PWA of $J/\psi \rightarrow K^+K^-\pi^0$ at BESII

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PWA Workshop
Beijing, Jan 25, 2007

Outline

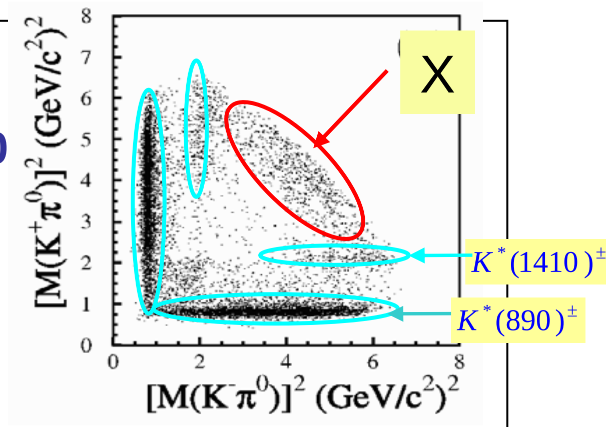
- Motivation
- Event selections and background study
- PWA of $J/\psi \rightarrow K^+K^-\pi^0$
  - Introduction to PWA method used
  - The official solution of PWA and various checks
  - Systematic uncertainties
- Conclusion and discussion

Motivation (I)

- Multi-quark states, glueballs and hybrids have been searched for experimentally for a very long time, **but none is established.**

- However, during the past few years, a lot of surprising experimental evidences showed the existence of hadrons that cannot (easily) be explained in the conventional quark model. Most of them are multi-quark candidates. **Searching for multi-quark states becomes one of the hottest topics in the hadron spectroscopy.**

Motivation (II)



- A broad resonance is observed in KK mass spectrum in $J/\psi \to K^+K^-\pi^0$ and the structure in Dalitz plot is complicated, so PWA is needed for this channel.

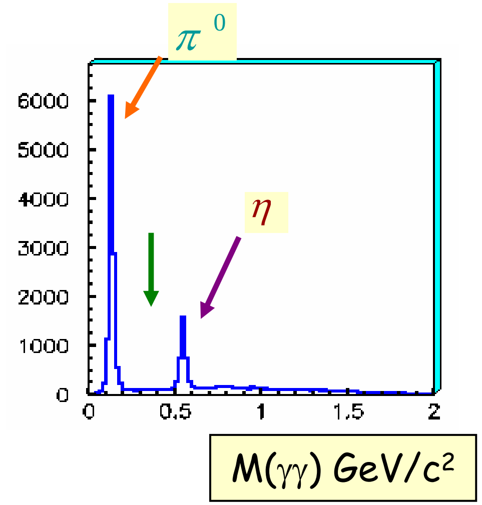
- The analysis of $J/\psi \to K^+K^-\pi^0$ is taken as an example to show how to perform PWA at BESII, especially how to deal with the signal statistical significance, background subtraction, systematic uncertainties, etc.

Event selections: final states: K^+, K^-, γ, γ

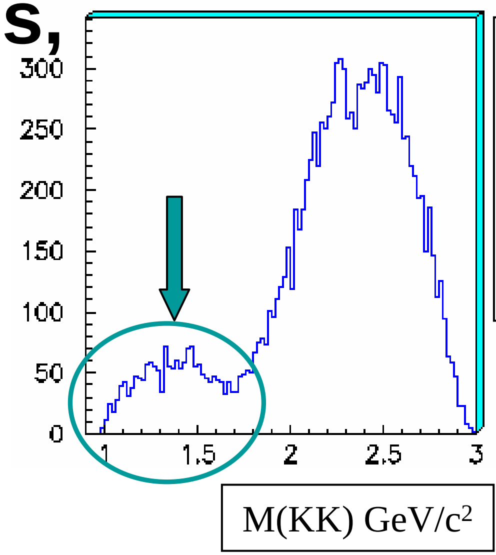
- Two good charged tracks
- Particle ID: two Kaons is required
- At least two good gammas
- Require good π^0 signal: $|M(\gamma\gamma) - 0.135| < 0.04$ GeV
- Kinematical fit (4c) : $\chi^2_{K^+K^-\gamma\gamma} < 10$
- To suppress $\gamma K^+K^-\pi^0$ background, $\chi^2_{\gamma K^+K^-\pi^0} > 50$ (5c fit) is required

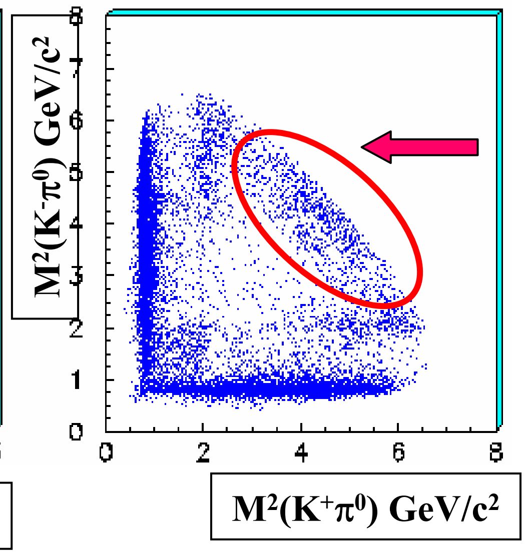
Distributions of candidate events

- The π^0 signal is very clear:



- After event selections, the broad resonant structure is evident:





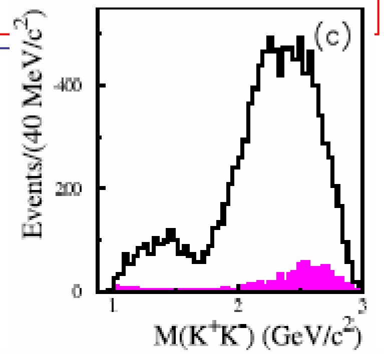
- **It is the first time that the structure is observed!**

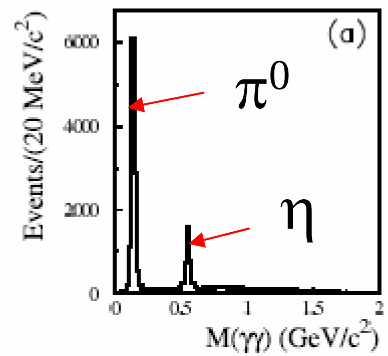
background study

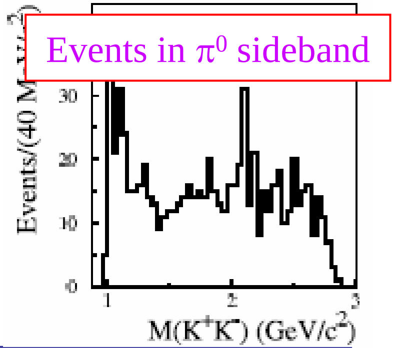
- **$\rho\pi$ background**
  - The contribution is about 5.6%,
  - ➢ **It is mainly in high mass region.**

- **Multi-gamma background : $(n\gamma)\ K^+K^-$**
  - **With clear π^0 signal**，**such as:**
  - **$J/\psi \rightarrow \gamma\eta_c$, $\eta_{c} \rightarrow K^+K^-\pi^0$, $a_0(980)\ \pi^0$, $a_2(1320)\ \pi^0$,**
  - **$J/\psi \rightarrow K^{*\pm} K^{\pm}\ \pi^0 \rightarrow K^+K^-\pi^0\ \pi^0$,**
  - ➢ **The contribution is small, and it can be negligible.**

  - **Without clear π^0 signal**，such as:
  - **γK^+K^-, $\gamma K^+K^-\pi^0$, $\gamma\gamma\phi$, ηK^+K^-, $K^+K^-\pi^0\ \pi^0$**
  - ➢ **It can be estimated by π^0 sideband.**
  - ➢ **About 1.6%.**
  - ➢ **The character of these events in KK mass spectrum is different to data.**







➔Backgrounds have been carefully studied. The structure in data is not from any background processes.

Introduction to PWA used

Introduction (I)- likelihood function

ξ is the physical quantity measured by experiment
$\omega(\xi)$ is the probability density to produce it $\omega(\xi) = \frac{d\sigma}{d\Phi_i}$
$\epsilon(\xi)$ is the efficiency， $P(\xi)$ is the probability to observe it ,
and $P(\xi)$ is defined as: $P(\xi) = \frac{\omega(\xi)\epsilon(\xi)}{\int d\xi \omega(\xi)\epsilon(\xi)}$

For n events, the probability density:

$$P(\xi_i, \xi_2, \cdots, \xi_n) = \prod_{i=1}^{n} P(\xi_i) = \prod_{i=1}^{n} \frac{\omega(\xi_i)\epsilon(\xi_i)}{\int d\xi \omega(\xi)\epsilon(\xi)}$$

$$\ln P(\xi_i, \xi_2, \cdots, \xi_n) = \sum_{i=1}^{n} \ln\left(\frac{\omega(\xi_i)}{\int d\xi \omega(\xi)\epsilon(\xi)}\right) + \underbrace{\sum_{i=1}^{n} \ln \epsilon(\xi_i)}_{constant}$$

Definition of likelihood function: $\mathcal{L} = P(\xi_i, \xi_2, \cdots, \xi_n)$

The logarithm of it: $\ln \mathcal{L} = \ln P(\xi_i, \xi_2, \cdots, \xi_n) = \sum_{i=1}^{n} \ln\left(\frac{d\sigma}{d\Phi_i}/\sigma\right)$

The total observed cross section: $\sigma = \int d\xi \omega(\xi)\epsilon(\xi)$

•In fact, s=- ln L is used, and minimum s is used to get the parameters be fitted.

Introduction (II) – cross section

The total observed cross section is calculated by MC integration:

$$\sigma = \int d\xi \omega(\xi)\epsilon(\xi) = \sum_i \Delta\xi_i \omega(\xi_i)\epsilon(\xi_i)$$
$$= \frac{1}{N_{gen}} \sum_i N_{gen} \Delta\xi_i \omega(\xi_i)\epsilon(\xi_i)$$
$$= \frac{1}{N_{gen}} \sum_i N_{\xi_i} \omega(\xi_i) = \frac{1}{N_{gen}} \sum_{i=1}^{N_{MC}} \omega(\xi_i)$$

The observed cross section:

$$\sigma = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} \left(\frac{d\sigma}{d\Phi}\right) = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} \left(\sum_j A_j\right)^2$$

The differential cross section: $\frac{d\sigma}{d\Phi_i} = |\sum_j A_j|^2$

The amplitude: $A = \sum_j A_j = \psi_\mu(m) A^\mu = \psi_\mu(m) \sum_j (\Lambda_j U_j^\mu)$

$\psi_\mu(m)$ is the polarization vector; m=1,2
U_j^μ is the j-th partial wave amplitude constructed in the covariant tensor formalism, in which BW appears;
Λ_j is the complex parameter for j-th partial wave.

Introduction (III) – Breit-Wigner formula

The decay process is modeled by a phase space contribution plus several sequential two-body decays:

$$J/\psi \rightarrow X\pi^0, X \rightarrow K^+K^-;$$
$$J/\psi \rightarrow \rho\pi^0, \rho \rightarrow K^+K^-;$$
$$J/\psi \rightarrow K^{*\pm}K^{\mp}, K^{*\pm} \rightarrow K^{\pm}\pi^0.$$

For broad resonance, X, the BW has a mass-dependent width:

$$\begin{cases} BW(R) = \frac{1}{s-M_R^2+i\sqrt{s}\Gamma_R(s)} \\ \Gamma_R(s) = \Gamma_R(\frac{M_R^2}{s})(\frac{p(s)}{p(M_R^2)})^{2l+1} \end{cases}$$

Used by the experiments:

- KLOE at DAΦNE, $e^+e^- \rightarrow \Phi \rightarrow \pi^+\pi^-\pi^0$, Phys. Lett. B561, 55(2003)
- SND at VEPP-2M, e+e- $\rightarrow \pi^+\pi^-\pi^0$, Phys. Rev. D68, 052006(2003); ibid. D65, 032002(2002)
- ALEPH at LEP, τ decays, Z. Phys. C76, 15(1997)

……

For other resonances, the BW has a constant width:

$$BW(R) = \frac{1}{s - M_R^2 + iM_R\Gamma_R}$$

Introduction (IV) – background subtraction

Since the log-likelihood is a sum over the number of data events, so background can be subtracted by subtracting log-likelihood value of background events.

$$
\begin{aligned}
N_{data} &= N_{signal} + N_{bkg} \\
-\ln \mathcal{L}_{data} &= -\ln \mathcal{L}_{signal} - \ln \mathcal{L}_{bkg} \\
-\ln \mathcal{L}_{signal} &= -\ln \mathcal{L}_{data} - (-\ln \mathcal{L}_{bkg}) \\
&= \sum_{i=1}^{N_{data}} \left(\frac{d\sigma}{d\Phi_i}/\sigma\right) - \sum_{i=1}^{N_{bkg}} \left(\frac{d\sigma}{d\Phi_i}/\sigma\right)
\end{aligned}
$$

Introduction (V) – statistical significance

For a resonance X,

$\mathcal{L}_0$ is the likelihood when X is not included in the fit;

$\mathcal{L}_1$ is the likelihood when X is included in the fit;

Δndf is the number of changed degree of freedom which equals the free parameters of X;

$\Delta\chi^2$ equals the twice difference of the log-likelihood.

$$\Delta\chi^2 = -2\ln\mathcal{L}_0 - (-2\ln\mathcal{L}_1)$$

The statistical significance of X will be given by:

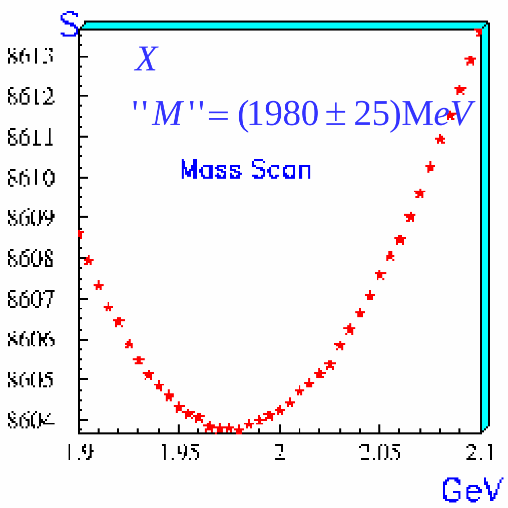
$$\boxed{prob(\Delta\chi^2, \Delta ndf)}$$

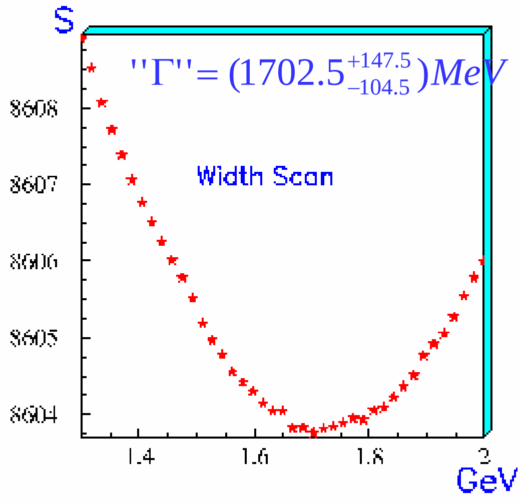
$$\boxed{dgausn(prob(\Delta\chi^2, \Delta ndf))}$$

PWA solution and various checks

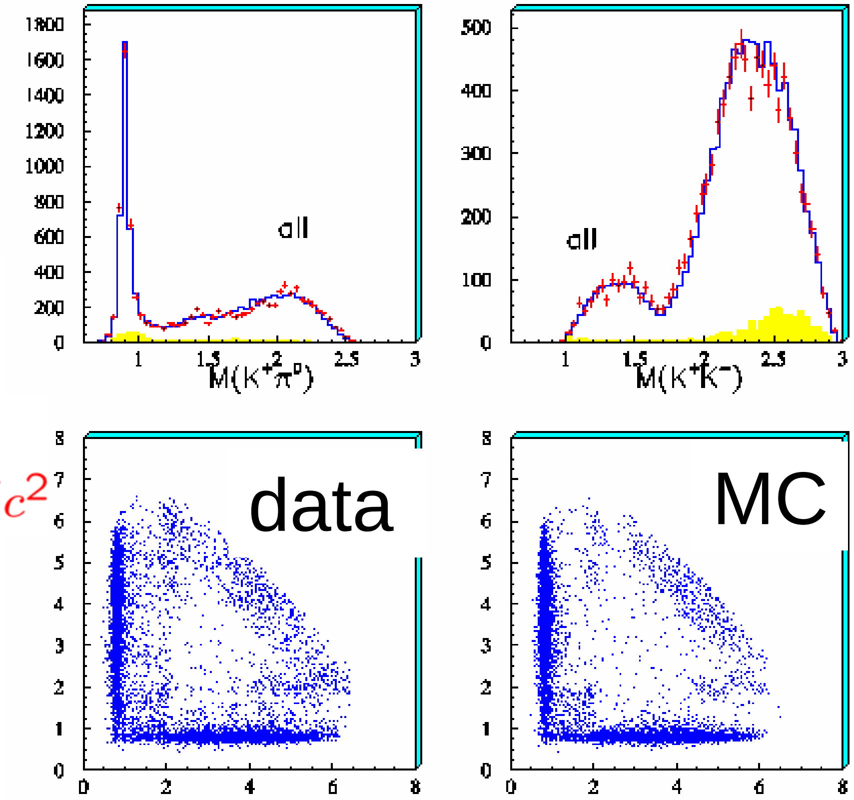
Official solution: Including K*(892), K*(1410), X, ρ (1700) and phase space(p.s.)

Scanning X with others fixed to PDG





error bar: data(red)
histogram: projection of fit result



Pole position:

$$M - \frac{i}{2}\Gamma$$
$$= (1576^{+49}_{-55}(stat)) - \frac{i}{2}(818^{+22}_{-23}(stat))MeV/c^2$$

Branching ratio:

$$BR(J/\psi \to X\pi^0).BR(X \to K^+K^-)$$
$$= (8.5 \pm 0.6(stat)) \times 10^{-4}$$

Spin-parity:

$$J^{pc} = 1^{--}$$

Various checks about the official solution:

- Try to describe the broad structure by known resonances or their interferences

- Statistical significance of each resonance in the official solution

- Statistical significances of other resonances

- J^{pc} of X

Describe it by known resonances

- **It is unlikely to be $\rho(1450)$, because:**
  - **The parameters of the X are incompatible with $\rho(1450)$.**
  - **$\rho(1450)$ has very small fraction to KK. From PDG:**

$BR(\rho(1450) \rightarrow K^+K^-) < 1.6\times10^{-3}$ (95%C.L.)

- **It cannot be fitted with the interference of $\rho(770)$, $\rho(1900)$ and $\rho(2150)$:**
  - **The log-likelihood value is worsen by 85.**

→ The broad resonant structure, X, can not be described by any known resonances or their interferences.

Statistical significance check

resonance	ΔS	Δndf	significance
$K^*(892)$, $K^*(1410)$, X, $\rho(1700)$, p.s.			
Remove $\rho(1700)$	28	2	7.2 σ
Remove X	533	4	$>>10\sigma$
Remove $K^*(892)$	11438	2	$>>10\sigma$
Remove $K^*(1410)$	465	2	$>>10\sigma$
Remove P.S.	254	2	$>>10\sigma$

→ **All above resonances have significances lager than 5σ.**

Other resonances

For other resonances, such as $K^*_2(1430)$, $K^*(1680)$, $K^*(2075)$, $\rho(770)$, $\rho(1900)$ and $\rho(2150)$, the statistical significances are less than 5σ,

➔These resonances are not included in the official solution, but their impacts will be considered into systematic uncertainty.

J^{pc} check

For parity conservation, the J^{pc} of X can only be 1^{--}, 3^{--}, 5^{--},…

If J^{pc} is changed from 1^{--} to 3^{--} , the log-likelihood is worsen by 325.

Even higher spin states are unlikely at such a low mass .

$\boxed{\rightarrow J^{pc}\ (X) \text{ should be } 1^{--}}$

Systematic uncertainties

Systematic uncertainties

- Uncertainty from each resonance in the fit.
- Uncertainties from of other resonances.
- Uncertainty from background level
- The impact from different BW form of X
- The impact from MDC wire resolution simulation
- Uncertainty from X’s mass and width

Systematic uncertainty from each resonance in the fit

resonance	Mass(MeV)(PDG)	Γ(MeV)(PDG)	Mass(MeV)(scanned)	ΓMeV) (scanned)
$K^{\star}(892)$	891.7±0.3	50.8±0.9	891.1±0.8	53.1±1.1
$K^{\star}(1410)$	1415±15	232±21	1438±6	158±5
$\rho(1700)$	1720±20	250±100	1705±9	239±17
X(fitted)	1576^{+49}_{-55}	818^{+22}_{-23}	1622±14	860±11

➔The differences caused by the parameters of $K^*(892)$, $K^*(1410)$ and $\rho(1700)$ are included into systematic uncertainties.

Systematic uncertainty from inclusions of other resonances

Inclusions of other resonances will cause the dominant uncertainties of the parameters of X.

resonance	ΔS	$\Delta M(MeV)$(pole)	$\Delta\Gamma$(MeV)(pole)	$(\Delta B.R./B.R.)\%$
best solution				
add $K_2^*(1430)$	-0.1	-8.0	-17.6	-1.4
add $K^*(1680)$	-14.0	52.4	9.2	-2.1
add $K^*(2075)$	-1.6	-26.6	-52.2	0.9
add $\rho(770)$	-13.0	-58.0	-111.6	-29.9
add $\rho(1900)$	-0.1	-7.0	40.4	2.8
add $\rho(2150)$	-2.3	51.4	9.4	26.8
total		$^{+73.4}_{-64.7}$	$^{+42.5}_{-124.5}$	$^{+27.0}_{-30.0}$

➔Their impacts are considered into the systematic uncertainties.

Systematic uncertainty from background level

bkg source	B.R.(%)	Δ Mass (MeV)	$\Delta\,\Gamma$ (MeV)
change $\rho\pi$ bkg by 10%	± 2.6	± 1.6	± 4.0
change π^0 sideband bkg by 50%	± 2.7	± 1.6	± 4.0
total affect	± 3.5	± 2.4	± 5.6

➔The influence from background uncertainty is small and will be taken into the systematic uncertainty.

Systematic uncertainty from different BW form

The BW form of X is changed by different forms :

$$\begin{cases} BW(R) = \frac{1}{s - M_R^2 + i\sqrt{s}\Gamma_R(s)} \\ \Gamma_R(s) = \Gamma_R(\frac{M_R^2}{s})(\frac{p(s)}{p(M_R^2)})^{2l+1} \end{cases} \longrightarrow BW(R) = \frac{1}{s - M_R^2 + iM_R\Gamma_R}$$

$$\boxed{M - \frac{i}{2}\Gamma = (1584 \pm 39(stat)) - \frac{i}{2}(821 \pm 72(stat)) MeV/c^2}$$

$$\boxed{BR(J/\psi \to X\pi^0).BR(X \to K^+K^-) = (8.3 \pm 0.6(stat)) \times 10^{-4}}$$

➔The difference caused by different BW form is small relatively and still will be considered into systematic uncertainty.

Systematic uncertainty from MDC wire resolution simulation

There are two different methods to simulate MDC wire resolution at BESII. The difference of them will cover the uncertainty of tracking and kinematical fit.

If two different MC samples are used to calculate total cross section, the parameters of X will be changed.

→The impact from MDC wire resolution simulation is considered into systematic uncertainty.

Systematic uncertainty from X's mass and width

The parameters of X given by official solution:

$$M-\frac{i}{2}\Gamma=(1576^{+49}_{-55}(stat))-\frac{i}{2}(818^{+22}_{-23}(stat))MeV/c^2$$

$$BR(J/\psi\rightarrow X\pi^0).BR(X\rightarrow K^+K^-)=(8.5\pm0.6(stat))\times10^{-4}$$

➔The BR will be changed when the pole position of X is changed by 1σ, and the impact will be considered into BR's systematic uncertainty.

Systematic uncertainties

sources	Mass (MeV) (pole)	Width (MeV) (pole)	B.R.(%)
Parameters of each resonance in the fit	46.0	42.0	7.0
Inclusions of other resonances	+73.4 -64.7	+42.5 -124.5	+27.0 -30.0
Background level	+2.4	5.6	3.5
BW formulae	8.0	-5.0	2.1
MDC wire resolution simulation	-44.2	-20.2	10.1
X's M,Γ uncertainty			+6.1 -25.6
photon efficiency			4.0
particle identification			2.0
$N_{J/\psi}$			4.7
Total	+98 -91	+64 -133	+31 -42

Considering the systematic uncertainties,
the parameters of X given by PWA are listed here:

$$M - \frac{i}{2}\Gamma$$
$$= (1576^{+49}_{-55}(stat)^{+98}_{-91}(syst)) - \frac{i}{2}(818^{+22}_{-23}(stat)^{+64}_{-133}(syst))MeV/c^2$$

$$BR(J/\psi \to X\pi^0).BR(X \to K^+K^-)$$
$$= (8.5 \pm 0.6(stat)^{+2.7}_{-3.6}(syst)) \times 10^{-4}$$

$$J^{pc} = 1^{--}$$

About ρ(1700)

In official solution, the significance of ρ(1700) is 7.2σ

Because data-MC inconsistencies are not included in the fit, the significance of the ρ(1700) may not be high enough to conclude that its inclusion in the fit is necessary. Therefore we list the results of the fit without ρ(1700).

$$M - \frac{i}{2}\Gamma = (1428^{+17}_{-18}(stat)) - \frac{i}{2}(1073^{+29}_{-24}(stat))MeV/c^2$$

$$BR(J/\psi \rightarrow X\pi^0).BR(X \rightarrow K^+K^-) = (6.3 \pm 0.6(\mathrm{stat})) \times 10^{-4}$$

Conclusion and discussion

- A broad 1^{--} resonant structure around $1.5 GeV/c^2$ is observed in the K^+K^- mass spectrum in $J/\psi \rightarrow K^+K^-\pi^0$

- The parameters of the resonance given by PWA are not compatible with any known meson resonances.

- Broad width is expected for a multi-quark state!

- To understand the nature of the broad 1^{--} peak, it is important to search for a similar structure in $J/\psi \rightarrow K_sK\pi$ decay to determine its isospin.

谢谢！

Thank you!