

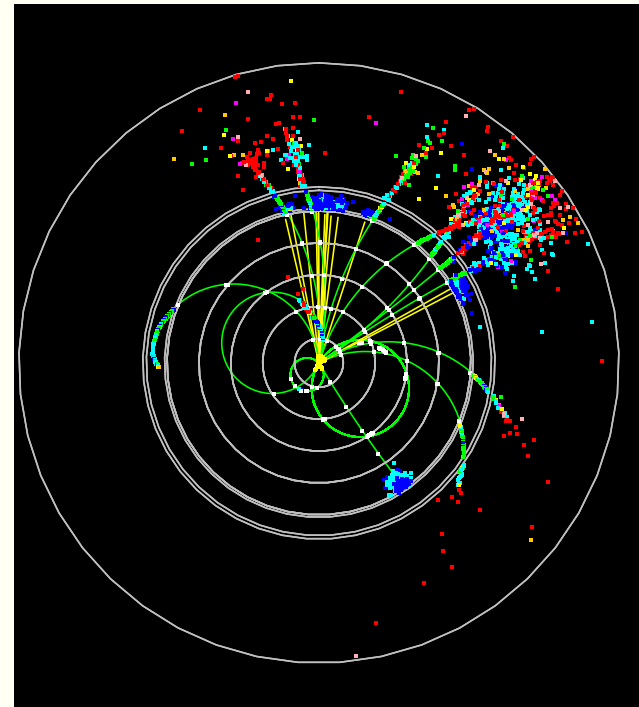
# Lecture 1. Supersymmetry: Introduction

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## ★ Outline

- Briefly, the Standard Model
- What is supersymmetry (SUSY)?
- Motivations for SUSY
- The Wess-Zumino model
- SUSY gauge theories
- SUSY breaking



# The Standard Model of Particle Physics

- ★ gauge symmetry:  $SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow g_{\mu A}, W_{\mu i}, B_\mu$
- ★ matter content: 3 generations quarks and leptons

$$\left( \begin{array}{c} u \\ d \end{array} \right)_L, u_R, d_R; \left( \begin{array}{c} \nu \\ e \end{array} \right)_L, e_R \quad (1)$$

- ★ Higgs sector  $\Rightarrow$  spontaneous electroweak symmetry breaking:

$$\phi = \left( \begin{array}{c} \phi^+ \\ \phi_0 \end{array} \right) \quad (2)$$

- ★  $\Rightarrow$  massive  $W^\pm, Z^0$ , massless  $\gamma$ , massive quarks and leptons
- ★  $\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} + \mathcal{L}_{Yuk.} + \mathcal{L}_{Higgs}$ : 19 parameters
- ★ good-to-excellent description of (almost) *all* accelerator data!

# Shortcomings of SM

## Data

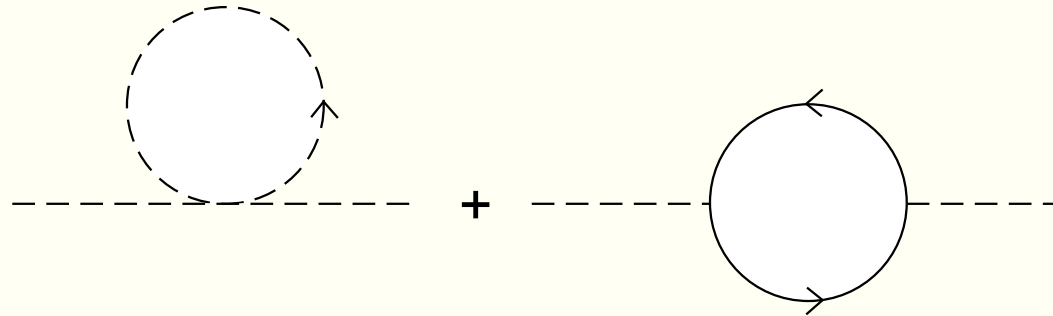
- ★ neutrino masses and mixing
- ★ baryogenesis (matter anti-matter asymmetry)
- ★ cold dark matter
- ★ dark energy

## Theory

- ★ quadratic divergences in scalar sector  $\Rightarrow$  fine-tuning
- ★ origin of generations
- ★ explanation of masses
- ★ origin of gauge symmetry/ quantum numbers
- ★ unification with gravity

## Problem of scalar quadratic divergences

### ★ Structure of scalar self energies



- $$\delta m_{H_{\text{SM}}}^2 = \langle H_{\text{SM}} | \frac{g^2 m_{H_{\text{SM}}}^2}{32M_W^2} H_{\text{SM}}^4 | H_{\text{SM}} \rangle = 12 \frac{g^2 m_{H_{\text{SM}}}^2}{32M_W^2} \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m_{H_{\text{SM}}}^2}$$

$$= \frac{12g^2 m_{H_{\text{SM}}}^2}{32M_W^2} \frac{1}{16\pi^2} \left( \Lambda^2 - m_{H_{\text{SM}}}^2 \ln \frac{\Lambda^2}{m_{H_{\text{SM}}}^2} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) \right)$$
- $$m_{H_{\text{SM}}}^2(\text{phys}) \simeq m_{H_{\text{SM}}}^2 + \frac{c}{16\pi^2} \Lambda^2$$
- If  $\Lambda \sim 10^{16}$  GeV, then fine-tuning to 1 part in  $10^{26}$  needed
- Alternatively, if  $\Lambda \sim 1$  TeV, then no fine-tuning, but then SM is valid only below 1 TeV scale

# Supersymmetry

- ★ Supersymmetry (SUSY) is a spacetime symmetry
  - most general extension of Poincare group
  - a quantum symmetry- no classical analogue
  - introduce spinorial generators  $Q$
  - relates fermions to bosons:  $Q|B\rangle = |F\rangle$
  - SUSY is a “square-root” of a translation
- ★ discovered gradually from  $\sim 1968$  to 1974
  - introduce fermions into string theory
  - non-linear realizations
  - Wess-Zumino 1974: first 4-d SUSY QFT

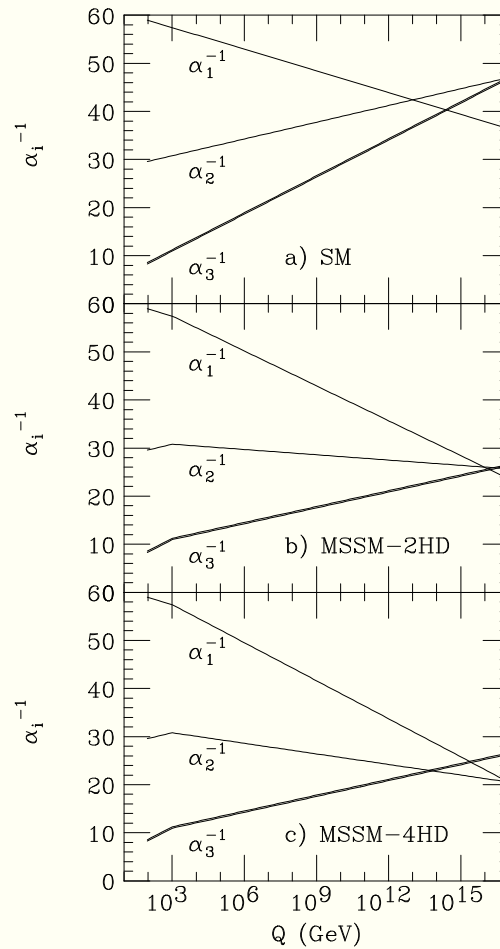
## Supersymmetry motivations (theory)

- Aesthetics
  - nature likely to use most general extension of Poincaré group at some level
- ultra-violet behavior and fine-tuning:  $m_{SUSY} \sim 1$  TeV!
- connection to gravity: local SUSY = supergravity
- ultraviolet completeness: desert all the way to  $M_{GUT}$ ?
- connection to strings: world-sheet SUSY vs. space-time SUSY

## Supersymmetry motivations (experiment)

- gauge coupling unification: connection to GUTs

# Gauge coupling evolution

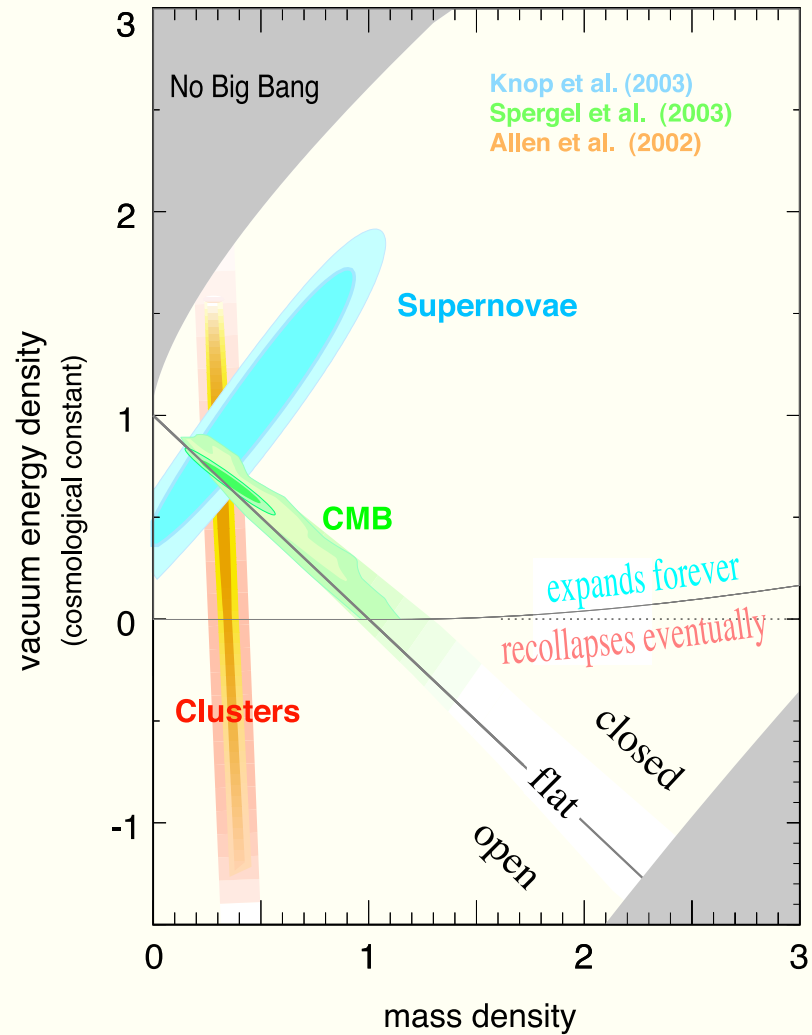




## Supersymmetry motivations (experiment)

- gauge coupling unification: connection to GUTs
- cold dark matter

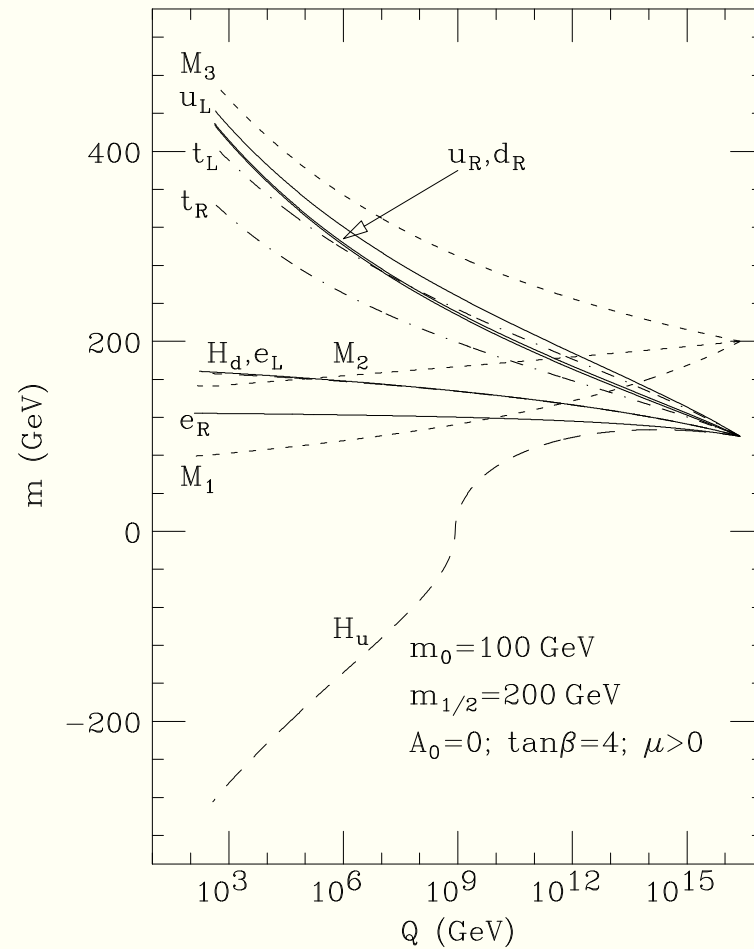
# Dark matter versus dark energy



## Supersymmetry motivations (experiment)

- gauge coupling unification: connection to GUTs
- cold dark matter
- radiative breakdown of EW symmetry (if  $m_t \sim 100 - 200$  GeV)

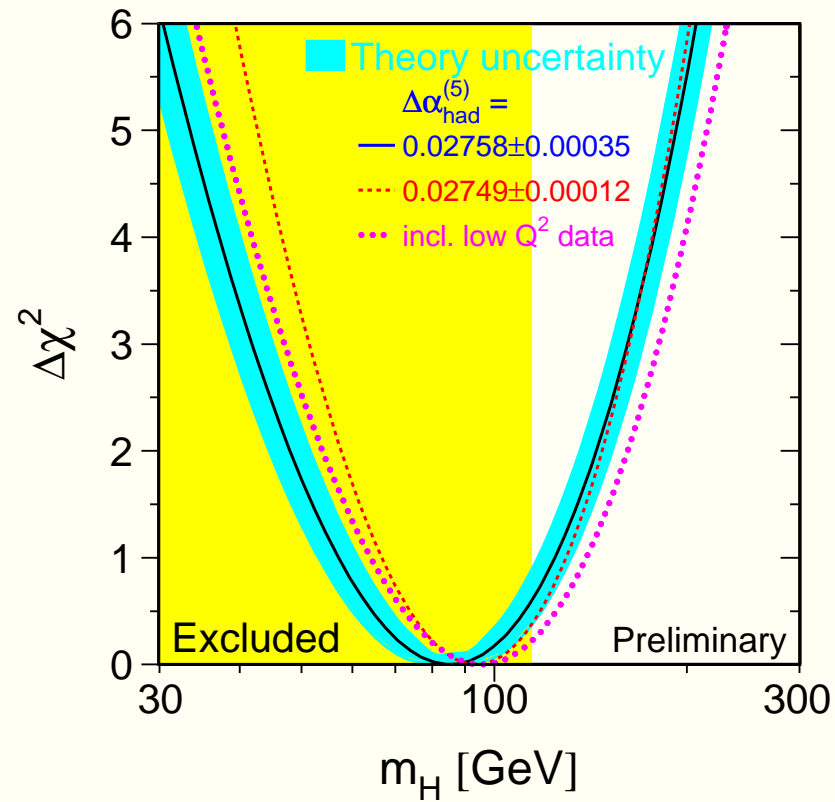
## Soft term evolution and radiative EWSB



## Supersymmetry motivations (experiment)

- gauge coupling unification: connection to GUTs
- cold dark matter
- radiative breakdown of EW symmetry (if  $m_t \sim 100 - 200$  GeV)
- decoupling in SUSY theories: tiny radiative corrections if  $m_{SUSY} \gtrsim 100$  GeV
- mass of the Higgs boson: MSSM  $\Rightarrow m_h \lesssim 135$  GeV

## Likelihood distribution of Higgs mass



## Supersymmetry motivations (experiment)

- gauge coupling unification: connection to GUTs
- cold dark matter
- radiative breakdown of EW symmetry (if  $m_t \sim 100 - 200$  GeV)
- decoupling in SUSY theories: tiny radiative corrections if  $m_{SUSY} \gtrsim 100$  GeV
- mass of the Higgs boson: MSSM  $\Rightarrow m_h \lesssim 135$  GeV
- mechanisms for baryogenesis: EW, leptogenesis, Affleck-Dine
- neutrino mass: stabilize see-saw scale; GUTs

**Focus of these lectures is on *supersymmetry***

“if we consider the main classes of new physics that are currently being contemplated... , it is clear that (supersymmetry) is the most directly related to GUTs. SUSY offers a well defined model computable up to the GUT scale and is actually supported by the quantitative success of coupling unification in SUSY GUTs. For the other examples... , all contact with GUTs is lost or at least is much more remote. ... the SUSY picture... remains the standard way beyond the Standard Model”

G. Altarelli and F. Feruglio, hep-ph/0306265



## Wess-Zumino toy SUSY model: 1974

- $\mathcal{L} = \mathcal{L}_{\text{kin.}} + \mathcal{L}_{\text{mass}}$ 
  - $\mathcal{L}_{\text{kin.}} = \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 + \frac{i}{2}\bar{\psi} \not{\partial}\psi + \frac{1}{2}(F^2 + G^2)$
  - $\mathcal{L}_{\text{mass}} = -m[\frac{1}{2}\bar{\psi}\psi - GA - FB]$
- $A$  and  $B$  are real scalar fields with  $[A] = [B] = 1$
- $\psi$  is a *Majorana* spinor with  $\psi = C\bar{\psi}^T$  and  $[\psi] = \frac{3}{2}$
- $F$  and  $G$  are *auxiliary* (non-propagating) fields with  $[F] = [G] = 2$ 
  - can be eliminated by E-L equations:  $F = -mB$ ,  $G = -mA$

## SUSY transformation in WZ model

- $A \rightarrow A + \delta A$  with  $\delta A = i\bar{\alpha}\gamma_5\psi$
- $\delta B = -\bar{\alpha}\psi,$
- $\delta\psi = -F\alpha + iG\gamma_5\alpha + \not{\partial}\gamma_5 A\alpha + i\not{\partial}B\alpha,$
- $\delta F = i\bar{\alpha}\not{\partial}\psi,$
- $G = \bar{\alpha}\gamma_5\not{\partial}\psi$

Using Majorana bilinear re-arrangements (e.g.  $\bar{\psi}\chi = -\bar{\chi}\psi$ ) and product rule  $\partial_\mu(f \cdot g) = \partial_\mu f \cdot g + f \cdot \partial_\mu g$  and algebra, can show that  $\mathcal{L} \rightarrow \mathcal{L} + \delta\mathcal{L}$  with

- $\delta\mathcal{L}_{\text{kin}} = \partial^\mu\left(-\frac{1}{2}\bar{\alpha}\gamma_\mu\not{\partial}B\psi + \frac{i}{2}\bar{\alpha}\gamma_5\gamma_\mu\not{\partial}A\psi + \frac{i}{2}F\bar{\alpha}\gamma_\mu\psi + \frac{1}{2}G\bar{\alpha}\gamma_5\gamma_\mu\psi\right),$
- $\delta\mathcal{L}_{\text{mass}} = \partial^\mu(mA\bar{\alpha}\gamma_5\gamma_\mu\psi + imB\bar{\alpha}\gamma_\mu\psi)$

Since Lagrangian changes by a total derivation, the *action*  $S = \int \mathcal{L}d^4x$  is invariant! (owing to Gauss' law in 4-d)  $\int_V d^4x\partial_\mu\Lambda^\mu = \int_{\partial V} d\sigma\Lambda^\mu n_\mu$  Thus, WZ transformation is a *symmetry* of the action!

## Aspects of the WZ model:

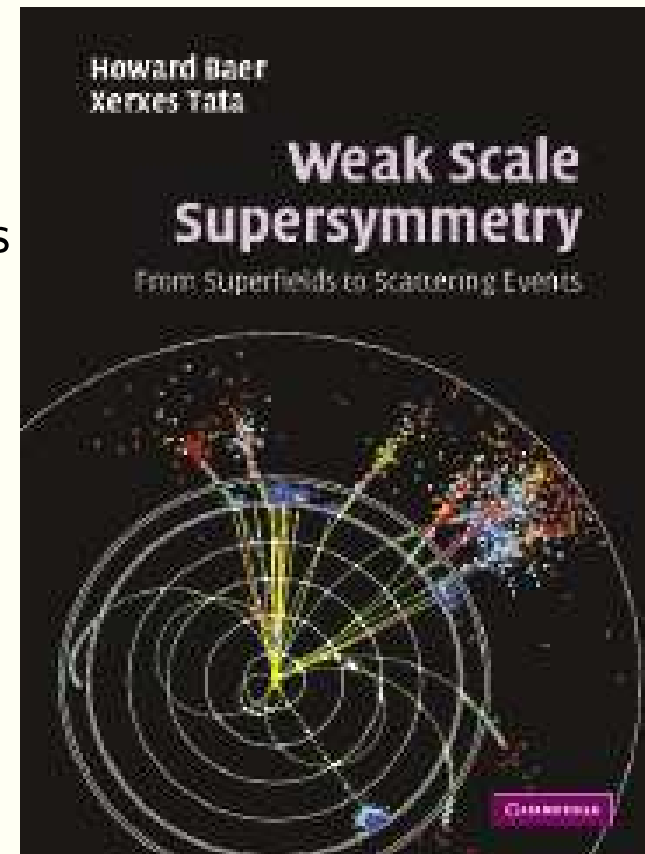
- Can add interactions:
- $\mathcal{L}_{\text{int}} = -\frac{g}{\sqrt{2}}A\bar{\psi}\psi + \frac{ig}{\sqrt{2}}B\bar{\psi}\gamma_5\psi + \frac{g}{\sqrt{2}}(A^2 - B^2)G + g\sqrt{2}ABF$
- Difficult calculation, but can show  $\delta\mathcal{L} \rightarrow$  total derivative
- Also, can show: quadratic divergences all *cancel!*
- If SUSY transformations expressed as  $\mathcal{S} \rightarrow \mathcal{S}' = e^{i\bar{\alpha}Q}\mathcal{S}e^{-i\bar{\alpha}Q} \approx \mathcal{S} + [i\bar{\alpha}Q, \mathcal{S}] = \mathcal{S} + \delta\mathcal{S} \equiv (1 - i\bar{\alpha}Q)\mathcal{S}$ , then can show that the generator  $Q$  obeys
  - $\{Q_a, \bar{Q}_b\} = 2(\gamma_\mu)_{ab}P_\mu$
  - SUSY is spacetime symmetry! (super-Poincaré algebra)

# Weak Scale Supersymmetry

HB and X. Tata

Spring, 2006; Cambridge University Press

- ★ Part 1: superfields/Lagrangians
  - 4-component spinor notation for exp'ts
  - master Lagrangian for SUSY gauge theories
- ★ Part 2: models/implications
  - MSSM, SUGRA, GMSB, AMSB, ...
- ★ Part 3: SUSY at colliders
  - production/decay/event generation
  - collider signatures
  - $R$ -parity violation



## Constructing supersymmetric models

- While the WZ model is interesting, it was essentially “taken out of a hat”, without rules of how to construct SUSY models in general
- Shortly after WZ model appeared in 1974, Salam and Strathdee developed *superfield formalism*, which *does* allow one to construct SUSY models in general.
- How does one combine scalar and spinor fields into a single entity?
- introduce *superspace*  $x^\mu \rightarrow (x^\mu, \theta_a)$  where  $\theta_a$  ( $a = 1 - 4$ ) are four *anti*-commuting dimensions arranged as a Majorana spinor

## Some types of superfields

- general superfield:

$$\hat{\Phi}(x, \theta) = \mathcal{S} - i\sqrt{2}\bar{\theta}\gamma_5\psi - \frac{i}{2}(\bar{\theta}\gamma_5\theta)\mathcal{M} + \frac{1}{2}(\bar{\theta}\theta)\mathcal{N} + \frac{1}{2}(\bar{\theta}\gamma_5\gamma_\mu\theta)V^\mu \\ + i(\bar{\theta}\gamma_5\theta)\left[\bar{\theta}\left(\lambda + \frac{i}{\sqrt{2}}\not{\partial}\psi\right)\right] - \frac{1}{4}(\bar{\theta}\gamma_5\theta)^2[\mathcal{D} - \frac{1}{2}\square\mathcal{S}]$$

- left chiral scalar superfield:  $\hat{\mathcal{S}}_L(x, \theta) = \mathcal{S}(\hat{x}) + i\sqrt{2}\bar{\theta}\psi_L(\hat{x}) + i\bar{\theta}\theta_L\mathcal{F}(\hat{x})$  where  $\hat{x}_\mu = x_\mu + \frac{i}{2}\bar{\theta}\gamma_5\gamma_\mu\theta$
- right chiral scalar superfield:  $\hat{\mathcal{S}}_R(x, \theta) = \mathcal{S}(\hat{x}^\dagger) - i\sqrt{2}\bar{\theta}\psi_R(\hat{x}^\dagger) - i\bar{\theta}\theta_R\mathcal{F}(\hat{x}^\dagger)$
- multiplication rules
  - LCSSF  $\times$  LCSSF = LCSSF
  - RCSSF  $\times$  RCSSF = RCSSF
  - LCSSF  $\times$  RCSSF = general superfield
- $D$ -term of general SF transforms to total derivative
- $F$ -term of LCSSF or RCSSF transforms into total derivative

## Supersymmetric Lagrangians

- Since  $D$  and  $F$  terms transform into total derivatives, they are candidates for SUSY Lagrangians!
- The *superpotential*  $\hat{f}$  is a function of LCSSFs only. Hence it is itself a LCSSF, and its  $F$  term is a candidate Lagrangian
- The *Kähler potential*  $K$  is a function of LCSSFs times RCSSFs. Hence it is a general superfield and its  $D$  term is a candidate Lagrangian
- Augmenting the superfields with *gauge* superfields  
 $\hat{\Phi}_A = \frac{1}{2}(\bar{\theta}\gamma_5\gamma_\mu\theta)V_A^\mu + i\bar{\theta}\gamma_5\theta \cdot \bar{\theta}\lambda_A - \frac{1}{4}(\bar{\theta}\gamma_5\theta)^2\mathcal{D}_A$  (in WZ gauge) or  
 $\hat{W}_A(\hat{x}, \theta) = \lambda_{LA}(\hat{x}) + \frac{1}{2}\gamma^\mu\gamma^\nu F_{\mu\nu A}(\hat{x})\theta_L - i\bar{\theta}\theta_L(\not{D}\lambda_R)_A - i\mathcal{D}_A(\hat{x})\theta_L$  allows one to write a *Master formula* for supersymmetric gauge theories!

## Master formula for SUSY gauge theories

$$\begin{aligned}
 \mathcal{L} = & \sum_i (D_\mu \mathcal{S}_i)^\dagger (D^\mu \mathcal{S}_i) + \frac{i}{2} \sum_i \bar{\psi}_i \not{D} \psi_i + \sum_{\alpha, A} \left[ \frac{i}{2} \bar{\lambda}_{\alpha A} (\not{D} \lambda)_{\alpha A} - \frac{1}{4} F_{\mu\nu\alpha A} F_{\alpha A}^{\mu\nu} \right] \\
 & - \sqrt{2} \sum_{i, \alpha, A} \left( \mathcal{S}_i^\dagger g_\alpha t_{\alpha A} \bar{\lambda}_{\alpha A} \frac{1 - \gamma_5}{2} \psi_i + \text{h.c.} \right) \\
 & - \frac{1}{2} \sum_{\alpha, A} \left[ \sum_i \mathcal{S}_i^\dagger g_\alpha t_{\alpha A} \mathcal{S}_i + \xi_{\alpha A} \right]^2 - \sum_i \left| \frac{\partial \hat{f}}{\partial \hat{\mathcal{S}}_i} \right|_{\hat{\mathcal{S}}=S}^2 \\
 & - \frac{1}{2} \sum_{i, j} \bar{\psi}_i \left[ \left( \frac{\partial^2 \hat{f}}{\partial \hat{\mathcal{S}}_i \partial \hat{\mathcal{S}}_j} \right)_{\hat{\mathcal{S}}=S} \frac{1 - \gamma_5}{2} + \left( \frac{\partial^2 \hat{f}}{\partial \hat{\mathcal{S}}_i \partial \hat{\mathcal{S}}_j} \right)_{\hat{\mathcal{S}}=S}^\dagger \frac{1 + \gamma_5}{2} \right] \psi_j,
 \end{aligned}$$

where the covariant derivatives are given by,

$$D_\mu \mathcal{S} = \partial_\mu \mathcal{S} + i \sum_{\alpha, A} g_\alpha t_{\alpha A} V_{\mu\alpha A} \mathcal{S},$$



$$D_\mu \psi = \partial_\mu \psi + i \sum_{\alpha, A} g_\alpha (t_{\alpha A} V_{\mu \alpha A}) \psi_L - i \sum_{\alpha, A} g_\alpha (t_{\alpha A}^* V_{\mu \alpha A}) \psi_R,$$

$$(\not{D}\lambda)_{\alpha A} = \not{\partial}\lambda_{\alpha A} + i g_\alpha \left( t_{\alpha B}^{adj} Y_{\alpha B} \right)_{AC} \lambda_{\alpha C},$$

$$F_{\mu\nu\alpha A} = \partial_\mu V_{\nu\alpha A} - \partial_\nu V_{\mu\alpha A} - g_\alpha f_{\alpha ABC} V_{\mu\alpha B} V_{\nu\alpha C}.$$

## Supersymmetry breaking

- Spontaneous breaking of global SUSY is possible:  
 $\langle 0|\mathcal{F}_i|0\rangle \neq 0$  or  $\langle 0|\mathcal{D}_A|0\rangle \neq 0$  ( $F$  or  $D$  type breaking)
- May also explicitly break SUSY by adding *soft* SUSY breaking terms to  $\mathcal{L}$ :
  - linear terms in the scalar field  $\mathcal{S}_i$  (relevant only for singlets of all symmetries),
  - scalar masses,
  - and bilinear or trilinear operators of the form  $\mathcal{S}_i\mathcal{S}_j$  or  $\mathcal{S}_i\mathcal{S}_j\mathcal{S}_k$  (where  $\hat{\mathcal{S}}_i\hat{\mathcal{S}}_j$  and  $\hat{\mathcal{S}}_i\hat{\mathcal{S}}_j\hat{\mathcal{S}}_k$  occur in the superpotential),
  - and finally, in gauge theories, gaugino masses, one for each factor of the gauge group,

## Recipe for SUSY model building

- Choose the gauge symmetry (adopting appropriate gauge superfields for each gauge symmetry)
- Choose matter and Higgs representations included as LCSSFs
- Choose the superpotential  $\hat{f}$  as a gauge invariant *analytic function* of LCSSFs; degree is  $\leq 3$  for renormalizable theory
- Adopt all allowed gauge invariant soft SUSY breaking terms; these are generally chosen to *parametrize our ignorance* of the mechanism of SUSY breaking
- The Master formula, augmented by the soft SUSY breaking terms, gives the final Lagrangian of the theory.

## Next lecture:

- Using the *Master formula*, we may write down the MSSM
- MSSM includes the SM as a sub-theory, but much more
- Since quadratic divergences cancel, MSSM may be a theory valid *e.g.* from  $M_{weak}$  to  $M_{GUT}$
- RGEs, gauge coupling evolution, REWSB, spectrum calculation
- supergravity
- Grand unified theories:
  - $SU(5)$
  - $SO(10)$