

# Electroweak Precision Tests and Physics Beyond the Standard Model II

*Topical Seminar on Frontier of Particle Physics 2006:  
Beyond the Standard Model, Beijing*

Masaharu Tanabashi (Tohoku U.)

*August 7, 2006*

## Plan of this talk

§.0 Power of precision

§.1 Weinberg-Salam model at tree level

§.2 Decoupling theorem and its violation in EW physics

§.3  $S$ - $T$  fit

§.4 Higgs mass

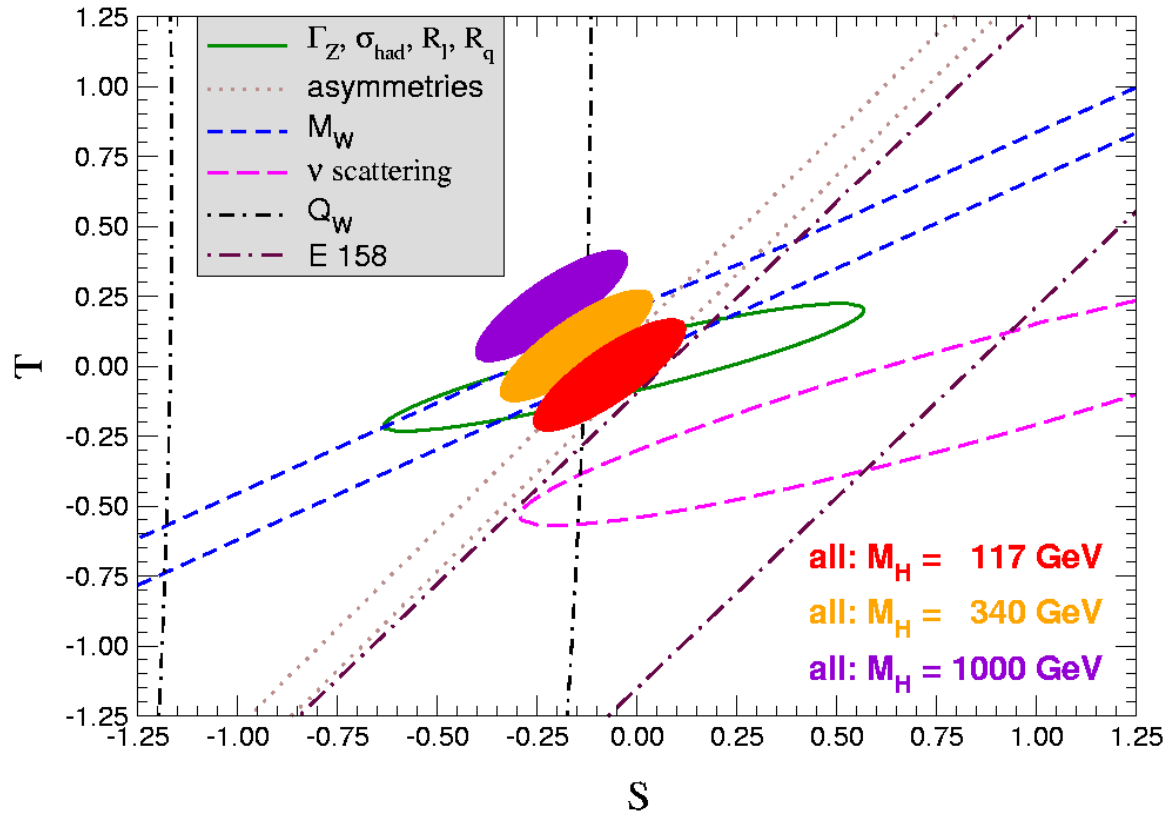
§.5 Estimate of  $S$  in technicolor models

§.6 Electroweak chiral perturbation theory

§.7 “Universal” non-oblique corrections

§.8 Summary

# S-T plot of Erler-Langacker review in RPP2006



90%CL

Erler and Langacker, in RPP2006

$$\begin{aligned}
 S &= -0.13 \pm 0.10 \\
 T &= -0.13 \pm 0.11 \\
 U &= 0.20 \pm 0.12
 \end{aligned}$$

for  $M_H = 117\text{GeV}$

$$\begin{aligned}
 S &= -0.21 \pm 0.10 \\
 T &= -0.04 \pm 0.11 \\
 U &= 0.21 \pm 0.12
 \end{aligned}$$

for  $M_H = 300\text{GeV}$

## §.5 Estimate of $S$ in technicolor models

### Dynamical chiral symmetry breaking in QCD

Chiral  $SU(2)_L \times SU(2)_R$  symmetry in 2-flavor massless QCD:

$$q_L \rightarrow g_L q_L, \quad q_R \rightarrow g_R q_R \quad g_{L,R} \in SU(2)_{L,R}, \quad q_{L,R} \equiv \begin{pmatrix} u_{L,R} \\ d_{L,R} \end{pmatrix}.$$

The  $\bar{q}q$  condensate is caused by non-perturbative QCD dynamics and breaks the chiral symmetry into diagonal  $SU(2)_V$ :

$$\langle \bar{q}q \rangle \neq 0, \quad SU(2)_L \times SU(2)_R \rightarrow SU(2)_V,$$

which looks similar to the EW symmetry breaking with custodial symmetry.

### Technicolor models

Attempts applying this mechanism for the EW symmetry breaking.

## The model

We consider  $SU(N_{\text{TC}})$  technicolor with one-doublet techni-fermion:

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \quad U_R, \quad D_R,$$

weak hypercharge:

$$Y_{Q_L} = 0, \quad Y_{U_R} = \frac{1}{2}, \quad Y_{D_R} = -\frac{1}{2}.$$

The  $SU(N_{\text{TC}})$  gauge coupling is assumed to become strong and causes TeV scale condensate:

$$\langle \bar{U}U \rangle = \langle \bar{D}D \rangle \sim (\text{TeV})^3 \neq 0.$$

Nambu-Goldstone bosons are eaten by  $W^\pm$  and  $Z$ . The mass of techni- $\rho$  ( $J^{PC} = 1^{--}$ ) can be estimated as

$$M_{T\rho} = \sqrt{\frac{3}{N_{\text{TC}}} \frac{v}{f_\pi} m_\rho} \simeq \sqrt{\frac{3}{N_{\text{TC}}}} \times 2 \text{ TeV}.$$

## Estimate of $S$

Definition:

$$\begin{aligned}
 S &= 16\pi \left( \tilde{\Pi}_{33}(0) - \tilde{\Pi}_{3Q}(0) \right) = 16\pi \frac{d}{dk^2} \left( \Pi_{33}(k^2) - \Pi_{3Q}(k^2) \right) \Big|_{k^2=0} \\
 &= -16\pi \frac{d}{dk^2} \Pi_{3Y}(k^2) \Big|_{k^2=0} = -16\pi \frac{d}{dk^2} \Pi_{L^3 R^3}(k^2) \Big|_{k^2=0} \\
 &= -4\pi \frac{d}{dk^2} \left( \Pi_{V^3 V^3}(k^2) - \Pi_{A^3 A^3}(k^2) \right) \Big|_{k^2=0},
 \end{aligned}$$

where

$$\Pi(k^2) = \Pi(0) + k^2 \tilde{\Pi}(k^2), \quad Q = I_3 + Y.$$

## Challenge

How can we calculate

$$\Pi(k^2) = \text{diagram}$$


reliably including *non-perturbative* technicolor correction?

Method working reasonably well in QCD hadron dynamics

## Vector Meson Dominance (VMD)

$$\Pi_{A^3 A^3}^{\mu\nu}(k) = \left( g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) \Pi_{A^3 A^3}(k^2)$$

Nambu-Goldstone boson and the axial-vector ( $J^{PC} = 1^{++}$ ) boson contribute to  $\Pi_{AA}$ :

$$\Pi_{A^3 A^3}(k^2) = v^2 + \frac{k^2}{M_{A_1}^2} \frac{g_{A_1}^2}{k^2 - M_{A_1}^2}, \quad v \simeq 246\text{GeV}.$$

The vector ( $J^{PC} = 1^{--}$ ) boson contribution to  $\Pi_{VV}$ :

$$\Pi_{V^3 V^3}(k^2) = \frac{k^2}{M_{V_1}^2} \frac{g_{V_1}^2}{k^2 - M_{V_1}^2}.$$

## Weinberg Sum Rules

We know

$$\Pi_{V^3V^3}(k^2) - \Pi_{A^3A^3}(k^2) = 4\Pi_{L^3R^3}(k^2) \propto \frac{(\langle\bar{Q}Q\rangle)^2}{k^4},$$

which implies

$$v^2 + \frac{g_{A1}^2}{M_{A1}^2} = \frac{g_{V1}^2}{M_{V1}^2}, \quad g_{A1}^2 = g_{V1}^2,$$

$$\Pi_{V^3V^3}(k^2) - \Pi_{A^3A^3}(k^2) = -\frac{v^2 M_{V1}^2 M_{A1}^2}{[M_{V1}^2 - k^2][M_{A1}^2 - k^2]},$$

under the VMD assumption.

$$S \simeq 4\pi \left( \frac{v^2}{M_{V1}^2} + \frac{v^2}{M_{A1}^2} \right) \simeq \frac{N_{TC}}{3} \times 0.25.$$



We include chiral logarithm correction so as to make the prediction more precise:

$$S \simeq 4\pi \left( \frac{v^2}{M_{V1}^2} + \frac{v^2}{M_{A1}^2} \right) - \frac{1}{6\pi} \ln \frac{M_{H,\text{ref}}}{M_{V1}}.$$

For  $N_{\text{TC}} = 3$ ,  $M_{H,\text{ref}} = 300\text{GeV}$ , we find

$$S \simeq 0.35,$$

which is ruled out by the present limit in the  $S$ - $T$  plane.

**Note:**

- The VMD estimate  $S \simeq 0.25$  is significantly larger than the one-loop free fermion estimate  $S = 3/(6\pi) \simeq 0.16$ . Non-perturbative technicolor correction is important.
- We don't have reliable way to estimate  $S$  in walking TC models. Naive conjecture: somewhere between 0.25 and 0.16?

Harada-Kurachi-Yamawaki  
Kurachi-Shrock

## §.6 Electroweak chiral perturbation theory

### Catalogue of *non-decoupling* corrections

#### Use of the gauged non-linear $\sigma$ model

T. Appelquist and C. Bernard, PRD22 (1980) 200.

Taking the  $M_H \rightarrow \infty$  limit, the original linear  $\sigma$  model Higgs field  $\Phi$  is replaced by the non-linear  $\sigma$  model field  $U$ :

$$\Phi = \sqrt{2} \begin{pmatrix} \varphi_0^* & \varphi_+ \\ -\varphi_+^* & \varphi_0 \end{pmatrix} \rightarrow vU = v \exp \left( \frac{i\tau^a w^a}{v} \right),$$

with  $w^a$  being the NG bosons eaten by  $W^\pm$ ,  $Z$ .

In this limit, the original Higgs Lagrangian is replaced by

$$\mathcal{L}_0 = \frac{1}{4} v^2 \text{tr}[(D_\mu U)^\dagger (D^\mu U)] - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu},$$

where

$$D_\mu U = \partial_\mu U + ig_W \frac{\tau_a}{2} W_\mu^a U - ig_Y U \frac{\tau_3}{2} B_\mu.$$

## Idea of Appelquist-Bernard paper

*We can catalogue all possible non-decouplings effects as coefficients of operators upto dimension 4.*

A. Longhitano, PRD22 (1980) 1166 ; NPB188 (1981) 118.  
T. Appelquist and G.-H. Wu, PRD48 (1993) 3235.

List of CP even dimension 4 operators.

$$\begin{aligned}
 \mathcal{L}_1 &\equiv \frac{1}{2}\alpha_1 g_W g_Y B_{\mu\nu} \text{tr}(TW^{\mu\nu}), & \mathcal{L}_6 &\equiv \alpha_6 \text{tr}(V_\mu V_\nu) \text{tr}(TV^\mu) \text{tr}(TV^\nu), \\
 \mathcal{L}_2 &\equiv i\alpha_2 g_Y B_{\mu\nu} \text{tr}(T[V^\mu, V^\nu]), & \mathcal{L}_7 &\equiv \alpha_7 \text{tr}(V_\mu V^\mu) \text{tr}(TV_\nu) \text{tr}(TV^\nu), \\
 \mathcal{L}_3 &\equiv i\alpha_3 g_W \text{tr}(W_{\mu\nu} [V^\mu, V^\nu]), & \mathcal{L}_8 &\equiv \frac{1}{4}\alpha_8 g_W^2 [\text{tr}(TW_{\mu\nu})]^2, \\
 \mathcal{L}_4 &\equiv \alpha_4 [\text{tr}(V_\mu V_\nu)]^2, & \mathcal{L}_9 &\equiv \frac{1}{2}i\alpha_9 g_W \text{tr}(TW_{\mu\nu}) \text{tr}(T[V^\mu, V^\nu]), \\
 \mathcal{L}_5 &\equiv \alpha_5 [\text{tr}(V_\mu V^\mu)]^2, & \mathcal{L}_{10} &\equiv \frac{1}{2}\alpha_{10} [\text{tr}(TV_\mu) \text{tr}(TV_\nu)]^2, \\
 & & \mathcal{L}_{11} &\equiv \alpha_{11} g \epsilon^{\mu\nu\rho\lambda} \text{tr}(TV_\mu) \text{tr}(V_\nu W_{\rho\lambda}).
 \end{aligned}$$

where

$$T \equiv U\tau_3 U^\dagger, \quad V_\mu \equiv (D_\mu U)U^\dagger, \quad \dim(\alpha_i) = 0.$$

One additional dimension 2 operator

$$\mathcal{L}'_1 \equiv \frac{1}{4} \beta_1 v^2 [\text{tr}(TV_\mu)]^2,$$

which violates custodial  $SU(2)_C$  symmetry even in the  $g_Y = 0$  limit.

	$\beta_1$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	$\alpha_6$	$\alpha_7$	$\alpha_8$	$\alpha_9$	$\alpha_{10}$	$\alpha_{11}$
$SU(2)_C$	×	○	○	○	○	○	×	×	×	×	×	×
#(gauge)	2	2	3	3	4	4	4	4	2	3	4	3

- From custodial symmetry, it is expected

$$\alpha_{1,2,3,4,5} \gg \alpha_{6,7,8,9,10,11}.$$

- $\beta_1$ ,  $\alpha_1$  and  $\alpha_8$  contribute to the electroweak oblique correction:

$$S = -16\pi\alpha_1, \quad \alpha T = 2\beta_1, \quad U = -16\pi\alpha_8.$$

They are constrained severely from the electroweak precision tests.

- $\alpha_2, \alpha_3, \alpha_9$  and  $\alpha_{11}$  contribute to the triple-gauge-vertices:

$$\begin{aligned}
\mathcal{L}_{\text{TGV}} = & -ie \frac{c_{M_Z}}{s_{M_Z}} [1 + \Delta\kappa_Z] W_\mu^+ W_\nu^- Z^{\mu\nu} \\
& -ie [1 + \Delta\kappa_\gamma] W_\mu^+ W_\nu^- A^{\mu\nu} \\
& -ie \frac{c_{M_Z}}{s_{M_Z}} [1 + \Delta g_1^Z] (W^{+\mu\nu} W_\mu^- - W^{-\mu\nu} W_\mu^+) Z_\nu \\
& -ie (W^{+\mu\nu} W_\mu^- - W^{-\mu\nu} W_\mu^+) A_\nu \\
& -e \frac{c_{M_Z}}{s_{M_Z}} g_5^Z \epsilon^{\mu\nu\rho\lambda} [W_\mu^+ (\partial_\rho W_\nu^-) - (\partial_\rho W_\mu^+) W_\nu^-] Z_\lambda
\end{aligned}$$

Hagiwara-Peccei-Zeppenfeld-Hikasa, NPB282 (1987) 253.

$$\Delta g_1^Z = \frac{1}{c^2 - s^2} \beta_1 + \frac{1}{c^2(c^2 - s^2)} e^2 \alpha_1 + \frac{1}{s^2 c^2} e^2 \alpha_3$$

$$\Delta \kappa_Z = \frac{1}{c^2 - s^2} \beta_1 + \frac{1}{c^2(c^2 - s^2)} e^2 \alpha_1 + \frac{1}{c^2} e^2 (\alpha_1 - \alpha_2) + \frac{1}{s^2} e^2 (\alpha_3 - \alpha_8 + \alpha_9),$$

$$\Delta \kappa_\gamma = \frac{1}{s^2} e^2 (-\alpha_1 + \alpha_2 + \alpha_3 - \alpha_8 + \alpha_9),$$

$$g_5^Z = \frac{1}{s^2 c^2} e^2 \alpha_{11}$$

If custodial symmetry violating term  $\alpha_9$  is negligible,

$$\Delta \kappa_Z = \Delta g_1^Z - \frac{s^2}{c^2} \Delta \kappa_\gamma.$$

Present experimental limits

$$\Delta g_1^Z = -0.016 + 0.022 - 0.019, \quad \Delta \kappa_\gamma = -0.027 + 0.044 - 0.045,$$

lead to bounds on  $\alpha_{2,3}$  at  $10^{-2}$  level.

- $\alpha_{4,5,6,7,10}$  contribute to  $WW \rightarrow WW$  process. Future colliders may be sensitive to these parameters:

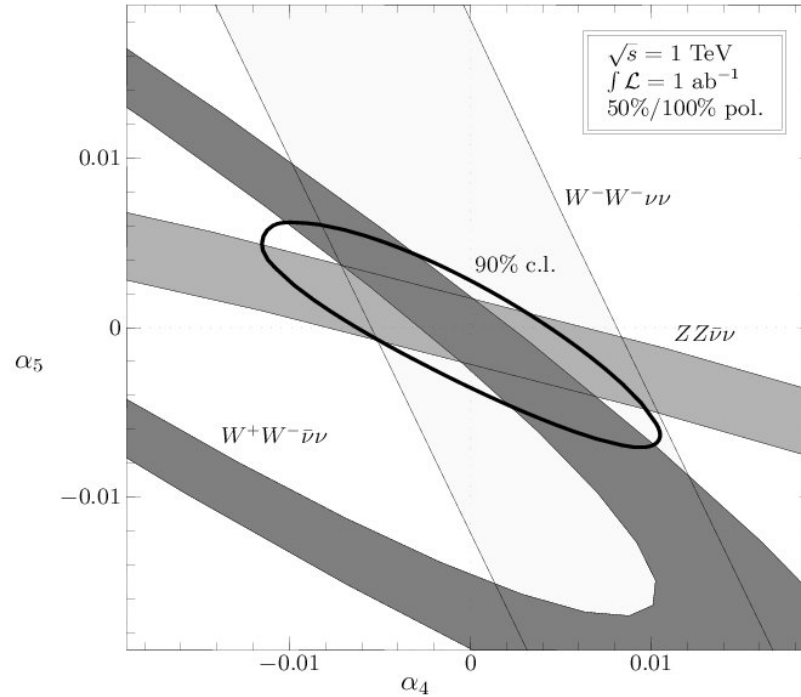


Figure 1: Exclusion contours for the hypothesis  $\alpha_{4,5} = 0$ , assuming  $\sqrt{s} = 1 \text{ TeV}$  and an integrated  $e^+e^-$  luminosity of  $\int \mathcal{L} = 1 \text{ ab}^{-1}$  (50%/100% polarization). The 90% exclusion line has been obtained by combining the  $W^+W^-$  and  $ZZ$  channels (dark gray). The contour for the  $W^-W^-$  channel (light gray) corresponds to an integrated  $e^+e^-$  luminosity of  $\int \mathcal{L} = 100 \text{ fb}^{-1}$  (100% polarization).

E. Boos, H.-J. He et al., PRD61 (2000) 077901.

## Effects of heavy fermion loop

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \quad U_R, \quad D_R$$

Heavy  $U$  and  $D \Rightarrow$  Large Yukawa coupling  
violation of decoupling theorem

From  $S$  and  $U$

$$\alpha_1 = -\frac{N}{96\pi^2} \left[ 1 - 2Y_{Q_L} \ln \frac{m_U^2}{m_D^2} \right],$$

$$\alpha_8 = -\frac{N}{96\pi^2} \left[ -\frac{5m_U^4 - 22m_U^2 m_D^2 + 5m_D^4}{3(m_U^2 - m_D^2)^2} + \frac{m_U^6 - 3m_U^4 m_D^2 - 3m_U^2 m_D^4 + m_D^4}{(m_U^2 - m_D^2)^3} \ln \frac{m_U^2}{m_D^2} \right].$$



From the TGV analysis,

$$\begin{aligned}
 \alpha_2 &= -\frac{N}{96\pi^2} \left[ 1 - Y_{QL} \left( \left( 2 + \frac{6m_U^2 m_D^2}{(m_U^2 - m_D^2)^2} \right) \ln \frac{m_U^2}{m_D^2} - 3 \frac{m_U^2 + m_D^2}{m_U^2 - m_D^2} \right) \right], \\
 \alpha_3 &= -\frac{N}{64\pi^2} \left[ \frac{m_U^2 m_D^2 (m_U^2 + m_D^2)}{(m_U^2 - m_D^2)^3} \ln \frac{m_U^2}{m_D^2} + \frac{m_U^4 - 6m_U^2 m_D^2 + m_D^4}{2(m_U^2 - m_D^2)^2} \right], \\
 \alpha_9 &= -\frac{N}{64\pi^2} \left[ \frac{m_U^2 m_D^2 (m_U^2 + m_D^2)}{(m_U^2 - m_D^2)^3} \ln \frac{m_U^2}{m_D^2} - \frac{m_U^4 + 10m_U^2 m_D^2 + m_D^4}{6(m_U^2 - m_D^2)^2} \right] + \alpha_8, \\
 \alpha_{11} &= -\frac{N}{64\pi^2} \left[ \frac{m_U^2 m_D^2}{(m_U^2 - m_D^2)^2} \ln \frac{m_U^2}{m_D^2} - \frac{m_U^2 + m_D^2}{2(m_U^2 - m_D^2)} \right]
 \end{aligned}$$

If  $U$  and  $D$  are almost degenerated,

$$|\Delta m| \ll \hat{m}, \quad \Delta m \equiv m_U - m_D, \quad \hat{m} \equiv \frac{m_U + m_D}{2}$$

we find

$$\beta_1 \simeq \frac{N}{24\pi^2} \frac{(\Delta m)^2}{v^2}$$

$$\alpha_1 \simeq -\frac{N}{96\pi^2}, \quad \alpha_2 \simeq -\frac{N}{96\pi^2}, \quad \alpha_3 \simeq -\frac{N}{96\pi^2} \left[ 1 - \frac{(\Delta m)^2}{10\hat{m}^2} \right],$$

$$\alpha_8 \simeq -\frac{N}{96\pi^2} \frac{4(\Delta m)^2}{5\hat{m}^2}, \quad \alpha_9 \simeq -\frac{N}{96\pi^2} \frac{7(\Delta m)^2}{10\hat{m}^2}, \quad \alpha_{11} \simeq \frac{N}{96\pi^2} \frac{\Delta m}{2\hat{m}}$$

Note:

- Custodial  $SU(2)_C$  symmetry in  $\Delta m = 0$  limit.
- The sizes of  $\alpha_{6,7,8,9,10,11}$  are extremely suppressed for  $(\Delta m)^2 \ll \hat{m}^2$ .
- Degenerated heavy 4th generation:  $\alpha_1 = \alpha_2 = \alpha_3 \simeq 4.2 \times 10^{-3}$ .

## Renormalization Group

T. Appelquist and C. Bernard, PRD22 (1980) 200.

Even if we start with  $\beta_1 = \alpha_i = 0$ , the loop diagram of the lowest order Lagrangian  $\mathcal{L}_0$  causes non-trivial running of  $\beta_1$  and  $\alpha_i$ :

$$\mu \frac{d}{d\mu} \beta_1^r(\mu) = \frac{\gamma_{\beta 1}}{(4\pi)^2}, \quad \mu \frac{d}{d\mu} \alpha_i^r(\mu) = \frac{\gamma_{\alpha i}}{(4\pi)^2}.$$

$\gamma_{\beta 1}$	$\gamma_{\alpha 1}$	$\gamma_{\alpha 2}$	$\gamma_{\alpha 3}$	$\gamma_{\alpha 4}$	$\gamma_{\alpha 5}$
$\frac{3}{4} g_Y^2$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$-\frac{1}{6}$	$-\frac{1}{12}$

Tree level matching conditions with the Standard Model Higgs sector

$$\beta_1^r(\mu \simeq M_H) = 0, \quad \alpha_1^r(\mu \simeq M_H) = 0$$

lead to

$$\beta^r(\mu) = -\frac{3g_Y^2}{4(4\pi)^2} \ln \frac{M_H}{\mu}, \quad \alpha_1^r(\mu) = -\frac{1}{6(4\pi)^2} \ln \frac{M_H}{\mu},$$

$$T \simeq \frac{8\pi}{e^2} \beta_1^r(\mu) \simeq -\frac{3}{8\pi c^2} \ln M_H, \quad S \simeq -16\pi \alpha_1^r(\mu) \simeq \frac{1}{6\pi} \ln M_H.$$

Alternative parametrization:

(motivated by the hadron chiral perturbation theory)

Gasser and Leutwyler

$$\begin{aligned}\mathcal{L}_{\text{GL}} = & L_1 \text{tr}(D_\mu U^\dagger D^\mu U) \text{tr}(D_\nu U^\dagger D^\nu U) + L_2 \text{tr}(D_\mu U^\dagger D_\nu U) \text{tr}(D^\mu U^\dagger D^\nu U) \\ & + i g_W L_{9L} \text{tr}(D_\mu U D_\nu U^\dagger W^{\mu\nu}) + i g_Y L_{9R} \text{tr}(D_\mu U^\dagger D_\nu U \frac{\tau_3}{2} B^{\mu\nu}) \\ & + g_W g_Y L_{10} \text{tr}(W^{\mu\nu} U \frac{\tau_3}{2} B_{\mu\nu} U^\dagger),\end{aligned}$$

$$\alpha_1 = L_{10},$$

$$\alpha_2 = -L_{9R}/2,$$

$$\alpha_3 = -L_{9L}/2,$$

$$\alpha_4 = L_2,$$

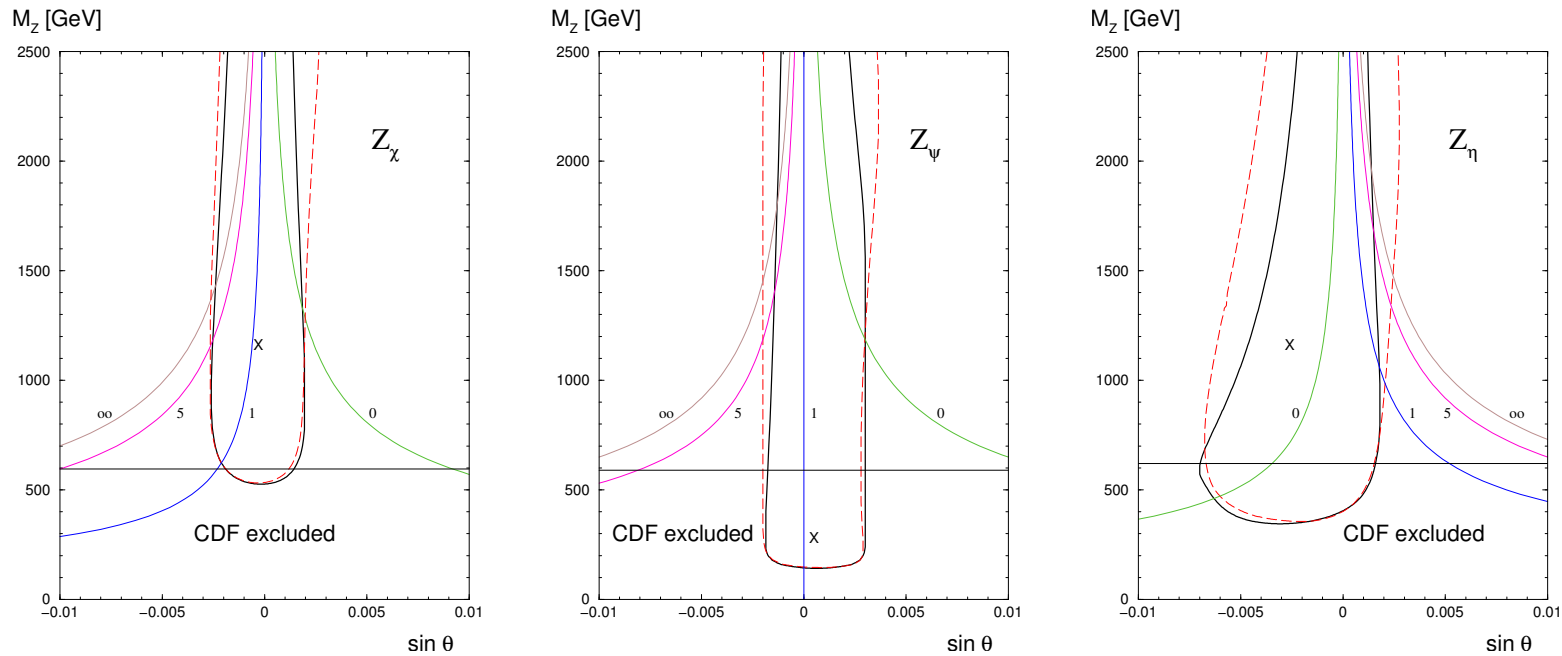
$$\alpha_5 = L_1.$$

## §.7 “Universal” non-oblique corrections

R. Barbieri, A. Pomarol, R. Rattazzi, and A. Strumia, NPB703 (2004) 127.  
 R.S.Chivukula, E.H.Simmons, H.-J. He, M. Kurachi, and M.T., PLB603 (2004) 210.

Effects of heavy  $Z'$  cannot be fully parametrized in the  $S,T$  and  $U$  framework.

For  $E_6$  models, dedicated study is required:



90% CL

Erlar-Langacker, PLB456 (1999) 68.

$Z'$  and  $W'$  couple with ordinary quarks/leptons only through  $J_W^{a\mu}$ ,  $J_Y^\mu$  in little-Higgs or Higgsless models. (“universal” models)

Fermion scattering amplitude in “universal” models:  $(S, T, \alpha\delta, \Delta\rho)$

- Charged current process

$$-\mathcal{M}_{\text{CC}} = \frac{(I_+ I'_- + I_- I'_+)/2}{-\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)k^2 + \frac{1}{4\sqrt{2}G_F} \left(1 + \frac{\alpha\delta}{4s^2 c^2}\right)} + \sqrt{2}G_F \frac{\alpha\delta}{s^2 c^2} \frac{(I_+ I'_- + I_- I'_+)}{2}.$$

- Neutral current process

$$-\mathcal{M}_{\text{NC}} = e^2 \frac{QQ'}{-k^2} + \frac{(I_3 - s^2 Q)(I'_3 - s^2 Q')}{-\left(\frac{s^2 c^2}{e^2} - \frac{S}{16\pi}\right)k^2 + \frac{1}{4\sqrt{2}G_F} \left(1 - \alpha T + \frac{\alpha\delta}{4s^2 c^2}\right)} + \sqrt{2}G_F \frac{\alpha\delta}{s^2 c^2} I_3 I'_3 + 4\sqrt{2}G_F (\Delta\rho - \alpha T)(Q - I_3)(Q' - I'_3).$$

R.S.Chivukula, E.H.Simmons, H.-J. He, M. Kurachi, and M.T., PLB603 (2004) 210.

Note:

- In the absence of  $W'$  and  $Z'$  induced four-fermion couplings,  $S$  and  $T$  agree with the Peskin-Takeuchi definitions of  $S$  and  $T$ .
- $\Delta\rho$  agrees with the original  $\rho$  parameter definition:

$$\rho = 1 + \Delta\rho = \frac{G_{\text{NC}}}{G_{\text{CC}}},$$

which has been measured through the low energy NC experiments.

- In the absence of  $\alpha\delta$ , even if  $\Delta\rho \neq \alpha T$ , limits on  $S$ - $T$  is identical to the existing  $S$ - $T$  limit derived from the Z-pole precision data.

## An example of “universal” model

$$SU(2)_W \times U(1)_{Y1} \times U(1)_{Y2}$$

Mass matrix of neutral gauge boson

$$\frac{1}{4} \begin{pmatrix} W_\mu^3 & B_{1\mu} & B_{2\mu} \end{pmatrix} \begin{pmatrix} g_W^2 v^2 & -g_W g_{Y1} v^2 & 0 \\ -g_W g_{Y1} v^2 & g_{Y1}^2 (v^2 + v_B^2) & -g_{Y1} g_{Y2} v_B^2 \\ 0 & -g_{Y1} g_{Y2} v_B^2 & g_{Y2}^2 v_B^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B_1^\mu \\ B_2^\mu \end{pmatrix}$$

Interaction with quarks and leptons:

$$\mathcal{L}_{\text{int}} = g_W J_W^{a\mu} W_\mu^a + g_{Y1} J_Y^\mu B_{1\mu}.$$

For  $g_W, g_{Y1} \ll g_{Y2}$ , we find

$$\alpha S = -4s^2 c^2 \frac{M_W^2 g_{Y1}^2}{M_{Z'}^2 g_{Y2}^2}, \quad \alpha T = -s^2 \frac{M_W^2 g_{Y1}^2}{M_{Z'}^2 g_{Y2}^2}, \quad \alpha \delta = 0, \quad \Delta \rho = 0.$$

Negative  $S$  !

Negative  $T$  !



Technicolor plus  $SU(2)_W \times U(1)_{Y_1} \times U(1)_{Y_2}$

- Technicolor large positive  $S$  may be canceled by the negative  $S$  from  $Z'$ .
- Negative  $T$  from extra  $Z'$  may be canceled by custodial symmetry breaking in the TC sector.

*Sounds very nice!*

This model predicts

$$\Delta\rho - \alpha T \sim 0.3 \times 10^{-2}.$$

## Alternative parametrization

R. Barbieri, A. Pomarol, R. Rattazzi, and A. Strumia, NPB703 (2004) 127.

Dimension 6 operators

$$\mathcal{L} = c_{WB} \mathcal{O}_{WB} + c_H \mathcal{O}_H + c_{WW} \mathcal{O}_{WW} + c_{BB} \mathcal{O}_{BB},$$

with

$$\mathcal{O}_{WB} = \frac{1}{g_W g_Y} (\phi^\dagger \tau^a \phi) W_{\mu\nu}^a B^{\mu\nu}, \quad \mathcal{O}_H = |\phi^\dagger D_\mu \phi|^2,$$

$$\mathcal{O}_{WW} = \frac{1}{2g_W^2} (D_\rho W_{\mu\nu}^a)^2, \quad \mathcal{O}_{BB} = \frac{1}{2g_Y^2} (\partial_\rho B_{\mu\nu})^2.$$

$$\hat{S} = \frac{c}{s} v^2 c_{WB}, \quad \hat{T} = -\frac{v^2}{2} c_H, \quad W = -\frac{g_W^2 v^2}{2} c_{WW}, \quad Y = -\frac{g_Y^2 v^2}{2} c_{BB}.$$

Dictionary between  $(S, T, \alpha\delta, \Delta\rho)$  and  $(\hat{S}, \hat{T}, W, Y)$

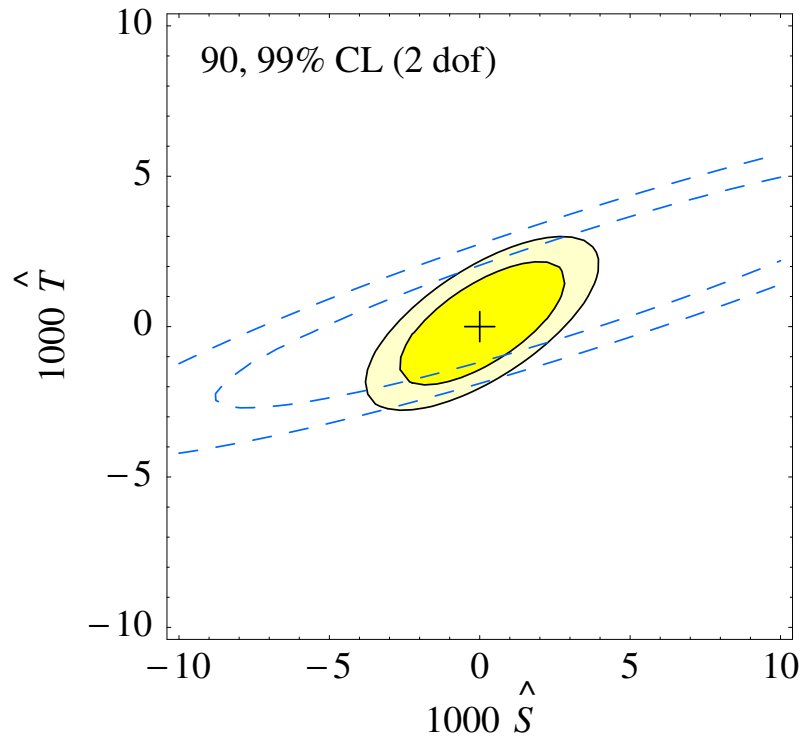
$$\begin{aligned}
 \alpha S &= 4s^2(\hat{S} - Y - W), & \hat{S} &= \frac{1}{4s^2} \left( \alpha S + 4c^2(\Delta\rho - \alpha T) + \frac{\alpha\delta}{c^2} \right), \\
 \alpha T &= \hat{T} - \frac{s^2}{c^2} Y, & \hat{T} &= \Delta\rho, \\
 \alpha\delta &= 4s^2 c^2 W, & W &= \frac{\alpha\delta}{4s^2 c^2}, \\
 \Delta\rho &= \hat{T}. & Y &= \frac{c^2}{s^2} (\Delta\rho - \alpha T).
 \end{aligned}$$

- In the absence of “universal” non-oblique corrections,  $W = Y = 0$  and

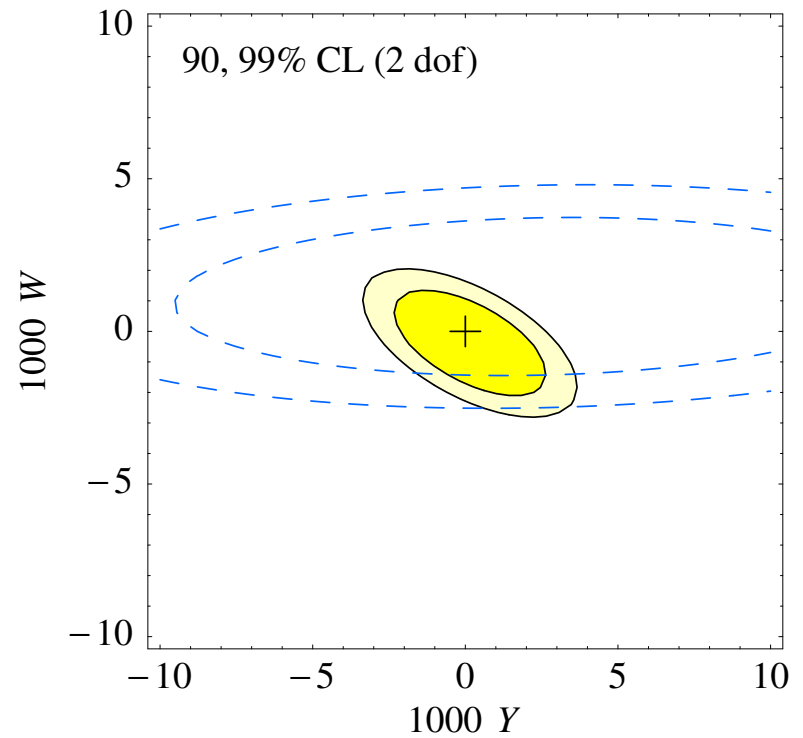
$$\alpha S = 4s^2 \hat{S}, \quad \alpha T = \hat{T}.$$

- Technicolor plus  $SU(2)_W \times U(1)_{Y1} \times U(1)_{Y2}$  model predicts  $\hat{S} \simeq 0$ ,  $\hat{T} \simeq 0$ ,  $W \simeq 0$ , but  $Y \simeq 10^{-2}$ .

$(\hat{S}, \hat{T}, W, Y)$ -fit with and without LEP2 data of  $\sigma(e^+e^- \rightarrow f\bar{f})$



$(\hat{S}, \hat{T})$  for generic  $(W, Y)$



$(W, Y)$  for generic  $(\hat{S}, \hat{T})$

Barbieri et al. [hep-ph/0405042](https://arxiv.org/abs/hep-ph/0405042)

## §.8 Summary

- New physics scenarios which affect EW physics through *non-decoupling* radiative corrections are severely constrained by the precision data:
  - Heavy 4th generation
  - Technicolor models
- The precision data suggest that, in the standard model framework, Higgs should be light  $M_H \leq 199\text{GeV}$ . Heavier Higgs thus indicates BSM.
- Electroweak chiral perturbation theory provides a powerful tool to catalogue all possible *non-decoupling* corrections:  
 $S$ - $T$ - $U$ , TGV,  $WW \rightarrow WW$
- In the “universal” models (little-Higgs, Higgsless, etc.),  $S$ - $T$  fit is not enough. We can do better by using  $(S, T, \alpha\delta, \Delta\rho)$  or  $(\hat{S}, \hat{T}, W, Y)$ .  
 $\Rightarrow$  Prof. Chivukula’s lecture for Higgsless models.

謝謝！

*Thank you very much !*