earlier results
  - quenched, several lattice spacings so $a \to 0$ limit taken
  - improved anisotropic gluon and fermion (clover) actions
  - good agreement with Born-Oppenheimer (but errors large)

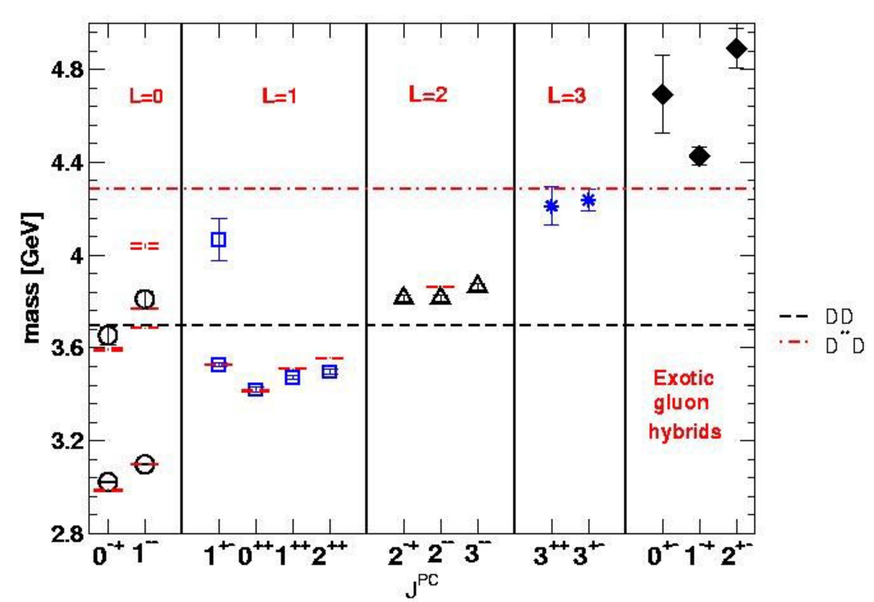
Liao, Manke, PRD**65**, 074508 (2002)



Charmonium hybrids

- recent determination of some charmonium hybrids
    - quenched, several lattice spacings for continuum limit
    - improved, anisotropic gluon and fermion (clover) actions
    - results suggest significant (but not large) corrections from LBO

Liao, Manke, hep-lat/0210030



Light-quark hybrids

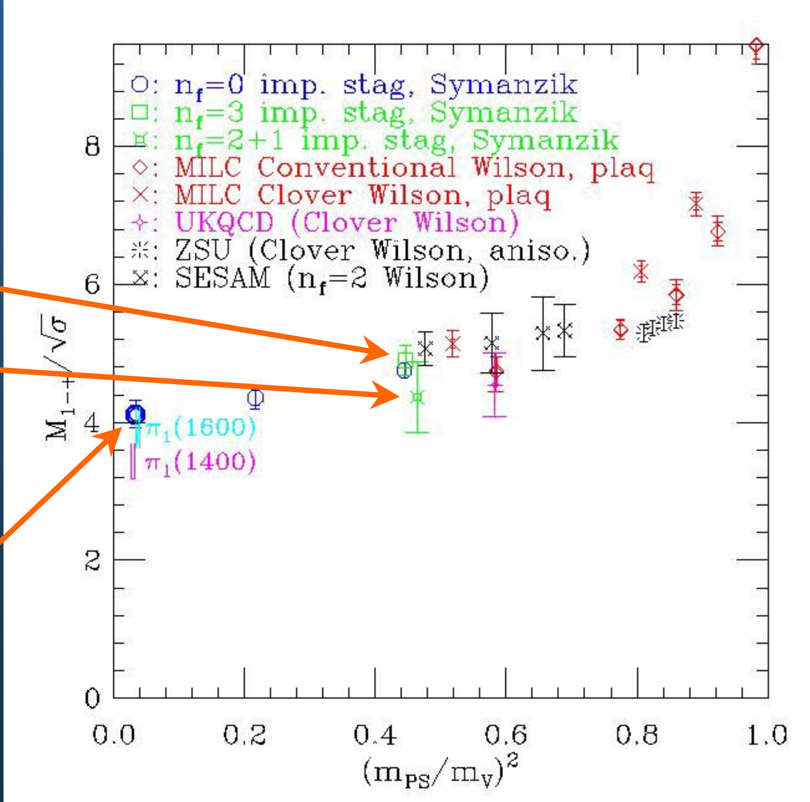
- recent new determination of exotic 1^{-+} hybrid meson
  - improved staggered fermions (lighter quark masses)
  - quenched and unquenched, Wilson gluon action
  - $a \approx 0.09$ fm
  - lightest mass still above experiment

MILC, hep-lat/0301024

$N_f = 3, \quad m_u = m_d = m_s$
(around strange quark mass)

$m_u = m_d = 0.4 m_s$

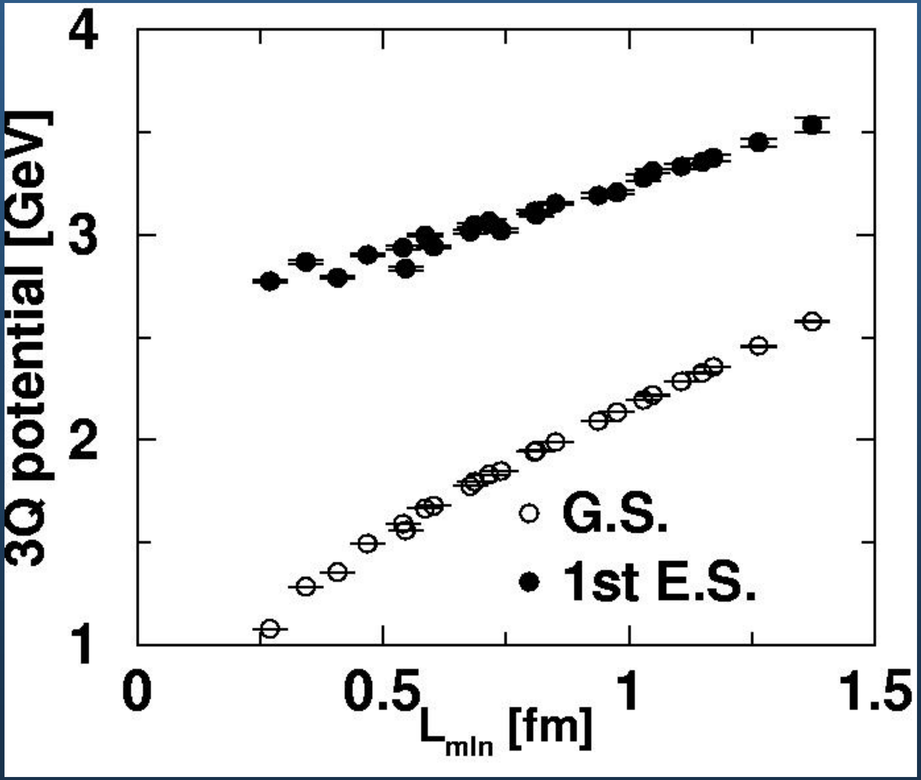
quenched continuum limit



Excitation of the static 3*q* system

- first excitation of the static 3*q* system recently determined
  - excitation energy about 1 GeV
  - finite spacing, finite volume errors still to be studied

Takahashi, Suganuma, hep-lat/0210024



Conclusion

- hadronic states bound by an *excited* gluon field
  - interesting new form of matter
  - promise to shed new light on confinement in QCD
- pure-gauge glueball spectrum well known, but needs polishing
- progress in including quark loops
  - *tentative* results so far→ little change to glueballs
- heavy-quark hybrid mesons
  - compelling physical picture from Born-Oppenheimer treatment
  - relationship to excitations of the static quark potential
  - quark spin effects do not seem to spoil BO
  - light quark loops → survival issue