Supersymmetry and Fermion Masses

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Abstract

[1] Phys. Lett. B 609 (2005) 111 (hep-ph/0501129);

[2] hep-ph/0507298.

If SUSY is not for stabilizing EW energy scale,

what is this beautiful mathematical physics used for in reality?

SUSY for flavors

Introduction

Originally, SUSY — for EW scale (Naturalness)

However, cosmological constant with 10^{120} fine tuning might be just so (Anthropic principle)

$\Downarrow$

No Naturalness ? $\Longrightarrow$ ♠ Standard Model

$\Downarrow$

No SUSY ? $\Longrightarrow$ ♠ Split SUSY

and

♠ SUSY for flavors

Flavor puzzle:

fermion masses, mixing & $\not{C}P$

Observation of the masses:

$$3rd \gg 2nd \gg 1st$$

$$\Downarrow$$

family symmetry: Z_{3L} of the $SU(2)_L$ doublets

$$L_1, Q_1 \to L_2, Q_2 \to L_3, Q_3 \to L_1, Q_1$$

$$\Downarrow$$

$$m_\tau \neq 0,\ m_t \neq 0,\ m_b \neq 0 \text{ only}$$

How Z_{3L} breaks? For the leptons, we have noted:

sneutrino VEV $v_i \neq 0$**,** $LLE^c \Longrightarrow$ **charged lepton masses.** (D.-S. Du and C.L., 1993)

But, $m_\nu \sim \dfrac{(g_2 v_i)^2}{M_{\tilde{Z}}} \sim 100$ MeV – too large!

We will make $M_{\tilde{Z}}$ large.

Model

Z_{3L} symmetry. It breaks in soft terms.

$$\mathcal{L} \supset \Big(H_1^\dagger H_1 + H_2^\dagger H_2 + \alpha L_i^\dagger L_i + \beta (L_1^\dagger L_2 + L_2^\dagger L_3 + L_3^\dagger L_1 + h.c.) + \frac{\gamma}{\sqrt{3}} (H_2^\dagger \sum_i L_i + h.c.) \Big) \Big|_{\theta\theta\bar{\theta}\bar{\theta}} , \tag{1}$$

$$\mathcal{W} = \frac{\tilde{y}_j}{\sqrt{3}} (\textstyle\sum_i L_i) H_2 E_j^c + \tilde{\lambda}_j (L_1 L_2 + L_2 L_3 + L_3 L_1) E_j^c + \tilde{\mu} H_1 H_2 + \tilde{\mu}' H_1 \textstyle\sum_i L_i , \tag{2}$$

$$\mathcal{L}_{soft1} = M_{\tilde{W}} \tilde{W} \tilde{W} + M_{\tilde{Z}} \tilde{Z} \tilde{Z} + m_h^2 h_1^\dagger h_1 + m_h^2 h_2^\dagger h_2 + m_{lL_{ij}}^2 \tilde{l}_i^\dagger \tilde{l}_j + m_{lR_{ij}}^2 \tilde{e}_i^* \tilde{e}_j + (B_{\tilde{\mu}} h_1 h_2 + B_{\tilde{\mu}_i} h_1 \tilde{l}_i + m_i'^2 h_2^\dagger \tilde{l}_i + h.c.) , \tag{3}$$

$$\mathcal{L}_{soft2} = \tilde{m}_{ij} \tilde{l}_i h_2 \tilde{e}_j + \tilde{m}_{ijk} \tilde{l}_i \tilde{l}_j \tilde{e}_k + h.c. . \tag{4}$$

Kinetic terms normalized,

$$\mathcal{L} \supset H_u^\dagger H_u + H_d'^\dagger H_d' + L_e^\dagger L_e + L_\mu^\dagger L_\mu + L_\tau'^\dagger L_\tau' \qquad (5)$$

by

$$\begin{aligned}
H_u &= H_1 \\
H_d' &= c_1 \left(H_2 + \frac{c_2}{\sqrt{3}} \sum_i L_i \right) \\
L_\tau' &= c_1' \left(H_2 - \frac{c_2}{\sqrt{3}} \sum_i L_i \right) \\
L_\mu &= \frac{c_3}{\sqrt{2}} (L_1 - L_2) \cos\theta + \frac{c_3}{\sqrt{6}} (L_1 + L_2 - 2L_3) \sin\theta \\
L_e &= -\frac{c_3}{\sqrt{2}} (L_1 - L_2) \sin\theta + \frac{c_3}{\sqrt{6}} (L_1 + L_2 - 2L_3) \cos\theta \,.
\end{aligned} \qquad (6)$$

The superpotential becomes

$$\begin{aligned}
\mathcal{W} = &-\sqrt{\textstyle\sum_j |y_j|^2} H_d L_\tau E_\tau^c + L_e L_\mu (\lambda_\tau E_\tau^c + \lambda_\mu E_\mu^c) \\
&+ \sqrt{\mu^2 + \mu'^2} H_u H_d \,.
\end{aligned} \qquad (7)$$

$H_d \rightarrow$ tau mass only
sneutrinos in L_e and $L_\mu \rightarrow$ muon mass
electron massless.

Assuming soft SUSY terms break Z_{3L}

$$\mathcal{L}_{soft} = M_{\tilde{W}}\tilde{W}\tilde{W} + M_{\tilde{Z}}\tilde{Z}\tilde{Z}$$
$$+m_{h_u}^2 h_u^\dagger h_u + m_{h_d}^2 h_d^\dagger h_d + m_{h_d}^2 \tilde{l}_\alpha^\dagger \tilde{l}_\alpha$$
$$+m_{lR_{\alpha\beta}}^2 \tilde{e}_\alpha^* \tilde{e}_\beta + (B_\mu h_u h_d + B_{\mu_e} h_u \tilde{l}_e$$
$$+\tilde{m}_{\alpha\beta}\tilde{l}_\alpha h_d \tilde{e}_\beta + \tilde{m}_{\alpha\beta\gamma}\tilde{l}_\alpha \tilde{l}_\beta \tilde{e}_\gamma + h.c.)\,, \quad (8)$$

where $\alpha = e, \mu, \tau$.

The key point: the scalar mass-squared matrix

$$\mathcal{M}^{(h_u, h_d^\dagger, \tilde{l}_\alpha^\dagger)} = \begin{pmatrix} m_{h_u}^2 & B_\mu & B_{\mu_e} & B_{\mu_\mu} & B_{\mu_\tau} \\ B_\mu & m_{h_d}^2 & 0 & 0 & 0 \\ B_{\mu_e} & 0 & m_{h_d}^2 & 0 & 0 \\ B_{\mu_\mu} & 0 & 0 & m_{h_d}^2 & 0 \\ B_{\mu_\tau} & 0 & 0 & 0 & m_{h_d}^2 \end{pmatrix} \tag{9}$$

Eigenvalues:

$$\begin{aligned} M_1^2 &= \bar{m} - \sqrt{\Delta^2 + (B_\mu)^2 + \textstyle\sum_\alpha (B_{\mu_\alpha})^2} \\ M_2^2 &= \bar{m} + \sqrt{\Delta^2 + (B_\mu)^2 + \textstyle\sum_\alpha (B_{\mu_\alpha})^2} \\ M_3^2 &= M_4^2 = M_5^2 = m_{h_d}^2 , \end{aligned} \tag{10}$$

where $\bar{m} = \frac{m_{h_u}^2 + m_{h_d}^2}{2}$, $\Delta = \frac{m_{h_u}^2 - m_{h_d}^2}{2}$.

By fine-tuning, $M_1^2 \sim -m_{EW}^2$, namely the EW symmetry breaking is achieved. The tuning is at the order of m_S^2/m_{EW}^2.

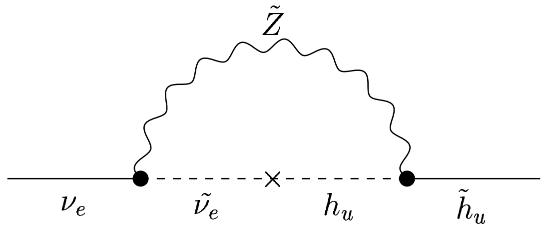
In addition to the Higgs doublets, $\tilde{l}_\alpha$ field also gets a VEV,

$$v_u \neq 0\,, \quad v_d \neq 0\,, \quad v_{l_\alpha} \neq 0 \quad (\alpha = e,\ \mu,\ \tau)\,. \tag{11}$$

The hierarchical charged lepton mass pattern is obtained,

$$\begin{aligned} m_\tau &\simeq y_\tau v_d + \frac{\lambda_\tau^2}{2y_\tau}\frac{v_{l_e}^2 + v_{l_\mu}^2}{v_d}\,, \\ m_\mu &\simeq \lambda_\mu \sqrt{v_{l_e}^2 + v_{l_\mu}^2}\,, \\ m_e &= 0\,. \end{aligned} \tag{12}$$

Numerically $v_d \sim 10$ GeV and $v_{l_\alpha} \sim 1$ GeV.



Is a large v_{l_e} safe? In addition, a huge B_{μ_e} induces a large lepton-Higgsino mixing at the loop-level, $m_{eh} = \dfrac{g_2^2 B_{\mu_e}}{16\pi^2 M_{\tilde{Z}}}$ which is about $10^{-3} m_S$. Neutralino mass matrix,

$$\begin{pmatrix}\nu_e \; \nu_\mu \; \nu_\tau \; \tilde{h}_d^0 \; \tilde{h}_u^0 \; \tilde{Z}\end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & m_{eh} & a v_{l_e} \\ 0 & 0 & 0 & 0 & m_{\mu h} & a v_{l_\mu} \\ 0 & 0 & 0 & 0 & m_{\tau h} & a v_{l_\tau} \\ 0 & 0 & 0 & 0 & -\bar{\mu} & a v_d \\ m_{eh} & m_{\mu h} & m_{\tau h} & -\bar{\mu} & 0 & -a v_u \\ a v_{l_e} & a v_{l_\mu} & a v_{l_\tau} & a v_d & -a v_u & M_{\tilde{Z}} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \tilde{h}_d^0 \\ \tilde{h}_u^0 \\ \tilde{Z} \end{pmatrix}, \tag{13}$$

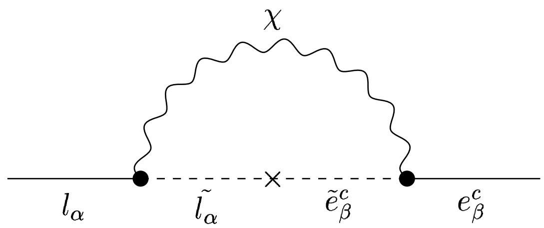
where $a = (\dfrac{g_2^2 + g_1^2}{2})^{1/2}$. Eigenvalues by taking $v_{l_\alpha} < v_d < v_u \ll \sqrt{\mu^2 + \mu'^2}, M_{\tilde{Z}}$:

$$\Lambda_1 \simeq M_{\tilde{Z}}\,, \quad \Lambda_2 \simeq \bar{\mu}\,, \quad \Lambda_3 \simeq -\bar{\mu}\,. \tag{14}$$

The three light neutrinos, a realization of the see-saw mechanism with the heavy higgsinos and gauginos playing the role of the right-handed neutrinos, the light Majorana neutrino mass matrix is

$$
\begin{aligned}
m^{\nu} &\simeq -m_{\rm Dirac} M_R^{-1} m_{\rm Dirac}^T\,, \\
&= -\frac{a^2}{M_{\tilde{Z}}} \begin{pmatrix} v_{l_e}v_{l_e} & v_{l_e}v_{l_\mu} & v_{l_e}v_{l_\tau} \\ v_{l_\mu}v_{l_e} & v_{l_\mu}v_{l_\mu} & v_{l_\mu}v_{l_\tau} \\ v_{l_\tau}v_{l_e} & v_{l_\tau}v_{l_\mu} & v_{l_\tau}v_{l_\tau} \end{pmatrix}\,.
\end{aligned}
\qquad (15)
$$

Democratic mass matrix for neutrinos, naturally large neutrino mixing. The nonvanishing mass is $m_{\nu_3} = \frac{a^2}{M_{\tilde{Z}}} v_{l_\alpha} v_{l_\alpha} \sim 10^{-1} - 10^{-2}$ eV when $M_{\tilde{Z}} \sim 10^{11} - 10^{12}$ GeV.



Electron mass:

$$\delta M^l_{\alpha\beta} \simeq \frac{\alpha}{\pi}\frac{\sqrt{\sum_j |y_j|^2}\tilde{m_S} v_d}{m_S}\,. \qquad (16)$$

Taking $\tilde{m_S}/m_S \simeq 0.1$, $\delta M^l_{\alpha\beta} \sim \mathcal{O}(\mathrm{MeV})$.

Neutrino oscillation

A SM singlet superfield N introduced,

$$\mathcal{W} \supset \kappa_\tau H_u L_\tau \bar{N} + \tilde{M} \bar{N} \bar{N} + \kappa_d H_u H_d \bar{N} + \tilde{\kappa}_3 \bar{N}^3 \,, \tag{17}$$

Full neutrino mass matrix is

$$\mathcal{M}^\nu = -\frac{a^2}{M_{\tilde{Z}}} \begin{pmatrix} v_{l_e} v_{l_e} & v_{l_e} v_{l_\mu} & v_{l_e} v_{l_\tau} \\ v_{l_\mu} v_{l_e} & v_{l_\mu} v_{l_\mu} & v_{l_\mu} v_{l_\tau} \\ v_{l_\tau} v_{l_e} & v_{l_\tau} v_{l_\mu} & v_{l_\tau} v_{l_\tau} + x \end{pmatrix} \tag{18}$$

with x being $\frac{M_{\tilde{Z}}}{\tilde{M}} \left(\frac{\kappa_\tau v_u}{a} \right)^2$.

Eigen values are

$$\begin{aligned} m_{\nu_3} &\simeq \frac{a^2}{M_{\tilde{Z}}} v_{l_\tau}^2 + \frac{(\kappa_\tau v_u)^2}{\tilde{M}} \,, \\ m_{\nu_2} &\simeq \frac{a^2}{M_{\tilde{Z}}} (v_{l_e}^2 + v_{l_\mu}^2) \frac{x}{x + v_{l_\tau}^2} \,, \\ m_{\nu_1} &= 0 \,. \end{aligned} \tag{19}$$

Solar neutrino problem requires: $m_{\nu_2} \simeq (10^{-2} - 10^{-3})$ eV which is achieved when $M_{\tilde{Z}} \sim 10^{13}$ GeV. Atmospheric neutrino problem requires a certain cancellation between the terms $\frac{a^2}{M_{\tilde{Z}}} v_{l_\tau}^2$ and $\frac{(\kappa_\tau v_u)^2}{\tilde{M}}$, in order to make $m_{\nu_3} \sim 10^{-1} - 10^{-2}$.

Lepton mixings:

$$|V_{e2}| = \frac{v_{l_\mu}^2 - v_{l_e}^2}{v_{l_e}^2 + v_{l_\mu}^2} \simeq O(1)\,. \tag{20}$$

$$|V_{\mu 3}| \simeq \frac{|\lambda_\tau| \sqrt{v_{l_e}^2 + v_{l_\mu}^2}}{\sqrt{y_\tau^2 v_d^2 + |\lambda_\tau|^2 (v_{l_e}^2 + v_{l_\mu}^2)}}\,. \tag{21}$$

A maximal mixing can be approached.

$$|V_{e3}| \simeq \frac{v_{l_\mu}^2 - v_{l_e}^2}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2}\, v_\tau}\,. \tag{22}$$

It is small ~ 0.1 if $\sqrt{v_{l_e}^2 + v_{l_\mu}^2}/v_\tau \sim 0.1$.

Quark masses also have three origins:

Higgs VEVs, sneutrino VEV, soft trilinear Z_{3L} *violating terms*

However, the roles of the sneutrino VEVs and the soft trilinear terms are switched. The sneutrino VEVs contribute to the first generation quark mass, and the soft trilinear Z_{3L} violating terms to the charm and strange quark masses.

The hierarchy between the second and first generation is not automatic.

Need a special structure of the soft breaking terms of the squarks.

$m_u < m_d$ can be understood.

Higgs mass $\simeq 145 \pm 7$ GeV.

Summary

SUSY is for flavor problems, which breaks at a high scale $\sim 10^{13}$ GeV. The electroweak energy scale is unnaturally small.

A family symmetry Z_{3L} which is broken by soft terms.

Under the family symmetry, only the third generation charged fermions get their masses. The τ mass is from the Higgs vacuum expectation value (VEV), the muon mass is due to the sneutrino VEVs, and the electron gains its mass via quantum corrections. The large neutrino mixing is naturally produced with neutralinos playing the role of right-handed neutrinos. For the quarks, the third generation masses are from the Higgs VEVs, the second generation masses are from quantum corrections, and the down quark mass due to the sneutrino VEVs.

Higgs mass is about 145 GeV.

θ_{13} **should be larger than** 0.1**.**