Electroweak Precision Tests and Physics Beyond the Standard Model I

Topical Seminar on Frontier of Particle Physics 2006:

Beyond the Standard Model, Beijing

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§.0 Power of precision

$$\alpha^{-1}(M_Z) = 128.91(2) \qquad 1.6 \times 10^{-4}$$

$$M_Z = 91.1876(21)\text{GeV} \qquad 2.3 \times 10^{-5}$$

$$G_F = 1.16637(1) \times 10^{-5}\text{GeV}^{-2} \qquad 9 \times 10^{-6}$$

	uncertainty
$\alpha^{-1}(M_Z) = 128.91(2)$	1.6×10^{-4}
$M_Z = 91.1876(21)\text{GeV}$	2.3×10^{-5}
$G_F = 1.16637(1) \times 10^{-5}\text{GeV}^{-2}$	9×10^{-6}

Erler-Langacker in RPP2006

We know three input parameters of the standard model ($SU(2)$ and $U(1)$ gauge couplings, g_W, g_Y, and the VEV of Higgs, v) within 10^{-4} accuracy.

Many values are precisely measured within 10^{-3} accurary

	uncertainty
$\Gamma_Z = 2.4952(23)\text{GeV}$	9.2×10^{-4}
$\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23153(16)$	6.9×10^{-4}
$M_W = 80.403(29)\text{GeV}$	3.6×10^{-4}

http://lepewwg.web.cern.ch/LEPEWWG/
RPP2006

at Z and W poles.

We can test the Standard Model at 10^{-3} accuracy!!

Implication of 10^{-3} accuracy to BSM: *a rule of thumb*

New particle mass at scale Λ

Three categories of BSM scenarios:

- New particle(s) contributing to EW physics at *one-loop level* through *non-decoupling* effects in the gauge boson vacuum polarization functions:

$$\frac{e^2}{(4\pi)^2} = \frac{\alpha}{4\pi} \simeq 10^{-3}$$

  e.g., technicolor, heavy 4th generation, $\cdots$

  In order to parametrize new physics effects in this class of models, we use

$$(S, T, U)$$

  Peskin and Takeuchi, PRL65 (1990) 964

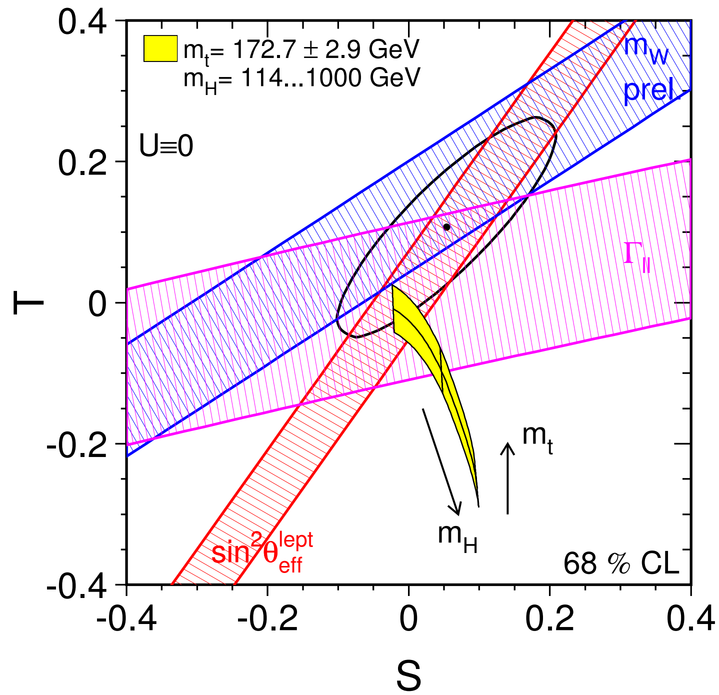
  or

$$(\epsilon_1, \epsilon_2, \epsilon_3)$$

  Alterelli and Barbieri, PLB253 (1991) 161

see also: Hagiwara, Matsumoto, Haidt and Kim, Z.Phys.C64 (1994) 559.

S-T plot



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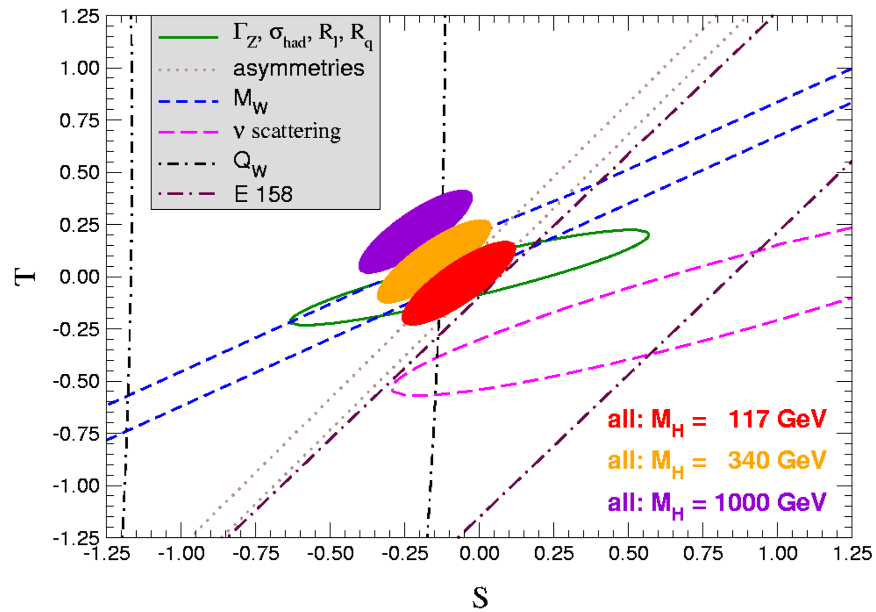


Erler and Langacker, in RPP2006

- New particle(s) contributing to EW physics at *tree level*:

$$\frac{M_Z^2}{\Lambda^2} \sim 10^{-3}$$

e.g., Z' models, Higgsless models, little Higgs models, $\cdots$

Precision EW measurements are sensitive to new physics at

$$\Lambda \sim 3\text{TeV}.$$

There is no simple parametrization to describe the effects of every type of new physics in this class, however.

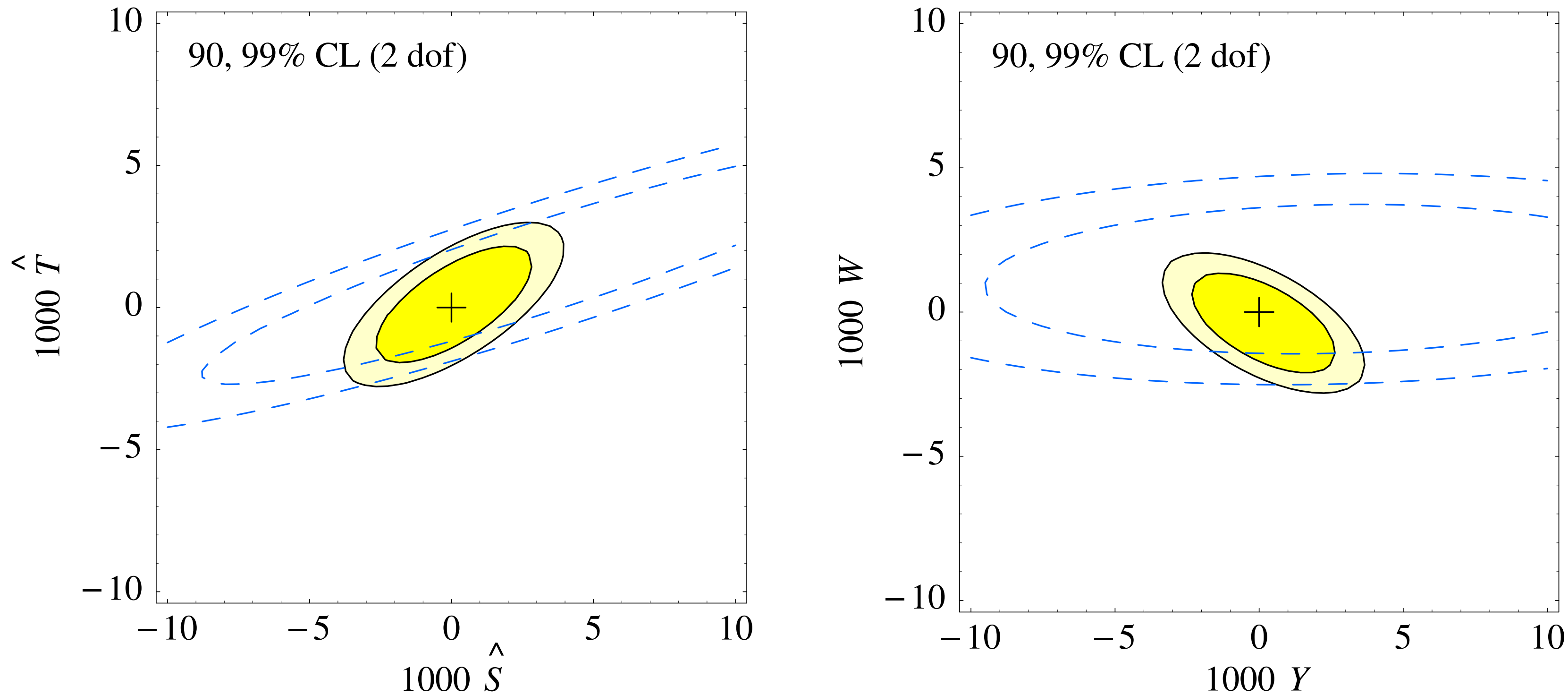
Recent proposal of parameters applicable to "universal" models (e.g., Higgsless models, little Higgs models):

$$(\hat{S}, \hat{T}, W, Y)$$ Barbieri et al. hep-ph/0405042

or

$$(S, T, \Delta\rho, \delta)$$ Chivukula et al. hep-ph/0408262

“Universal” model:



R. Barbieri, A. Pomarol, R. Rattazzi, and A. Strumia, NPB703 (2004) 127.

- New particle contributes to EW physics at *one-loop level* through *decoupling* effects:

$$\frac{1}{(4\pi)^2}\frac{M_Z^2}{\Lambda^2} \sim 10^{-3}.$$

e.g., TeV scale SUSY, little Higgs models with T-parity, $\cdots$

Precision EW measurements are sensitive to new physics at

$$\Lambda \sim 300\text{GeV}$$

Plan of this talk

§.0 Power of precision

§.1 Weinberg-Salam model at tree level

§.2 Decoupling theorem and its violation in EW physics

§.3 S-T fit

§.4 Higgs mass

§.5 Estimate of S in technicolor models

§.6 Electroweak chiral perturbation theory

§.7 “Universal” non-oblique corrections

§.8 Summary

§.1 Weinberg-Salam model at tree level

WS model is a chiral $SU(2)_W \times U(1)_Y$ gauge theory:

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \qquad a = 1, 2, 3$$

with

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g_W \epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.$$

Interaction with quarks and leptons:

$$\mathcal{L}_{\rm int} = g_W J^{a\mu}_W W^a_\mu + g_Y J^\mu_Y B_\mu, \qquad J^{a\mu}_W = \sum_\psi \bar{\psi} I_a \gamma^\mu \psi, \quad J^\mu_Y = \sum_\psi \bar{\psi} Y \gamma^\mu \psi.$$

ψ	ℓ_L	e_R	q_L	u_R	d_R
I_a	$\frac{\tau_a}{2}$	0	$\frac{\tau_a}{2}$	0	0
Y	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$

$$\ell_L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

We need Higgs field ϕ so as to make the weak gauge bosons massive:

$$\phi = \begin{pmatrix} \varphi_+ \\ \varphi_0 \end{pmatrix}, \qquad \begin{array}{l} \text{weak } SU(2)_W \text{ doublet} \\ \text{weak hypercharge } Y = 1/2 \end{array}$$

$$\mathcal{L}_{\text{Higgs}} = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi), \qquad V(\phi) = \lambda \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2,$$

with

$$D_\mu \phi = \left(\partial_\mu + i g_W \frac{\tau^a}{2} W_\mu^a + i g_Y \frac{1}{2} B_\mu \right) \phi.$$

Thanks to the wine bottle shape of the Higgs potential $V(\phi)$, Higgs field acquires its VEV:

$$V(\phi) \simeq$$

$$\Rightarrow \qquad \langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix},$$

and breaks $SU(2)_W \times U(1)_Y$ into $U(1)_Q$ spontaneously.

Neutral current

W^3_μ and B^μ mix with each other:

$$\mathcal{L}_{\rm mass} = \frac{1}{8}(W^3_\mu \; B_\mu) \begin{pmatrix} g_W^2 v^2 & -g_W g_Y v^2 \\ -g_W g_Y v^2 & g_Y^2 v^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix},$$

Mass diagonalization:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}, \qquad s = \sin\theta_W, \quad c = \cos\theta_W,$$

with Weinberg angle θ_W being given by

$$\sin\theta_W = \frac{g_Y}{\sqrt{g_W^2 + g_Y^2}}, \qquad \cos\theta_W = \frac{g_W}{\sqrt{g_W^2 + g_Y^2}}.$$

Mass eigenvalues

$$M_\gamma^2 = 0, \qquad M_Z^2 = \frac{g_W^2 + g_Y^2}{4} v^2.$$

Z and photon interactions with quarks/leptons:

$$\mathcal{L}_{\rm int}^{\rm NC} = e \sum_{\psi} \bar{\psi} \mathcal{Q} \gamma^{\mu} \psi A_{\mu} + \frac{e}{sc} \sum_{\psi} \bar{\psi} (I_3 - s^2 \mathcal{Q}) \gamma^{\mu} \psi Z_{\mu},$$

with

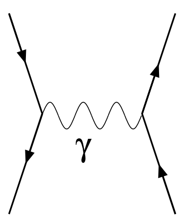
$$e^2 = \frac{g_W^2 g_Y^2}{g_W^2 + g_Y^2}, \qquad \frac{e^2}{s^2 c^2} = g_W^2 + g_Y^2, \qquad M_Z^2 = \frac{e^2}{s^2 c^2} \frac{v^2}{4}.$$

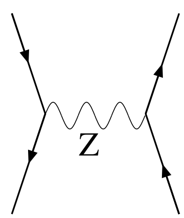
$$\mathcal{Q} \equiv I_3 + Y$$

(vector-like)

ψ	ν_L	e_L	e_R	u_L	u_R	d_L	d_R
$\mathcal{Q}$	0	-1	-1	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$

Neutral current $f\bar{f} \to f'\bar{f}'$ amplitude:

$\mathcal{M}_{\rm NC} =$ 

$+$ 

$$= e^2 \frac{\mathcal{Q}\mathcal{Q}'}{k^2} + \frac{e^2}{s^2 c^2} \frac{(I_3 - s^2 \mathcal{Q})(I_3' - s^2 \mathcal{Q}')}{k^2 - M_Z^2}.$$

Charged current

W boson mass term:

$$\mathcal{L}_{\text{mass}} = \frac{g_W^2 v^2}{4} W_\mu^+ W^{-\mu}, \qquad W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2).$$

Mass of W:

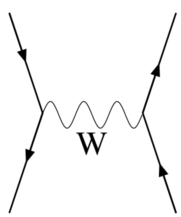
$$M_W^2 = \frac{g_W^2}{4} v^2 = \frac{e^2}{4s^2} v^2, \qquad \frac{e^2}{s^2} = g_W^2.$$

W boson interaction with quarks/leptons:

$$\mathcal{L}_{\text{int}}^{\text{CC}} = \frac{1}{\sqrt{2}} \frac{e}{s} \sum_\psi \bar{\psi} I_- \gamma^\mu \psi W_\mu^+ + \text{h.c.}, \qquad I_\pm \equiv I_1 \mp i I_2.$$

Charged current $f\bar{f} \to f'\bar{f}'$ amplitude:

$$\mathcal{M}_{\text{CC}} =$$



$$= \frac{e^2}{s^2} \frac{(I_+ I'_- + I_- I'_+)/2}{k^2 - M_W^2}.$$

Custodial $SU(2)$ symmetry

Low energy four-fermion couplings from W and Z exchanges:

$$4\sqrt{2}G_{\mathrm{CC}} = \frac{e^2}{s^2}\frac{1}{M_W^2} = \frac{4}{v^2}, \qquad 4\sqrt{2}G_{\mathrm{NC}} = \frac{e^2}{s^2c^2}\frac{1}{M_Z^2} = \frac{4}{v^2}.$$

$$\rho \equiv \frac{G_{\mathrm{NC}}}{G_{\mathrm{CC}}} = 1. \qquad \text{(a distinctive property of doublet Higgs)}$$

What is the physics underlying $\rho = 1$?

Higgs Lagrangian can be rewritten as

$$\mathcal{L}_{\mathrm{Higgs}} = \frac{1}{4}\mathrm{tr}\left((D_\mu\Phi)^\dagger(D_\mu\Phi)\right) - V(\Phi),$$

with $$\Phi = \sqrt{2}\left(\tilde{\phi}\phi\right) = \sqrt{2}\begin{pmatrix} \varphi_0^* & \varphi_+ \\ -\varphi_+^* & \varphi_0 \end{pmatrix}, \qquad \tilde{\phi} \equiv i\tau_2\phi^*.$$

$$D_\mu\Phi = \partial_\mu\Phi + ig_W\frac{\tau_a}{2}W_\mu^a\Phi - ig_Y\Phi\frac{\tau_3}{2}B_\mu.$$

Symmetry of the Higgs Lagrangian is enhanced to $SU(2)_W \times SU(2)_R$ in the $g_Y \to 0$ limit:

$$\Phi \to U_L \Phi U_R^\dagger, \qquad U_L \in SU(2)_W, \quad U_R \in SU(2)_R$$

VEV of Higgs breaks $SU(2)_W \times SU(2)_R$ symmetry to diagonal $SU(2)_C$:

$$SU(2)_W \times SU(2)_R \to SU(2)_C,$$

$$\langle\Phi\rangle = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}, \qquad \langle\Phi\rangle \to \langle U_C \Phi U_C^\dagger\rangle = U_C \langle\Phi\rangle U_C^\dagger = \langle\Phi\rangle, \quad U_C \in SU(2)_C.$$

$SU(2)_C$: custodial $SU(2)$ symmetry under which NG bosons eaten by W and Z behave as a triplet.

It will turn out that the cutdodial $SU(2)$ symmetry is a useful concept in the parametrization of new physics in the precision EW tests.

Number of free parameters

Fermi coupling G_F is determined as

$$4\sqrt{2}G_F = \frac{e^2}{s^2}\frac{1}{M_W^2} = \frac{4}{v^2}.$$

At the tree-level, structure of fermion scattering amplitude is determined completely once *three parameters* (e, s, G_F) are all fixed:

- Charged current process

$$-\mathcal{M}_{\mathrm{CC}} = \frac{(I_+I'_- + I_-I'_+)/2}{-\dfrac{s^2}{e^2}k^2 + \dfrac{1}{4\sqrt{2}G_F}}, \qquad M_W^2 = \frac{e^2}{s^2}\frac{1}{4\sqrt{2}G_F},$$

- Neutral current process

$$-\mathcal{M}_{\mathrm{NC}} = e^2\frac{\mathcal{Q}\mathcal{Q}'}{-k^2} + \frac{(I_3 - s^2\mathcal{Q})(I'_3 - s^2\mathcal{Q}')}{-\dfrac{s^2c^2}{e^2}k^2 + \dfrac{1}{4\sqrt{2}G_F}}, \qquad M_Z^2 = \frac{e^2}{s^2c^2}\frac{1}{4\sqrt{2}G_F}.$$

What is $\sin\theta_W$? (extraction of $\sin\theta_W$ from EW observables)

- (e, M_W, M_Z) scheme:

$$s_W^2 \equiv 1 - \frac{M_W^2}{M_Z^2}, \qquad c_W^2 \equiv 1 - s_W^2$$

- (e, G_F, M_W) scheme:

$$s_{M_W}^2 \equiv \frac{e^2}{4\sqrt{2} G_F M_W^2}, \qquad c_{M_W}^2 \equiv 1 - s_{M_W}^2$$

- (e, G_F, M_Z) scheme:

$$s_{M_Z}^2 c_{M_Z}^2 \equiv \frac{e^2}{4\sqrt{2} G_F M_Z^2}, \qquad c_{M_Z}^2 \equiv 1 - s_{M_Z}^2$$

These definitions are all equivalent at the tree-level.

§.2 Decoupling theorem and its violation in EW physics

What we eager to do:

Hunting new physics through the precision tests of the standard model

Bad News: decoupling theorem

T. Appelquist and J. Carazzone, PRD11 (1975) 2856.

If the new physics remains perturbative in the heavy particle limit, all effects of the heavy particle are suppressed by powers of the heavy particle mass.

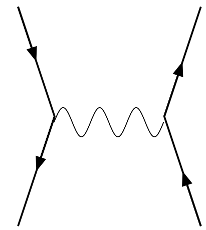
Good News: violation of decoupling theorem

The standard model is a spontaneously broken chiral gauge theory. There is a class of new physics scenarios in which heavy particles' masses are proportional to their couplings. (e.g., technicolor, heavy 4th generation, $\cdots$)

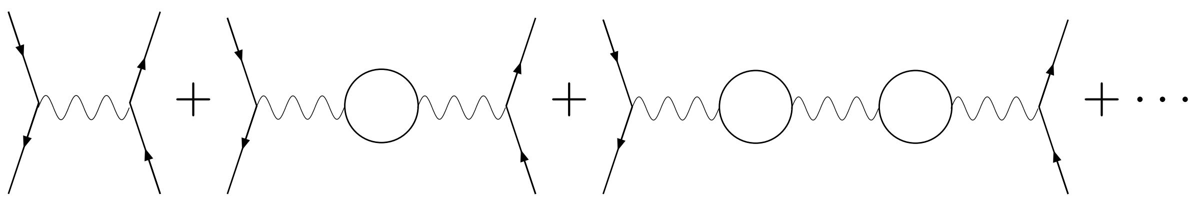
Question: How can we parametrize such non-decoupling effects?
How many parameter do we have?

Peskin-Takeuchi parameters S, T, U for oblique correction.

Decoupling theorem in QED

Born amplitude of $f\bar{f} \to f'\bar{f}'$ scattering in QED:  $= \mathcal{Q}\dfrac{e_0^2}{k^2}\mathcal{Q}'$.

Consider new particle of mass M_{new} contributing to the photon vacuum polarization function (*oblique* correction)



$$= \mathcal{Q}\frac{e_0^2}{k^2}\mathcal{Q}' + \mathcal{Q}\frac{e_0^2}{k^2}\Pi_{\text{new}}\frac{e_0^2}{k^2}\mathcal{Q}' + \mathcal{Q}\frac{e_0^2}{k^2}\Pi_{\text{new}}\frac{e_0^2}{k^2}\Pi_{\text{new}}\frac{e_0^2}{k^2}\mathcal{Q}' + \cdots$$

$$= \mathcal{Q}\frac{e_0^2}{k^2 - e_0^2\Pi_{\text{new}}}\mathcal{Q}' = \frac{\mathcal{Q}\mathcal{Q}'}{\frac{1}{e_0^2}k^2 - \Pi_{\text{new}}}$$

Radiative corrections from *known* physics are ignored for simplicity.

QED gauge invariance

$$\Pi^{\mu\nu}_{\rm new}(k^2) = \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\Pi_{\rm new}(k^2) = \left(g^{\mu\nu}k^2 - k^\mu k^\nu\right)\widetilde{\Pi}_{\rm new}(k^2)$$

reads

$$\Pi_{\rm new}(0) = 0.$$

Simple consideration based on the mass dimension

$$\dim\widetilde{\Pi}_{\rm new}(k^2) = 0$$

suggests $\widetilde{\Pi}_{\rm new}$ scales like

$$\widetilde{\Pi}_{\rm new}(0) \propto (M_{\rm new})^0$$

Radiative correction $\widetilde{\Pi}_{\rm new}(k^2)$ seems to be nonvanishing even in the $M_{\rm new} \to \infty$ limit.

We should be careful about the renormalization.

Acutually, the charge renormalization procedure

$$\frac{1}{e^2} = \frac{1}{e_0^2} - \widetilde{\Pi}_{\rm new}(k^2 = 0),$$

absorbs the nonvanishing $\widetilde{\Pi}_{\rm new}(0)$. Improved Born $f\bar{f} \to f'\bar{f}'$ scattering amplitude can then be written as

$$\mathcal{M}_{\rm QED} = \frac{\mathcal{Q}\mathcal{Q}'}{\left(\frac{1}{e_0^2} - \widetilde{\Pi}_{\rm new}(k^2)\right) k^2} = \frac{\mathcal{Q}\mathcal{Q}'}{\left(\frac{1}{e^2} - k^2\widetilde{\Pi}'_{\rm new}(0) + \mathcal{O}(k^4)\right) k^2},$$

with

$$\widetilde{\Pi}_{\rm new}(k^2) = \widetilde{\Pi}_{\rm new}(0) + k^2\widetilde{\Pi}'_{\rm new}(0) + \cdots .$$

Simple analysis based on mass dimension:

$$\text{dim}\widetilde{\Pi}'_{\rm new} = -2$$

suggests

$$\widetilde{\Pi}'_{\rm new} \sim \frac{1}{M_{\rm new}^2},$$

with $M_{\rm new}^2$ being the mass scale of new physics.

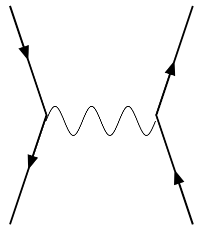
New physics thus decouples from the low energy QED $f\bar{f} \to f'\bar{f}'$ scattering amplitude in $M_{\rm new} \to \infty$ limit,

$$\mathcal{M}_{\rm QED} = \frac{\mathcal{Q}\mathcal{Q}'}{\left(\frac{1}{e^2} + \mathcal{O}\left(\frac{k^2}{M_{\rm new}^2}\right)\right) k^2},$$

if we write the amplitude in terms of appropriately renormalized couplings.

Appelquist-Carazzone decoupling theorem

Spontaneously broken $U(1)$ gauge theory

Born amplitude:  $= \mathcal{Q} \dfrac{g_0^2}{k^2 - g_0^2 v_0^2} \mathcal{Q}'.$

New physics contribution to the vacuum polarization $\Pi_{\rm new}$.

Improved Born amplitude

$$\mathcal{M} = \frac{\mathcal{Q}\mathcal{Q}'}{\frac{1}{g_0^2} k^2 - v_0^2 - \Pi_{\rm new}(k^2)}.$$

The $U(1)$ gauge invariance is broken spontanesously

$$\Pi_{\rm new}(0) \neq 0.$$

We define

$$\Pi_{\rm new}(k^2) = \Pi_{\rm new}(0) + k^2 \widetilde{\Pi}_{\rm new}(k^2)$$

Non-zero $\Pi_{\rm new}(0)$ and $\widetilde{\Pi}_{\rm new}(0)$ are absorbed into the renormalization of v and g, respectively:

$$v^2 = v_0^2 + \Pi_{\rm new}(0), \qquad \frac{1}{g^2} = \frac{1}{g_0^2} - \widetilde{\Pi}_{\rm new}(0).$$

Remaining correction $\widetilde{\Pi}'_{\rm new}(0)$ in

$$\mathcal{M} = \frac{\mathcal{Q}\mathcal{Q}'}{\left(\dfrac{1}{g^2} - k^2\widetilde{\Pi}'_{\rm new}(0) + \cdots\right)k^2 - v^2}$$

behaves as

$$\widetilde{\Pi}'_{\rm new}(0) \sim \frac{1}{M_{\rm new}^2}$$

and decouples from the low energy amplitude in the $M_{\rm new} \to \infty$ limit.

Violation of decoupling theorem in EW physics

$$SU(2)_W \times U(1)_Y \to U(1)_Q$$

Number of vacuum polarization functions (*oblique corrections*):

- Charged current: $$\Pi_{11}(k^2) = \Pi_{22}(k^2)$$
- Neutral current: $$\Pi_{33}(k^2), \quad \Pi_{3Q}(k^2), \quad \Pi_{QQ}(k^2).$$

Thanks to the unbroken QED gauge invariance

$$\Pi_{11}^{\text{new}}(k^2) = \Pi_{11}^{\text{new}}(0) + k^2 \widetilde{\Pi}_{11}^{\text{new}}(0) + \mathcal{O}(\frac{k^4}{M_{\text{new}}^2})$$

$$\Pi_{33}^{\text{new}}(k^2) = \Pi_{33}^{\text{new}}(0) + k^2 \widetilde{\Pi}_{33}^{\text{new}}(0) + \mathcal{O}(\frac{k^4}{M_{\text{new}}^2})$$

$$\Pi_{3Q}^{\text{new}}(k^2) = \qquad k^2 \widetilde{\Pi}_{3Q}^{\text{new}}(0) + \mathcal{O}(\frac{k^4}{M_{\text{new}}^2})$$

$$\Pi_{QQ}^{\text{new}}(k^2) = \qquad k^2 \widetilde{\Pi}_{QQ}^{\text{new}}(0) + \mathcal{O}(\frac{k^4}{M_{\text{new}}^2})$$

- 6 seemingly non-decoupling degree of freedoms:

$$\Pi_{11}^{\rm new}(0), \Pi_{33}^{\rm new}(0), \widetilde{\Pi}_{11}^{\rm new}(0), \widetilde{\Pi}_{33}^{\rm new}(0), \widetilde{\Pi}_{3Q}^{\rm new}(0), \widetilde{\Pi}_{QQ}^{\rm new}(0)$$

- 3 renormalization:

$$g_W\,, \quad g_Y\,, \quad v, \qquad\qquad \text{or } (e, s, G_F)$$

- $6 - 3 = 3$ non-decoupling parameters left unabsorbed after the renormalization (Peskin-Takeuchi parameters):

$$\begin{aligned}
\alpha S &= 4e^2 \left(\widetilde{\Pi}_{33}^{\rm new}(0) - \widetilde{\Pi}_{3Q}^{\rm new}(0)\right), \\
\alpha T &= 4\sqrt{2} G_F \left(\Pi_{11}^{\rm new}(0) - \Pi_{33}^{\rm new}(0)\right), \\
\alpha U &= 4e^2 \left(\widetilde{\Pi}_{11}^{\rm new}(0) - \widetilde{\Pi}_{33}^{\rm new}(0)\right).
\end{aligned}$$

Fermion scattering amplitude

- Charged current process

$$-\mathcal{M}_{\rm CC} = \frac{(I_+ I'_- + I_- I'_+)/2}{-\left(\frac{s^2}{e^2} - \frac{S+U}{16\pi}\right)k^2 + \frac{1}{4\sqrt{2}G_F}}$$

- Neutral current process

$$-\mathcal{M}_{\rm NC} = e^2 \frac{\mathcal{Q}\mathcal{Q}'}{-k^2} + \frac{(I_3 - s^2\mathcal{Q})(I'_3 - s^2\mathcal{Q}')}{-\left(\frac{s^2c^2}{e^2} - \frac{S}{16\pi}\right)k^2 + \frac{1}{4\sqrt{2}G_F}\left(1 - \frac{e^2}{4\pi}T\right)}$$

What is $\sin\theta_W$?

- (e, M_W, M_Z) scheme:

$$s_W^2 \equiv 1 - \frac{M_W^2}{M_Z^2}, \qquad c_W^2 \equiv 1 - s_W^2$$

- (e, G_F, M_W) scheme:

$$s_{M_W}^2 \equiv \frac{e^2}{4\sqrt{2} G_F M_W^2}, \qquad c_{M_W}^2 \equiv 1 - s_{M_W}^2$$

- (e, G_F, M_Z) scheme:

$$s_{M_Z}^2 c_{M_Z}^2 \equiv \frac{e^2}{4\sqrt{2} G_F M_Z^2}, \qquad c_{M_Z}^2 \equiv 1 - s_{M_Z}^2$$

$$M_W^2 = \frac{1}{4\sqrt{2}G_F}\frac{e^2}{s^2}\left[1+\frac{1}{4s^2}(\alpha S+\alpha U)\right],$$

$$M_Z^2 = \frac{1}{4\sqrt{2}G_F}\frac{e^2}{s^2c^2}\left[1+\frac{1}{4s^2c^2}\alpha S-\alpha T\right], \qquad \alpha \equiv \frac{e^2}{4\pi}.$$

$$s_W^2 = s^2+\Delta_W, \qquad \Delta_W = \frac{\alpha}{4}S - c^2\alpha T - \frac{c^2}{s^2}\frac{\alpha}{4}U,$$

$$s_{M_W}^2 = s^2+\Delta_{M_W}, \qquad \Delta_{M_W} = -\frac{\alpha}{4}S - \frac{\alpha}{4}U,$$

$$s_{M_Z}^2 = s^2+\Delta_{M_Z}, \qquad \Delta_{M_Z} = \frac{1}{c^2-s^2}\left[-\frac{\alpha}{4}S+s^2c^2\alpha T\right].$$

$$s^2 \neq s_W^2 \neq s_{M_W}^2 \neq s_{M_Z}^2$$

Lesson: *We need to be careful about the definition of* $\sin\theta_W$ *under the presence of* S, T, U *(or at the loop-level).*

Effects of heavy fermion loop

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \qquad U_R, \quad D_R$$

Heavy U and D $\Rightarrow$ Large Yukawa coupling

violation of decoupling theorem

$$S = \frac{N}{6\pi}\left[1 - 2Y_{Q_L} \ln \frac{m_U^2}{m_D^2}\right], \qquad Y_{Q_L}: \text{ weak hypercharge of } Q_L$$

$$T = \frac{N}{16\pi s^2 c^2 M_Z^2}\left[m_U^2 + m_D^2 - \frac{2m_U^2 m_D^2}{m_U^2 - m_D^2} \ln \frac{m_U^2}{m_D^2}\right],$$

$$U = \frac{N}{6\pi}\left[-\frac{5m_U^4 - 22m_U^2 m_D^2 + 5m_D^4}{3(m_U^2 - m_D^2)^2} + \frac{m_U^6 - 3m_U^4 m_D^2 - 3m_U^2 m_D^4 + m_D^4}{(m_U^2 - m_D^2)^3} \ln \frac{m_U^2}{m_D^2}\right].$$

If U and D are almost degenrated,

$$|\Delta m| \ll \hat{m}, \qquad \Delta m \equiv m_U - m_D, \qquad \hat{m} \equiv \frac{m_U + m_D}{2}$$

we find

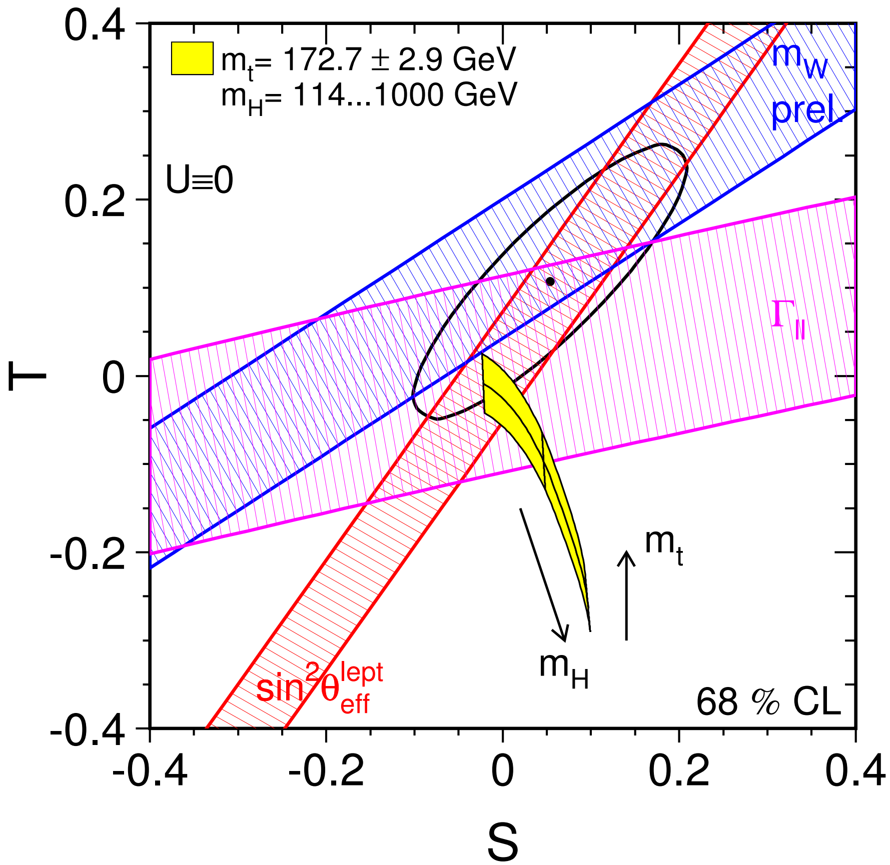
$$S \simeq \frac{N}{6\pi}, \quad T \simeq \frac{N}{12\pi s^2 c^2} \frac{(\Delta m)^2}{M_Z^2}, \quad U \simeq \frac{2N}{15\pi} \frac{(\Delta m)^2}{\hat{m}^2}.$$

Note:

- If custodial $SU(2)$ symmetry is exact, $\Delta m = 0$ and thus $T = U = 0$. Nonzero T (and U) should be regarded as a consequence of the custodial $SU(2)$ violation.
- The size of U is extremely suppressed for $(\Delta m)^2 \ll \hat{m}^2$.
- Sizable contribution to T is possible for $(\Delta m)^2 \sim M_Z^2 \ll \hat{m}^2$.
- Degenerated heavy 4th generation: $S = \dfrac{4}{6\pi} \simeq 0.21$, $T = 0$, $U = 0$

§.3 S-T fit

LEPEWWG2005



http://lepewwg.web.cern.ch/LEPEWWG/

- $U = 0$ is assumed.
- $m_t^{\rm ref} = 175 {\rm GeV}$
- $m_H^{\rm ref} = 150 {\rm GeV}$

The value of $\sin^2\theta_{\rm eff}^{\rm lept}$ is extracted from asymmetries on Z-pole:

$e^-e^+ \to \ell^-\ell^+$ forward-backward asymmetry:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_\ell, \qquad \mathcal{A}_f = \frac{g_{Lf}^2 - g_{Rf}^2}{g_{Lf}^2 + g_{Rf}^2} = \frac{2g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2}$$

$$\frac{g_{V\ell}}{g_{A\ell}} = \frac{-\frac{1}{4} - Q_\ell \sin^2\theta_{\rm eff}^{\rm lept}}{-\frac{1}{4}} = 1 - 4\sin^2\theta_{\rm eff}^{\rm lept},$$

$$\begin{aligned}
\sin^2\theta_{\rm eff}^{\rm lept} &= s^2 + (\text{SM correction}) \\
&= s_{M_Z}^2\left(1 + \frac{1}{4s^2(c^2 - s^2)}\alpha S - \frac{c^2}{c^2 - s^2}\alpha T + (\text{SM correction})\right) \\
&= s_{M_Z}^2 \times (1 + 2.01 \times \alpha S - 1.43 \times \alpha T + (\text{SM correction}))
\end{aligned}$$

$\Gamma(Z \to \ell^+\ell^-) = 83.985(86)\mathrm{MeV}$

$Z\bar{e}_L e_L$ coupling: $(I_3 = -1/2,\ \mathcal{Q} = -1)$

$$g^2_{Le} = \frac{e^2}{s^2c^2}\left(-\frac{1}{2} + s^2\right)^2 \left(1 + \frac{\alpha}{4s^2c^2}S\right)$$

$Z\bar{e}_R e_R$ coupling: $(I_3 = 0,\ \mathcal{Q} = -1)$

$$g^2_{Re} = \frac{e^2}{s^2c^2}\left(0 + s^2\right)^2 \left(1 + \frac{\alpha}{4s^2c^2}S\right)$$

$$\Gamma(Z \to \ell^+\ell^-) \propto g^2_{L\ell} + g^2_{R\ell}$$

$$= \frac{e^2}{s^2c^2}\left((-\frac{1}{2} + s^2)^2 + s^4\right)\left(1 + \frac{1}{4s^2c^2}\alpha S\right)$$

$$= \frac{e^2}{s^2_{M_Z}c^2_{M_Z}}\left((-\frac{1}{2} + s^2_{M_Z})^2 + s^4_{M_Z}\right)(1 - 0.281 \times \alpha S + 1.20 \times \alpha T)$$

M_W^2

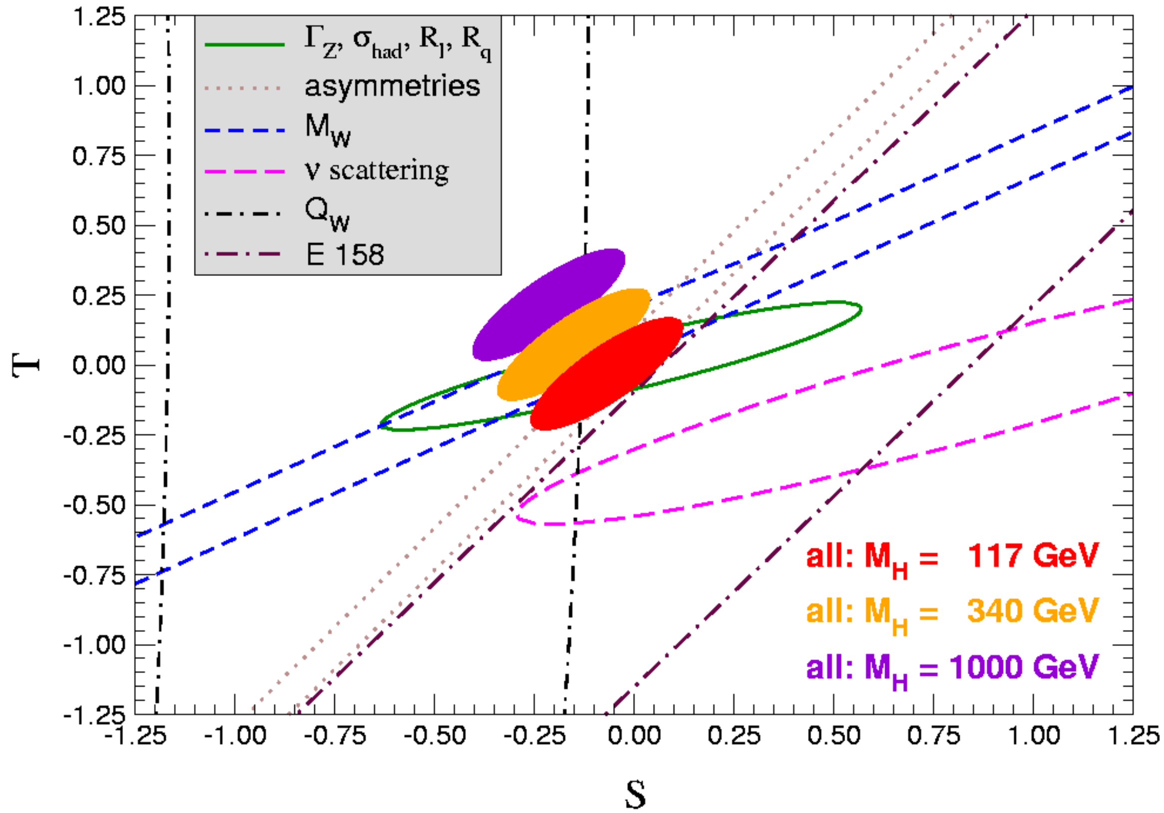
$$
\begin{aligned}
M_W^2 &= \frac{1}{4\sqrt{2}G_F}\frac{e^2}{s^2_{M_W}} \\
&= \frac{1}{4\sqrt{2}G_F}\frac{e^2}{s^2_{M_Z}}\left[1 - \frac{1}{s^2}(\Delta_{M_W} - \Delta_{M_Z}) + (\text{SM correction})\right] \\
&= \frac{1}{4\sqrt{2}G_F}\frac{e^2}{s^2_{M_Z}}\left[1 - 0.930 \times \alpha S + 1.43 \times \alpha T + 1.08 \times \alpha U + (\text{SM corr.})\right]
\end{aligned}
$$

SM ambiguities

S, T are defined as deviations from the SM: we should be careful...

$$
\begin{aligned}
S &\simeq \frac{1}{12\pi}\ln\frac{M_H^2}{M_{H,\mathrm{ref}}^2} + \frac{1}{6\pi}\ln\frac{m_t^2}{m_{t,\mathrm{ref}}^2} \\
T &\simeq -\frac{3}{16\pi c^2}\ln\frac{M_H^2}{M_{H,\mathrm{ref}}^2} + \frac{3}{16\pi s^2 c^2}\frac{m_t^2 - m_{t,\mathrm{ref}}^2}{M_Z^2} \\
U &\simeq \frac{1}{2\pi}\ln\frac{m_t^2}{m_{t,\mathrm{ref}}^2}
\end{aligned}
$$

S-T plot of Erler-Langacker review in RPP2006



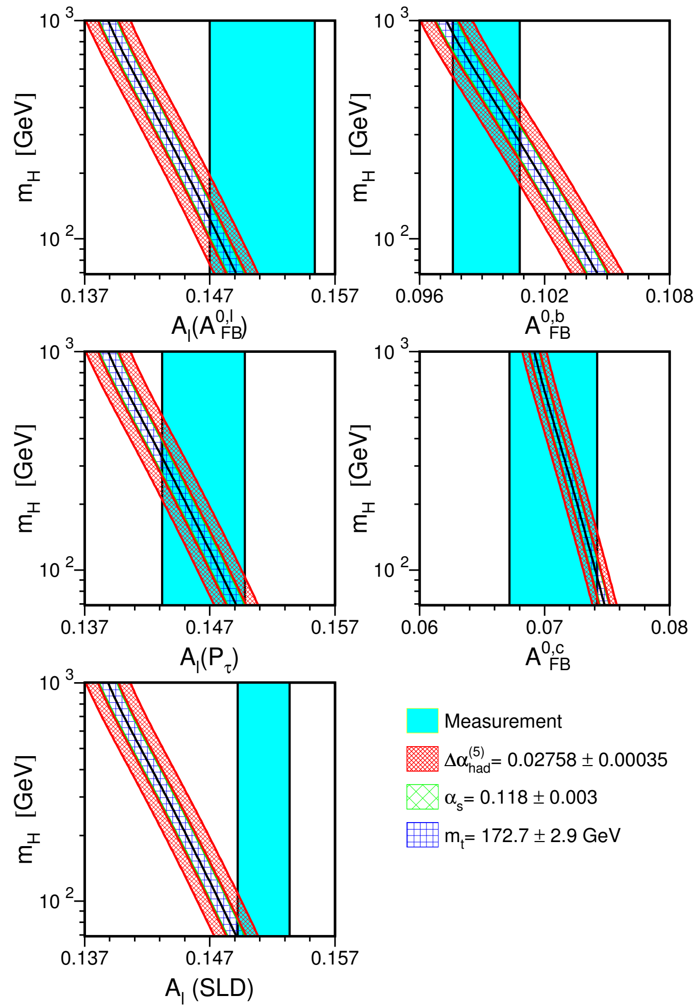
$$
\begin{aligned}
M_H^{\rm ref} &= 117\text{GeV} \\
S &= -0.13 \pm 0.10 \\
T &= -0.13 \pm 0.11 \\
U &= 0.20 \pm 0.12
\end{aligned}
$$

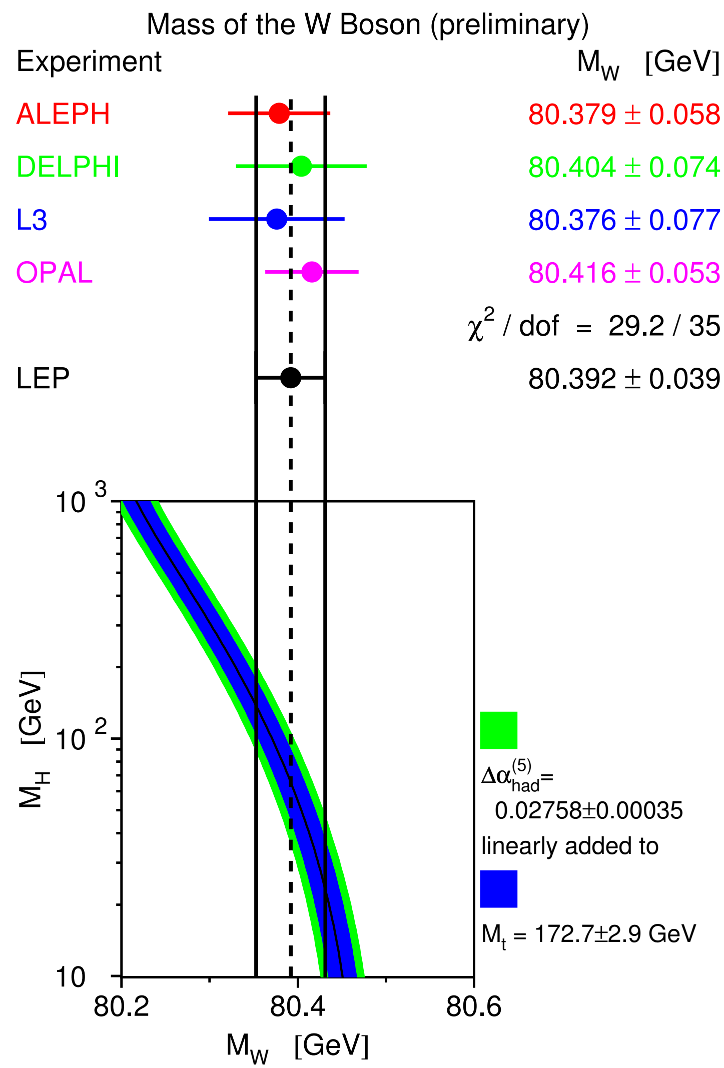
$$
\begin{aligned}
M_H^{\rm ref} &= 300\text{GeV} \\
S &= -0.21 \pm 0.10 \\
T &= -0.04 \pm 0.11 \\
U &= 0.21 \pm 0.12,
\end{aligned}
$$

90%CL

Erler and Langacker, in RPP2006

§.4 Higgs mass





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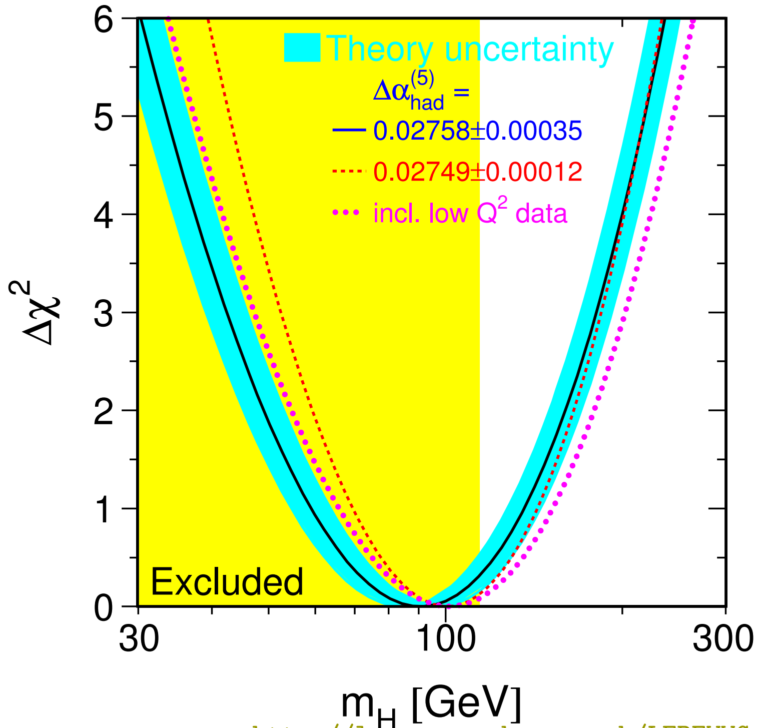

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$M_H \leq 199\text{GeV}$ (95% CL)

Heavy Higgs looks inconsistent with precision data!

We need to be careful: this bound is relaxed significantly if there exists other positive (negative) contribution to the T-parameter (S-parameter). Higgs mass heavier than this bound indicates the presence of BSM.

References

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- J. Erler and P. Langacker, "Electroweak model and constraints on new physics", in Review of Particle Physics, J. of Physics G33, 1 (2006).
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