Constraints from chiral symmetry to PWA

Hanqing Zheng

Peking University

Workshop on PWA, Jan. 25 – 26, IHEP, Beijing

The Breit–Wigner description of resonances

In principal the Breit–Wigner description of resonance only works for infinitely small width.

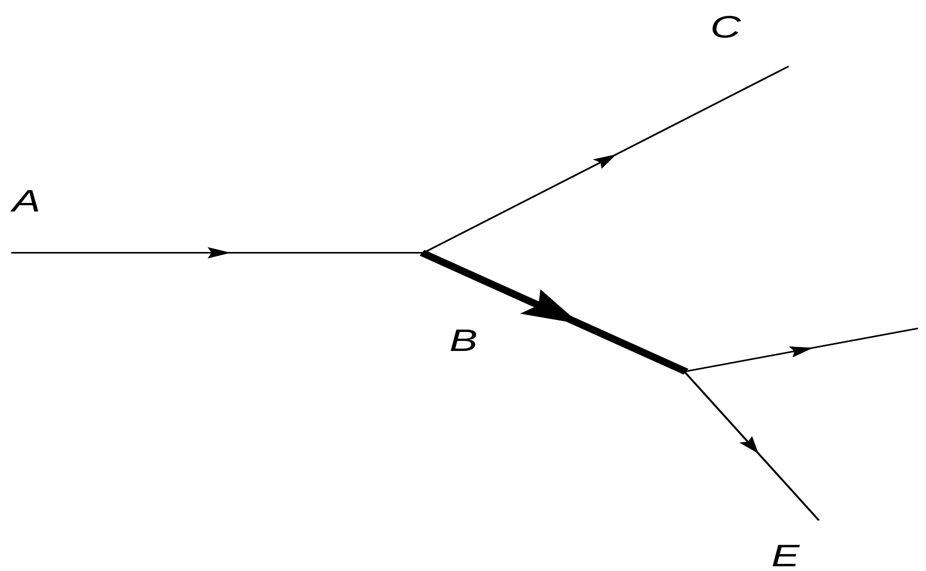


Figure 1:

$$\Gamma(A \to BC) \equiv \Gamma(A \to CDE) \Rightarrow \triangle_B(p^2) = \frac{1}{p^2 - m_B^2 + i m_B \Gamma_B}$$

For stable particle,

$$\triangle(p^2) = \frac{1}{p^2 - m^2 + i\epsilon} \;, \tag{1}$$

$$A \to B + C, \;\; B \to D + E \;. \tag{2}$$

$$H_{eff} = f ABC + g BDE \;, \tag{3}$$

$$\Rightarrow \Gamma_A = \frac{f^2}{16\pi}\frac{m_A^2 - m_B^2}{m_A^3} \;, \;\; \Gamma_B = \frac{g^2}{16\pi}\frac{1}{m_B} \tag{4}$$

Ansatz:

$$\triangle_B(p^2) = \frac{1}{p^2 - \alpha + i\beta} \;. \tag{5}$$

Equivalently, Eq. (2)≡ $A \to C + D + E$,

Now calculating $\Gamma_{A\to B+D+E}$ (in the calculation $m_A >> \Gamma_A$ is assumed to simplify the phase space integration of 3-body final state)

$$\Gamma_A = \frac{f^2}{16\pi}\frac{g^2}{16\pi}\frac{m_A^2 - \alpha}{m_A^3 \beta}. \tag{6}$$

$$\Rightarrow \triangle_{BW}(p^2) = \frac{1}{p^2 - m^2 + im\Gamma} \,. \tag{7}$$

Not very good analytical property!

$$\Rightarrow \triangle_{BW}(p^2) = \frac{1}{p^2 - m^2 + i\rho(s)G} \,. \tag{8}$$

Inspired by perturbative calculation to propagator.

Not applicable to a broad object like the σ and κ!

How to parametrize an unstable particle's propagator?

1. (Real) analyticity (micro-causality), not necessarily for narrow width!

$$T(s)^* = T(s^*)$$

2. No spurious poles nearby
3. respecting known constraints from (chiral) theory
4. No unambiguous separation of pole contribution and the continuum contributions
5. Threshold effects (including couple channel effects) have to be included if there is a nearby threshold

The problem of an old description to the light and broad resonances

A very common parametrization form in the literature

$$S = \frac{M^2 - s + i\rho(s)g}{M^2 - s - i\rho(s)g} \,, \tag{9}$$

For a sufficiently large M^2 and small g it contains a resonance and a virtual state. (The latter is not found by χPT! and violates the validity of chiral expansions) for equal mass scatterings and two resonances for unequal mass scatterings. The scattering length of the two poles are additive and are both positive. The companions can have larger contributions than the resonance itself if the resonance is light and broad!

Used both by E791 and BES Experiments

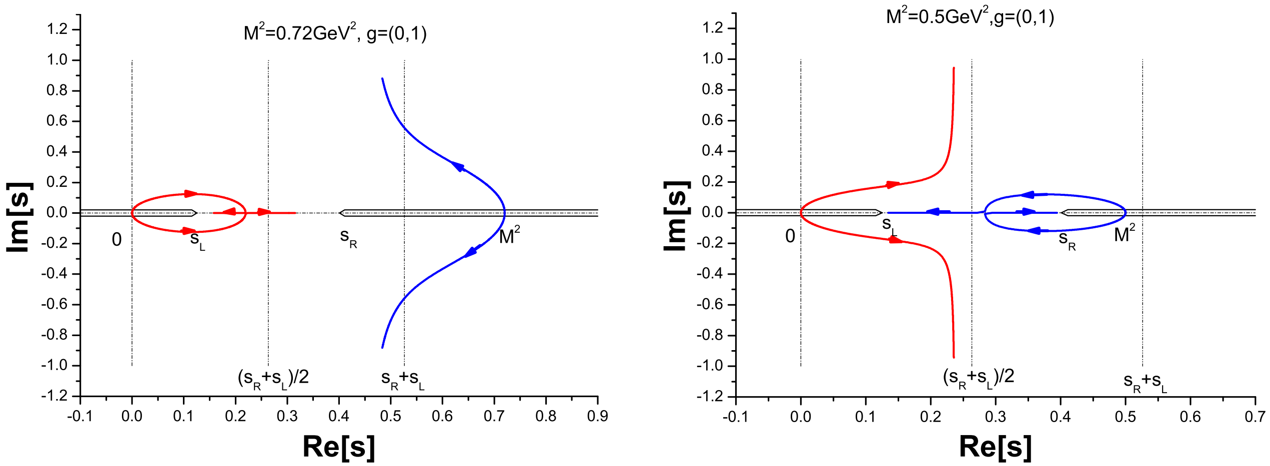


Figure 2: The traces of two pairs of resonances when increasing g for different M^2. We give two typical figures: left) figure for $M^2 > (s_R + s_L)$; right) $M^2 < (s_R + s_L)$.

The PKU dispersive parametrization form

A resonance located at z_0 (Im$[z_0] > 0$) and z_0^*: The S matrix can be expressed as:

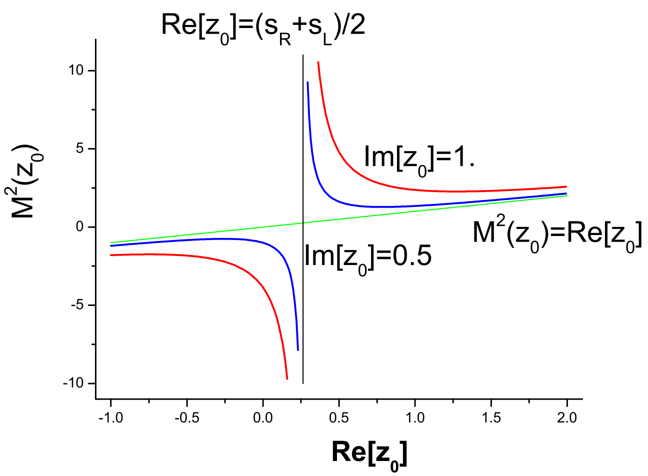
$$S(s) = \frac{M^2(z_0) - s + i\rho(s)sG[z_0]}{M^2(z_0) - s - i\rho(s)sG[z_0]}\ , \tag{10}$$

where

$$M^2(z_0) = \mathrm{Re}[z_0] + \frac{\mathrm{Im}[z_0]\,\mathrm{Im}[z_0\,\rho(z_0)]}{\mathrm{Re}[z_0\,\rho(z_0)]}\ ,\ G[z_0] = \frac{\mathrm{Im}[z_0]}{\mathrm{Re}[z_0\,\rho(z_0)]}\ , \tag{11}$$

and the scattering length is,

$$a(z_0) = \frac{\mathrm{Im}[z_0]\mathrm{Re}[z_0\,\rho(z_0)]}{\mathrm{Im}[z_0]^2 + \mathrm{Re}[z_0\rho(z_0)]^2}\frac{2\sqrt{s_R}\,\left(M^2(z_0) - s_R\right)}{(s_R - z_0)\,(s_R - z_0^*)}\ . \tag{12}$$



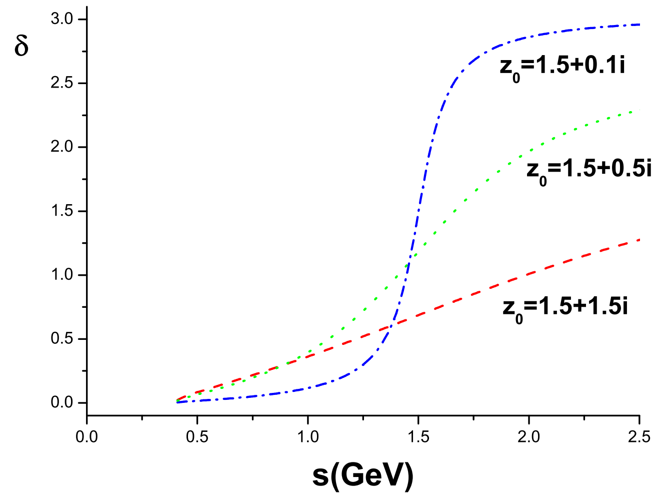


Figure 3: Left: $M^2(z_0)$ as a function of $\mathrm{Re}[z_0]$, for fixed $\mathrm{Im}[z_0]$.

At $s = M^2(z_0)$ resonance contribution to the phase shift passes $\pi/2$. However, a light and broad resonance can have a very large $M(z_0)^2$. When $\mathrm{Re}[z_0] \leq (s_L + s_R)/2$, the phase shift never reaches $\pi/2$!

The factorization of the single channel scattering S matrix

$$S^{phy.} = \prod_i S^{R_i} \cdot S^{cut} \ . \qquad (13)$$

NO LOSS OF GENERALITY!

S^{cut}: no longer contains any pole:

$$\begin{aligned} S^{cut} &= e^{i\rho f(s)} \\ f(s) &= \frac{s}{\pi}\int_{-\infty}^{0} \frac{\mathrm{Im}_L f(s')}{s'(s'-s)} + \frac{s}{\pi}\int_{R}^{+\infty} \frac{\mathrm{Im}_R f(s')}{s'(s'-s)} \end{aligned} \qquad (14)$$

Couple channel effects (or physics at III, IV,... sheets, etc.)
$\Rightarrow$ The phase is additive,

$$\delta(s) = \sum_i \delta_{R_i} + \delta_{b.g.} \qquad (\delta_{b.g.}(s) = \rho(s) f(s)) \qquad (15)$$

The violation of Levinson's theorem

If $M^2 > 4$, the phase pass $\pi/2$ when $s = M^2$ and

$$\delta(\infty) = \pi - \tan^{-1}(\frac{\mathrm{Im}[z_0]}{\mathrm{Re}[\sqrt{z_0(z_0-4)}]}) < \pi \ . \tag{16}$$

Actually,

$$\delta(\infty) - \delta(-\infty) = \pi \ . \tag{17}$$

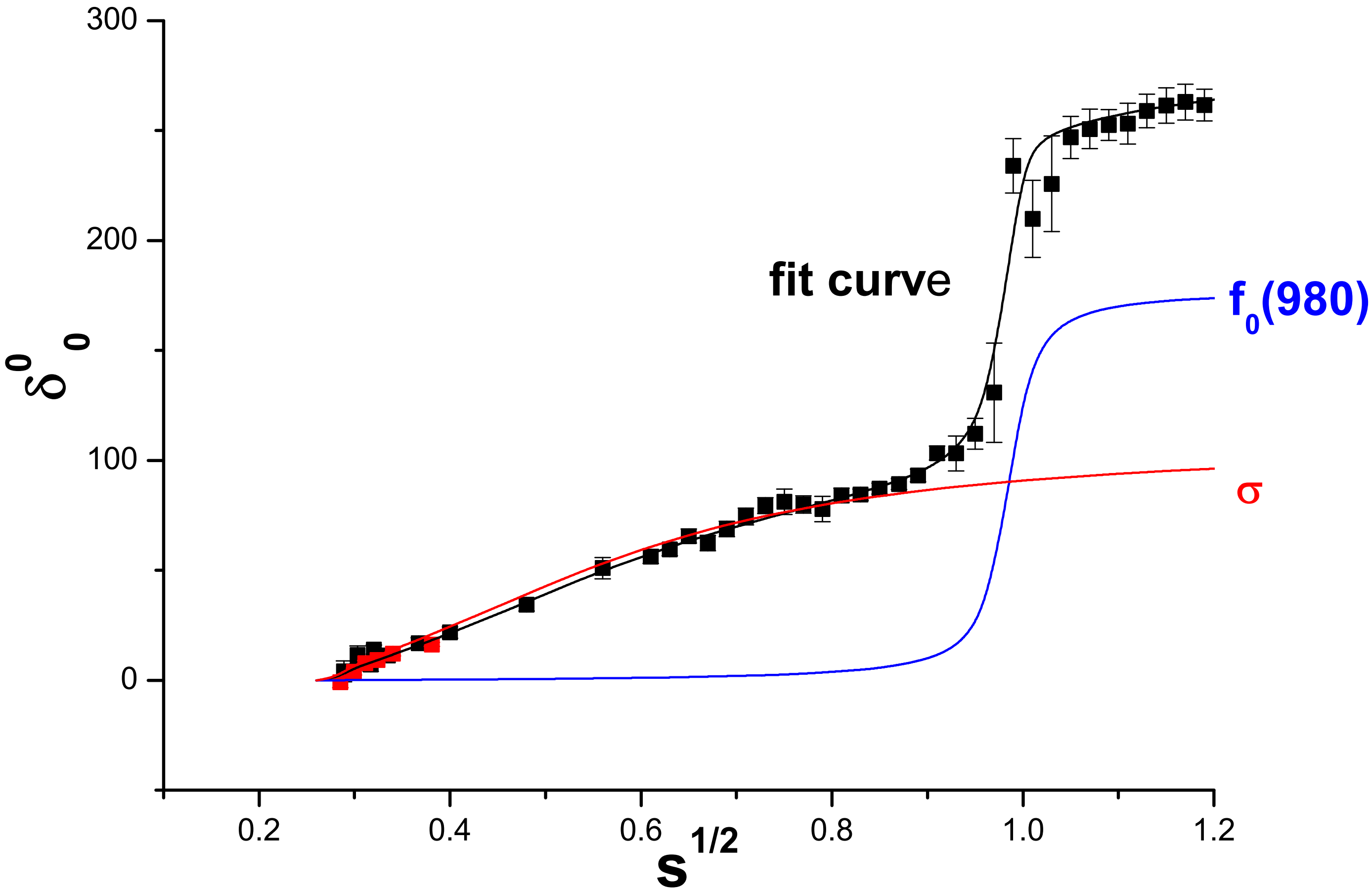


Figure 4: The σ pole contribution to the phase shift δ_0^0. (Zhou 05)

Evidence for the existence of σ from BES experiments: $J/\psi \to \omega\pi^+\pi^-$

(Phys. Lett. B598: 149-158,2004.) (also E791)

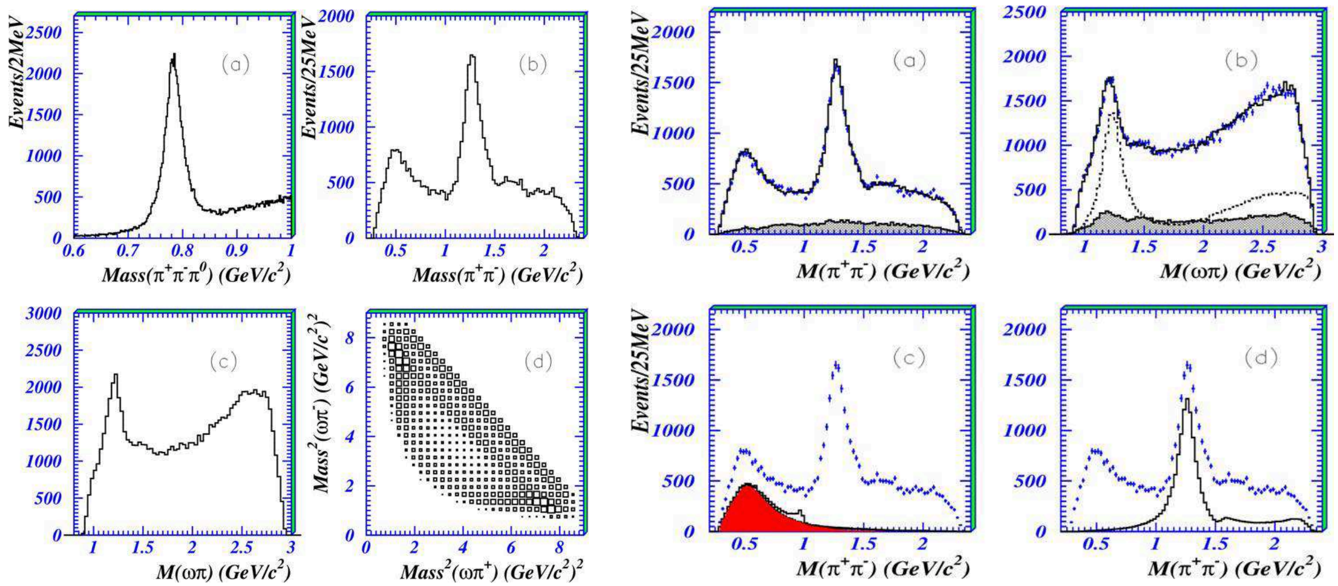


Figure 5: Dalitz plot and $\pi^+\pi^-$ invariant mass spectrum.

$$M_\sigma - i\Gamma_\sigma/2 = (541 \pm 39) - i(252 \pm 42)\text{MeV}$$

The σ meson in $\Psi' \to J/\Psi\pi\pi$ process

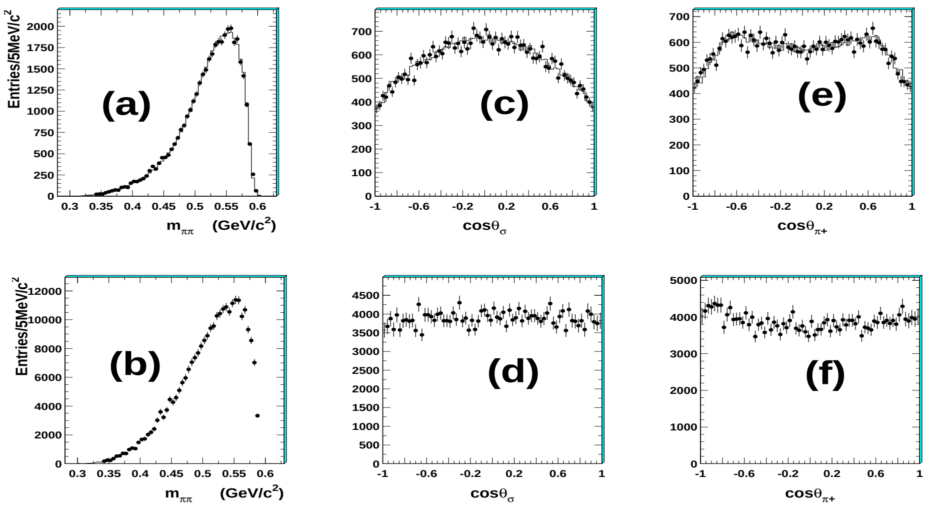


Figure 6: Fit results of $\psi(2S) \to \pi^+\pi^- J/\psi$ (P.K.U. ansatz). Dots with error bars are data and the histograms are the fit results. (a) and (b) are the $\pi^+\pi^-$ invariant mass, (c) and (d) the cosine of the σ polar angle in the lab frame, and (e) and (f) the cosine of the π^+ polar angle in the σ rest frame.

(BES Collaboration, hep-ex/0610023)

Universality of pole location

Example: Assuming there is a resonance in the channel I,J of $\pi\pi \longrightarrow \pi\pi$ scattering. Then in any process $i \longrightarrow (\pi\pi)_{IJ} + f$, does the same pole exist as in the elastic process $\pi\pi \longrightarrow \pi\pi$?

Considering any process $i \longrightarrow f + (\pi\pi)_{IJ}$, transition amplitude $A(s_1, s_2, \cdots, s_n)$, $s_i = (p_{i_1} + p_{i_2})^2$ ($s_1 = (p_{\pi_1} + p_{\pi_2})^2$).
A real analytic function of complex variable s_1:

$$A(s_1^*, s_2 \cdots, s_n) = A^*(s_1, s_2 \cdots, s_n) \tag{18}$$

can be proved to maintain the following property, using Cutkosky rule:

$$\begin{aligned} \text{disc } A(s_1, s_2 \cdots, s_n) &= A(s_1, s_2 \cdots, s_n) - A(s_1^*, s_2 \cdots, s_n) \\ &= 2iA(s_1, s_2 \cdots, s_n)\rho_{\pi\pi}(s_1)T^*_{\pi\pi\to\pi\pi}(s_1) \ . \end{aligned} \tag{19}$$

$T^*(s_1) = T^{II}(s_1) = T(s_1)/S(s_1) \Rightarrow$

$$A^{II}(s_1, \cdots) = A^*(s_1, \cdots) = A(s_1, \cdots)/S(s_1) \ . \tag{20}$$

$\kappa(700)$

The κ resonance also has a rather long history and the status is more controversial. Data also contain some problems.

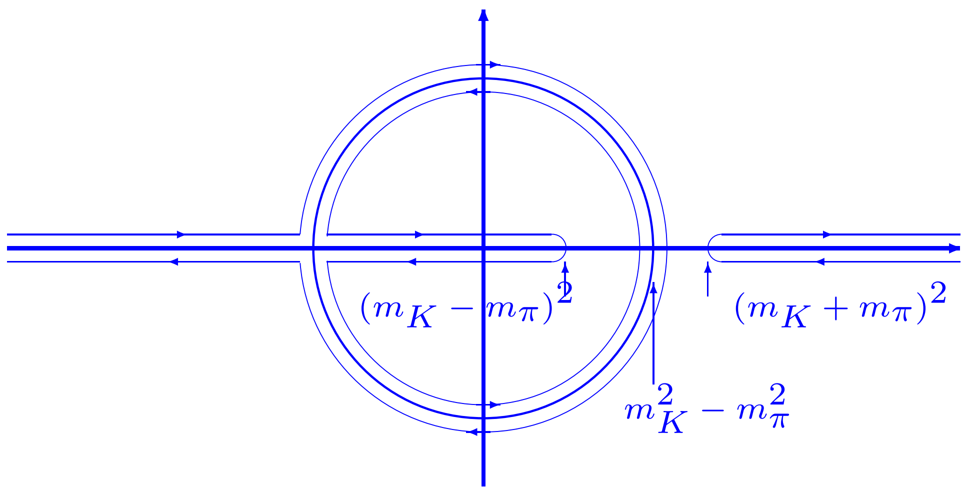
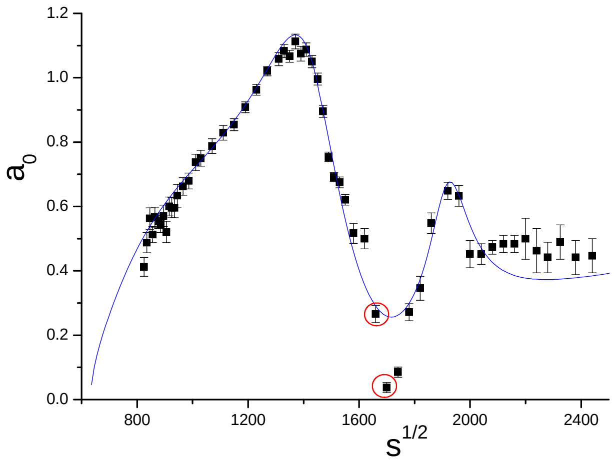
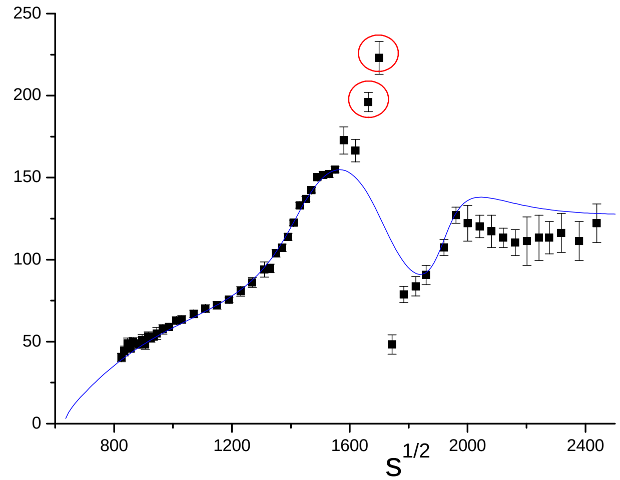


Figure 7: The left, circular and right hand cut of πK scatterings.

κ exists if the scattering length is not far from the value obtained from χPT. Conclusions (almost) model independent.
(H.Q. Zheng, et. al., Nucl.Phys.A733:235-261,2004)

Taking $f(0) = 0$ into account:
Z. Y. Zhou and H. Q. Zheng, Nucl. Phys. **A755** (2006) 212.





Fit up to 1430MeV of LASS data

$$\begin{aligned}
&\chi^2_{d.o.f.} = 80.3/(60-6)\ ; \\
&M_\kappa = 694 \pm 53 MeV\ ,\ \ \Gamma_\kappa = 606 \pm 59 MeV\ ; \\
&M_{K^*} = 1443 \pm 15 MeV\ ,\ \ \Gamma_{K^*} = 199 \pm 35 MeV \\
&\Lambda^2_{1/2} = -13.6 \pm 40.0 Gev^2\ ,\ \ \Lambda^2_{3/2} = -11.4 \pm 1.7 Gev^2\ ,
\end{aligned} \tag{21}$$

$$\begin{aligned}
&a_{1/2} = 0.219 \pm 0.034 \qquad b_{1/2} = 0.075 \pm 0.023; \\
&a_{3/2} = -0.042 \pm 0.002\ \ b_{3/2} = -0.0271
\end{aligned} \tag{22}$$

Scattering lengths no longer free parameters. If freezing kappa, $\chi^2_{d.o.f.} = 1055.0/(60-6)$.

Fit up to 2100 MeV of LASS data

$$\chi^2_{d.o.f.} = 229.5/(96-9) \ ;$$
$$M_\kappa = 649 \pm 63 MeV \ , \ \ \Gamma_\kappa = 602 \pm 22 MeV \ ;$$
$$M_{K^*} = 1435 \pm 6 MeV \ , \ \ \Gamma_{K^*} = 288 \pm 22 MeV$$
$$\Lambda^2_{1/2} = -16.9 \pm 5.6 Gev^2 \ , \ \ \Lambda^2_{3/2} = -10.2 \pm 1.2 Gev^2 \ ,$$
$$M_{K(1950)} = 1917 \pm 12 MeV \ , \ \ \Gamma^{tot}_{K(1950)} = 145 \pm 38 MeV \ ;$$
$$\Gamma^{K\pi}_{K(1950)} = 87 \pm 14 MeV \qquad (23)$$

To be compared with the BES-II results:

$$M - \frac{i}{2}\Gamma = (841 \pm 30^{+81}_{-73}) - i(309 \pm 45^{+48}_{-72}) MeV. \qquad (24)$$

The Roy–Steiner equation analysis (S. Descotes-Genon and B. Moussallam 06)

$$m_\kappa = 658 \pm 13 MeV \ , \ \ \Gamma_\kappa = 557 \pm 24 MeV \qquad (25)$$

Watson's final state theorem

Assuming there exist a group of eigenstates of strong interaction and total angular momentum J (J is a good quantum number) $|1>, |2>, \ldots, |n>$. The strong interaction S matrix diagonal:

$$S_0 = diag(e^{2i\delta_1}, \cdots, e^{2i\delta_n})$$

Introducing the "weak" interaction

$$S = S_0 + i\Sigma \tag{26}$$

$i\Sigma \sim O(\epsilon)$ and no longer diagonal. Unitarity requires $SS^+ = S^+S = 1$, or

$$1 \simeq S_0^+ S_0 + i(\Sigma S_0^+ - S_0 \Sigma^+) + O(\epsilon^2) \; . \tag{27}$$

This leads to $\Sigma S_0^+ = S_0 \Sigma^+$. From **time reversal invariance** $\Sigma_{mn} = \Sigma_{nm}$ we get

$$\Sigma_{mn} = |\Sigma_{mn}| e^{i(\delta_n + \delta_m)} \; . \tag{28}$$

$D^+ \to K^-\pi^+ l\nu_l$ decays

Leptons are spectator for strong interactions, the strong phase generated from πK rescattering, when the invariant mass of πK system is less than the $\pi\eta'$ threshold, are exactly the same as the one appeared in πK elastic scatterings, according to the final state theorem.

The $D^+ \to K^-\pi^+ l\nu_l$ process is p wave dominant ($D^+ \to \bar{K}^{*0} l\nu_l$).

However evidence exist for a small, even spin $K\pi$ amplitude that interferes with the dominant $\bar{K}^{*0}$ component (Focus Collaboration 02):

Data can be described by $\bar{K}^{*0}$ interference with either a constant amplitude or a broad spin 0 resonance.

The final state theorem is also confirmed by the Focus Collaboration (Focus 05).
(see also Edera and Pennington 06)

The Omnés solution

The spectral function of the form-factor satisfies

$$\mathrm{Im}A = \rho A T^* \ , \tag{29}$$

where T is the $\pi\pi$ (partial wave) scattering amplitude. The Eq. (29) has a simple solution, called the Omnés solution:

$$A(s+i\epsilon) = P_n(s)\exp\{\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\delta_\pi(s')ds'}{s'(s'-s-i\epsilon)}\} \ , \tag{30}$$

From Eq. (29) one make analytic continuation of $A(s)$ we get the following relation

$$A(s-i\epsilon) = A^{II}(s+i\epsilon) = A(s+i\epsilon)/S \ . \tag{31}$$

Next we remove all possible zeros of on the complex plane by dividing a polynomial: $A \rightarrow A/P_n(s)$. Then $\log(A/P_n(s))$ is also analytic on the entire cut plane and obeys a simple dispersion relation:

$$
\begin{aligned}
\log(A/P_n(s)) &= \frac{1}{2\pi i}\oint_C \frac{\log(A(s')/P_n(s'))}{s'-s}ds' \\
&= \frac{1}{2\pi i}\int_{4m_\pi^2}^{\infty} \frac{\log(A(s'+i\epsilon)/A(s'-i\epsilon))}{s'-s}ds' = \frac{1}{2\pi i}\int_{4m_\pi^2}^{\infty} \frac{\log(S(s'))}{s'-s}ds' \\
&= \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\delta_\pi(s')}{s'-s}ds' \ , \qquad (32)
\end{aligned}
$$

which reproduces Eq. (30).

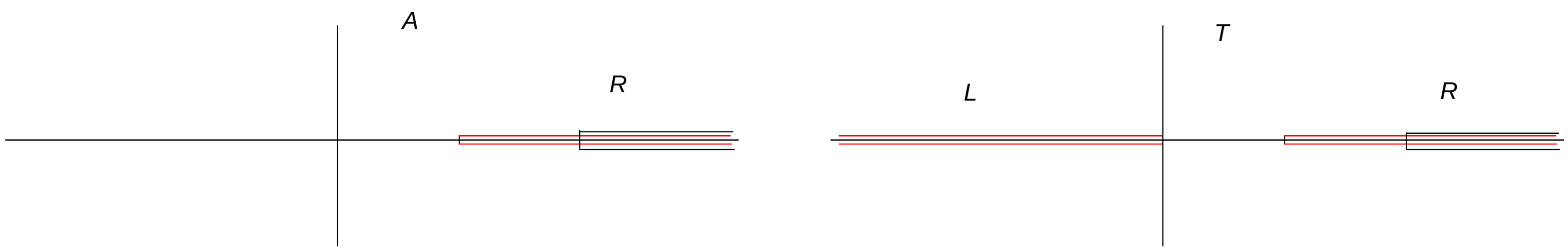


Figure 8: Cut structure of elastic scattering amplitude and the form-factor.

Final state phases in the $J/\Psi \to \omega\pi\pi$ process

The final state phases in $J/\Psi \to \omega\pi\pi$ process is also discussed in the literature. (Bugg 06)
Subtracting the $\omega\pi$ and 4π final state interactions, Watson theorem fixes the final state phases:

$$A_0(s) = |A_0(s)| e^{i\delta_\pi} \ , \tag{33}$$

similar to the scalar form factor

$$F_0(s) = |F_0(s)| e^{i\delta_\pi} \ . \tag{34}$$

$\Rightarrow R(s) = A_0(s)/F_0(s)$ real when $s < 4M_K^2$. Furthermore, since the poles both in A_0 and F_0 cancel each other and the cut in R is distant (starting from $4M_K^2$), R has to be a slowly varying function, at least at low energies (i.e., when $s << 4M_K^2$):

$$R(s) = R_0 + R_1 s + \frac{(s-4M_K^2)^2}{\pi} \int_{4M_K^2} \frac{\mathrm{Im} R(t)}{(t-4M_K^2)^2 (t-s)} \ . \tag{35}$$

A_0 from $J/\Psi \to \omega\pi\pi$ process extracted (Bugg 06). Also Lahde and Meissner 06

$\Rightarrow A_0(s) = R_0(1 - s/s_0)F_0(s)$ and $s_0 \simeq 1.65\text{GeV}^2$ (Leutwyler 06).

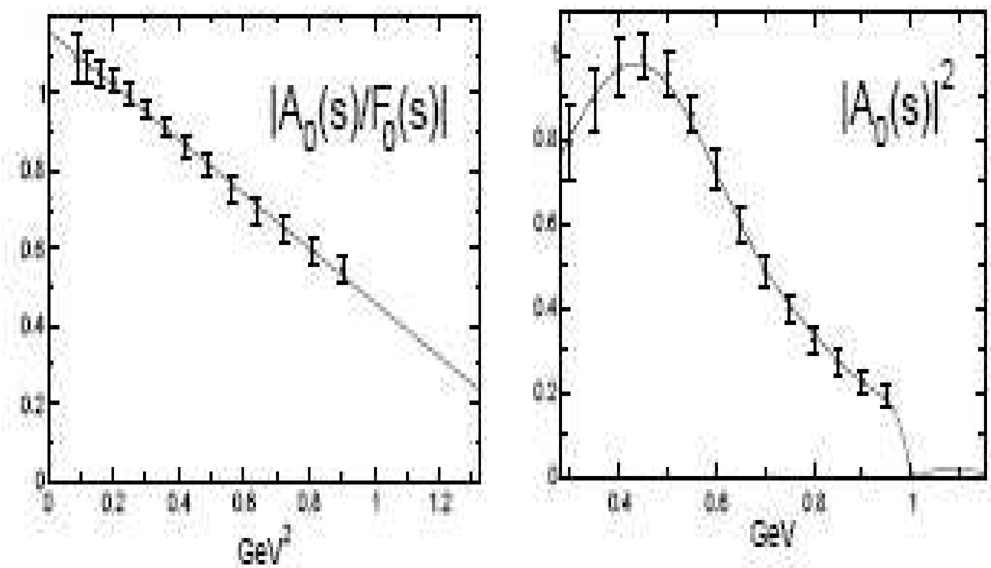


Figure 9: The comparison between the scalar form factor and the scalar amplitude extracted from BES $J/\Psi \to \omega\pi\pi$ process.

$D^+ \to K^-\pi^+\pi^+$ decays

In (E791 2002), an isobar model is used to parameterize the partial wave amplitude. In this model, the decay amplitude $\mathcal{A}$ is described by a sum of quasi two-body terms $D \to R + k$, $R \to i + j$, in each of the three channels $k = 1, 2, 3$: (Bugg 06, Lahde and Meissner 06)

$$\mathcal{A} = d_0 e^{i\delta_0} + \sum_{n=1}^{N} d_n e^{i\delta_n} \frac{F_R(p, r_R, J)}{m_{R_n}^2 - s_{ij} - i m_{R_n} \Gamma_{R_n}(s_{ij})} \times F_D(q, r_D, J)\, M_J(p, q)\,, \tag{36}$$

s_{ij}: squared invariant mass of the ij system. J: the spin
m_{R_n} the mass and $\Gamma_{R_n}(s_{ij})$ the width of each of the N resonances R_n.
F_R and F_D: form factors with effective radius parameters r_R and r_D, for all R_n and for the parent D meson, respectively.
p and q are momenta of i and k, respectively, in the ij rest frame.
$M_J(p, q)$: a factor introduced to describe spin conservation in the decay (E791 01).

The complex coefficients $d_n e^{i\delta_n}$ $(n = 0, N)$ are determined by the D decay dynamics and are parameters estimated by a fit to the data.

The E791 kappa

The first, non-resonant (NR) term describes direct decay to $i + j + k$ with no intermediate resonance, and d_0 and δ_0 are assumed to be independent of s_{ij}. In E791 2002 it is noticed that the NR term was small, and that a further term, parameterized as a new $J = 0$ resonance $\kappa(800)$ with $m_R = (797 \pm 19 \pm 43)$MeV and $\Gamma_R = (410 \pm 43 \pm 87)$MeV, gave a much better description of the data.

Apparently the above parametrization forms can be improved for the purpose of exploring wide and broad resonance and for testing final state phases.

All phases are generated by cuts as dictated by real analyticity and hence are of dynamical origin!

The E791 test of FS theorem

Meadows 05: using a generalized isobar picture of two body interactions. Higher $K\pi$ waves are described by sums of known resonances, the s-wave amplitude and phase are determined bin-by-bin in $K\pi$ mass. The phase variation is found not to be that of $K^-\pi^+$ elastic scattering obtained from LASS Collaboration.

The applicability of Watson theorem in the 3 body hadronic decay is examined in Meadows 05. In the 3 body D decays, $K^-\pi^+$ forms both isospin 1/2 and 3/2 components. It is not clear, however, how to estimate the $I = \frac{3}{2}$ component in s-wave.

Simple quark spectator model of D decay to $K\pi\pi \Rightarrow$ the $K\pi$ system has only $I = 1/2$. However, it is found that if $I = \frac{1}{2}$ dominates, then the Watson theorem does not describe these data well.

This question is re-examined in Edera and Pennington 05. Suggests that in $D \to K\pi\pi$ decays there exists a different mixture of $I = 1/2$ and $I = 3/2$ s-wave interactions than in elastic scattering. Applying Watson's theorem to this generalized isobar model allows one to estimate the $I = 3/2$ $K\pi$ s-wave component, and it is found to be larger than in hadronic scattering or semi-leptonic processes.

Conclusions:

The BEPC-II experiment provides a unique opportunity in the era of precise hadron physics!