The 3-site Higgsless Model

Elizabeth H. Simmons
Michigan State University

- Review of General Principles
- A Simple 3-Site Model
- Unitarity Delay in the 3-site model
- The 3-site model and Experiment
- Conclusions

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Refs: hep-ph/0607124 ; 0607191 ; 0608020

Higgsless Models and Ideal Delocalization:

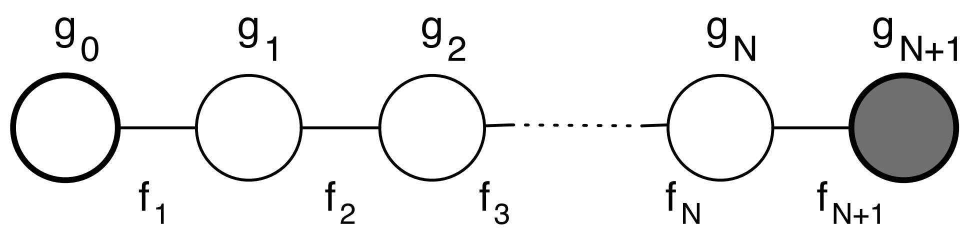
Review of General Principles

Previously discussed :

Higgsless models are low-energy effective theories of dynamical electroweak symmetry breaking including the following elements

- massive 4-d gauge bosons arise in the context of a 5-d gauge theory with appropriate boundary conditions
- WW scattering unitarized through exchange of KK modes (instead of Higgs exchange)
- language of Deconstruction allows a 4-d "Moose" representation of the model

Deconstructed Higgsless Models

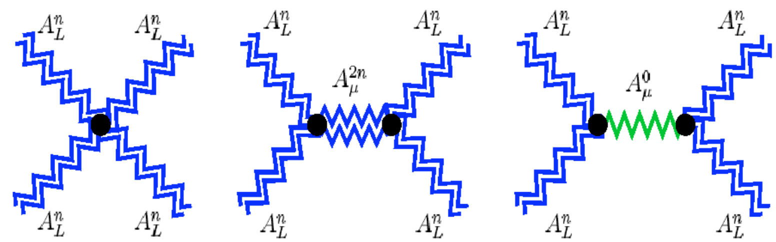


- 5th dimension discretized
- $SU(2)^N \times U(1)$; general f_j and g_k encompass spatially-dependent couplings, warping
- Localized fermions sit on "branes" [sites 0 and N+1] but these present difficulties

Foadi, *et. al.* & Chivukula *et. al.*

Conflict of S & Unitarity

Heavy resonances must unitarize WW scattering (since there is no Higgs!)



This bounds lightest KK mode mass: $m_{Z_1} < \sqrt{8\pi}\, v$

... and yields a value of the S-parameter that is

$$\alpha\, S \geq \frac{4 s_Z^2 c_Z^2 M_Z^2}{8\pi v^2} = \frac{\alpha}{2}$$

too large by a factor of a few!

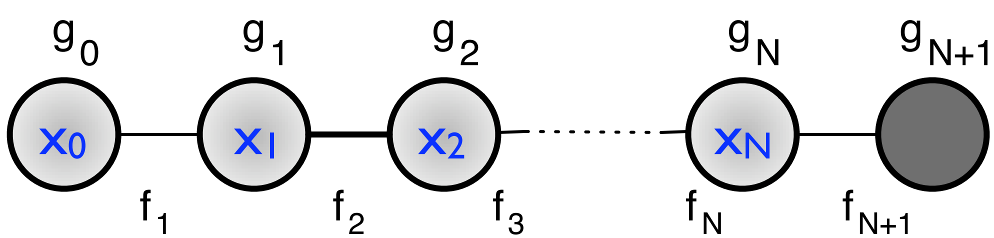
Independent of warping or gauge couplings chosen...

Delocalized Fermions

Delocalized Fermions, .i.e., mixing of "brane" and "bulk" modes

$$\mathcal{L}_f = \vec{J}^\mu_L \cdot \left(\sum_{i=0}^{N} x_i \vec{A}^i_\mu\right) + J^\mu_Y A^{N+1}_\mu$$

Can Reduce Contribution to S!



Cacciapaglia, Csaki, Grojean, & Terning

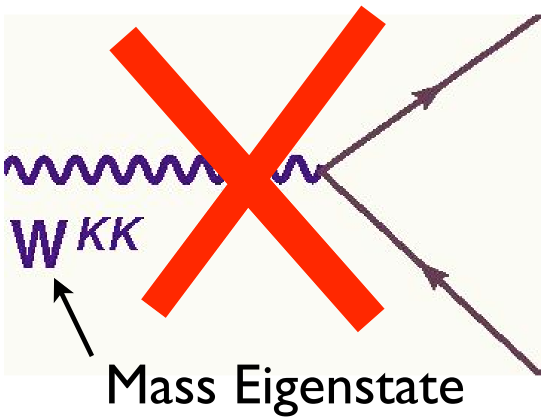
Foadi, Gopalkrishna, & Schmidt

Ideal Fermion Delocalization

- Recall that the light W's wavefunction is orthogonal to wavefunctions of KK modes
- Choose fermion delocalization profile to match W wavefunction profile along the 5th dimension:

$$g_i x_i \propto v_i^W$$

- No (tree-level) fermion couplings to KK modes!



$$\hat{S} = \hat{T} = W = 0$$

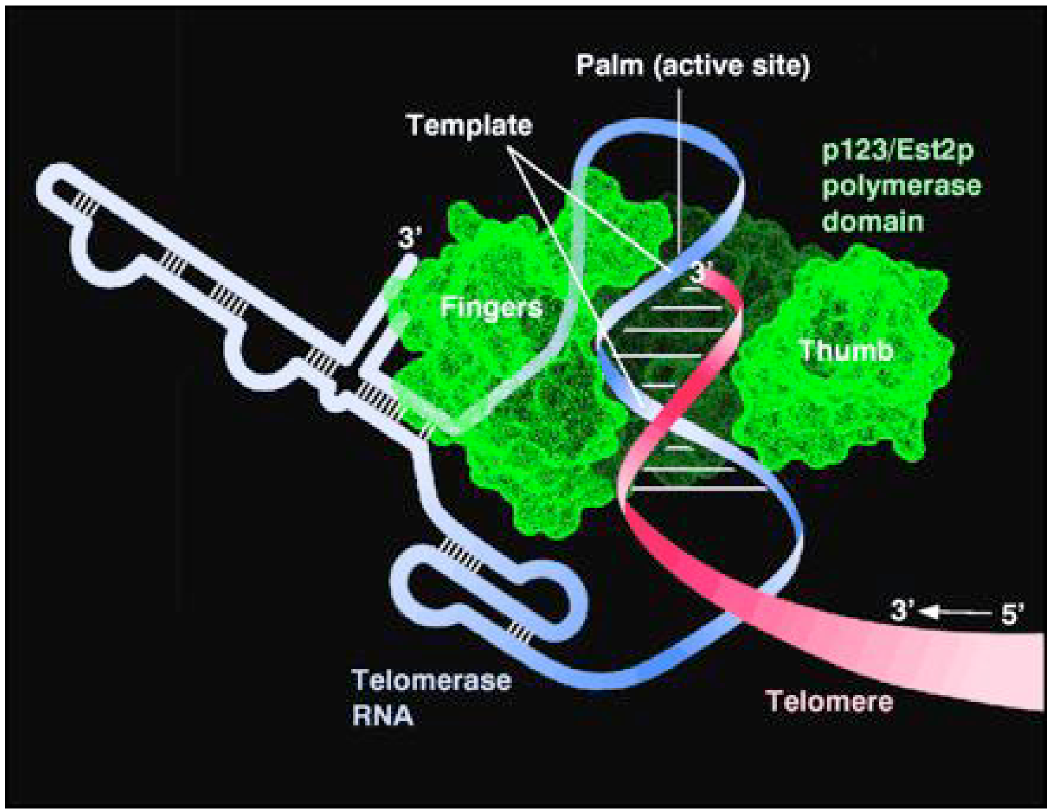
$$Y = M_W^2(\Sigma_W - \Sigma_Z)$$

RSC, HJH, MK, MT, EHS hep-ph/0504114

The 3-site Model:

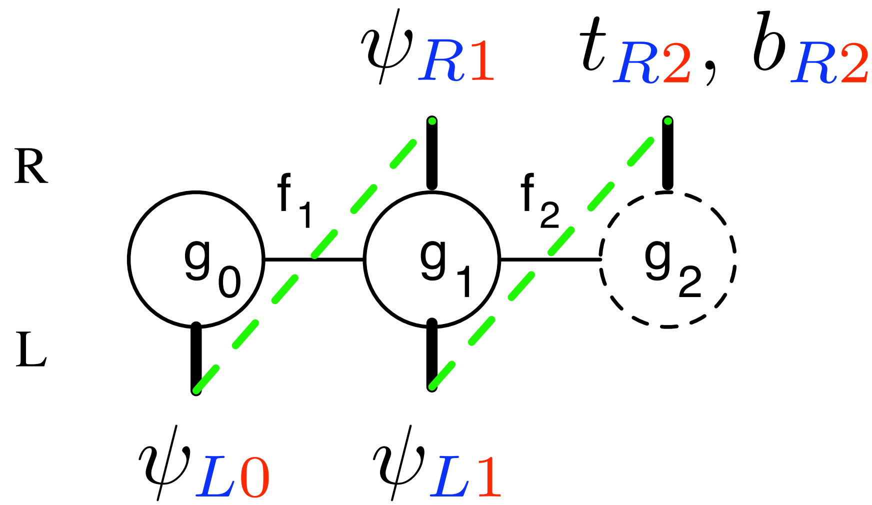
general principles in action

Three-site model in biology



3-Site Model: basic structure

$SU(2) \times SU(2) \times U(1)$ $g_0,\ g_2 \ll g_1$



Gauge boson spectrum: photon, Z, Z', W, W'

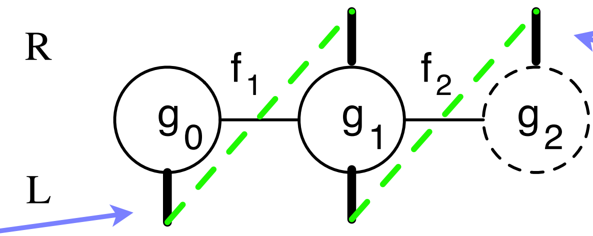
Fermion spectrum: t, T, b, B (ψ is an SU(2) doublet)

and also c, C, s, S, u, U, d, D plus the leptons

3-Site Model: fermion details

$SU(2) \times SU(2) \times U(1)$

$g_0,\ g_2 \ll g_1$



RH Boundary Fermion

LH Boundary Fermion

"Bulk Fermion"

$$\mathcal{L}_f = \varepsilon_L M\, \bar{\psi}_{L0}\Sigma_1\psi_{R1} + M\, \bar{\psi}_{R1}\psi_{L1} + M\, \bar{\psi}_{L1}\Sigma_2 \begin{pmatrix} \varepsilon_{tR} & \\ & \varepsilon_{bR} \end{pmatrix} \begin{pmatrix} t_{R2} \\ b_{R2} \end{pmatrix} + h.c.$$

related to degree of delocalization

Fermion Structure Motivated by 5-D

Flavor Structure Identical to Standard Model

3-Site Wavefunctions

Diagonalize W mass matrix $x = g_0/g_1 \ll 1$

$$\frac{\tilde{g}^2 v^2}{2}\begin{pmatrix} x^2 & -x \\ -x & 2 \end{pmatrix}$$

to obtain

$$x^2 = 2\frac{M_W^2}{M_{W'}^2}$$

$$\begin{aligned} W^\mu &= v_W^0\, W_0^\mu + v_W^1\, W_1^\mu \\ &= \left(1 - \frac{x^2}{8} - \frac{5x^4}{128} + \ldots\right) W_0^\mu + \left(\frac{x}{2} + \frac{x^3}{16} - \frac{9x^5}{256} + \ldots\right) W_1^\mu \end{aligned}$$

Diagonalize fermion mass matrix

$$M_{u,d} = \sqrt{2}\tilde{\lambda} v \begin{pmatrix} \varepsilon_L & 0 \\ 1 & \varepsilon_{uR,dR} \end{pmatrix} \qquad m_f \approx \frac{\epsilon_L \epsilon_{fR} M}{\sqrt{1+\epsilon_{fR}^2}}$$

$$\begin{aligned} t_L &= t_L^0\, \psi_{L0}^t + t_L^1\, \psi_{L1}^t \\ &= \left(-1 + \frac{\varepsilon_L^2}{2(1+\varepsilon_{tR}^2)^2} + \frac{(8\varepsilon_{tR}^2-3)\varepsilon_L^4}{8(\varepsilon_{tR}^2+1)^4} + \ldots\right)\psi_{L0}^t + \left(\frac{\varepsilon_L}{1+\varepsilon_{tR}^2} + \frac{(2\varepsilon_{tR}^2-1)\varepsilon_L^3}{2(\varepsilon_{tR}^2+1)^3} + \ldots\right)\psi_{L1}^t \end{aligned}$$

$$\begin{aligned} t_R &= t_R^1\, \psi_{R1}^t + t_R^2\, t_{R2} \\ &= \left(-\frac{\varepsilon_{tR}}{\sqrt{1+\varepsilon_{tR}^2}} + \frac{\varepsilon_{tR}\,\varepsilon_L^2}{(1+\varepsilon_{tR}^2)^{5/2}} + \ldots\right)\psi_{R1}^t + \left(\frac{1}{\sqrt{1+\varepsilon_{tR}^2}} + \frac{\varepsilon_{tR}^2\,\varepsilon_L^2}{(1+\varepsilon_{tR}^2)^{5/2}} + \ldots\right) t_{R2} \end{aligned}$$

3-Site Ideal Delocalization

General ideal delocalization condition $g_i(\psi_i^f)^2 = g_W v_i^w$

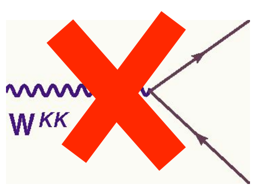
becomes $\frac{g_0(\psi_{L0}^f)^2}{g_1(\psi_{L1}^f)^2} = \frac{v_W^0}{v_W^1}$ in 3-site model

From W, fermion eigenvectors, solve for

$$\epsilon_L^2 \to (1+\epsilon_{fR}^2)^2 \left[\frac{x^2}{2} + \left(\frac{1}{8} - \frac{\epsilon_{fR}^2}{2}\right) x^4 + \frac{5\,\epsilon_{fR}^4\, x^6}{8} + \ldots\right]$$

For all but top, $\epsilon_{fR} \ll 1$ and $\varepsilon_L^2 = 2\left(\frac{M_W^2}{M_{W'}^2}\right) + 6\left(\frac{M_W^2}{M_{W'}^2}\right)^2 + \ldots$

insures W' and Z' are fermiophobic!



$$\hat{S} = \hat{T} = W = 0$$

$$Y = M_W^2(\Sigma_W - \Sigma_Z)$$

Use WW scattering to see W': Birkedal, Matchev, Perelstein hep-ph/0412278

Unitarity Delay in the 3-site Model

Elastic Unitarity

$$S^\dagger S = \mathcal{I} \quad \Rightarrow \quad |s_l|^2 = 1$$

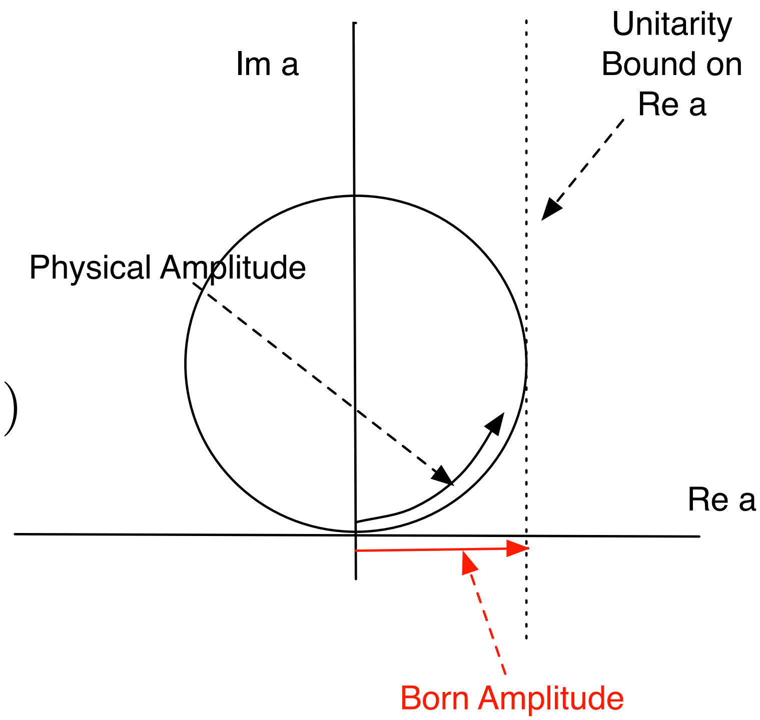
$$s_l = 1 + 2ia_l \quad \Rightarrow \quad \mathrm{Im}(a_l) = |a_l|^2$$

$$a_l = e^{i\delta_l} \sin\delta_l$$

$$a_l = \frac{1}{32(2)\pi} \left[\frac{4k^2}{s}\right]^{1/2} \int_{-1}^{1} d\cos\theta\, \mathcal{M}(s,\theta)$$

Identical Particle Factor

Feynman Amplitude

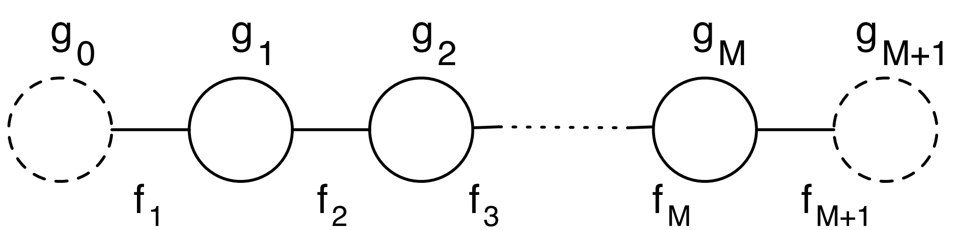


These formulae apply to the elastic scattering of pairs of particles of fixed helicity

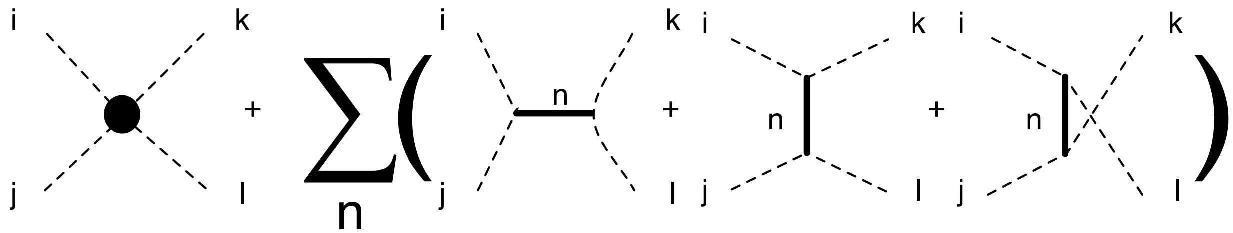
Jacob and Wick, 1959

Elastic $W_L W_L$ Scattering

- Longitudinal W's associated with bad high-energy behavior
- Work in a pure SU(2) model
- Equivalence Theorem: W_L properties same as "eaten" Goldstone Bosons
- Work in flat space



Pion Scattering in Flat Space



$$i\mathcal{A}(\pi^i \pi^j \to \pi^k \pi^l) = A(s,t,u)\delta^{ij}\delta^{kl} + A(t,s,u)\delta^{ik}\delta^{jl} + A(u,t,s)\delta^{il}\delta^{jk}$$

$$A(s,t,u) = (s-u)D(-t) + (s-t)D(-u)$$

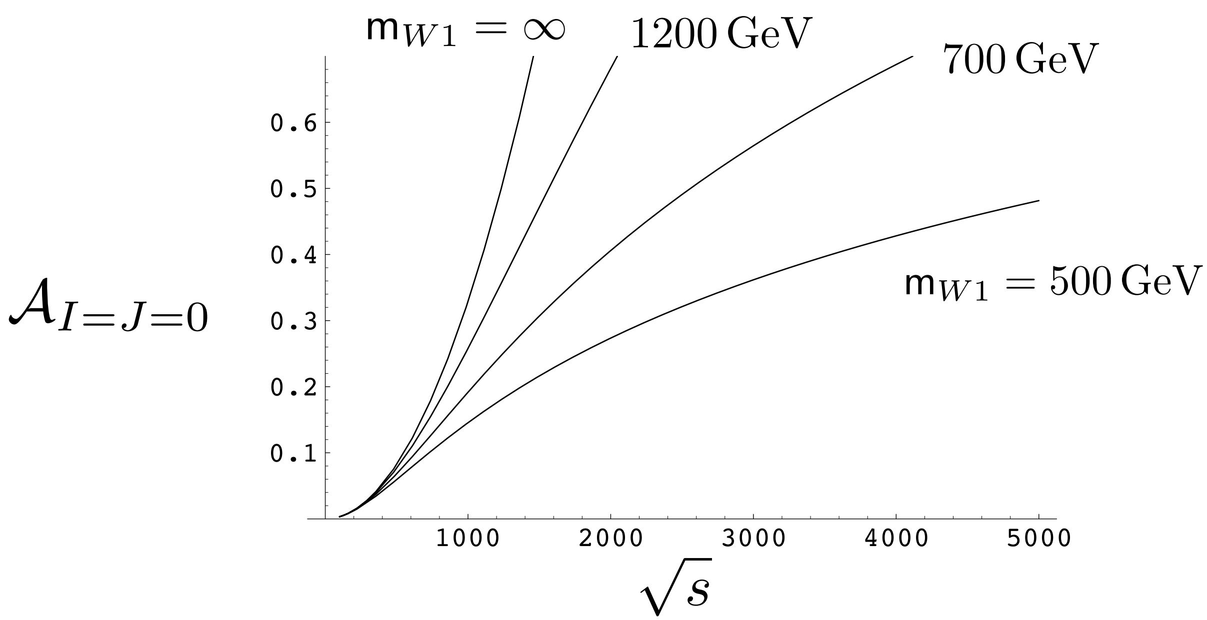
$$D(Q^2) = \sum_{n=1}^{M} \frac{g_{\pi\pi n}^2}{Q^2 + \mathsf{m}_{Wn}^2}$$

$$D(x) = \frac{4}{x^2\, v^2}\left[1 + \frac{2}{x\ \sinh x} - \frac{2\cosh x}{x\ \sinh x}\right] \qquad\qquad x = \frac{Q\pi}{\mathsf{m}_{W1}}$$

Isosinglet S-Wave Scattering

$$\mathcal{A}_{I=0}(s,\cos\theta) = 3A(s,t,u) + A(t,s,u) + A(u,t,s)$$
$$= 2(s-u)D(-t) + 2(s-t)D(-u)$$

$$\mathcal{A}_{I=J=0}(s) = \frac{1}{64\pi}\int_{-1}^{+1} d\cos\theta\, \mathcal{A}_{I=0}(s,\cos\theta) P_0(\cos\theta)$$



Elastic Unitarity Delay!

MultiChannel Unitarity

Spin-j: $S_j = \mathcal{I} + 2i\mathcal{T}_j \quad , \quad \mathcal{T}_j = \left[a_j^{\alpha\to\beta}\right]$

Rotation Coefficient

$$a_j^{\alpha\to\beta} = \frac{1}{32(\sqrt{2}_i)(\sqrt{2}_f)\pi}\left[\frac{4k_ik_f}{s}\right]^{1/2}\int_{-1}^{1} d\cos\theta\,\mathcal{M}^{\alpha\to\beta}(s,\cos\theta)d^j_{\Delta\lambda_i\Delta\lambda_f}(\theta)$$

Identical Particle Factors

Feynman Amplitude

$$S_j^\dagger S_j = \mathcal{I} \quad \Rightarrow \quad \mathcal{T}_j = U^\dagger\, e^{i\Delta_j}\sin\Delta_j\, U$$

Eigenvalues of $\mathcal{T}_j$ bounded : $\max\left[\left|\mathrm{Re}\left(e^{i\Delta_j}\sin\Delta_j\right)\right|\right] < \frac{1}{2}$

These formulae apply to the scattering of pairs of particles of fixed helicity

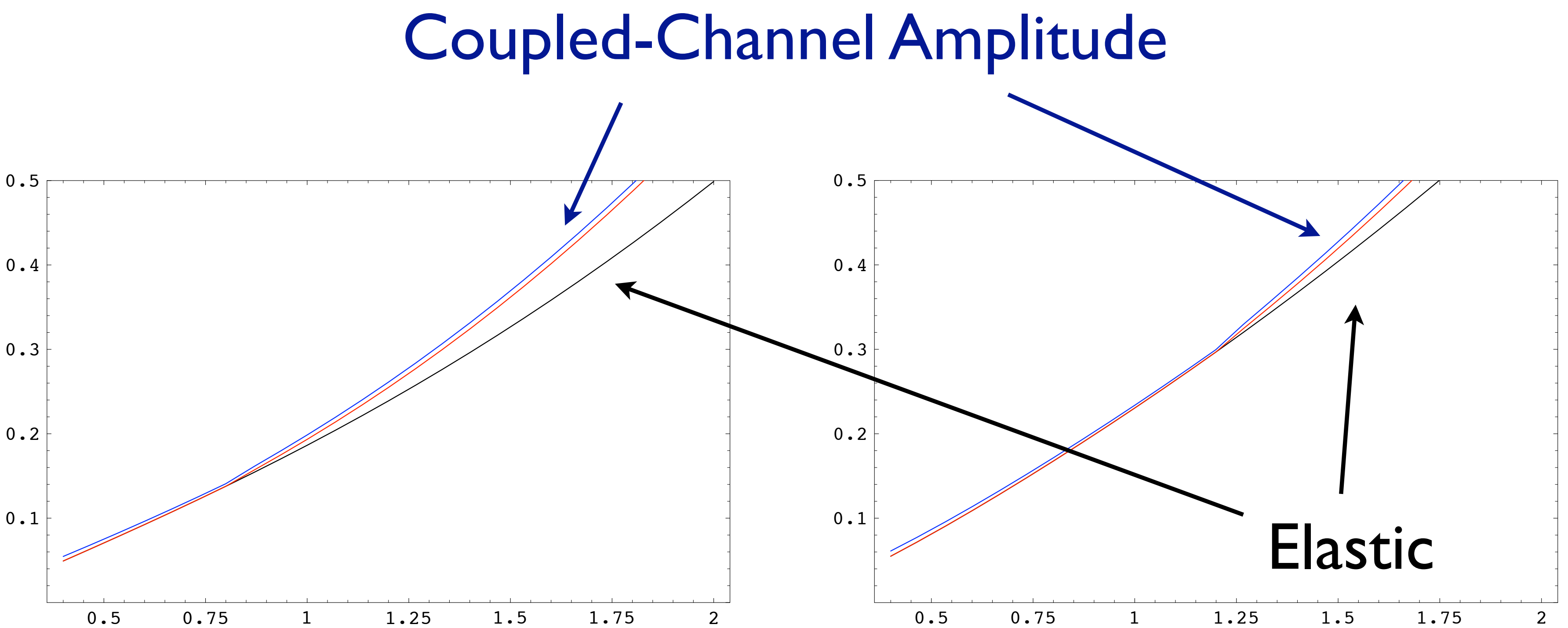
see, for example Durand and Lopez, PRD 40, 207 (1989)

MultiChannel Unitarity in 3-site Model

- Consider $W_L W_L \longrightarrow W_L W_L$, $W_{L1} W_{L1}$
- Longitudinal Helicity
- Pairs of KK modes of same KK level
- spin 0
- isospin 0

Tree-level amplitude unitarity violation implies limiting scale of effective theory

3-Site Unitarity Delay



$M_{W'} = 400\,\text{GeV}$ $\qquad$ $M_{W'} = 600\,\text{GeV}$

Modest Enhancement of Scale of Unitarity Violation

The 3-site Model and Experiment

Triple Gauge Vertices

Hagiwara, *et al.* define:

$$\begin{aligned}\mathcal{L}_{TGV} &= -ie\frac{c_Z}{s_Z}[1+\Delta\kappa_Z]\,W^+_\mu W^-_\nu Z^{\mu\nu} - ie\,[1+\Delta\kappa_\gamma]\,W^+_\mu W^-_\nu A^{\mu\nu}\\ &- ie\frac{c_Z}{s_Z}[1+\Delta g_1^Z]\,(W^{+\mu\nu}W^-_\mu - W^{-\mu\nu}W^+_\mu)Z_\nu\\ &- ie(W^{+\mu\nu}W^-_\mu - W^{-\mu\nu}W^+_\mu)A_\nu\ ,\end{aligned}$$

In 3-site model: $\Delta g_1^Z = \Delta\kappa_Z = \dfrac{M_W^2}{2c^2 M_{W'}^2}$ $\quad\Delta\kappa_\gamma = 0$

LEP II measurement: $\Delta g_1^Z \le 0.028 \quad @ \quad 95\%\mathrm{CL}$

places lower bound on W' mass:

$$M_{W'} \ge 380\,\mathrm{GeV}\sqrt{\frac{0.028}{\Delta g_1^Z}}$$

... plus unitarity

and recalling $$\varepsilon_L^2 = 2\left(\frac{M_W^2}{M_{W'}^2}\right) + 6\left(\frac{M_W^2}{M_{W'}^2}\right)^2 + \ldots$$

this translates into $$\epsilon_L \approx 0.30 \left(\frac{380\,\mathrm{GeV}}{M_{W'}}\right)$$

As mentioned earlier, maintaining unitarity of WW scattering requires

$$m_{W'} < \sqrt{8\pi}\, v \approx 1.2\,\mathrm{TeV}$$

We conclude: $$0.095 \leq \epsilon_L \leq 0.30$$

$b \to s\gamma$

To keep this within bounds requires* $g_R^{Wtb}/g_L^W < .004$

which translates into the bound $\varepsilon_{tR} < 0.67$

Since $m_f \approx \frac{\epsilon_L \epsilon_{fR} M}{\sqrt{1+\epsilon_{fR}^2}}$ and b-quark mass is small,

we can leverage ϵ_{tR} to show $\epsilon_{bR} < .015$

hence, b is ideally delocalized like light fermions

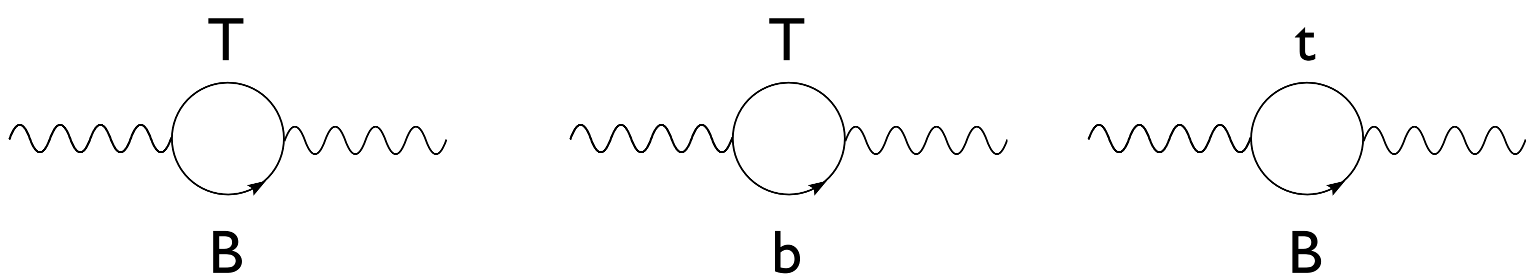
* Larios, Perez, Yuan, hep-ph/9903394

$\Delta\rho$ at one loop

In $\epsilon_L \to 0$ limit, can calculate leading "new" contribution

- SM contribution vanishes since $m_t, m_b \propto \epsilon_L$
- ϵ_L is custodially symmetric

From the following W diagrams (and related Z diagrams)



leading contribution is:

$$\Delta\rho \approx \frac{1}{16\pi^2} \frac{\varepsilon_{tR}^4 M^2}{v^2}$$

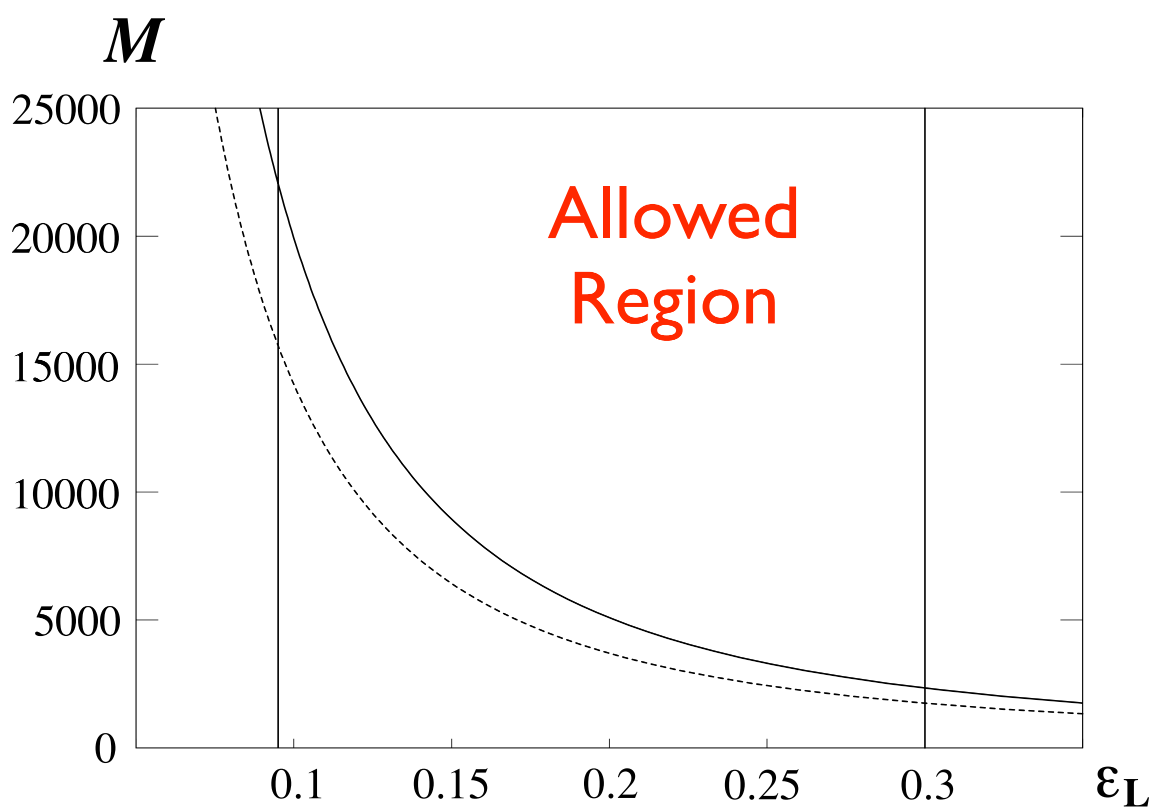
3-Site Parameter Space

Conditions setting boundaries:

$$m_f \approx \frac{\epsilon_L \epsilon_{fR} M}{\sqrt{1+\epsilon_{fR}^2}}$$

$$\Delta\rho \approx \frac{1}{16\pi^2}\frac{\varepsilon_{tR}^4 M^2}{v^2}$$

$$0.095 \leq \epsilon_L \leq 0.30$$

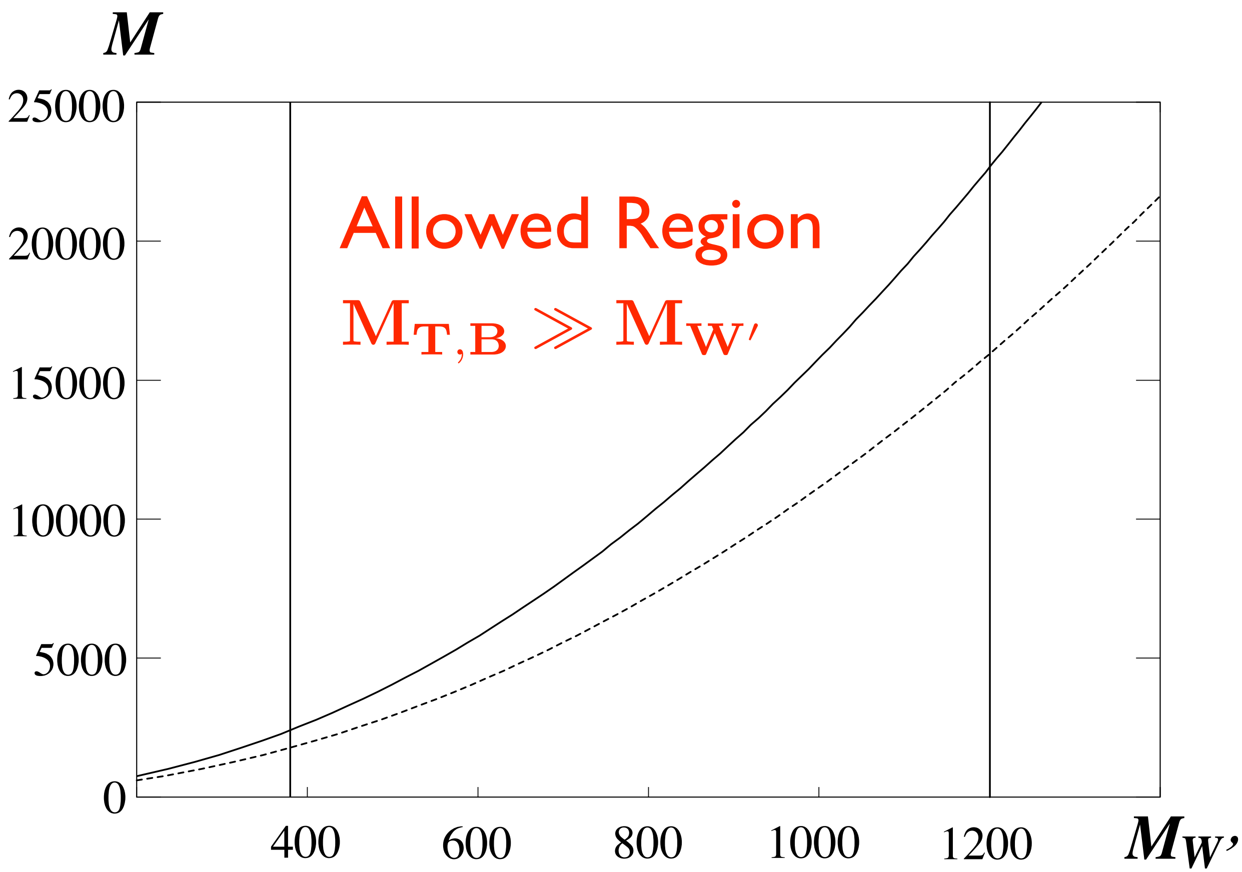


3-Site Parameter Space

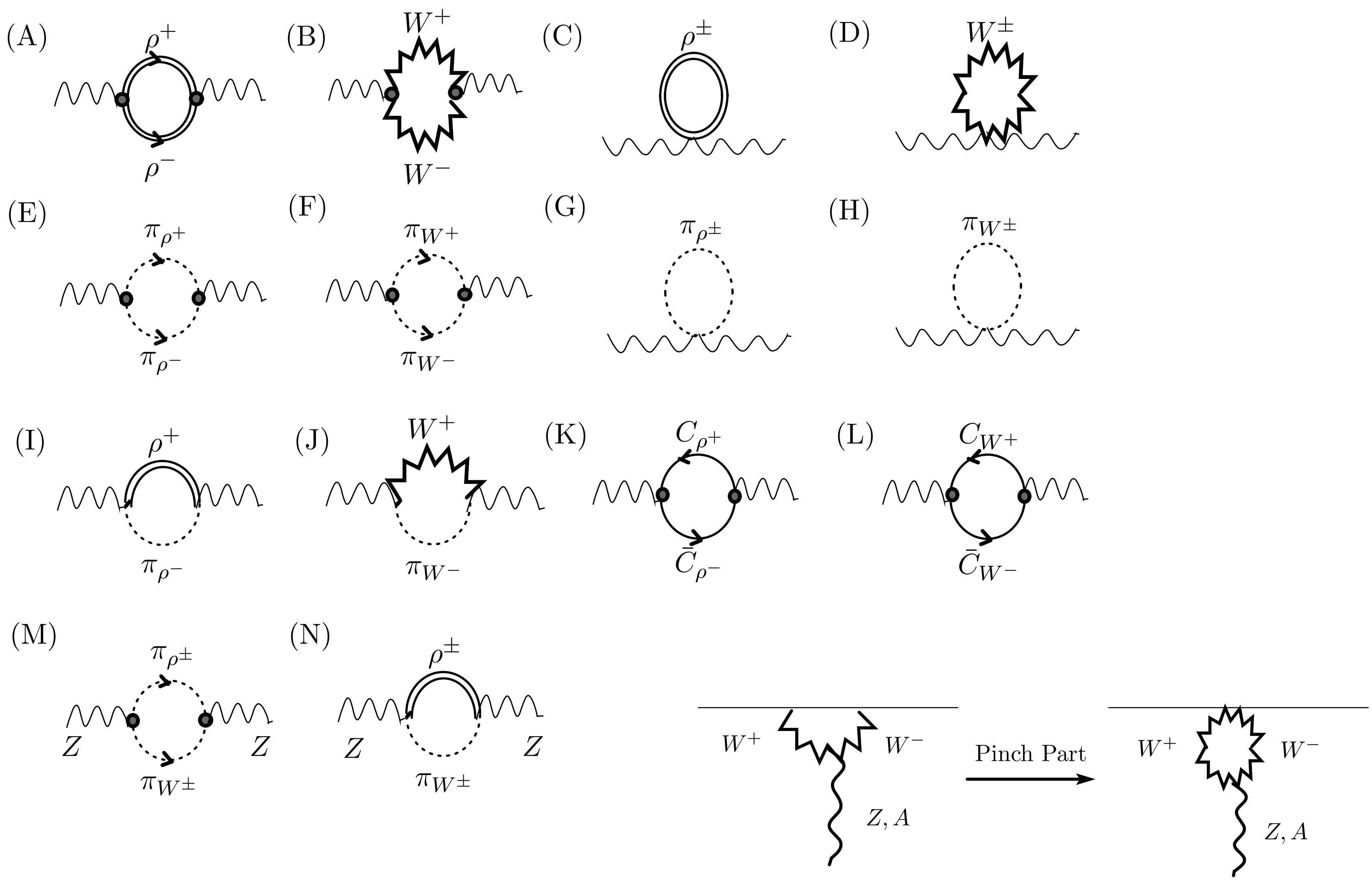
Rewrite in terms of W' mass:

$$\epsilon_L \approx 0.30 \left(\frac{380\,\mathrm{GeV}}{M_{W'}}\right)$$

$$380\,\mathrm{GeV} \leq M_{W'} \leq 1.2\,\mathrm{TeV}$$



S at one loop


(A)
ρ^+
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W^+
W^-
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W^+
W^-
Z, A

S at one loop

$$\alpha S_{3-site} = \frac{4s^2 M_W^2}{M_{W'}^2}\left(1 - \frac{x_1 M_{W'}^2}{2M_W^2}\right)$$

tree; involves ideal delocalization (x_1)

$$+ \frac{\alpha}{12\pi}\ln\frac{M_{W'}^2}{M_{Href}^2}$$

one-loop; up to W' mass

$$- \frac{17\alpha}{24\pi}\ln\frac{\Lambda^2}{M_{W'}^2}$$

one-loop; up to cutoff

$$- 8\pi\alpha\left(c_1(\Lambda) + c_2(\Lambda)\right)$$

counterterms; cf. L_{10}

$$c_2 g\tilde{g} Tr(W_1^{\mu\nu}\Sigma_1 W_{2\,\mu\nu}\Sigma_1^\dagger) + c_1 g\tilde{g} Tr(W_2^{\mu\nu}\Sigma_2 B_{\mu\nu}\Sigma_2^\dagger)$$

link 1 link 2

Perelstein hep-ph/0408072

T at one loop

$$\alpha T_{3-site} = -\frac{3\alpha}{16\pi c^2}\log\frac{M_{W'}^2}{M_{Href}^2}$$

one-loop;
up to W' mass

$$-\frac{15\alpha}{64\pi c^2}\log\frac{\Lambda^2}{M_{W'}^2}$$

one-loop;
up to cutoff

$$+\frac{4\pi\alpha\, c_0(\Lambda)}{c^2}$$

isospin-violating
counterterm

$$\frac{4\pi\alpha\, c_0}{c^2} f^2 \left(\mathrm{tr}[D_\mu \Sigma_{(2)} \frac{\tau_3}{2} \Sigma_{(2)}^\dagger]\right)^2$$

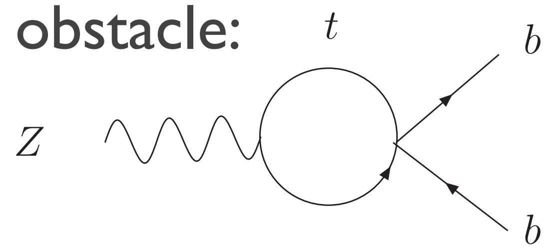
$Z \to b\bar{b}$ at one loop

Involves heavy fermions whose mass (M) is above the reach of the effective theory. We invoke a benchmark UV completion to estimate the size of effects:

$$\frac{\delta g_{Zbb}^{1-loop,3-site}}{g_{Zbb}^{SM}} \sim \frac{m_t^2}{16\pi^2 M^2} \approx \frac{v^2}{M^2} \frac{\delta g_{Zbb}^{1-loop,SM}}{g_{Zbb}^{SM}}$$

This is acceptably small.

Note: ideal delocalization removes a possible obstacle:



$$\frac{\delta g_{Zbb}}{g_{Zbb}^{SM}} \approx \frac{g^2 v^2}{16\pi^2 M_{W'}^2} \ln\left(\frac{M_{W'}^2}{m_t^2}\right)$$

Conclusions:

The 3-site model yields a viable effective theory of electroweak symmetry breaking valid up to 1.5 - 2 TeV

- incorporates / illustrates general principles
  [Higgsless models, deconstruction, ideal delocalization]

- accommodates flavor [e.g. heavy t quark]

- observables [e.g., S, T] calculable at one loop

- extra gauge bosons can be relatively light
  [hard to find at LHC/ILC since they are fermiophobic]