



# $D^0$ mixing and $CP$ violation in $D^0$ decays from Belle

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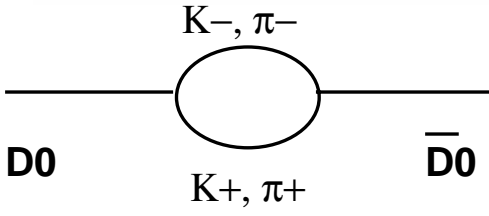
University of Science & Technology of China

International Workshop on Tau-Charm Physics  
Charm 2006 (June 5-7, Beijing, China)

## Topic

- $D^0(t) \rightarrow K^+ \pi^-$ ,  $400 \text{ fb}^{-1}$
- $D^0(t) \rightarrow K^{(*)+} e^- \nu$ ,  $253 \text{ fb}^{-1}$
- $D^0 \rightarrow K^+ (n\pi)^-$ ,  $281 \text{ fb}^{-1}$

# $D^0-\bar{D}^0$ mixing formalism:



- Only mixing via light quark intermediate states
- doubly-Cabibbo-suppressed with respect to  $\Gamma_D$
- long-distance contributions

	c	d,s,(b)	u
D0	W		W
	$\bar{u}$	d,s,(b)	$\bar{c}$
			$\bar{D}^0$

Flavor eigenstates are not mass eigenstates:

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle \quad |D_{1,2}(t)\rangle = e^{-(\Gamma_{1,2}/2 + im_{1,2})t}|D_{1,2}\rangle$$

$$\bar{m} \equiv \frac{1}{2}(m_1 + m_2) \quad \bar{\Gamma} \equiv \frac{1}{2}(\Gamma_1 + \Gamma_2) \quad \Delta m \equiv m_2 - m_1 \quad \Delta\gamma \equiv \Gamma_2 - \Gamma_1$$

$$\mathcal{A}_f \equiv \langle f|H|D^0\rangle \quad \bar{\mathcal{A}}_f \equiv \langle f|H|\bar{D}^0\rangle \quad \mathcal{A}_{\bar{f}} \equiv \langle \bar{f}|H|D^0\rangle \quad \bar{\mathcal{A}}_{\bar{f}} \equiv \langle \bar{f}|H|\bar{D}^0\rangle$$

For  $\Delta mt \ll 1$  and  $\Delta\gamma t \ll 1$ :

$$R(D^0(t) \rightarrow \bar{f}) \approx |\bar{\mathcal{A}}_{\bar{f}}|^2 \left| \frac{q}{p} \right|^2 e^{-\bar{\Gamma}t} \left\{ |\bar{\lambda}|^2 + [y\text{Re}(\bar{\lambda}) + x\text{Im}(\bar{\lambda})](\bar{\Gamma}t) + \frac{1}{4}(x^2 + y^2)(\bar{\Gamma}t)^2 \right\}$$

$$R(\bar{D}^0(t) \rightarrow f) \approx |\mathcal{A}_f|^2 \left| \frac{p}{q} \right|^2 e^{-\bar{\Gamma}t} \left\{ |\lambda|^2 + [y\text{Re}(\lambda) + x\text{Im}(\lambda)](\bar{\Gamma}t) + \frac{1}{4}(x^2 + y^2)(\bar{\Gamma}t)^2 \right\}$$

<b>Direct</b>	<b>Interference</b>	<b>Mixing</b>
$x \equiv \frac{m_2 - m_1}{\bar{\Gamma}}$	$y \equiv \frac{\Gamma_2 - \Gamma_1}{2\bar{\Gamma}}$	$\lambda \equiv \frac{q \bar{\mathcal{A}}_f}{p \mathcal{A}_f} \quad \bar{\lambda} \equiv \frac{p \mathcal{A}_{\bar{f}}}{q \bar{\mathcal{A}}_{\bar{f}}}$

**MIXING PARAM.**

**CPV enters here**

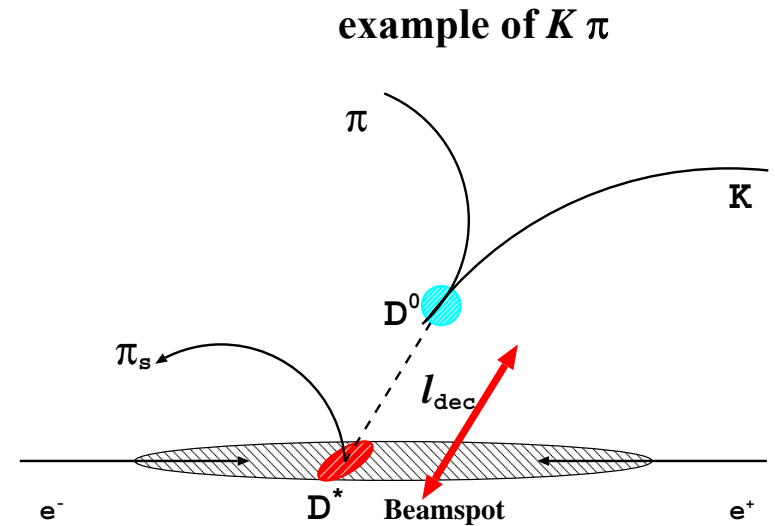
Expect in SM:  $|x| \approx |y| \sim 10^{-6} - 10^{-3}$  (short distance)  
 $10^{-3} - 10^{-2}$  (long distance)

Nelson, hep-ex/9908021  
 Burdman & Shipsey, Annu.Rev. Nucl.Part.Sci.53 431(2003)

• If obtain  $|x| \gg |y|$  or CPV  $\Rightarrow$  New Physics

# Common features of analyses

- Initial flavor of  $D^0$  is determined from  $D^{*+} \rightarrow D^0 \pi^+$  or  $D^{*-} \rightarrow D^0 \pi^-$ 
  - Background largely reduced:  $Q = m_{K\pi\pi^-} - m_{K\pi^-} - m_\pi$  only 6 MeV (very near threshold)
- Common backgrounds:
  - Random  $\pi$  combined with real (Cabibbo-favored)  $\bar{D}^0$  decays
  - Combinatorial (random combinations of tracks)
- $p_{D^*}$  cut to eliminate  $D^*$ 's from  $B$  decays
- Signal/background yields obtained from  $m-Q$  fit
- Signal shapes/resolution functions taken from Cabibbo-favored modes
- (Unbinned) ML fit to  $t = (l_{\text{dec}}/p)(m/c)$



# Wrong-sign $D^0(t) \rightarrow K^+ \pi^-$ decays

$$\lambda = \frac{q \bar{\mathcal{A}}_f}{p \mathcal{A}_f} \equiv \left| \frac{q}{p} \right| \sqrt{R_D^-} e^{i(-\delta+\phi)}$$

$\delta =$  strong phase

$$\bar{\lambda} = \frac{p \mathcal{A}_{\bar{f}}}{q \bar{\mathcal{A}}_{\bar{f}}} \equiv \left| \frac{p}{q} \right| \sqrt{R_D^+} e^{i(-\delta-\phi)}$$

$\phi =$  weak phase

$$\frac{\Gamma(D^0(t) \rightarrow K^+ \pi^-)}{\Gamma(\bar{D}^0 \rightarrow K^+ \pi^-)} = e^{-\bar{\Gamma}t} \left\{ R_D^+ + \left| \frac{q}{p} \right| \sqrt{R_D^+} (y' \cos \phi - x' \sin \phi) (\bar{\Gamma}t) + \left| \frac{q}{p} \right|^2 \frac{x'^2 + y'^2}{4} (\bar{\Gamma}t)^2 \right\}$$

$$\frac{\Gamma(\bar{D}^0(t) \rightarrow K^- \pi^+)}{\Gamma(D^0 \rightarrow K^- \pi^+)} = e^{-\bar{\Gamma}t} \left\{ R_D^- + \left| \frac{p}{q} \right| \sqrt{R_D^-} (y' \cos \phi + x' \sin \phi) (\bar{\Gamma}t) + \left| \frac{p}{q} \right|^2 \frac{x'^2 + y'^2}{4} (\bar{\Gamma}t)^2 \right\}$$

**Mixing:**

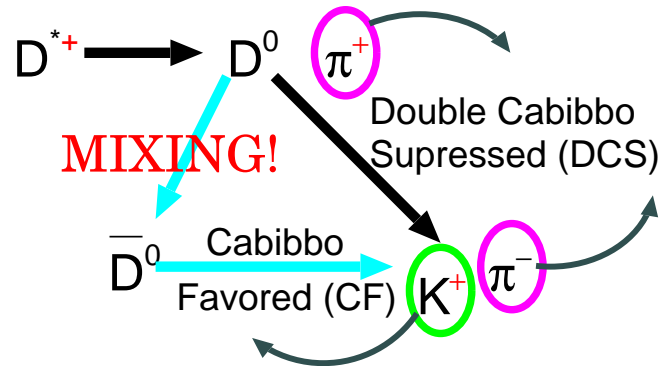
$$x' \equiv x \cos \delta + y \sin \delta \quad y' \equiv y \cos \delta - x \sin \delta$$

**CPV:**

$A_M \equiv (1 -  q/p ^4)/(1 +  q/p ^4)$	CPV in mixing
$A_D \equiv (R_D^+ - R_D^-)/(R_D^+ + R_D^-)$	CPV in the decay amplitude (direct CPV)
$\phi$	CPV in mixed/direct interference

No CPV ( $R_D^+ = R_D^-$ ,  $|q/p| = 1$ , and  $\phi = 0$ ):

$$r_{ws}(t) = e^{-\bar{\Gamma}t} \left\{ R_D + \sqrt{R_D} y' (\bar{\Gamma}t) + \frac{x'^2 + y'^2}{4} (\bar{\Gamma}t)^2 \right\}$$

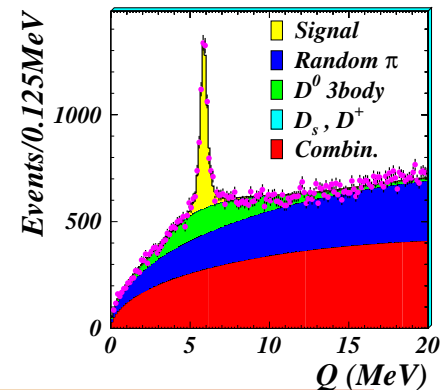
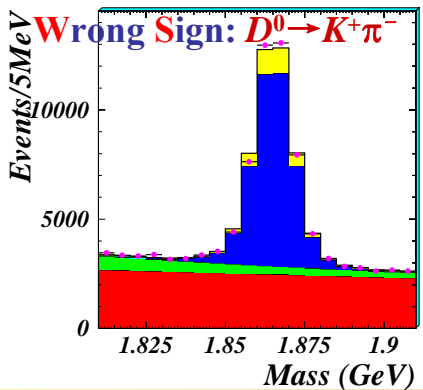
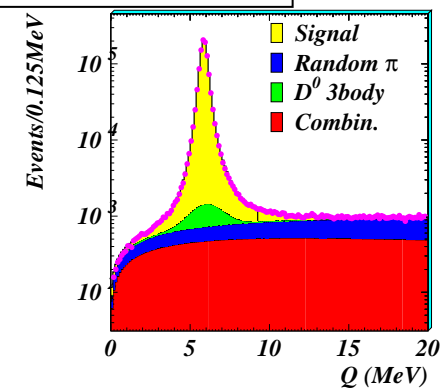
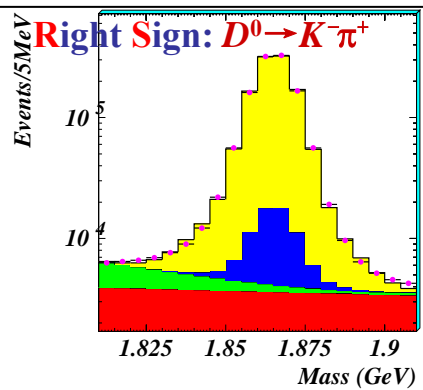


# $D^0(t) \rightarrow K^+ \pi^-$ , Data distribution

E. M. Aitala <i>et al.</i> (E791), PRD 57, 13 (1998):	36 WS events
R. Godang <i>et al.</i> (CLEO), PRL 84, 5038 (2000):	45 WS events
J. M. Link <i>et al.</i> (FOCUS), PRL 86, 2955 (2001), PLB 618, 23 (2005):	234 WS events
B. Aubert <i>et al.</i> (Babar), PRL 91, 171801 (2004):	430 WS events
J. Li <i>et al.</i> (Belle), PRL 94, 071801 (2005):	845 WS events
L. M. Zhang <i>et al.</i> (Belle), PRL 96, 151801 (2006):	4024 WS events

- RS signal shape: double-Gaussian ( $m_{K\pi}$ ) + sum of bifurcated Student and Gaussian ( $Q$ )
- background shape fixed to MC
- WS signal shape fixed to RS signal's

$N_{RS} = 1073993 \pm 1108$   
 $N_{WS} = 4024 \pm 88$   
 $R_{WS} = N_{WS}/N_{RS} = (0.375 \pm 0.008)\%$



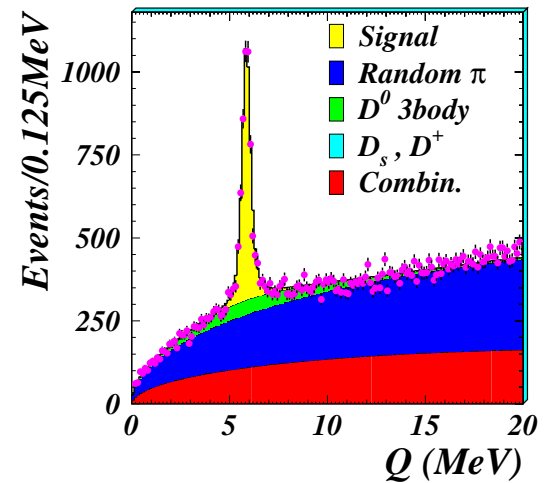
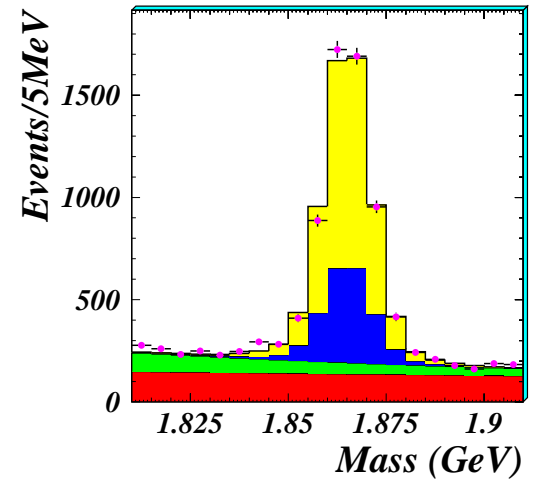
# $D^0(t) \rightarrow K^+ \pi^-$ , *WS* signal

- No. of events in  $m_{K\pi}$ ,  $Q$   $3\sigma$  region

Signal	$3509 \pm 77$
Random $\pi$	$1595 \pm 13$
$D^0$ 3 body	$443 \pm 17$
$D^+_{(s)}$	$1.66 \pm 0.99$
Combin.	$1103.1 \pm 9.4$

Signal/Background = 1.1

Data@400/fb



# $D^0(t) \rightarrow K^+ \pi^-$ , Proper-time fit

## Probability density function

$$P_i = \int_0^\infty dt' \left\{ f_{\text{sig}}^i P_{\text{sig}}(t'; R_D, x'^2, y') R_{\text{sig}}(t_i - t') + \sum_{\text{bkg}} f_{\text{bkg}}^i P_{\text{bkg}}(t') R_{\text{bkg}}(t_i - t') \right\}$$

- Signal resolution function and lifetime fixed to RS fit

– Check:  $\tau_{D^0}(\text{RS}) = 409.9 \pm 0.7 \text{ fs}$   
(PDG:  $410.3 \pm 1.5 \text{ fs}$ )

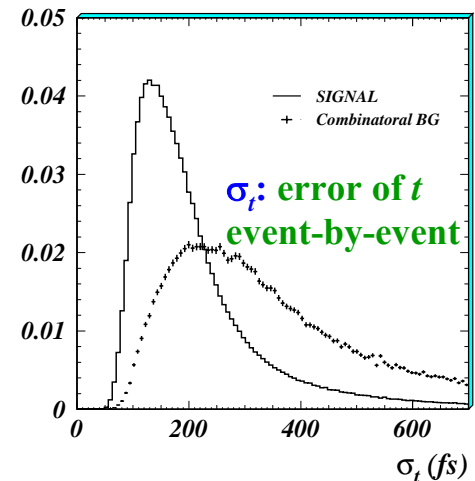
- BG param.'s fixed from sideband fit or MC
- Fit  $4\sigma$  signal region
- $(R_D, x'^2, y')$  the only free parameters
- Event fractions include  $\sigma_t$  dependent

$$f^i(M, Q) \Rightarrow f^i(M, Q, \sigma_t)$$

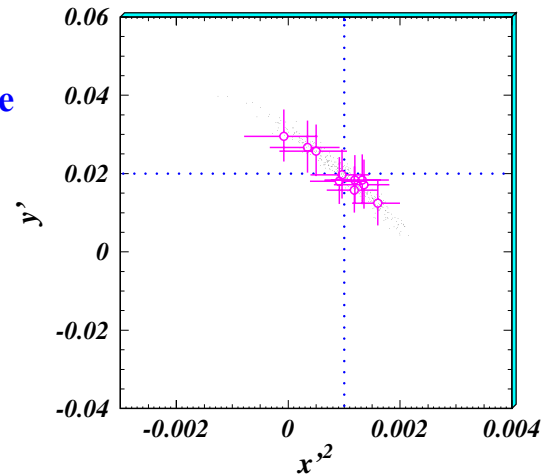
(old fit method in previous analysis)      (new fit method)

- Checked with 450 full MCs

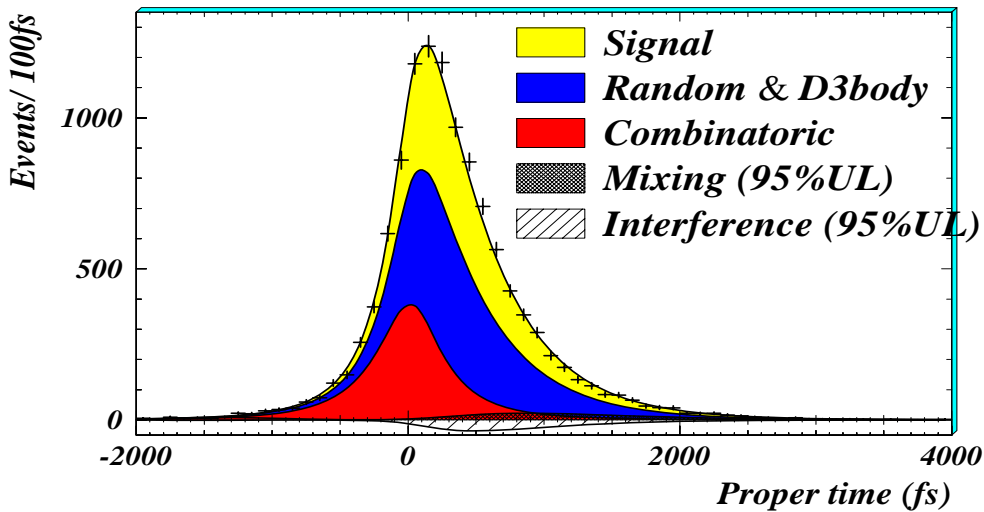
Large difference in  $\sigma_t$  between signal and BG



One example  
10 full MCs  
400 “Toy”



# $D^0(t) \rightarrow K^+ \pi^-$ , Fit result



## CP test

Fit  $D^0$  and  $\bar{D}^0$  separately:

$$\Rightarrow \{R_D^\pm, x'^{\pm 2}, y'^{\pm}\}$$

## CP Asymmetry

$$A_D = \frac{R_D^+ - R_D^-}{R_D^+ + R_D^-}$$

$$A_M = \frac{R_M^+ - R_M^-}{R_M^+ + R_M^-}$$

$$\phi = [9.4 \text{ (or } 84.5) \pm 25.3]^\circ$$

where

$$R_M^+ = \frac{x'^{+2} + y'^{+2}}{2}$$

$$R_M^- = \frac{x'^{-2} + y'^{-2}}{2}$$

Fit Case	Parameter	Fit Result ( $\times 10^{-3}$ )
No CPV	$x'^2$	$0.18^{+0.21}_{-0.23}$
	$y'$	$0.6^{+4.0}_{-3.9}$
	$R_D$	$3.64 \pm 0.17$
CPV allowed	$A_D$	$23 \pm 47$
	$A_M$	$670 \pm 1200$
No mixing or CPV	$R_D$	$3.77 \pm 0.08 \text{ (stat.)} \pm 0.05 \text{ (syst.)}$

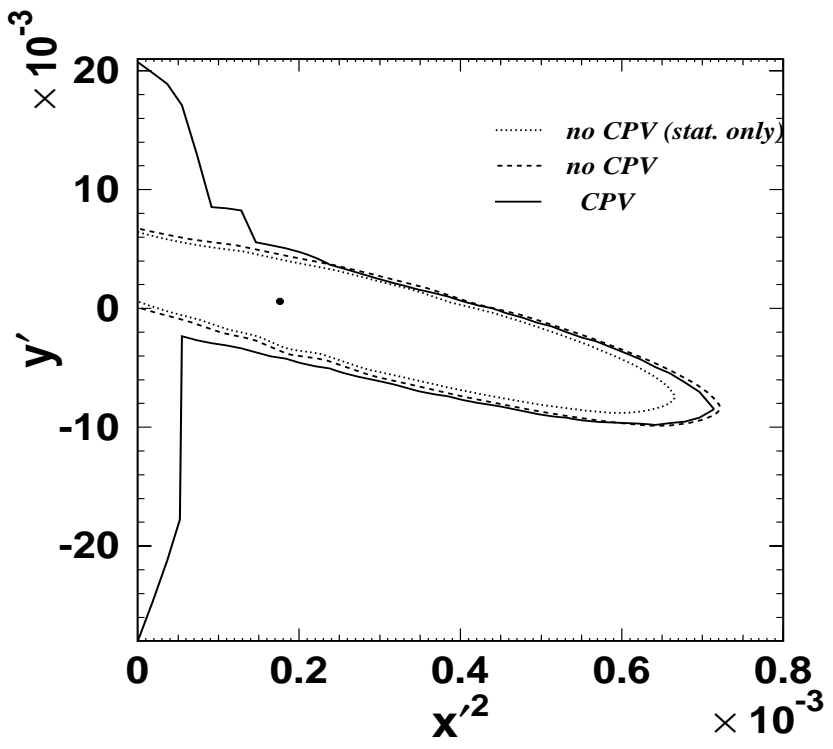


# $D^0(t) \rightarrow K^+ \pi^-$ , 95% C.L. contour

Use toy MC to obtain frequentist (Feldman-Cousins) confidence region (95% C.L.):

two solutions of  $(x'^2, y')$  in CPV allowed case

$$x'^{\pm} = \left( \frac{1 \pm A_M}{1 \mp A_M} \right)^{1/4} (x' \cos \phi \pm y' \sin \phi)$$

$$y'^{\pm} = \left( \frac{1 \pm A_M}{1 \mp A_M} \right)^{1/4} (y' \cos \phi \mp x' \sin \phi)$$


Fit Case	Parameter	95% CL interval ( $\times 10^{-3}$ )
No CPV	$x'^2$	$x'^2 < 0.72$
	$y'$	$-9.9 < y' < 6.8$
	$R_D$	$3.3 < R_D < 4.0$
	$R_M$	$0.63 \times 10^{-5} < R_M < 0.40$
CPV allowed	$A_D$	$-76 < A_D < 107$
	$A_M$	$-995 < A_M < 1000$
	$x'^2$	$x'^2 < 0.72$
	$y'$	$-28 < y' < 21$
	$R_M$	$R_M < 0.40$

no CPV:  $(x'^2, y') = (0,0)$  has 1-C.L. of 3.9% (out of 95% C.L. contour)

# Wrong-sign $D^0(t) \rightarrow K^{(*)+} e^- \nu$ decays

E. M. Aitala *et al.* (E791), PRL 77, 2384 (1996): 2504 RS events

B. Aubert *et al.* (Babar), PRD 70, 091102 (2004): 49620 RS events

U. Bitenc *et al.* (Belle), PRD 72, 071101 (2005): 229452 RS events

**Method:** flavor at production tagged via  $D^{*+} \rightarrow D^0 \pi^+$  (pion charge)

flavor at decay tagged via  $D^0 \rightarrow K^{(*)+} e^- \nu$  (lepton charge)

$\Rightarrow$  mixing signal is  $\pi^+ e^-$  or  $\pi^- e^+$  (Wrong Sign)

normalize sens. to  $\pi^+ e^+$  or  $\pi^- e^-$  (Right Sign)

● Only mixing term, no DCS contaminant in WS

● To measure mixing rate  $R_M \approx \frac{(x^2 + y^2)}{2} = \frac{N_{WS}}{N_{RS}}$

**Neutrino Reconstruction:**  $\nu$  momentum:  $P_\nu = P_{\text{cms}} - P_{\pi K e} - P_{\text{rest}}$

●  $|P_{\text{rest}}|$  adjusted to give  $(P_{\text{cms}} - P_{\text{rest}})^2 = m_{D^*}^2$

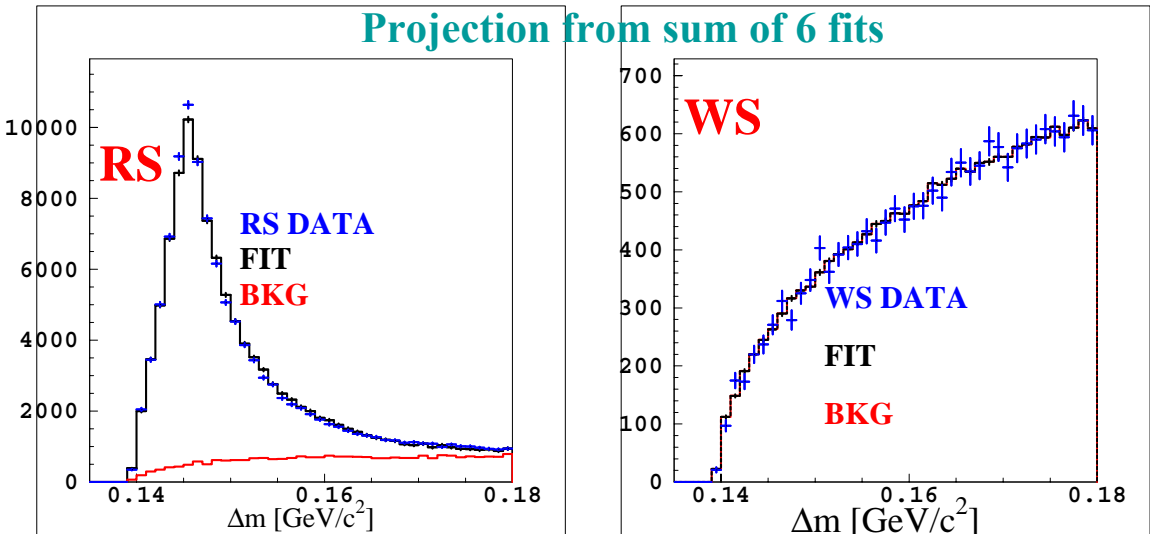
●  $\vec{p}_{\text{rest}}$  direction adjusted to satisfy  $m_\nu^2 = 0$

# $D^0(t) \rightarrow K^{(*)+} e^- \nu$ fit

- Proper decay time used to reduce background and increase sensitivity:

$$\frac{dN_{\text{sig}}}{dt} \propto \frac{x^2 + y^2}{4} t^2 e^{-t} \qquad \frac{dN_{\text{bkg}}}{dt} \propto \delta(t) + e^{-t}$$

- Select  $1 < t < 10 [\tau_{D^0}]$  to fit  $\Delta m = m_{K\pi e \nu^-} - m_{K e \nu}$  in 6 bins of  $t$



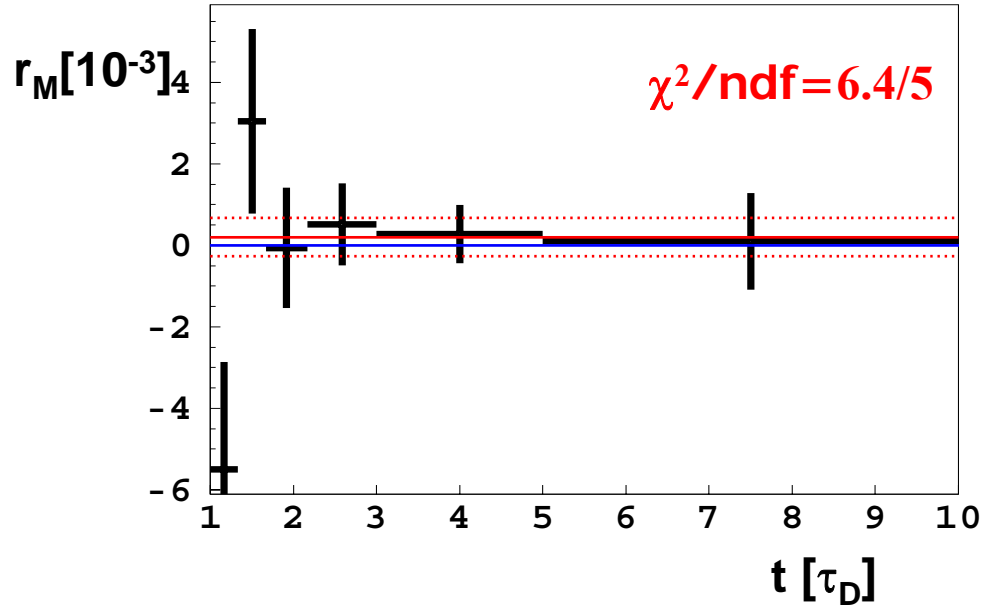
- main RS/WS BKG is combin./random
- Their shape obtained from combination of uncorrelated tracks/event in data

$$R_M = \frac{N_{\text{WS}} \epsilon_{\text{RS}}^t}{N_{\text{RS}} \epsilon_{\text{WS}}^t}$$

RS and WS signals differ in proper decay time distribution

# $D^0(t) \rightarrow K^{(*)+} e^- \nu$ result

- Result: Fit RS and WS in 6  $t$  bins



$$R_M = (x^2 + y^2)/2 = [0.20 \pm 0.47 \text{ (stat.)} \pm 0.14 \text{ (syst.)}] \times 10^{-3}$$

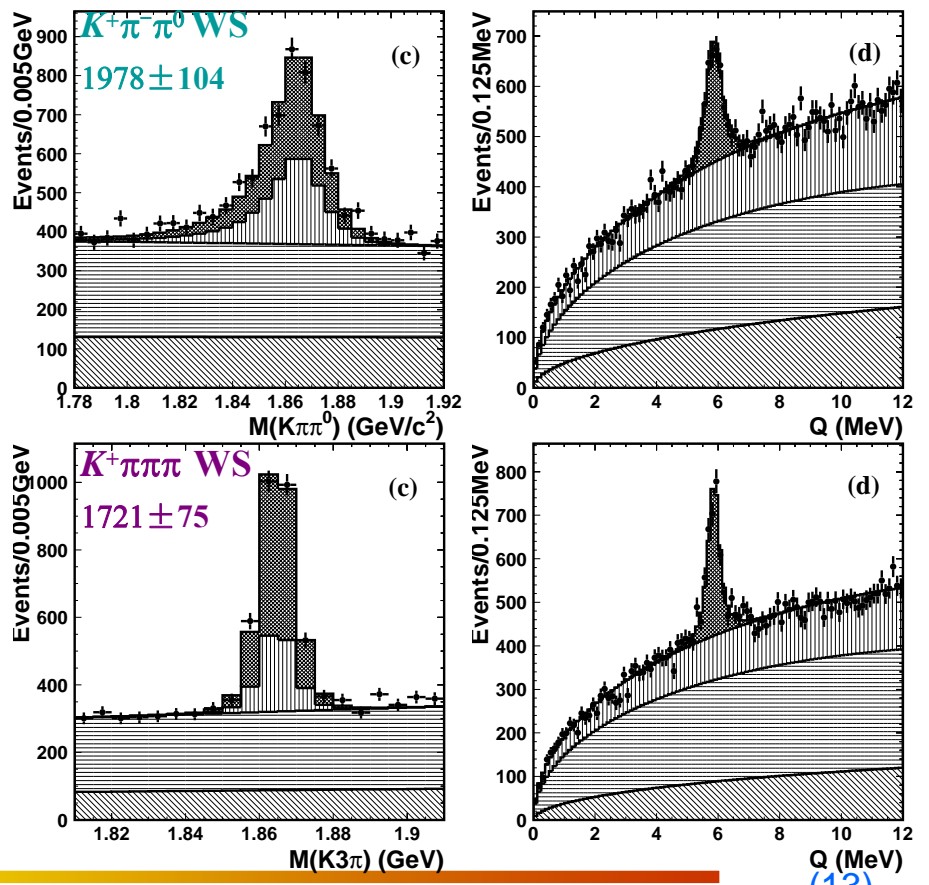
95% C.L. upper limit:  $R_M < 1.2 \times 10^{-3}$

(Currently best limit in semileptonic decays)

# Wrong-sign $D^0 \rightarrow K^+ \pi^- \pi^0, K^+ \pi^- \pi^+ \pi^-$ decays

E. M. Aitala <i>et al.</i> (E791), PRD 57, 13 (1998):	7 WS $K^+ \pi \pi \pi$
G. Brandenburg <i>et al.</i> (CLEO), PRL 87, 071802 (2001):	38 WS $K^+ \pi \pi^0$
S. A. Dytman <i>et al.</i> (CLEO), PRD 64, 111101 (2001):	54 WS $K^+ \pi \pi \pi$
X. C. Tian <i>et al.</i> (Belle), PRL 95, 231801 (2005):	3699 WS $K^+ \pi \pi^0 + K^+ \pi \pi \pi$

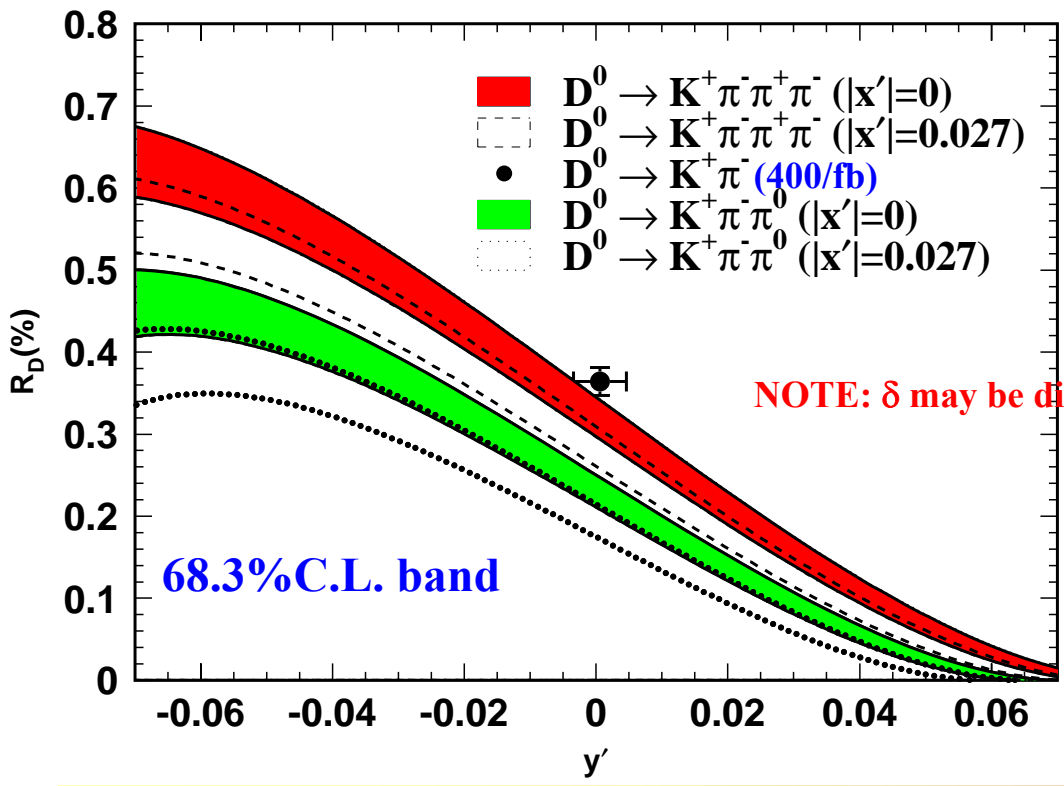
- $p_{D^*} > 2.5 \text{ GeV}/c$  to reject  $D^*$ 's from  $B$  decays
  - veto Cabibbo-suppressed  $D^0 \rightarrow K^+ K_S \pi^-$
  - $m_{K(n\pi)} - Q$  fit to obtain wrong-sign and right-sign yield  $N_{K(n\pi)}$ , similar to  $D^0 \rightarrow K^+ \pi^-$  mode
  - acceptance for RS and WS events do not cancel: resonant substructure is different and acceptance varies over the Dalitz plot
- => bin events in  $m_{K\pi} m_{\pi\pi}$  etc., calculate acceptance for each bin, then average acceptance for RS and WS
- $\langle \epsilon_{RS} \rangle / \langle \epsilon_{WS} \rangle = 1.01 \pm 0.05, 0.98 \pm 0.04$



# Results of $WS D^0 \rightarrow K^+ \pi^- \pi^0, K^+ \pi^- \pi^+ \pi^-$

$$R_{WS} \equiv \frac{\int \mathcal{P}[D^0 \rightarrow K^+ \pi^- (n\pi)] dt}{\int \mathcal{P}[D^0 \rightarrow K^- \pi^- (n\pi)] dt} \approx R_D + \sqrt{R_D} y' + \frac{x^2 + y^2}{2}$$

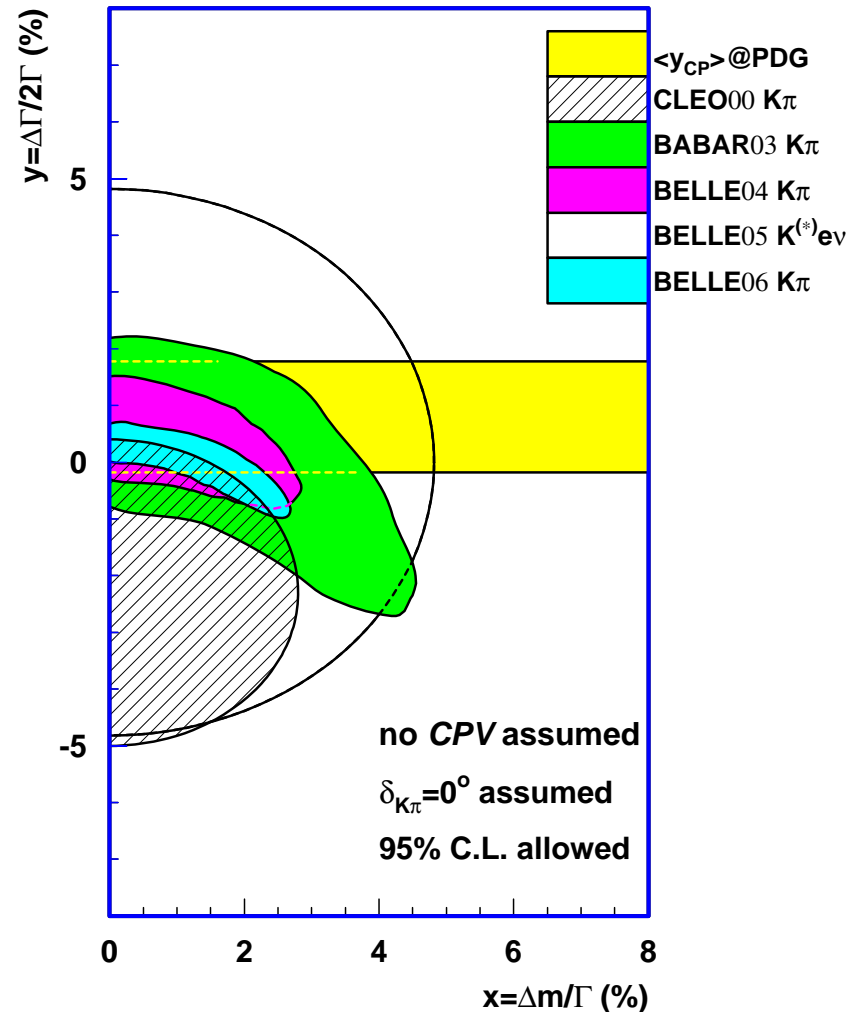
$$= \begin{cases} [2.29 \pm 0.15 \text{ (stat.) } {}^{+0.13}_{-0.09} \text{ (syst.)}] \times 10^{-3} & K^+ \pi^- \pi^0 \\ [3.20 \pm 0.18 \text{ (stat.) } {}^{+0.18}_{-0.13} \text{ (syst.)}] \times 10^{-3} & K^+ \pi^- \pi^+ \pi^- \end{cases}$$



$|x'|=0.027$  is 95% C.L. upper limit from 400  $\text{fb}^{-1}$   $K^+ \pi^-$  analysis

- $D^0(t) \rightarrow K^+ \pi^-$ ,  $400 \text{ fb}^{-1}$   
 $|x'| < 2.7\%$   
 $-0.99\% < y' < 0.68\%$  @95% C.L., no CPV  
 $0.01\% < \sqrt{x^2 + y^2} < 2.8\%$   
 † No-mixing point (0, 0) corresponds to a 1-C.L.=3.9% (no CPV)  $\Rightarrow 2.1\sigma$   
 † No CPV observed
- $D^0(t) \rightarrow K^{(*)+} e^- \nu$ ,  $253 \text{ fb}^{-1}$   
 $\sqrt{x^2 + y^2} < 4.9\%$  @95% C.L.
- $D^0 \rightarrow K^+ (n\pi)^-$ ,  $281 \text{ fb}^{-1}$   
 $A_{CP}(K\pi\pi^0) = -0.006 \pm 0.053$   
 $A_{CP}(K3\pi) = -0.018 \pm 0.044$

$$A_{CP} = \frac{N_{WS}^{D^0 \rightarrow K^+ (n\pi)^-} - N_{WS}^{\bar{D}^0 \rightarrow K^- (n\pi)^+}}{N_{WS}^{D^0 \rightarrow K^+ (n\pi)^-} + N_{WS}^{\bar{D}^0 \rightarrow K^- (n\pi)^+}}$$



# Summary II

- Current sensitivity by experiments

- $|x|, |y| \sim 10^{-2}$
- $R_M \sim 10^{-4}$

- Strong phase difference  $\delta_{K\pi}$ :

From  $K\pi$  result and world average  $\langle y_{CP} \rangle = (1.09 \pm 0.46)\%$  [hep-ex/0605032]

$$\begin{cases} x' = x \cos \delta + y \sin \delta \\ y' = y \cos \delta - x \sin \delta \end{cases}$$

Assuming no  $CPV$

$$\Rightarrow \delta_{K\pi} = (52 \pm 22)^\circ \text{ or } (122 \pm 39)^\circ$$

Larger  $SU(3)$  breaking?

- $D^0$ -mixing may be within our reach with present statistics (hints of positive signals in  $y_{CP}$ )
- Need much larger samples to precisely pin down the mixing phenomena in  $D^0$  system

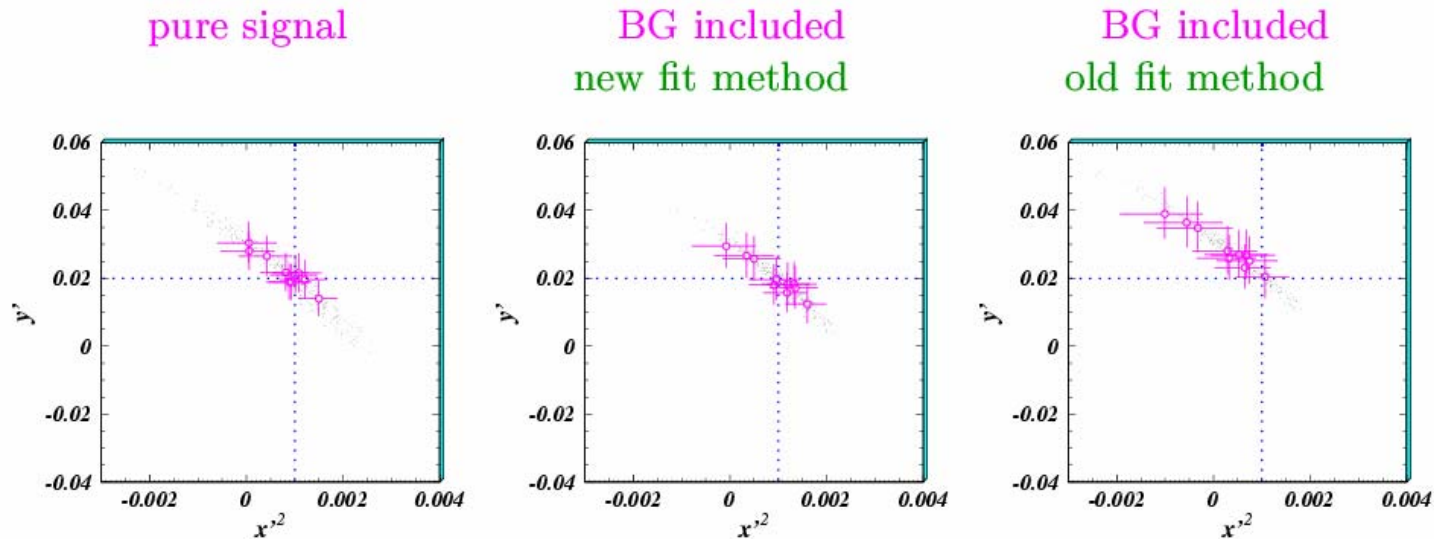


*Backup*

# $D^0(t) \rightarrow K^+ \pi^-$ , full and Toy MC tests

- Full (Geant simulation) MC show **no bias** in large (M,Q) region
- And consistent with Toy (simple generator) MC
- Old fit method give **0.5-1 $\sigma$  bias** shift to unphysical region
- We didn't know the bias source in the previous study, because  $\sigma_t$  difference wasn't included in Toy MC, even some bias appeared in the Full MC.

## One example of Full MC



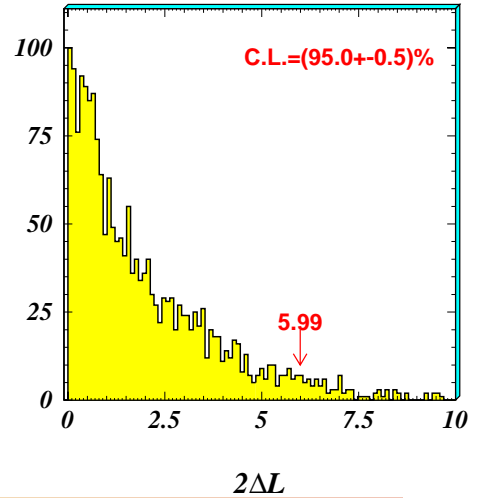
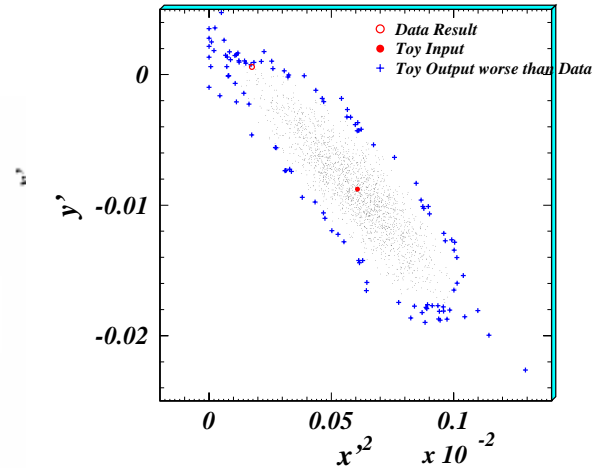
# $D^0(t) \rightarrow K^+ \pi^-$ , Method to set C.L. contour

Using Toy MC to obtain frequentist (Feldman-Cousins) confidence region.

- choose  $\vec{\alpha} = (x'^2, y')$ , generate Toy MC
- fit toy MC sample, calculate  $\Delta \ln \mathcal{L}(\vec{\alpha}) = \ln \mathcal{L}_{max} - \ln \mathcal{L}(\vec{\alpha})$  for each toy experiment
- find fraction  $p$  with values  $< \Delta \ln \mathcal{L}_{data}(\vec{\alpha})$
- contour is locus of  $(x'^2, y')$  points with  $p = 0.95$

All fits require  $x'^2$  in physical region.

Not standard Feldman-Cousins method





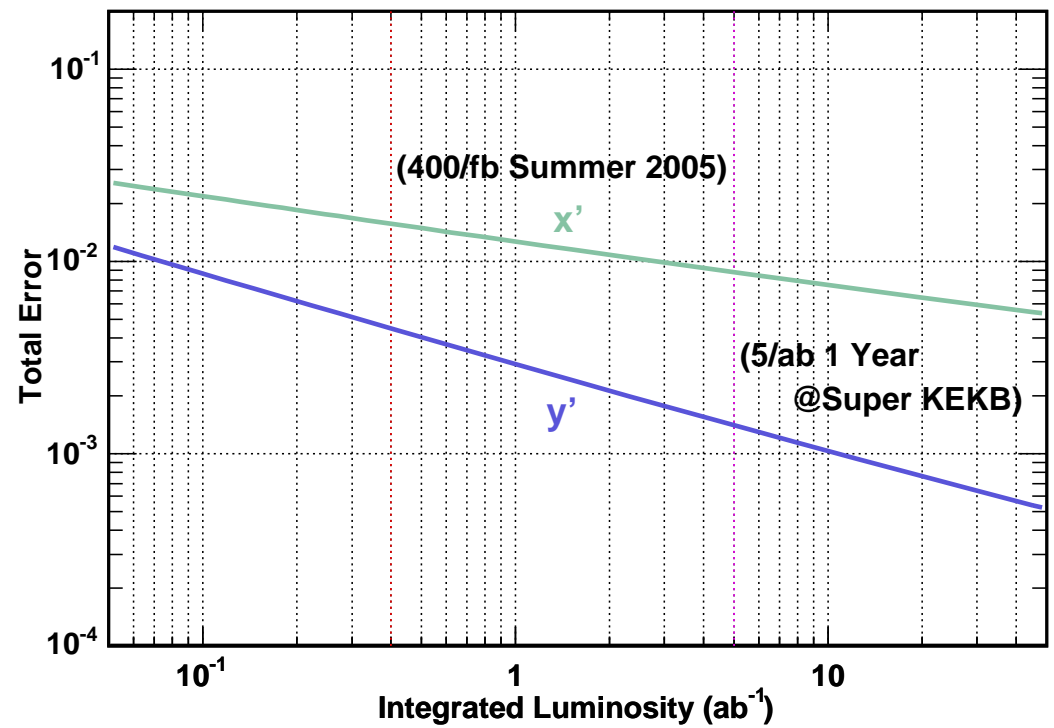
# $D^0(t) \rightarrow K^+ \pi^-$ , systematics

Source	$\Delta y' / \sigma_{y'}$ (%)	$\Delta x'^2 / \sigma_{x'^2}$ (%)	$\Delta(-2 \ln L)$ (%)
KID cut	25.5	-26.7	7.37
PID cut	-22.9	16.2	6.94
$\chi^2$ cut	23.0	-21.2	5.31
$p^*(D^*)$ cut	34.0	-20.7	19.2
$\sigma_t$ PDF	24.4	-18.3	7.0
resolution function	9.2	-9.4	0.90
resolution para.'s	-	-	6.90
BG yields	-	-	0.44
M,Q PDFs	-	-	1.06
Lifetime bias	-7.0	7.0	0.78
Total	-	-	55.9

$$\text{Scaling factor} = \sqrt{1 + 0.559/2.3} = 1.12$$

# $D^0(t) \rightarrow K^+ \pi^-$ , Prospect of future

- Log-likelihood  $\propto \mathcal{L}_{\text{int}}$   
 $\Rightarrow 3\sigma$  ( $\sim 750 \text{ fb}^{-1}$ )
- Better precision:
  - Stat. error  $\propto (\mathcal{L}_{\text{int}})^{-0.5}$
  - Relative syst. error  $\propto (\mathcal{L}_{\text{int}})^{0.15}$
  - For small enough  $x'$  :  
 $\sigma_{x'} \approx \sqrt{\sigma_{x'^2}}$



Sensitivity	Total Error on $x'$			Total Error on $y'$		
	1 $\text{ab}^{-1}$	5 $\text{ab}^{-1}$	50 $\text{ab}^{-1}$	1 $\text{ab}^{-1}$	5 $\text{ab}^{-1}$	50 $\text{ab}^{-1}$
Value ( $\times 10^{-3}$ )	12.7	8.8	5.3	2.9	1.4	0.52



# Neutrino reconstruction

Get  $P_{\nu_e} = P_{\text{CMS}} - P(\pi_s K e) - P_{\text{REST}}$   
 (4-momentum conservation).

Correction for better  $\nu$  4-momentum:

1. **rescale**  $P_{\text{REST}}$  ( $M_{D^*}$  constraint).

$$P_{\text{REST}} \rightarrow x \cdot P_{\text{REST}}$$

$$|P_{\text{CMS}} - x \cdot P_{\text{REST}}|^2 \equiv M_{D^{*+}}^2$$

2. **correct direction** ( $M_\nu = 0$  constraint).

$$\text{new } \angle(\vec{P}_{\text{REST}}, \vec{P}_{\pi_s K e})$$

for  $M_\nu^2 \equiv 0$ .

