Interference between resonance and continuum aplitudes in e^+e^- experiments

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Outline

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The importance of the inteference
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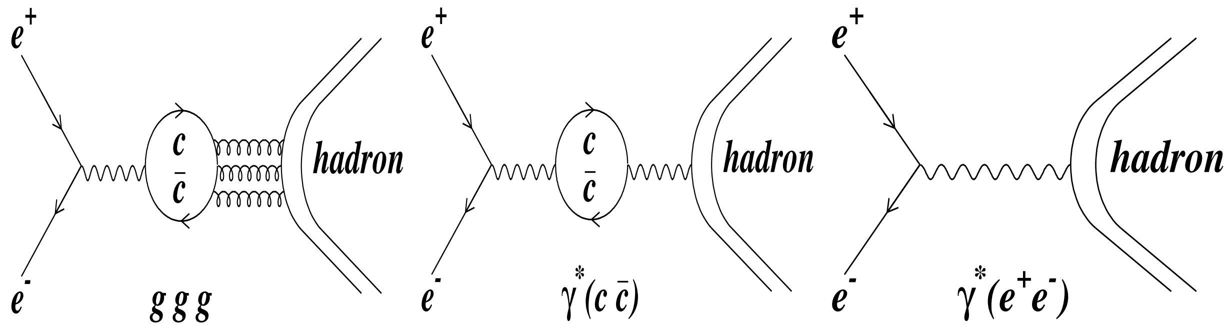
Summery and discussions

Quarkonium in e^+e^- experiments

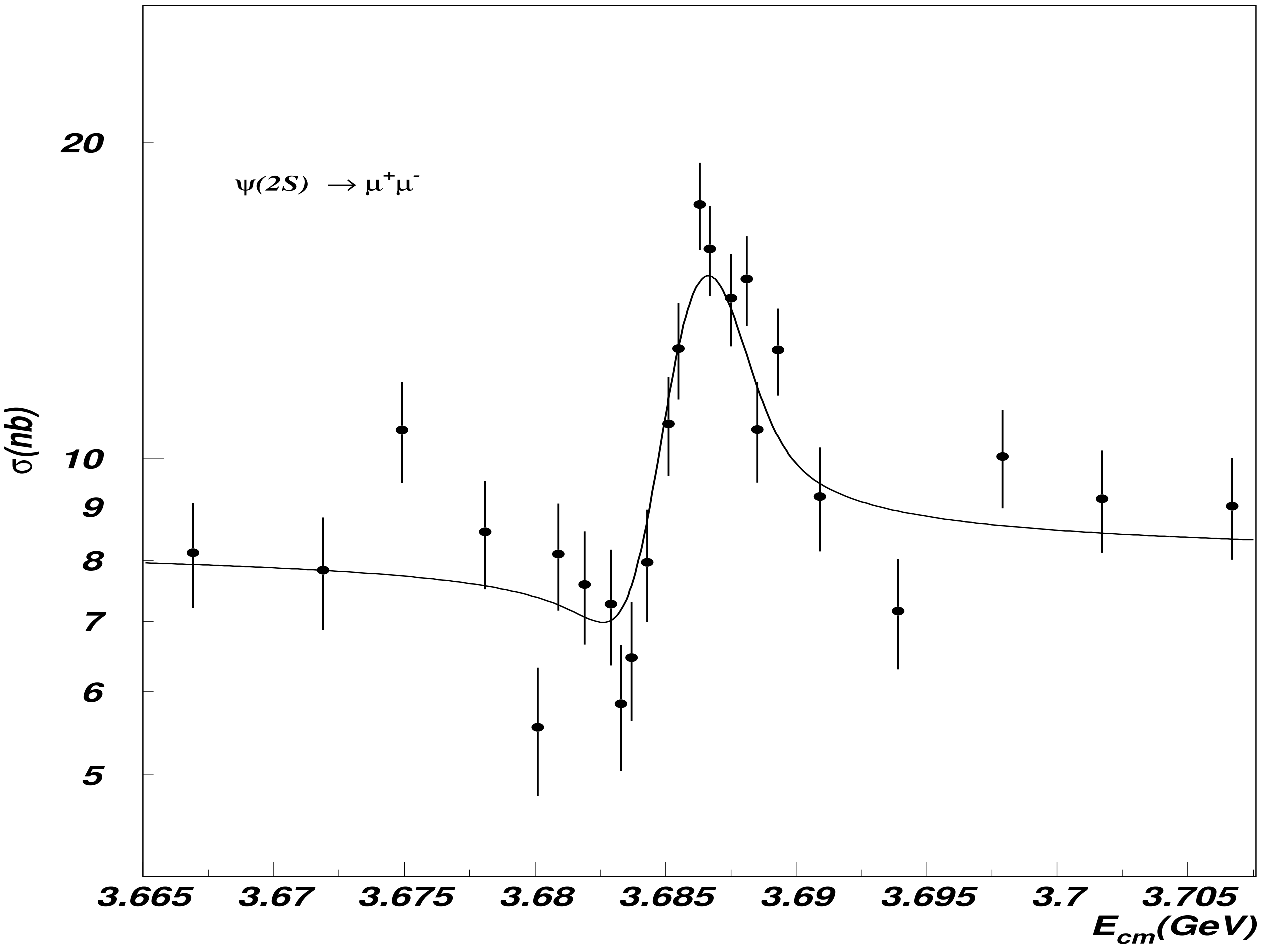
For the quarkonium produced in e^+e^- experiments, there are three diagrams which contribute to the production of a final state.

S.Rudaz, Phys.Rev.D14,298(1976);
P. Wang, C. Z. Yuan, X. H. Mo and D. H. Zhang, Phys. Lett. **B** 593; 89-94 (2004)



The importance of the inteference

The importance of non-resonance $e^+e^- \to \gamma^*$ diagram can be seen from the scanned curve of $e^+e^- \to \mu^+\mu^-$ in the vicinity of $\psi(2^3S_1)$.



The importance of the inteference

For pure elctromagnetic interaction, i.e. $\omega\pi^0$, $\pi^+\pi^-$, if the data is taken on top of the resonance,

the electromagnetic deccay amplitude is expressed as

$$a_\gamma(s) = \frac{3\Gamma_{ee}\mathcal{F}(s)/(\alpha\sqrt{s})}{s - M^2 + iM\Gamma_t}, \qquad (1)$$

while the continuum amplitude is expressed as

$$a_c(s) = \frac{\mathcal{F}(s)}{s}, \qquad (2)$$

At the energy of the resonance mass, i.e. $s = M^2$, the phase between the two amplitudes is -90°. So the interference can be neglected.

The importance of the inteference

For these decay modes, we can simply subtract the off resonance cross section scaled for s dependence from the cross section measured on top of the resonance to get the resonance cross section. The ratio of the resonance decay cross section of a particular final state to the total resonance cross section gives the branching fraction of this mode.

The importance of the inteference

Under such circumstance, the interference is still non-vanishing due to two reasons: first, for the practical reason, in the experiments the data is usually taken at where the maximum inclusive hadron cross section is, which in general does not coincide with the mass of the resonance; second, even if the data is collected at the energy of the resonance mass, the interference is non-vanishing because of radiative correction. For the narrow resonances with widthes smaller than the energy spread of the e^+e^- colliders, the smearing of the C.M. energy also results in a non-vanishing interference term. Although these are beyond the concern for the accuracy of current experiments, but one day we may need to take it into account in very high precision measurement.

The importance of the inteference

But for other final states which have contributions from both strong and electromagnetic interactions, there could be interference between the strong decay amplitude a_{3g} and the continuum amplitude a_c as well as between strong amplitude a_{3g} and electromagnetic amplitude a_γ.

Based on the analysis of the experimental data, we have suggested that the phase between strong and EM amplitudes θ_g is universally -90° in quarkonium decays. Since at the energy of resonance mass, the phase of the electromagnetic amplitude a_γ is -90° relative to continuum amplitude a_c, so the relative phase between the strong amplitude a_{3g} and continuum amplitude a_c is either 180° or 0°, depending on whether the relative sign between strong interaction g and electromagnetic interaction e, is plus or minus.

The importance of the inteference

The interference between a_{3g} and a_c is destructive for the final states $\rho\pi$, $\omega\eta$, $\omega\eta'$, $K^{*+}K^- + c.c.$, $b_1\pi$, and K^+K^-, but constructive for $\phi\eta$, $\phi\eta'$, and $K^{*0}\overline{K^0} + c.c.$

Destructive interference between strong amplitude a_{3g} and continuum amplitude a_c means that the observed cross section at the the resonance can be smaller than the off resonance cross section measured at nearby energy and scaled for s dependence. The experimental results on $\rho\pi$ and $\omega\eta$ modes on and off $\psi(3770)$ resonance resonance by CLEOc demonstrate this interference pattern.

The importance of the inteference

Mode	Amplitude	σ(3.671 GeV) [pb]	σ(3.773 GeV) [pb]
		VP	
$\rho\pi$	$g+e$	$8.0^{+1.7}_{-1.4}\pm 0.9$	$4.4\pm 0.3\pm 0.5$
$\omega\pi^0$	$3e$	$15.2^{+2.8}_{-2.4}\pm 1.5$	$14.6\pm 0.6\pm 1.5$
$\phi\pi^0$	0	<2.2	<0.2
$\rho\eta$	$3eX_\eta$	$10.0^{+2.2}_{-1.9}\pm 1.0$	$10.3\pm 0.5\pm 1.0$
$\omega\eta$	$(g+e)X_\eta$	$2.3^{+1.8}_{-1.1}\pm 0.5$	$0.4^{+0.2}_{-0.2}\pm 0.1$
$\phi\eta$	$[g(1-2s_g)-2e]Y_\eta$	$2.1^{+1.9}_{-1.2}\pm 0.2$	$4.5\pm 0.5\pm 0.5$
$\rho\eta'$	$3eX_{\eta'}$	$2.1^{+4.7}_{-1.6}\pm 0.2$	$3.8^{+0.9}_{-0.8}\pm 0.4$
$\omega\eta'$	$(g+e)X_{\eta'}$	<17.1	$0.6^{+0.8}_{-0.3}\pm 0.6$
$\phi\eta'$	$[g(1-2s_g)-2e]Y_{\eta'}$	<12.6	$2.5^{+1.5}_{-1.1}\pm 0.4$
$K^{*0}\overline{K^0}$	$g(1-s_g)-2e$	$23.5^{+4.6}_{-3.9}\pm 3.1$	$23.5\pm 1.1\pm 3.1$
$K^{*+}K^-$	$g(1-s_g)+e$	$1.0^{+1.1}_{-0.7}\pm 0.5$	<0.6
		AP	
$b_1\pi$	$g+e$	$7.9^{+3.1}_{-2.5}\pm 1.8$	$6.3\pm 0.7\pm 1.5$

Measurement in the circumstance of inteference

The interference changes the distribution of invariant mass of the final state hadron systems. Here is an example for $\psi(3770)$.

Measurement in the circumstance of inteference

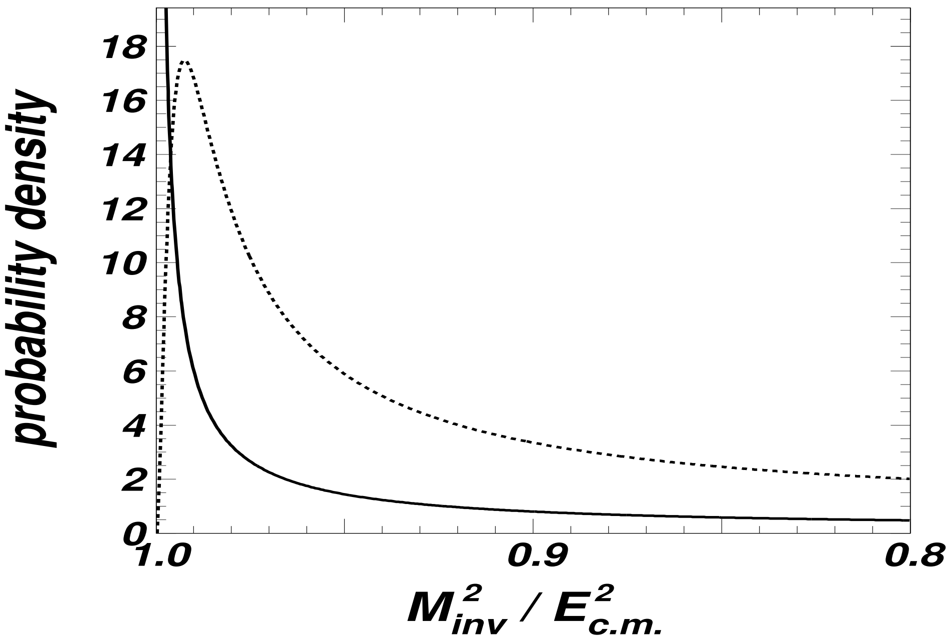


Figure 1: The distribution of the probability density as a function of M^2_{inv}/s with M_{inv} the invariant mass for a VP final state. The solid line is for the continuum process. The dashed line shows an example of destructive interference between a_{3g} and a_c with the complete cancellation of the two amplitudes. The probability is normalized to $\sigma_{r.c.}(M^2_{\psi(3770)})$ with $s_m = 0.8s$.

Measurement in the circumstance of inteference

For continuum process, as well as for the process with no interference or constructive interference between the resonance and continuum amplitudes, the maximum probability density occurs (actually diverges) at $M_{inv}^2 = s$, which corresponds to $E_\gamma \to 0$ with E_γ the energy of the emitted photons; but if there is destructive interference between the strong amplitude a_{3g} and continuum amplitude a_c which leads to almost complete cancellation of the two amplitudes, the probability density may have a minimum point near $M_{inv}^2 = s$.

Measurement in the circumstance of inteference

Quantitatively, for a detector with energy-momentum resolution $x_0/2 = 0.5\%$, among the data taken off resonance, 78% of the events have invariant mass equal to the C.M. energy of the incoming e^+e^-; while among the data taken at the $\psi(3770)$ resonance, we may find that majority of the events (85%) have invariant mass smaller than the C.M. energy of the incoming e^+e^-. If this happens, it indicates destructive interference between the resonance and continuum amplitudes.

Measurement in the circumstance of inteference

The Monte Carlo simulation requires the knowledge of the phase between strong amplitude a_{3g} and EM amplitude a_γ as well as their relative strength, so does the calculated efficiency depend on such phase.

Measurement in the circumstance of inteference

The above discussions lead to a profound feature of the experimental measurement in the presence of interference between the resonance and continuum amplitudes a_c: the complete determination of the branching fraction must come together with the determination of the phase between the resonance and continuum ϕ by scanned data around the resonance peak. The data must be taken at least at four energy points, because there are three quantities which must be determined simultaneously: $\mathcal{F}(M^2)$, $|\mathcal{C}|$ and ϕ. At the same time, the form of dependence of the observed cross section on $|\mathcal{C}|$ is quadric. Herein the continuum amplitude a_c or equivalently $\mathcal{F}(M^2)$ is determined by the data taken off the resonance and scaled for s dependence.

Measurement in the circumstance of inteference

In the treatment of the data taken around the resonance, if both strong and electromagnetic interactions exist, the Monte Carlo generator requires the input of $|\mathcal{C}|$ and ϕ, so the data analysis is an iterative process. The usual procedure is to fix ϕ, and varying $|\mathcal{C}|$, until the calculated efficiency $\epsilon(s_m)$ by the Monte Carlo and the radiatively corrected cross section $\sigma_{r.c.}(s, s_m)$ satisfy

$$\frac{N}{\mathcal{L}} = \sigma_{r.c.}(s, s_m) \cdot \epsilon(s_m). \quad (3)$$

Then the partial width or branching fraction can be obtained by the relative strength and phase between strong and EM amplitudes.

For detailed discussion, see our paper P.Wang, X.H.Mo and C.Z.Yuan, hep-ph/0512329

Measurement in the circumstance of inteference

If the data are taken at two points, i.e. one off resonance and the other on top of the resonance, only a relation between $|\mathcal{C}|$ and ϕ can be obtained. In such case, the partial width and branching fraction cannot be uniquely determined. They can only be determined within a range.

Measurement in the circumstance of inteference

The detectors of BES-III and CLEOc have a CsI(TI) calorimeter and a magnetic field of 1 Tesla or more, measure the energy-momentum with resolution of 1%, which is comparable to the ratio Γ/M for $\psi(3770)$ and $\Upsilon(5S)$ resonances. So the invariant mass of the final state hadrons in a event can be determined to the accuracy of 1%.

This makes it possible to verify the destructive interference between $\psi(3770)$ and continuum with only the data on top of $\psi(3770)$ peak.

Measurement in the circumstance of inteference

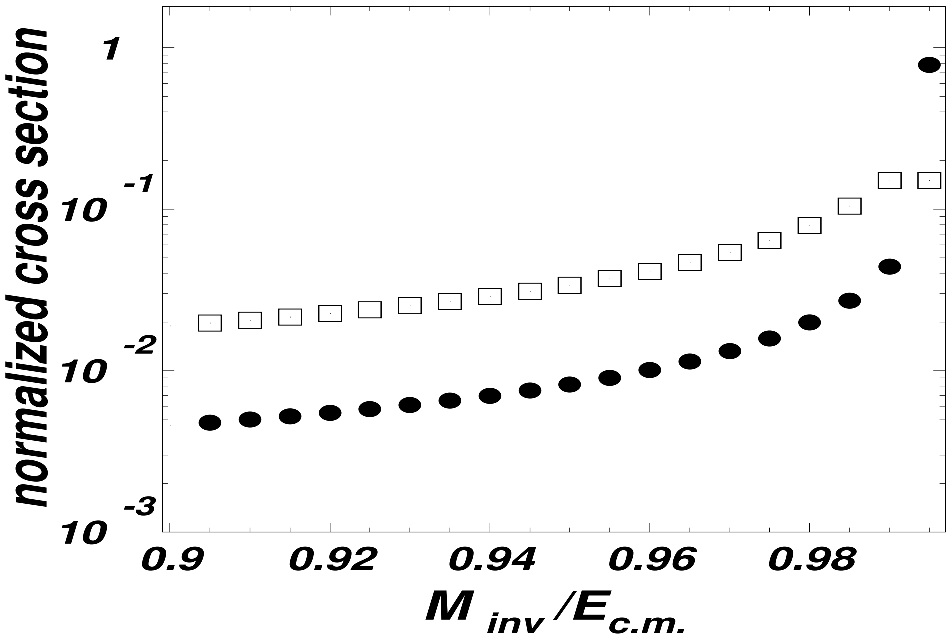


Figure 2: The interval of the invariant mass of the final state hadron system from 100% to 90% of the e^+e^- C.M. energy is divided into 20 bins, the probabilities of the hadron event in each bin are plotted for continuum process (dots) and for the process with the destructive interference between strong decay amplitude a_{3g} and continuum amplitude which leads complete cancellation of the two amplitudes.

Measurement in the circumstance of inteference

Here in the continuum process, the events are highly concentrated in the last bin with $M_{inv}/E_{c.m.} \approx 100\%$; if there is no interference or if the interference is constructive, the events are even more concentrated in the last bin; while the destructive interference could weaken such tendency significantly.

This method works for the exclusive decays of $\psi(3770)$ and $\Upsilon(5S)$.

Implication to physics

From pQCD, we would expect for any final states in $\psi(2^3S_1)$ decay,

$$\frac{\mathcal{B}(\psi(2^3S_1) \to f)}{\mathcal{B}(J/\psi \to f)} = \frac{\mathcal{B}(\psi(2^3S_1) \to e^+e^-)}{\mathcal{B}(J/\psi \to e^+e^-)} = (12.7 \pm 0.6)\%.$$

This is called the 12% rule. (It was called 14% rule, or 13%, due to the change of the experimental $\mathcal{B}(\psi(2^3S_1) \to e^+e^-)$ data.)

But the $\rho\pi$ mode defies this rule.

While $\mathcal{B}(J/\psi \to \rho\pi) = (1.27 \pm 0.09)\%$;

$\mathcal{B}(\psi(2^3S_1) \to \rho\pi) = (5.1 \pm 0.7 \pm 0.8) \times 10^{-5}$. (BES in ICHEP04)

So,

$$\frac{\mathcal{B}(\psi(2^3S_1) \to \rho\pi)}{\mathcal{B}(J/\psi \to \rho\pi)} \approx 0.4\%.$$

Also

$\mathcal{B}(J/\psi \to K^{*+}K^- + c.c.) = (5.0 \pm 0.4) \times 10^{-3}$

compared with

$\mathcal{B}(\psi(2^3S_1) \to K^{*+}K^- + c.c.) = (2.9^{+1.3}_{-1.7} \pm 0.4) \times 10^{-5}$.

So

$$\frac{\mathcal{B}(\psi(2^3S_1) \to K^{*+}K^-)}{\mathcal{B}(J/\psi \to K^{*+}K^-)} \approx 0.58\%.$$

Implication to physics

Similar supression was found for VT final states by BES.

This is called $\rho\pi$ puzzle which has been puzzling us for more than two decades.

Since both J/ψ and $\psi(2^3S_1)$ decay through three gluon and one-photon annihilations, the only difference between J/ψ and $\psi(2^3S_1)$ decays is the mass scale which does not differ very much.

$M_{J/\psi} = 3.0969$GeV and $M_{\psi(2^3S_1)} = 3.686$GeV.

One must find a reason for the strong suppression of these modes in $\psi(2^3S_1)$ decays.

Implication to physics

Rosner assumes that the pQCD rule works only between 1^3S_1 and 2^3S_1 states, i.e.

$$\frac{\langle 2^3S_1|\rho\pi\rangle}{\langle J/\psi|\rho\pi\rangle} = \frac{\langle 2^3S_1|e^+e^-\rangle}{\langle J/\psi|e^+e^-\rangle}.$$

We know that J/ψ is a pure 1^3S_1 state, but $\psi(2^3S_1)$ is not a pure 2^3S_1 state.

Implication to physics

Rosner proposed that the missing of $\rho\pi$ mode in $\psi(2^3S_1)$ decay is due to the mixing between $\psi(2^3S_1)$ and $\psi(1^3D_1)$ states:
E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. D **17**, 3090 (1978); D **21**, 313(E) (1980); D **21**, 203(E) (1980); Y.-P. Kuang and T. M. Yan, Phys. Rev. D **41**, 155 (1990)

$$|\psi(3686)\rangle = |2^3S_1\rangle \cos\theta - |1^3D_1\rangle \sin\theta \ ,$$
$$|\psi(3770)\rangle = |2^3S_1\rangle \sin\theta + |1^3D_1\rangle \cos\theta \ ,$$

where $\theta = 12^\circ$ is the mixing angle.

Implication to physics

The missing of $\rho\pi$ in $\psi(2^3S_1)$ decay is due to the cancellation of the two terms in $\langle\rho\pi|\psi(2^3S_1)\rangle$. J. L. Rosner, Phys. Rev. **D64** (2001) 094002.

$$\langle\rho\pi|\psi(3686)\rangle = \langle\rho\pi|2^3S_1\rangle\cos\theta - \langle\rho\pi|1^3D_1\rangle\sin\theta \ ,$$
$$\langle\rho\pi|\psi(3770)\rangle = \langle\rho\pi|2^3S_1\rangle\sin\theta + \langle\rho\pi|1^3D_1\rangle\cos\theta \ .$$

If

$$\langle\rho\pi|2^3S_1\rangle\cos\theta = \langle\rho\pi|1^3D_1\rangle\sin\theta,$$

then $\langle\rho\pi|\psi(2^3S_1)\rangle = 0$;
while

$$\langle\rho\pi|\psi(3770)\rangle = \langle\rho\pi|2^3S_1\rangle/\sin\theta.$$

Implication to physics

So the missing mode of $\rho\pi$ in $\psi(3686)$ decays shows up in $\psi(3770)$ decays.

This scenario is simple, and with the $\langle\rho\pi|2^3S_1\rangle$ calculated from

$$\frac{\langle 2^3S_1|\rho\pi\rangle}{\langle J/\psi|\rho\pi\rangle} = \frac{\langle 2^3S_1|e^+e^-\rangle}{\langle J/\psi|e^+e^-\rangle},$$

it predicts with little uncertainty that

$$\mathcal{B}_{\psi(3770)\to\rho\pi} = (6.8 \pm 2.3) \times 10^{-4} \quad .$$

If measured in e^+e^- experiments,

$$\sigma^{Born}_{e^+e^-\to\psi(3770)\to\rho\pi} = (7.9 \pm 2.7)\text{pb} \tag{4}$$

Implication to physics

Mode	Amplitude	σ(3.671 GeV) [pb]	σ(3.773 GeV) [pb]
		VP	
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$\phi\pi^0$	0	< 2.2	< 0.2
$\rho\eta$	$3eX_\eta$	$10.0^{+2.2}_{-1.9}\pm 1.0$	$10.3\pm 0.5\pm 1.0$
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		AP	
$b_1\pi$	$g+e$	$7.9^{+3.1}_{-2.5}\pm 1.8$	$6.3\pm 0.7\pm 1.5$

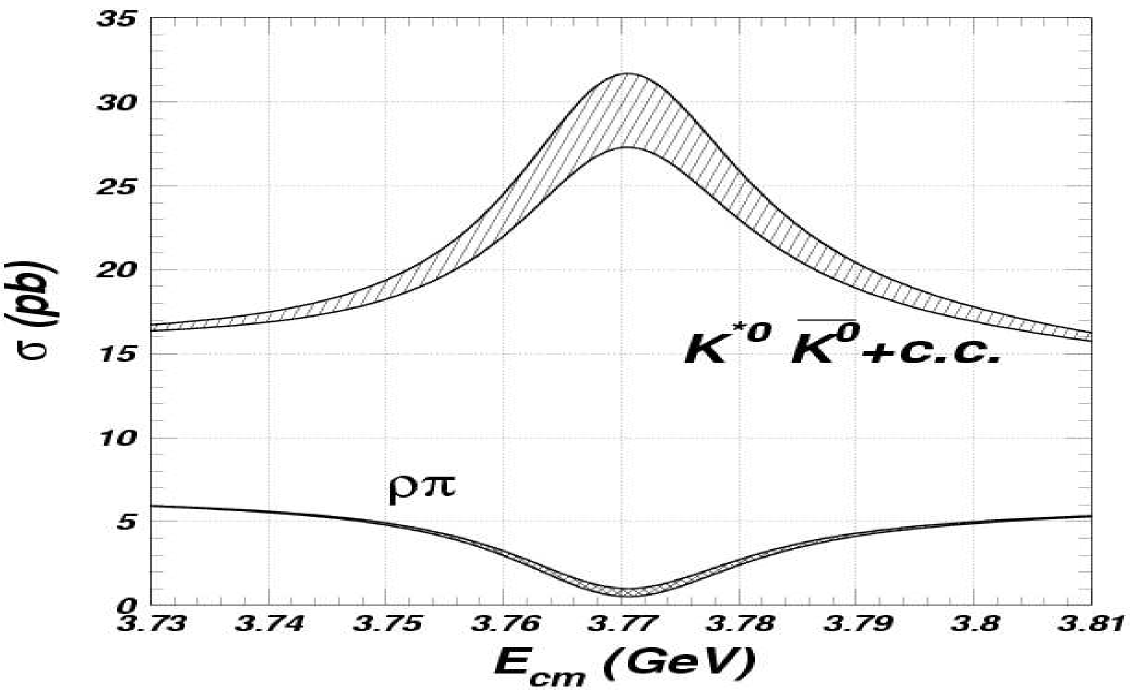
Implication to physics

This means (P. Wang, C. Z. Yuan and X. H. Mo, Phys. Lett. **B574**, 41 (2003))

- There must be destructive interference between resonance and continum, i.e. the phase between the strong and EM amplitudes is around -90°.
- $10^{-6} < \mathcal{B}(\psi(3770) \to \rho\pi) < 10^{-3}$, Rosner's prediction is in the range.

Implication to physics

If we scan $\psi(3770)$, we shall find the cross sections of $e^+e^- \to \rho\pi$ and $e^+e^- \to K^{*0}\overline{K^0} + c.c.$ versus energy like the curves



In the figure, the shaded area is due to an unknown phase between the 2^3S_1 and 1^3D_1 matrix elements.

The $K^{*+}K^- + c.c.$ cross section is similar to $\rho\pi$.

Implication to physics

It is known that in J/ψ decays, the three gluon amplitude a_{3g} and one-photon amplitude a_γ are orthogonal for

- 1^+0^- 90° M. Suzuki, Phys. Rev. **D63**, 054021 (2001)
- 1^-0^- $(106 \pm 10)^\circ$ J. Jousset *et al.*, Phys. Rev. **D41**, 1389 (1990); D. Coffman *et al.*, Phys. Rev. **D38**, 2695 (1988); J. Jousset *et al.*, *Phys. Rev.* D **41**, 1389 (1990); A. Bramon, R. Escribano and M. D. Scadron, *Phys. Lett.* B **403**, 339 (1997); M. Suzuki, *Phys. Rev.* D **58**, 111504 (1998); N.N.Achasov, Talk at Hadron2001; G. López Castro *et al.*, in CAM-94, Cancum, Mexico.
- 1^-1^- $(138 \pm 37)^\circ$ L. Köpke and N. Wermes, Phys. Rep. **174**, 67 (1989).
- 0^-0^- $(89.6 \pm 9.9)^\circ$ M. Suzuki, Phys. Rev. **D60**, 051501(1999); G. López Castro *et al.*, ibid; L. Köpke and N. Wermes, ibid.
- $N\overline{N}$ $(89 \pm 15)^\circ$ R. Baldini, *et al.* Phys. Lett.**B444**, 111 (1998); G. López Castro *et al.*, ibid.

J. M. Gérard and J. Weyers Phys. Lett. **B462**, 324 (1999) argued that this large phase follows from the orthogonality of three-gluon and one-photon virtual processes.

Is this phase universal for quarkonium decays?

How about

- $\psi(2^3S_1)$
- $\psi(3770)$
- $\Upsilon(nS)$

Implication to physics

We have found that the interference pattern explains the small signal of $\rho\pi$ and $K^{*+}K^-$ but large signal of $K^{*0}\overline{K^0}$ observed by BES and CLEOc at $\psi(2^3S_1)$ BES and CLEOc report at ICHEP04.

We have suggested that in $\psi(2^3S_1) \to VP$ decays, the strong and EM amplitudes are still orthogonal and the sign of the phase must be negative. That is, the phase is $-90°$. P. Wang, C. Z. Yuan, X. H. Mo, Phys. Rev. **D** 69; 057502 (2004)

In the e^+e^- experiments, becasue there is the virtual photon ampltude, we can determine the sign of this phase.

Implication to physics

For $\psi(2^3S_1) \to 0^-0^-$ decays, CLEOc gives

Table 1: The branching fractions of $\psi(2^3S_1)$ decay into $\pi^+\pi^-$, K^+K^- and $K_S^0K_L^0$ by CLEOc. Also listed is the parametrization of $\psi(3770)$ decays into PP final states. These amplitudes are parametrized in terms of electromagnetic amplitude E and strong amplitude $\mathcal{M}$ which breaks SU(3) symmetry. Similar parametrization can be applied to VV final states.

final state	BR(10^{-5})	amplitude
$\pi^+\pi^-$	$0.8 \pm 0.8 \pm 0.2$	E
	$\leq$ 2.1 (90%C.L.)	
K^+K^-	$6.3 \pm 0.6 \pm 0.3$	$E + \frac{\sqrt{3}}{2}\mathcal{M}$
$K_S^0K_L^0$	$(5.8 \pm 0.8 \pm 0.4$	$\frac{\sqrt{3}}{2}\mathcal{M}$

CLEOc extracts the relative phase between strong interaction $\mathcal{M}$ and EM interaction E to be $(95 \pm 15)^\circ$. S.Dobbs etc, hep-ph/0603020

Implication to physics

Combined with the data of $\psi(3770)$ to VP decays, there is increased evidence to suport our suggestion that this phase beteen strong and EM interaction is universal.

We need to measure this phase in Υ decays

Implication to physics

Another implication is that if Rosner's scenario is correcct, then the matrix element $\langle \rho\pi | 1^3D_1 \rangle$ would be much larger than $\langle \rho\pi | 2^3S_1 \rangle$. This would defy our understanding of charmonium (quarkonium) decays.

Summery

We have shown that the continuum amplitude must be taken into account in the analysis of e^+e^- experiments. This interference could be inportant, and it could complicate our data analysis. For high precision measurement, we need to develop more sophisticated method to treat the experimental data. This could lead to new physics beyond standard model.

Discussions

In the next a few years, when most of us in the high energy physics community will move into one experiment LHC, and see if we are fortunate to find some thing new, we should ask:

Have we worked hard enough to explore new physics from exisiting experiments?

The discovery of X, Y, Z by BELLE, BaBar, CLEO and BES has shown that there is the chance to find new physics from existing experimental data.

The possible universal phase between strong and EM interactions, if confirmed by experimental data, could be a new empirical law in physics. Is it the physics beyond the standard model?

We should keep searching new physics from exisitng experiments, like BES, CLEO, BELLE, BaBar and the ϕ factories at INFN and BINP.