Beijing, August 2005

Flavour Dynamics & ~~C~~P in the SM*: A Tale of Great Successes, Little Understanding -- and Promise for the Future!

Ikaros Bigi, Notre Dame du Lac

Lecture IV (6)

Adding High Accuracy to High Sensitivity

Recap from Lecture III

Novel & unprecedented success for the SM:
predicted `Paradigm of large CP' in B decays confirmed:
indirect, direct CP & T

- `demystification of CP':
  if dynamics can support CP, it can be large!
- CKM ansatz promoted to a tested theory

✍ only now reaching territory where significant deviations from CKM can `realistically' hoped for

✍ B_s transitions independent chapter in Nature's book none of the SM's successes weaken the case for it being incomplete & for New Physics being even `near-by'

- instability of Higgs dynamics
- Strong CP problem of QCD
- `heavenly data'
  - baryon # of Universe
  - neutrino oscillations
  - dark matter
  - dark energy

-- they mainly tell us that manifestations of it might be subtle rather than numerically massive

among indirect searches for New Physics are those based on

- high accuracy; e.g., g-2
- high sensitivity; e.g., ϵ_K, ΔM_B, EDM's

The New Paradigm of Heavy Flavour Studies:

- impact of New Physics likely to be subtle
- goal goes beyond `merely' establishing existence of NP -- one wants to identify salient features of the latter

➥ numerical accuracy & reliability on both the experimental & theoretical side required!

Heavy flavour transitions like B^0 oscillations always recognized as allowing high sensitivity searches for New Physics

now high accuracy added, since { neccessary / possible }

➻ need a comprehensive, coherent & dedicated program

… like for building cathedrals …

~~CP~~ Observables

❶ $|T(B \to f)| \neq |T(\bar{B} \to \bar{f})|$ $\Delta B = 1$

strong F(inal)S(tate)I(nteractions) necessary evil!

❷ $|q| \neq |p|$ $\Delta B = 2$

❸ $\mathrm{Im}(q/p)\bar{\rho}(f)$, $\bar{\rho}(f) = T(\bar{B} \to f)/T(B \to f)$ $\Delta B = 1$ & $\Delta B = 1$

strong F(inal)S(tate)I(nteractions) evil!

Note (prompted by Prof. Giorgi):

- $B \to PP, PV$ size of amplitude only dynamical info
  - ➥ only partial rate asymmetries
- $B \to VV, 3P, \ldots$ much more dynamical info than size

asymm. also in distributions -- Dalitz plots, T-odd moments

☞ Strong FSI can fake a T-odd moment, not ~~CP~~!

☞ type ❸ observables particular powerful!

`K and B decays -- exactly the same, only different!'

K	D	B_d	B_s
$\Delta M_K \sim \Gamma_K$	$\Delta M_D \ll \Gamma_D$	$\Delta M_B \sim \Gamma_B$	$\Delta M_B \gg \Gamma_B$
$\Delta\Gamma_K \sim \Gamma_K$	$\Delta\Gamma_D \ll \Gamma_D$	$\Delta\Gamma_B \ll \Gamma_B$	$\Delta\Gamma_B \sim \Gamma_B$
$\Delta\Gamma_K \sim \Delta M_K$	$\Delta\Gamma_D \sim \Delta M_D$	$\Delta\Gamma_B \ll \Delta M_B$	$\Delta\Gamma_B \ll \Delta M_B$

indeed there
are some domains
with large ~~CP~~ in
$K^{neut} \to \pi\pi$
& in $K_L \to \pi^+\pi^- e^+ e^-$

but: Mass ES ~ CP ES Mass ES ≠ CP ES

Theorem: $M^{neut}(t) \to f_{CP} = Ke^{-\Gamma t}$, unless ~~CP~~

$K^{neut} \to \pi\pi \sim Ke^{-\Gamma t}$ but $B_d \to \pi\pi \neq Ke^{-\Gamma t}$

➥ statement `CP in B decays is much larger than in K decays'
is an empirically verified fact

Menu for Lecture IV

I Heavy Quark Theory

II H(eavy) Q(uark) E(xpansions), Fundamentals

III First Tests: Weak Lifetimes & SL BR's

IV H(eavy) Q(uark) E(xpansions), Refinements & Applications

A Case Study in Accuracy: Extracting V(cb), m_b etc. from $B \to l\nu X$ & $B \to \gamma X$

V Summary of Lecture IV

I Heavy Quark Theory

✍ the goal: to treat nonperturbative dynamics quantitatively

☺ the hope: $m_b \gg \Lambda_{QCD}$

2-Step-Methodology

symmetry principle asymptotic limit	dynamical approach pre-asymptotia

e.g.:

Chiral dynamics = QCD at `low' energies

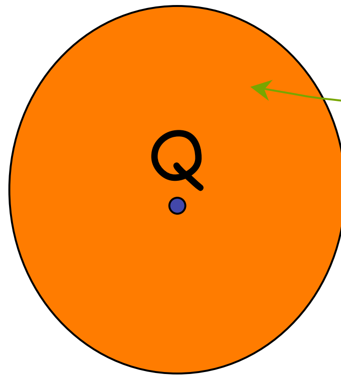
Chiral perturbation theory

➾ chiral invariance: $m_q=0$, $m_\pi=0$ + chiral pert.theory: $(m_q/4\pi f_\pi)^{2n}$

Heavy Quark Symmetry + Heavy Quark Expans.

H_Q=[Qq]

energy-X with soft
medium ~ Λ ~ 1/2 GeV
~ no QQ fluctuations



cloud of light d.o.f.

Q static
➥ QM for Q

➥ QFTh for light d.o.f.:
$(\Lambda/m_Q)^n$

Heavy Quark Symmetry ⊕ Heavy Quark Expans.

$$\sim H_{Pauli} = -A_0 + (i\partial - \mathbf{A})^2/2m_Q + \sigma \mathbf{B}/2m_Q \quad \rightarrow -A_0 \quad \text{as} \quad m_Q \rightarrow \infty$$

i.e.,

infinitely heavy static quark, without spin dynamics,
only colour Coulomb potential!

*

- hadrons H_Q labeled by total spin S and by $j_q = l_q + s_q$:

- ground states: $[S|l_q|j_q] = [0,1|0|1/2]$:

PS -- B or D -- & V -- B* or D*

- 1st excit. states: [0,1|1|1/2] & [1,2 |1|3/2]

4 P wave states: 2 $j_q=3/2$ narrow states
2 $j_q=1/2$ broad states

Comment on H(eavy)Q(uark)E(ffective)T(heory)

*

Heavy Quark Theory = Heavy Quark Symmetry ⊕ $1/m_Q$ expan.

...

...

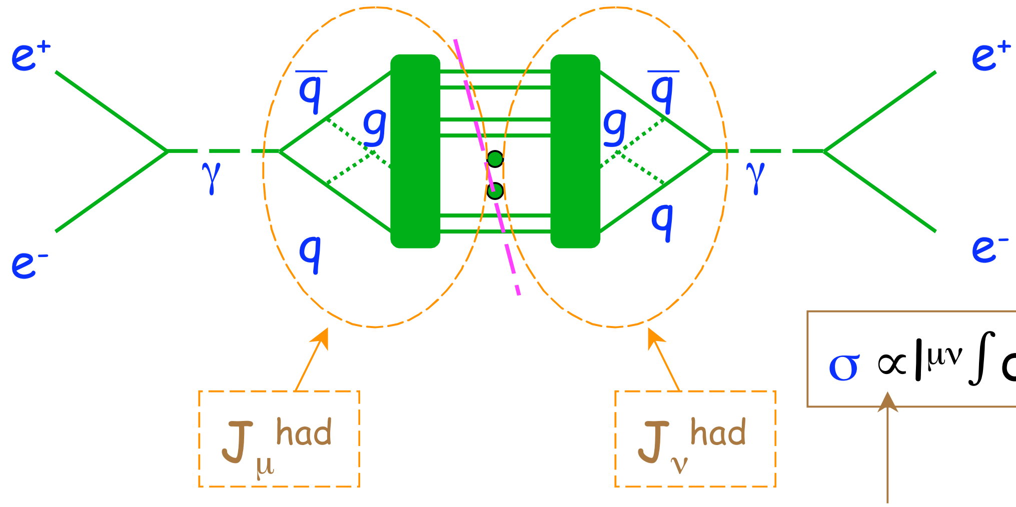
HQET

a theory of hadronic form factors for exclusive channels

- form factors normalized: $B \to l\,\nu\,D^*$ at zero recoil
- number of independent FF reduced: $\Lambda_b \to l\,\nu\,\Lambda_c$

	$1/m_Q$ expan. f. inclus.	vs. HQET in exclus.
B decays		
$m_b \gg m_c \gg \Lambda$	√, $1/(m_b - m_c)$	√, $1/m_c$
$m_c \to m_b$	↯	√ (yet trivial)
$m_c \to 0$	√√	↯

II H(eavy) Q(uark) E(xpansions), Fundamentals

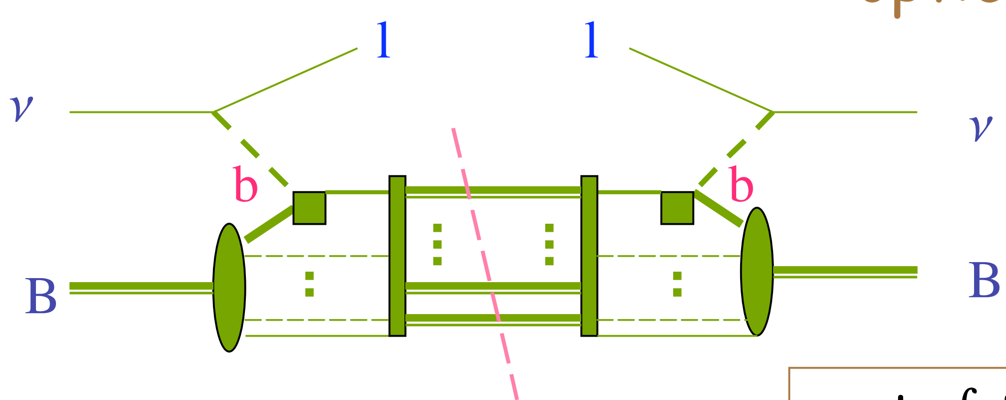


$$\sigma \propto l^{\mu\nu} \int d^4x\, e^{-iQx} \langle 0 | \{ J_\mu^{had}(x) J_\nu^{had}(0) \}_T | 0 \rangle$$

optical thm

$\Sigma_i f_i(x) O_i(0)$

as $Q \to \infty$



$$\Gamma \propto l^{\mu\nu} \int d^4x\, e^{-iQx} \langle B | \{ J_\mu^{had}(x) J_\nu^{had}(0) \}_T | B \rangle$$

optical thm

$\Sigma_i f_i(x) O_i(0)$

as $m_b \to \infty$

- conjecture

$$\text{rate}(H_Q \to f_{incl}) = K(CKM, m_Q) \Sigma_{i=0}\, c_i(\alpha_S)(\Lambda/m_Q)^i$$

complication in weak decays:

$$\Gamma(H_Q) \propto m_Q^{5}(\mu)\,[\, z_3(m,\mu,\alpha_S) + \ldots]$$

tool chest to implement idea:

- operator product expansion (OPE)
- dispersion relations (see later)
- sum rules

when is a quark heavy -- when light?

u & d quarks clearly light, b quarks heavy!

need a yard stick

- 1st guess: $\Lambda_{NonPD} \sim \Lambda_{QCD} \sim 200$ MeV
  - c quarks clearly heavy, yet s quarks ??
- 2nd guess: $\Lambda_{NonPD} \sim 700$ MeV ($\sim N_C \times \Lambda_{QCD}$?)
  - s quarks light by default;

    c quarks: iffy -- need case-by-case study!

2.1 Basics of O(perator)P(roduct)E(xpansion)

$$\Gamma(H_Q \to f) = \Sigma_i\, c_i^{(f)}\,(KM, M_W, m_Q, \alpha_S, \mu)\, \langle H_Q \mid O_i \mid H_Q \rangle_{(\mu)}$$

- short distance dynamics → coeff. $c_i^{(f)}$
- universal cast of local operators O_i
- $\langle H_Q|O_i|H_Q\rangle$ inferred from other observables or lattice QCD!
- expansion parameter

$$1/E_{release} \sim \begin{cases} 1/(m_b - m_c) & \text{for } b \to c \\ 1/m_b & \text{for } b \to u \end{cases}$$

- Wilson: auxiliary scale μ s.t.

short distance < μ^{-1} < long distance

- c_i ⇔ short distance dynamics
- O_i active fields - long distance dynamics

☞ choose judiciously!

Scylla & Charybdis: $\Lambda_{QCD} \ll \mu \ll m_Q$ *

➻ matrix elements calculable

➻ $\alpha_S \ll 1$

$\mu \sim 1$ GeV √ for Q = b! yet: Q = c?

leads to `smart' pert. treatment

☞ treat as physical parameter (s. sum rules)

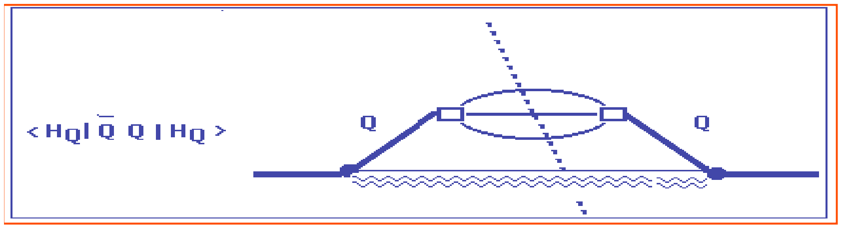
`OPE' is an age-meter:

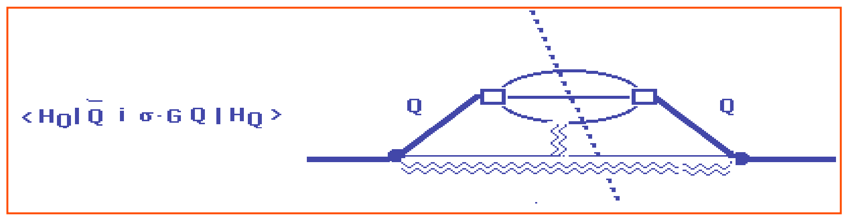
Answer A: OPE = One Pion Exchange
Answer B: OPE = One Pomeron Exchange
Answer C: OPE = Operator Product Expansion

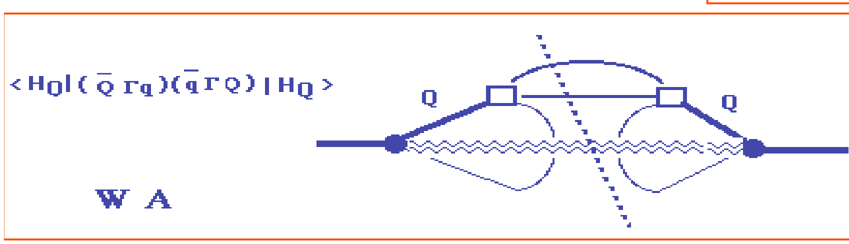
$$\Gamma(H_Q \to f_{incl}) = \frac{G_F^2 \, |KM|^2 \, m_Q^5}{192\,\pi^3} \times$$

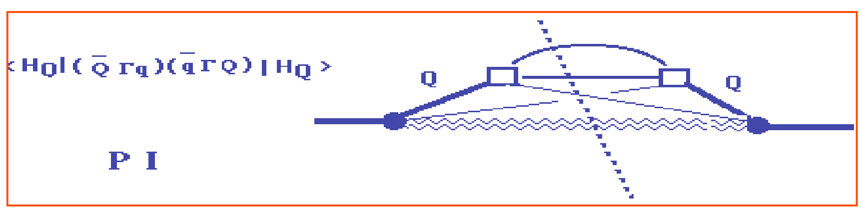
$$\times \left\{ c_3(f) \langle H_Q | \bar{Q} Q | H_Q \rangle + c_5(f) \frac{\langle H_Q | \bar{Q} i\sigma \cdot G Q | H_Q \rangle}{m_Q^2} + c_6(f) \frac{\langle H_Q | (\bar{Q}\Gamma q) \cdot (\bar{q}\Gamma Q) | H_Q \rangle}{m_Q^3} + \ldots \right\}$$

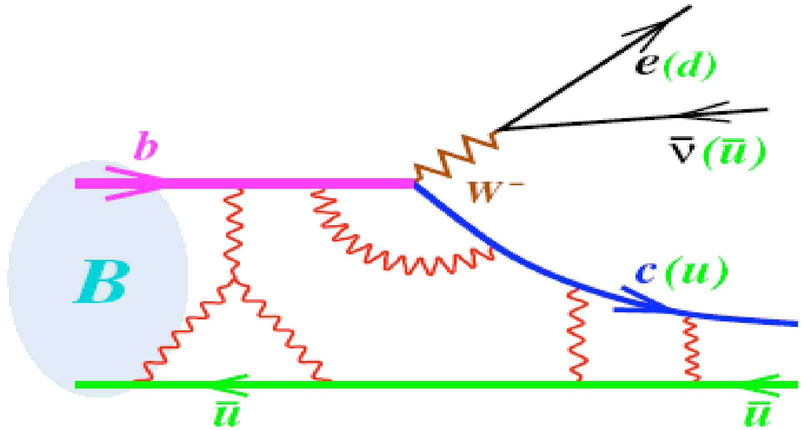
*











Quark level:

$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3} \cdot z(m_c, m_b) \cdot N_c \cdot \left(1 - \frac{\alpha_s}{\pi} \ldots\right) \cdot \ldots \cdot \left\{ \begin{array}{c} |V_{cb}|^2 \\ |V_{ub}|^2 \end{array} \right\}$$

No $\Lambda_{\rm QCD}/m_b$ corrections to inclusive widths of heavy flavor hadrons

Bigi, Shifman, Uraltsev, Vainshtein 1992

Applies to all types: semileptonic, nonleptonic, $b \to s + \gamma$, $b \to s\, \ell^+ \ell^-$, ...

$B,\ B_s,\ \Lambda_b, \ldots \qquad \frac{\Delta M}{M} \sim \frac{\Lambda_{\rm QCD}}{m_b}$ yet $\frac{\Delta \Gamma}{\Gamma} \sim \frac{\Lambda^2_{\rm QCD}}{m_b^2}$

$$M_B = m_b + \overline{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2}{2m_b} + \frac{\mu_3^2}{m_b^2} + \ldots$$

yet $\overline{\Lambda}$ does not affect the width!

local colour gauge symmetry of QCD essential

Exact cancellation of the bound state effects with the final state interaction

- leading nonperturb. contrib. ~ $O(1/m_Q^2)$:

  ~ O(5 %) for Q = b

not true for exclusive modes!

weight of nonpert. effects greatly reduced in beauty decays

- perturb. contributions numerically important
  - with `smart' pert. treatment Γ_{parton} often good estimate

Novel symbiosis between different theoretical technologies for heavy flavour nonperturbative dynamics -- in particular between HQE and LQCD

$$\text{observables} = \Sigma_i \, c_i(CKM, m_Q, \alpha_S) \, \langle H_Q | O_i | H_Q \rangle$$

HQE HQP LQCD

- it enhances the power of and confidence in both technologies by
  - increasing the range of applications &
  - providing more benchmarks

- duality ≠ additional ad-hoc assumption
- duality violation in $\Gamma_{SL}(B)$ < 0.5 %!

IB & N.Uraltsev, Int.J.Mod.Phys.A16(01)5201, `Vademecum ...' (48 p!)

III First Tests: Weak Lifetimes & SL BR's

mostly inclusive, yet some semi-inclusive transitions

3.1 (Weak) Lifetimes

- Since $\Gamma_{SL}(P^0) = \Gamma_{SL}(P^\pm)$ + ~~SU(2)~~ ⟹ $\tau(P^\pm)/\tau(P^0) \approx BR_{SL}(P^\pm)/BR_{SL}(P^0)$
- Yet $\Gamma_{SL}(P_Q) \neq \Gamma_{SL}(\Lambda_Q) \neq \Gamma_{SL}(\Xi_Q)$
  - ➥ baryonic SL BR's do not reflect lifetime ratios!
- evaluation of absolute value of $BR_{SL}(P)$ less reliable than ratios of $BR_{SL}(P)$

$\tau(D^+) > \tau(D^0) \sim \tau(D_s) \geq \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\Omega_c^0)$ *

	$1/m_c$ expectations	theory comment	data
$\tau(D^+)/\tau(D^0)$	$\sim 1+(f_D/200\ \mathrm{MeV})^2 \sim 2.4$	PI dominant	2.54 ± 0.01
$\tau(D_s)/\tau(D^0)$	1.0 - 1.07 0.9 - 1.3	without WA with WA	1.22 ± 0.02
$\tau(\Lambda_c^+)/\tau(D^0)$	~ 0.5	Quark Model ME	0.49 ± 0.01
$\tau(\Xi_c^+)/\tau(\Lambda_c^+)$	$\sim 1.3 - 1.7$	"	2.2 ± 0.1
$\tau(\Lambda_c^+)/\tau(\Xi_c^0)$	$\sim 1.6 - 2.2$	"	2.0 ± 0.4
$\tau(\Xi_c^+)/\tau(\Xi_c^0)$	~ 2.8	"	4.5 ± 0.9
$\tau(\Xi_c^+)/\tau(\Omega_c^0)$	~ 4	"	5.8 ± 0.9
$\tau(\Xi_c^0)/\tau(\Omega_c)$	~ 1.4	"	1.42 ± 0.14

$\tau(D^+)/\tau(\Omega_c^0) \sim 14 >> 1!$

- yes, apply expansion in $1/m_c$ at your own risk, but ...

*

- saving grace: leading correction of order $1/m_c^2$ rather than $1/m_c$
- observed pattern reproduced/predicted semiquantitatively -- with $\tau(D^+)/\tau(\Omega_c^0) \sim 14$!
- destructive PI main engine driving lifetime differences among mesons, yet WA -- while not leading -- still significant in D decays
  - more theoretical work needed on WA in meson decays
  - impact of WA on exclusive final states in meson decays: constructive in D^0 and/or destructive in D_s?
- baryons present complex challenge
- description for baryonic widths helped by sizeable errors
- sole sign for significant discrepancy emerges in $\tau(\Xi_c^+)$ -- observed lifetime 50 % longer than predicted!
- Γ_{SL}(baryons) highly non-universal

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Success in describing observed lifetime ratios one of the best confirmations for charm being a heavy quark whenever leading nonperturb. contributions ~ $O(1/m^2_Q)$

- SELEX has reported candidates for weakly decaying double-charm baryons

  my judgement:

  whatever SELEX has observed -- I do not believe its peculiar events can be double-charm baryons:

  - mass splittings too large
  - lifetimes too short without expected hierarchy

	$1/m_b$ predict.	comment	data
$\tau(B^-)/\tau(B_d)$	$1{+}0.05(f_B/0.2GeV)^2$ '92 1.06 ±0.02 '98-'03	PI in $\tau(B^-)$ fact. at low scale ~ 1 GeV	1.076 ± 0.008 ☺
$<\tau(B_s)>/\tau(B_d)$	$1 \pm \mathcal{O}(0.01)$ '94		0.920 ± 0.030 😐
$\tau(\Lambda_b)/\tau(B_d)$	~0.9 - 1.0 '93 0.88 - 0.97 '98	quark model ME	0.806 ± 0.047 😐
$\tau(B_c)$	~ 0.5 psec '94	largest lifetime difference! no $1/m_Q$ crucial	0.45±0.12 psec ☺
$\Delta\Gamma(B_s)/\Gamma(B_s)$	$0.18(f_B/0.2GeV)^2$ '87 0.12±0.04 '04	less reliable than $\Delta M(B_s)$	0.65±0.3 CDF 0.23±0.17 D0

- Predictions for meson life times generally on the mark!
- ☺ $\tau(B^-)$ exceeds $\tau(B_d)$ by a few % -- as predicted in '92
- ☺ $\tau(B_c)/\tau(B_d) \sim 1/3$ -- as predicted;largest lifetime difference
- 😐 concern for reasonable (though not sacrosanct) prediction of $\langle\tau(B_s)\rangle/\tau(B_d) = 1 \pm \sim 0.01\text{-}0.02$
- 😐 appearance of Feynman diagrams can be deceiving
  - ☞ quark box diagram less reliable for $\Delta\Gamma(B)$ than for $\Delta M(B)$
  - ☞ $\Delta\Gamma(B_s)/\Gamma(B_s) > 0.25$ inconsistent with $\langle\tau(B_s)\rangle/\tau(B_d) = 1 \pm \sim 0.01\text{-}0.02$
- issue of $\tau(\Lambda_b)/\tau(B_d)$ not settled yet
  - ☞ memento: one predicts $1 - \tau(\Lambda_b)/\tau(B_d) = 0.03 - 0.12$ vs.
  - ☞ need $\tau(\Xi_b^-)$, $\tau(\Xi_b^0)$ to { confirm success or diagnose failure! }

3.2 BR($B \to l \nu X$)

*

from early on: $BR(B \to l\nu X)|_{meas} < BR(b \to l\nu c)$

in '93:

- measurements became more conclusive
- $BR(B \to l\nu X) = BR(b \to l\nu c) + 1/m_Q^2$

➡ 2 observables: $BR(B \to l\nu X)$, charm content n_c

☞ suggested n_c larger than assumed

last analysis from '95

✍ should be redone -- likely to remove discrepancy -- in particular with large value for n_c from BABAR

$n_c(B^-) = 1.313 \pm 0.037|_{stat} \pm 0.062|_{syst}{}^{+0.63}_{-0.042}|_{charm\ BR}$

$n_c(B_d) = 1.276 \pm 0.062|_{stat} \pm 0.058|_{syst}{}^{+0.66}_{-0.046}|_{charm\ BR}$

IV H(eavy) Q(uark) E(xpansions), Refinements & Applications

Can we answer the ~ % level accuracy challenge?

\`Trust is good -- control is better' (Lenin)

Status '04

$m_b(1\ \mathrm{GeV}) = (4.61 \pm 0.068)\ \mathrm{GeV}$ ⟸ 1.5 %

$m_c(1\ \mathrm{GeV}) = (1.18 \pm 0.092)\ \mathrm{GeV}$ ⟸ 7.8 %

$m_b(1\ \mathrm{GeV}) - 0.74\ m_c(1\ \mathrm{GeV}) = (3.74 \pm 0.017)\ \mathrm{GeV}$ ⟸ 0.5 %

$|V(cb)| = (41.390 \pm 0.870) \times 10^{-3}$ ⟸ 2.1 %

vs.

$|V(us)|_{KTeV} = 0.2252 \pm 0.0022$ ⟸ 1.1 %

You can ignore recent PDG \`review' on V(cb)!

A Case Study in Accuracy: Extracting V(cb), m_b etc.

$B \rightarrow l\nu X_c$

2 step procedure for quantitative results

❶ express observable in terms of HQP through explicit OPE

Benson,ibi,Mannel, Uraltsev Bauer,Ligeti,Luke,Manohar

❷ determine HQP from independent observable

both with commensurate accuracy & reliability!

Gambino,Uraltsev Trott; Bauer,Ligeti,Luke,Manohar

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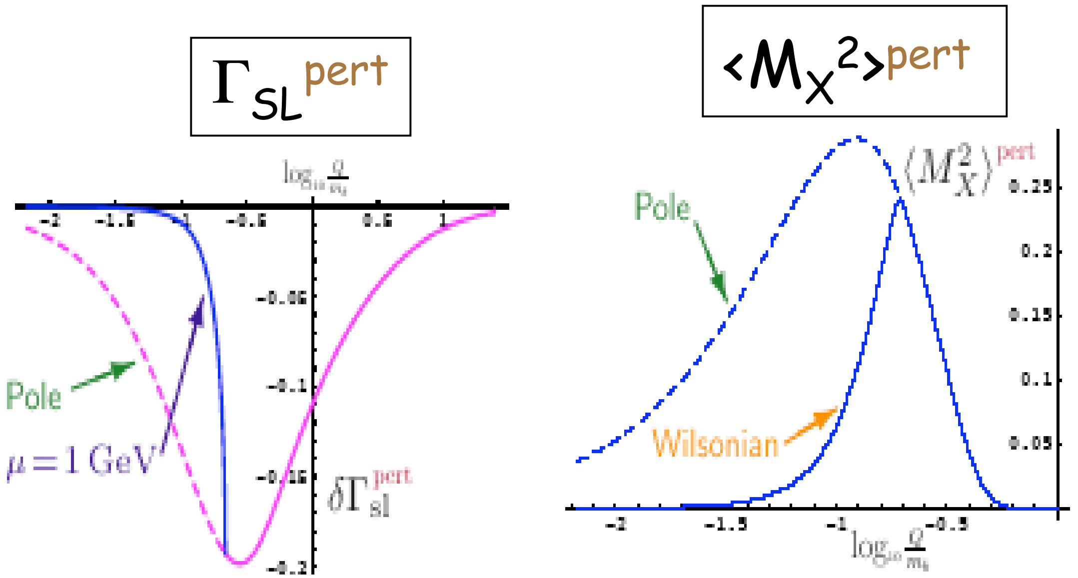


Figure 1: The role of the gluons with different momenta in Γ_{sl} and in $\langle M_X^2\rangle$, for $b \to c\ell\nu$.

Uraltsev

"not all OPE's are created equal!"

4.1 Master Formulae for SL Width

$\Gamma_{SL}(B) = \Gamma_0(b)$

Benson et al., Nucl.Phys.B665('03)367

$$\{f(z)[1-(2/3)(\alpha_S/\pi)(g(z)/f(z))+c_2\alpha_S^2+c_3\alpha_S^3+..]\times[1-(\mu_\pi^2(\mu)-\mu_G^2(\mu))/2m_b^2]$$
$$-2(1-z)^4\mu_G^2(\mu)/m_b^2$$
$$+[d(z)\rho_D^3(\mu)+l(z)\rho_{LS}^3(\mu)]/m_b^3+O(1/m_b^4)\}$$

$\Gamma_0(b) = G_F^2 m_b^5(\mu)\,|V(cb)|^2/192\pi^3$

$f(z), g(z), d(z), l(z)$: phase space function of $z=m_c^2/m_b^2$

c_2: BLM $\alpha_S^2\beta_0$ + estimate for non-BLM, $\beta_0 = 11N_C/3 - 2N_f/3 = 9$

c_3: BLM $\alpha_S^3\beta_0^2$, [BLM known to all orders $\alpha_S^n\beta_0^{n-1}$]

$\mu_\pi^2(\mu) = \langle B|b(i\mathbf{D})^2 b|B\rangle|_\mu/2M_B$ kinetic energy

$\mu_G^2(\mu) = \langle B|b(i/2)\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle|_\mu/2M_B$ chromomagn. moment

$\rho_D^3(\mu) = \langle B|b(-1/2\mathbf{D}\cdot\mathbf{E})b|B\rangle|_\mu/2M_B$ Darwin term

$\rho_{LS}^3(\mu) = \langle B|b(\boldsymbol{\sigma}\cdot\mathbf{E}\times\boldsymbol{\pi})b|B\rangle|_\mu/2M_B$ LS term

➥ $\Gamma_{SL}(B \to l\nu X_c) =$

$$F(HQP) \pm 1\%|_{pert} \pm 2.4\%|_{hWc} \pm 0.8\%|_{hpc} \pm 1.4\%|_{IC} =$$

$$F(HQP) \pm 3\%|_{th}$$

$$|V(cb)|/0.0417 = (1+\delta_{th}) \times [1+0.30(\alpha_S(m_b) -0.22)] \times$$

$$[1-0.66x(m_b(1\ GeV) -4.6\ GeV) + 0.39x(m_c(1\ GeV) -1.15\ GeV) +$$

$$+0.013x(\mu_\pi^2 -0.4\ GeV^2) + 0.05x(\mu_G^2 -0.35\ GeV^2) +$$

$$+0.09x(\rho_D^3 -0.2\ GeV^3) + 0.01x(\rho_{LS}^3 +0.15\ GeV^3)];$$

$$\delta_{th} = \pm 0.5\ \%|_{pert} \pm 1.2\%|_{hWc} \pm 0.4\%|_{hpc} \pm 0.7\%|_{IC}$$

4.2 Heavy Quark Parameters

need definitions of HQP that can pass muster by quantum field theory!

through $\mathcal{O}(1/m_Q^3)$: 6 HQP

2 different classes of HQP

- m_b, m_c -- external to QCD, i.e. can never be calculated by LQCD without experimental input

  caveat: quark masses depend on renorm. scheme & scale

- μ_π^2, μ_G^2, ρ_D^3, ρ_{LS}^3, ... internal to QCD, i.e. can be calculated by LQCD without experimental input

  caveat: $\mu_\pi^2 \neq -\lambda_1$, $\mu_G^2 \neq -\lambda_2$

Quark masses

complication in weak decays:

$\Gamma(H_Q) \propto m_Q^5(\mu)\,[\,z_3(m,\mu,\alpha_S) + \ldots]$

*

unlike for QED due to confinement no `natural' definition

- pole mass (position of pole in perturb. Green fct.)
  - ☺ IR finite & gauge invariant
  - ☹ IR unstable in complete theory

$\Delta_{intrinsic}\, m_{pole} \sim \mathcal{O}(\Lambda_{QCD})$ `renormalons'

$m^5_{Q,pole} = (m_{Q,pert} + \Delta_{intrinsic} m_{pole})^5 = m^5_{Q,pert}\,(1 + \Delta_{intrinsic} m_{pole}/m_{Q,pert})$

$1/m_Q$ contrib.!

☞ intrinsic uncert. parametrically > nonpert. contrib.!

➥ use running mass with IR cut-off μ to `freeze out' renormalons (pole mass: $\mu \to 0$)

* $\overline{MS}$ mass

*

not a parameter in the effective Lagrangian --
rather an ad-hoc combination convenient in perturb. calculations

$m_Q(\mu) = m_Q(m_Q)[1+2(\alpha_S/\pi)\log(m_Q/\mu)] \quad \to \infty \quad$ as $\quad \mu/m_Q \to 0$!

☞ appropriate when relevant scale $\mu > m_Q$

production process, e.g.: $Z \to b\,\bar{b}$

☞ inadequate for decays where $\mu < m_Q$

- `kinetic mass'

$$dm_Q(\mu)/d\mu = - 16\alpha_S(\mu)/3\pi - 4\alpha_S(\mu)/3\pi(\mu/m_Q) + \ldots$$

i.e., linear scale dependence in IR

m_b

$\Lambda_{HQET} \approx \Lambda_{kin}$ (1 GeV) - 0.255 GeV   to one-loop

before 2002 $\Upsilon(4S) \to b\bar{b}$:

$m_{b,kin}$(1 GeV) =

4.56 ±0.06 GeV	MeYe
4.57 ±0.05 GeV	Ho
4.59 ±0.06 GeV	BeSi
4.58 ±0.05 GeV	KuSt

➥ $\langle m_{b,kin}(1\ GeV)\rangle|_{bb}$ = 4.57 ±0.08 GeV

$m_b - m_c$, m_c

*

$$m_b - m_c = \langle M_B\rangle - \langle M_D\rangle + \mu_\pi^2(1/2m_c - 1/2m_b) + \ldots$$
$$= 3.50\ \text{GeV} \ + \quad 40\ \text{MeV}(\mu_\pi^2 - 0.5\ \text{GeV}^2)/0.1\ \text{GeV}^2$$

vulnerable relation since

- ☛ expansion in $1/m_c$
- ☛ nonlocal correlators at $1/m_c^2$

➥ do not impose a priori this relation on $m_b - m_c$!

$$m_c(m_c) = \begin{cases} 1.19 \pm 0.11\ \text{GeV} & \text{charmonium sum rules I} \\ 1.30 \pm 0.03\ \text{GeV} & \text{charmonium sum rules II} \end{cases}$$

Hadronic Expectation Values

- *dim 3*

$$\langle H_Q | \bar{Q} Q | H_Q \rangle = 1 - \frac{\langle H_Q | \bar{Q} (i\vec{D})^2 Q | H_Q \rangle}{2m_Q^2} + \frac{\langle H_Q | \bar{Q} \frac{i}{2} \sigma \cdot G Q | H_Q \rangle}{2m_Q^2} + \ldots$$

- *dim 5*

chromomagnetic moment μ_G^2

$\mu_G^2 = \langle H_Q | Q i/2 \sigma_{\mu\nu} G_{\mu\nu} Q | H_Q \rangle / 2M(H_Q) = (3/2)\,[M^2(V_Q) - M^2(P_Q)]$

for b = Q: $\mu_G^2 \approx 0.35^{+0.03}_{-0.02}$ GeV^2

kinetic energy μ_π^2

$\mu_\pi^2 = \langle H_Q | Q \pi^2 Q | H_Q \rangle / 2M(H_Q) \approx -\lambda_1 + 0.18$ GeV^2 to one-loop

SV SR: $\mu_\pi^2 > \mu_G^2$;

`QCD' SR: $\mu_\pi^2 = 0.45 \pm 0.1$ GeV^2

❑ *dim 6* *

Darwin term ρ_D^3

$\rho_D^3(\mu) = \langle B|b(-1/2\mathbf{D}\cdot\mathbf{E})b|B\rangle|_\mu/2M_B$, $\rho_D^3(\mu) \sim 0.1\ \text{GeV}^3$

LS term ρ_{LS}^3

$\rho_{LS}^3(\mu) = \langle B|b(\sigma\cdot\mathbf{E}\times\pi)b|B\rangle|_\mu/2M_B$, $-\rho_{LS}^3(\mu) \leq \rho_D^3(\mu)$

hardly relevant, since very reduced contribution

"intrinsic charm"

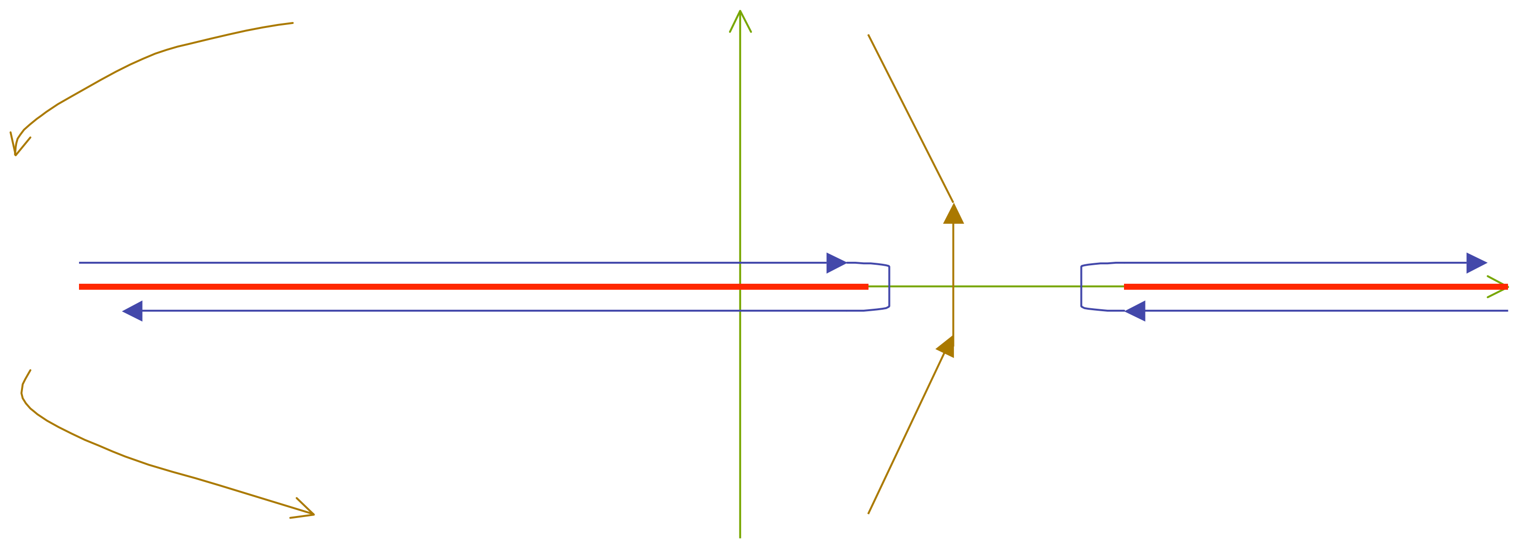
$\langle B|b\Gamma c c\Gamma b|B\rangle|_\mu/2M_B$

4.3 Dispersion relations

*

OPE defined and constructed in Euklidean regime

➥ extrapolate to Minkowskian regime via dispersion relations



$\Gamma(H_Q)|_{OPE} \leftrightarrow \Gamma(H_Q)|_{observ}$

dispersion integral

- assume: QCD creates no unphysical singularity
- intrinsic limitation of algorithm
  - ➥ limitations to duality!

4.4 Extracting Heavy Quark Parameters

Second task --

determine HQP without compromising advantages of OPE

V(cb) & HQP ⟹ $\Gamma_{SL}(B \to l\nu X_c)$, i.e. integrated spectrum

~~V(cb) &~~ HQP ⟹ shape of (E_l&M_X) spectrum

normalized moments ⟺ shape of spectrum

➥ normalized moments ⟹ HQP

Lepton energy and hadronic mass moments:

$$M_1(E_l) = \Gamma^{-1}\int dE_l\, E_l\, d\,\Gamma/d\,E_l$$

$$M_n(E_l) = \Gamma^{-1}\int dE_l\, [E_l - M_1(E_l)]^n d\,\Gamma/d\,E_l \quad , n > 1$$

$$M_1(M_X) = \Gamma^{-1}\int dM_X^2(M_X^2 - M_D^2)d\,\Gamma/dM_X^2$$

$$M_n(M_X) = \Gamma^{-1}\int dM_X^2(M_X^2 - \langle M_X^2\rangle)^n d\,\Gamma/dM_X^2 \, , n > 1$$

➻ aim for overconstraints

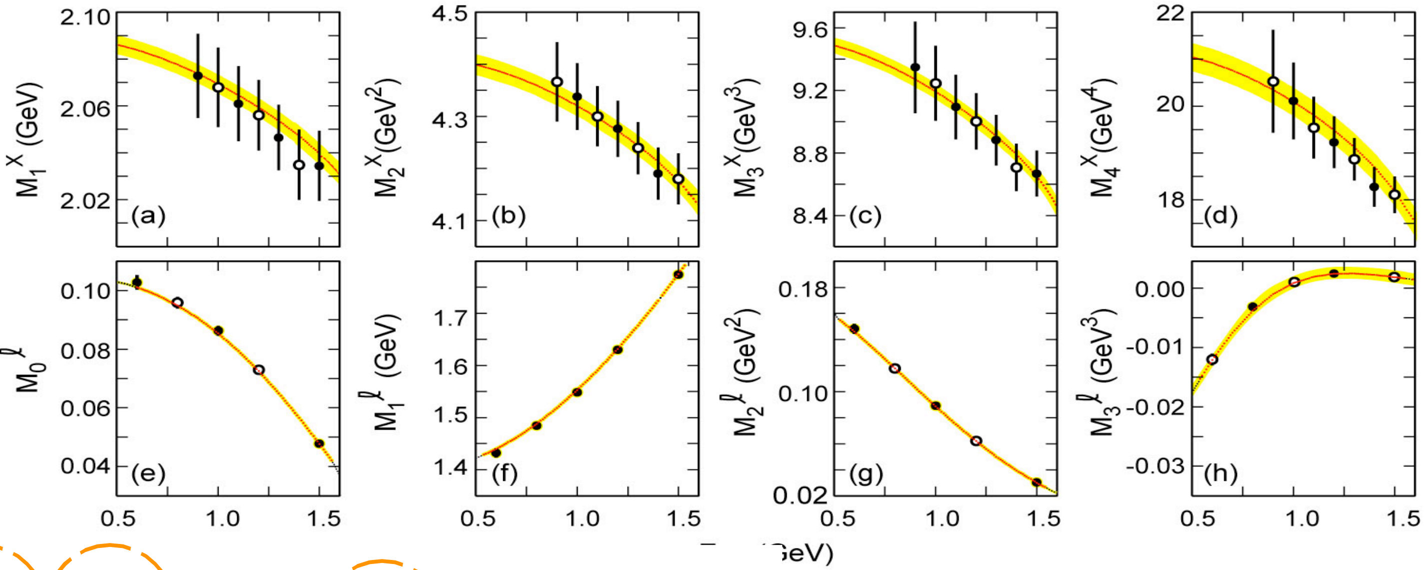
short comment on history: *

- CLEO did lots of pioneering work -- also on moments

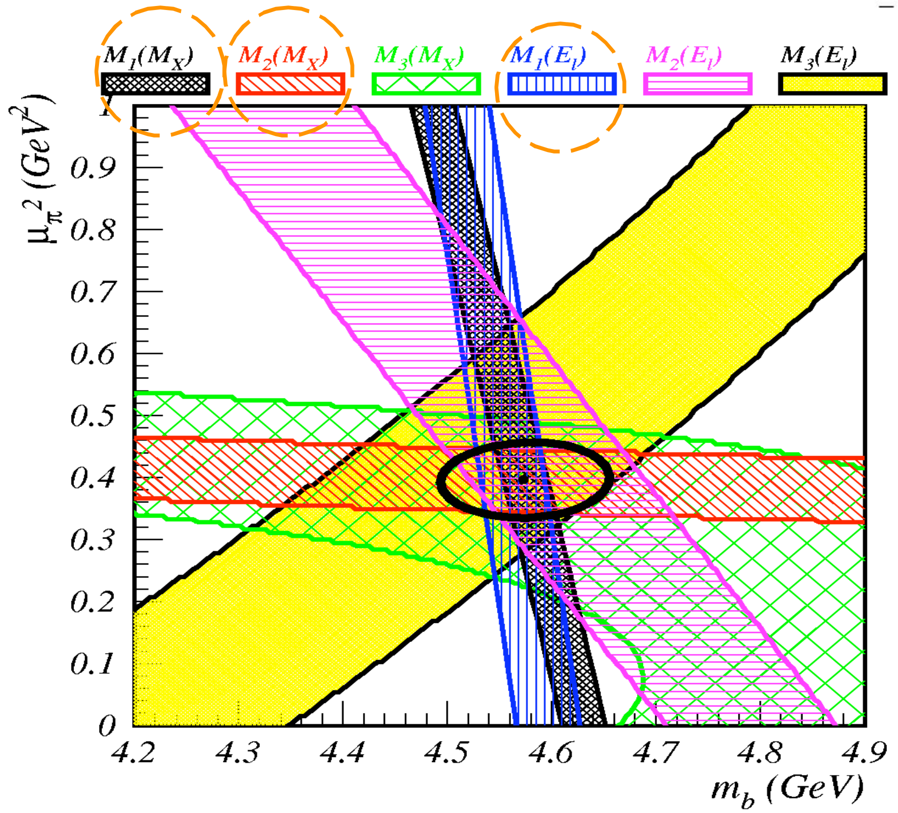
  measured originally 2 lepton energy moments in $B \to l\nu X_c$ & photon energy moment in $B \to \gamma X_c$ with severe lower cuts on E_l & E_γ.

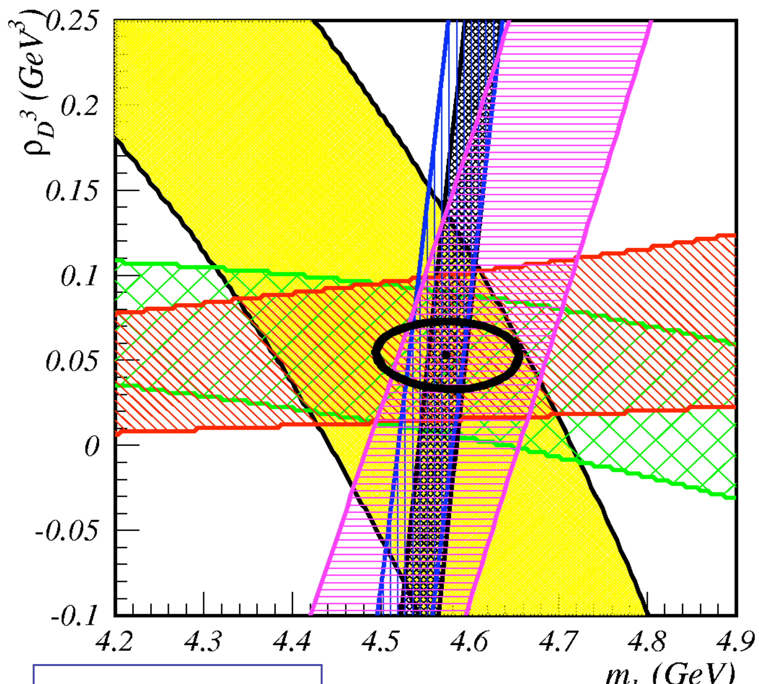
- Analysis by Battaglia et al. on DELPHI data in 2002 was the first to establish the present working paradigm:

  - measure 3 lepton energy and 3 hadronic mass moments in $B \to l\nu X_c$ with acceptance over the full range of E_l.



BABAR

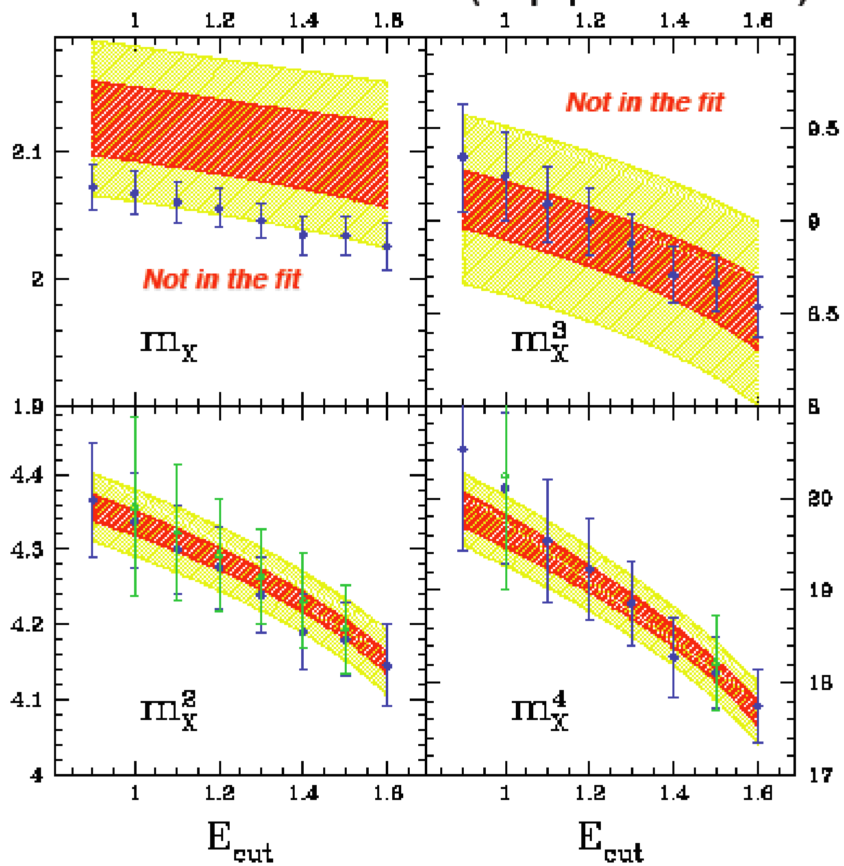




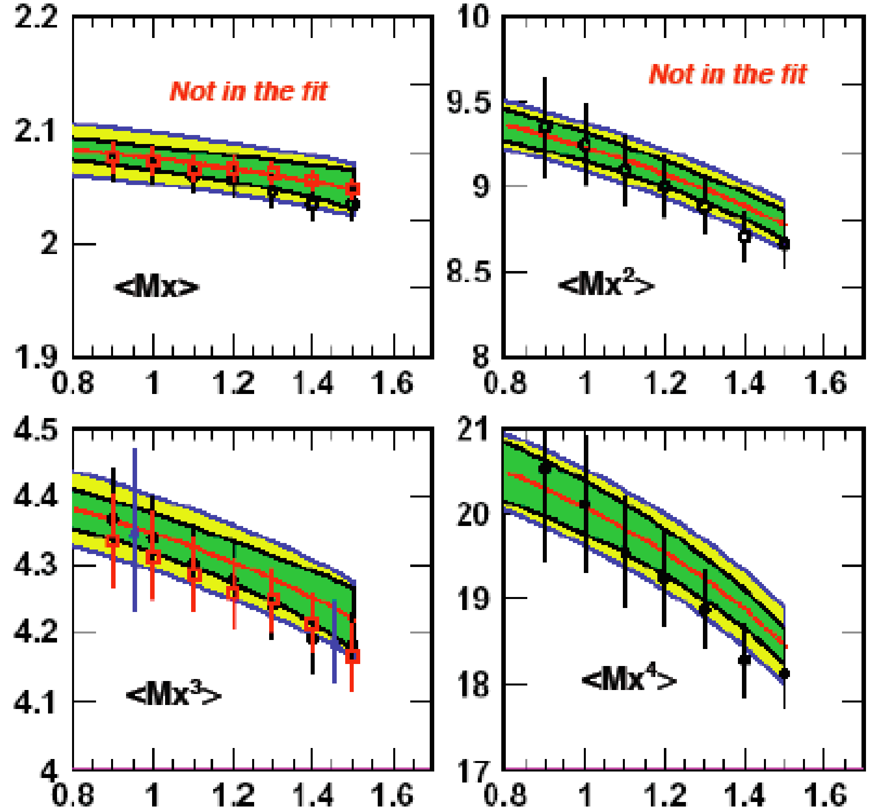
DELPHI

Hadron Moments: <Mx> and $<Mx^3>$ *

Bauer et al. 1s scheme (hep-ph/0408002)



Our results - kinetic scheme



While the lepton energy moments lead to very similar results, especially the non-integer hadron mass moments *<Mx> and $<Mx^3>$* are different …

from O. Buchmueller

- excellent description of large set of data points in terms of 6 or even merely 4 parameters: m_b, m_c, μ_π^2, ρ_D^3, (μ_G^2, ρ_{LS}^3)
- a priori free fit parameters assume values obeying various theoretical constraints and knowledge!

$m_b(1\ \text{GeV}) = (4.61 \pm 0.068)\ \text{GeV}$ $m_{b,kin}(1\ \text{GeV})|_{bb} = 4.57 \pm 0.08\ \text{GeV}$

$m_c(1\ \text{GeV}) = (1.18 \pm 0.092)\ \text{GeV}$

$m_b(1\ \text{GeV}) - m_c(1\ \text{GeV}) = (3.436 \pm 0.032)\ \text{GeV}$

$m_b(1\ \text{GeV}) - m_c(1\ \text{GeV})|_{MB-MD} = (3.48 \pm 0.02 \pm\ ?)\ \text{GeV}$

$m_b(1\ \text{GeV}) - 0.74 m_c(1\ \text{GeV}) = (3.737 \pm 0.017)\ \text{GeV}$

→ challenge for LQCD!

$\mu_G^2(1\ \text{GeV}) = (0.267 \pm 0.067)\ \text{GeV}^2$ $\mu_G^2|_{HF} \approx 0.35 \pm 0.03\ \text{GeV}^2$

$\mu_\pi^2(1\ \text{GeV}) = (0.447 \pm 0.053)\ \text{GeV}^2$ $\mu_\pi^2|_{QCDSR} = 0.45 \pm 0.1\ \text{GeV}^2$

$\rho_D^3(1\ \text{GeV}) = (0.195 \pm 0.029)\ \text{GeV}^3$ $\rho_D^3(1\ \text{GeV}) \sim 0.1\ \text{GeV}^3$

$m_b(1\ \text{GeV})\|_{B \to l\nu X_c}$ = 4.61 ± 0.068 GeV		BaBar
$m_b(1\ \text{GeV})\|_{H_b \to l\nu X_c}$ = 4.575±0.069±0.043±0.005 GeV		DELPHI
$m_b(1\ \text{GeV})\|_{\Upsilon(4S) \to bb}$ = 4.57±0.08 GeV		
$m_c(1\ \text{GeV})\|_{B \to l\nu X_c}$ = 1.18 ± 0.092 GeV		BaBar
$m_c(1\ \text{GeV})\|_{H_b \to l\nu X_c}$ = 1.144±0.106±0.071±0.020 GeV		DELPHI
$m_c(1\ \text{GeV})\|_{cc\ SR}$ = 1.19 ± 0.11 GeV		
$m_c(1\ \text{GeV})\|_{cc\ SR}$ = 1.30 ± 0.03 GeV		
$m_b(1\ \text{GeV}) - m_c(1\ \text{GeV})\|_{B \to l\nu X_c}$ = 3.436±0.032 GeV		BaBar
$m_b(1\ \text{GeV}) - m_c(1\ \text{GeV})\|_{H_b \to l\nu X_c}$ = 3.431 ± ? GeV		DELPHI
$m_b(1\ \text{GeV}) - m_c(1\ \text{GeV})\|_{MB-MD}$ = 3.48±0.02± ? GeV		
$\mu_\pi^2(1\ \text{GeV})\|_{B \to l\nu X_c}$ = 0.447 ± 0.053 GeV^2		BaBar
$\mu_\pi^2(1\ \text{GeV})\|_{H_b \to l\nu X_c}$ = 0.399±0.047±0.039±0.020 GeV^2		DELPHI
$\mu_\pi^2(1\ \text{GeV})\|_{QCDSR}$ = 0.45 ±0.1 GeV^2		
$\mu_G^2(1\ \text{GeV})\|_{HF}$ = 0.35 ±0.03 GeV^2		

*

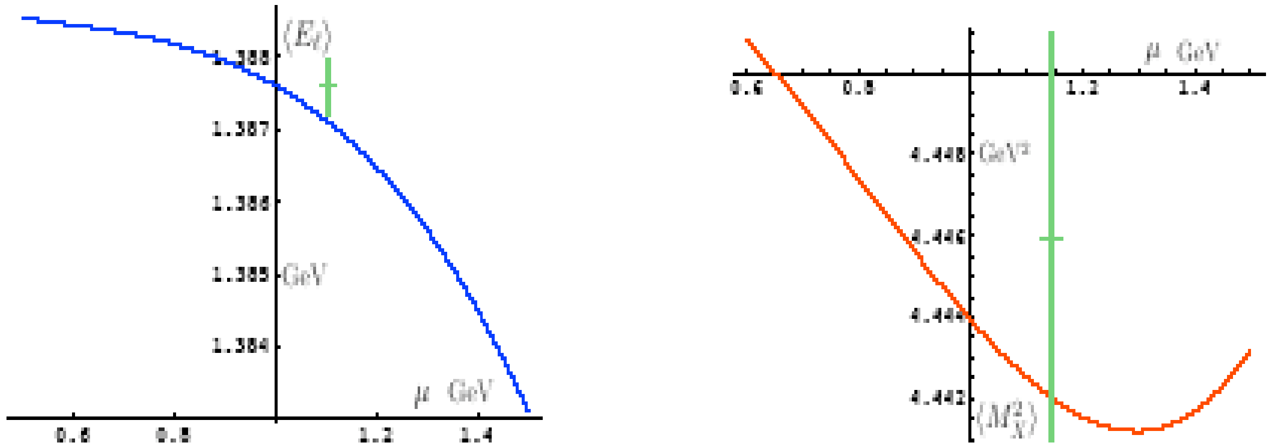


Figure 5: Dependence of $\langle E_\ell \rangle$ (left) and of $\langle M_X^2 \rangle$ (right) on the separation scale μ. The green vertical bars show the change in the moments when m_b is varied by $\pm 1\,\mathrm{MeV}$

Uraltsev

… CLEO … DELPHI …

BABAR '04:

$V(cb)|_{incl}=(41.390 \pm 0.870) \times 10^{-3}=41.390\times(1 \pm 0.021) \times 10^{-3}$

DELPHI '04 preliminary

$V(cb)|_{incl}=(42.1 \pm 1.1) \times 10^{-3} =42.1\times(1 \pm 0.025) \times 10^{-3}$

Comment: impressive consistency of all measured moments yields best bounds on $b \to c$ being purely left-handed!

4.4 Lessons for $B \to l\nu X_u$

no need to `re-invent the wheel' -- for $B \to l\nu X_u$ use the same values of the HQP as determined in $B \to l\nu X_c$

- in principle $\Gamma(B \to l\nu X_u)$ under better theoretical control than $\Gamma(B \to l\nu X_c)$

Lepton energy endpoint spectrum ?

- ☹ model dependent!
- 😐 can get heavy quark distribution function from $B \to \gamma X$
  - 😐 but only to leading order in $1/m_b$
- 😐 endpoint spectrum different for SL B_u and B_d decays (WA)

Hadronic recoil mass spectrum !

➥ |V(ub)| within 10 % likely, 5 % possible

4.5 Issue of `biases' due to experimental cuts

*

Experimental cuts on energy etc. applied for practical reasons yet they degrade `hardness' Q of transition

∃ `exponential' contributions $\exp[-cQ/\mu_{had}]$ missed in usual OPE expressions

- quite irrelevant for $Q \gg \mu_{had}$
- yet relevant for $Q \sim \mu_{had}$!

Test case: $B \rightarrow \gamma X_q$

for $B \rightarrow \gamma X_q$: $Q = m_b - 2\, E_{cut}$

e.g.: for $E_{cut} \sim 2$ GeV, $Q \sim 1$ GeV !

*

- noted before: usual OPE expression for $B \to \gamma X_q$ somewhat indifferent to impact of experimental cuts

- early CLEO analyses showed a systematic shift in values of HQP extracted from $B \to \gamma X_q$

Pilot study (Uraltsev, IB, PL B 579 ('04) 340)

Detailed study (Benson,Uraltsev, IB, hep-ph/0410080) impact of cuts depends on spectrum *

- evaluate $(1/\Gamma)(d\Gamma/dE_\gamma)(B \to \gamma X_q)$ with Wilsonian prescription *ab initio*

- employ 2 quite different ansaetze for *b* quark distribution functions

$$F_1(k_+) = N_1\,(\overline{\Lambda}-k_+)^\alpha\, e^{ck_+}\, \theta(\overline{\Lambda}-k_+),$$
$$F_2(k_+) = N_2\,(\overline{\Lambda}-k_+)^\beta\, e^{-d(\overline{\Lambda}-k_+)^2}\, \theta(\overline{\Lambda}-k_+)$$

- not a unique choice
- expect smooth function, since B ground state
- only interested in fairly global properties

*

$$\langle E_\gamma\rangle \simeq \langle E_\gamma\rangle^{\rm pert} + \langle E_\gamma\rangle^{\rm power} + \tfrac{1}{2}\tilde{\delta} m_b,$$

$$\langle E_\gamma^2 - \langle E_\gamma\rangle^2\rangle \simeq \langle E_\gamma^2 - \langle E_\gamma\rangle^2\rangle^{\rm pert} + \langle E_\gamma^2 - \langle E_\gamma\rangle^2\rangle^{\rm power} - \tfrac{1}{12}\tilde{\delta}\mu_\pi^2.$$

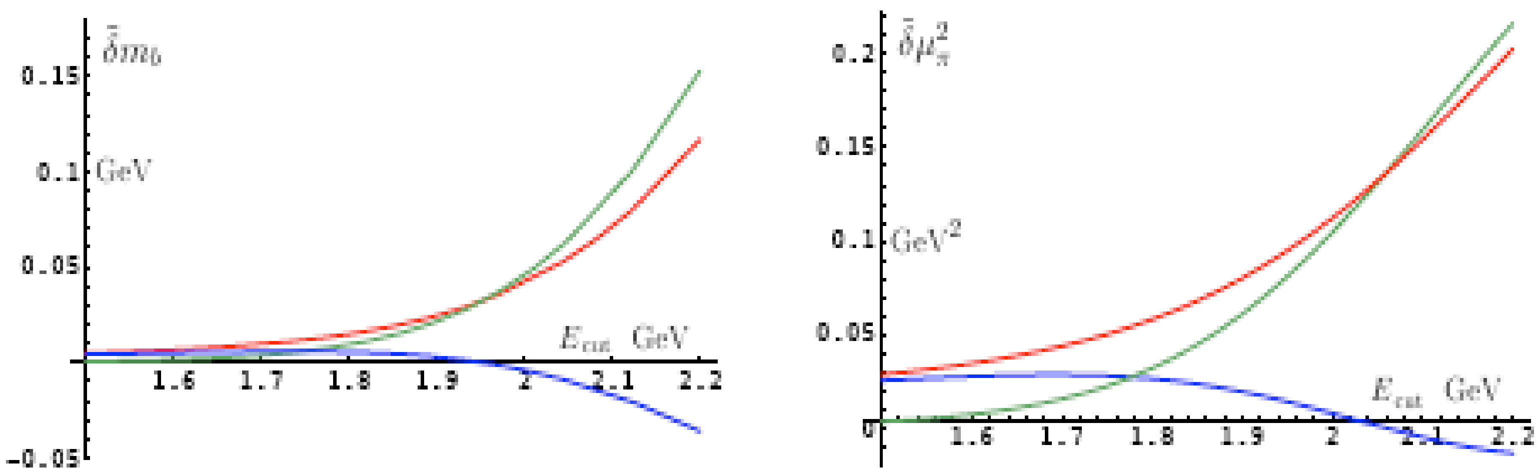


Figure 2: The perturbative effects on the biases $\tilde{\delta} m_b$ and $\tilde{\delta}\mu_\pi^2$. The red curve is the complete bias using the full spectrum. The green curve is the bias calculated without perturbative corrections, and the blue line shows the difference between the two curves.

bias corrections depend on values of HQP *

for BABAR's central values of the HQP we get

$\langle E_\gamma \rangle|_{1.8}^{biased}$=2.305 GeV → $\langle E_\gamma \rangle|_{1.8}^{corr}$=2.312 GeV

$\langle E_\gamma \rangle|_{1.8}^{BELLE}$=2.292±0.026±0.034 GeV

$\langle E_\gamma \rangle|_{1.9}^{biased}$=2.313 GeV → $\langle E_\gamma \rangle|_{1.9}^{corr}$=2.325 GeV

$\langle E_\gamma \rangle|_{2.0}^{biased}$=2.321 GeV → $\langle E_\gamma \rangle|_{2.0}^{corr}$=2.342 GeV

$\langle E_\gamma \rangle|_{2.0}^{CLEO}$=2.346±0.032±0.011 GeV

$\langle E_\gamma \rangle|_{2.1}^{biased}$=2.329 GeV → $\langle E_\gamma \rangle|_{2.1}^{corr}$=2.364 GeV

*

$$\langle(E_\gamma-\langle E_\gamma\rangle)^2\rangle|_{1.8}^{bias}=0.0357\ GeV^2 \rightarrow \langle(E_\gamma-\langle E_\gamma\rangle)^2\rangle|_{1.8}^{corr}=0.0309\ GeV^2$$

$$\langle(E_\gamma-\langle E_\gamma\rangle)^2\rangle|_{1.8}^{BELLE}=0.0305\pm0.0074\pm0.0063\ GeV^2$$

$$\langle(E_\gamma-\langle E_\gamma\rangle)^2\rangle|_{1.9}^{bias}=0.0321\ GeV^2 \rightarrow \langle(E_\gamma-\langle E_\gamma\rangle)^2\rangle|_{1.9}^{corr}=0.0255\ GeV^2$$

$$\langle(E_\gamma-\langle E_\gamma\rangle)^2\rangle|_{2.0}^{bias}=0.0293\ GeV^2 \rightarrow \langle(E_\gamma-\langle E_\gamma\rangle)^2\rangle|_{2.0}^{corr}=0.020\ GeV^2$$

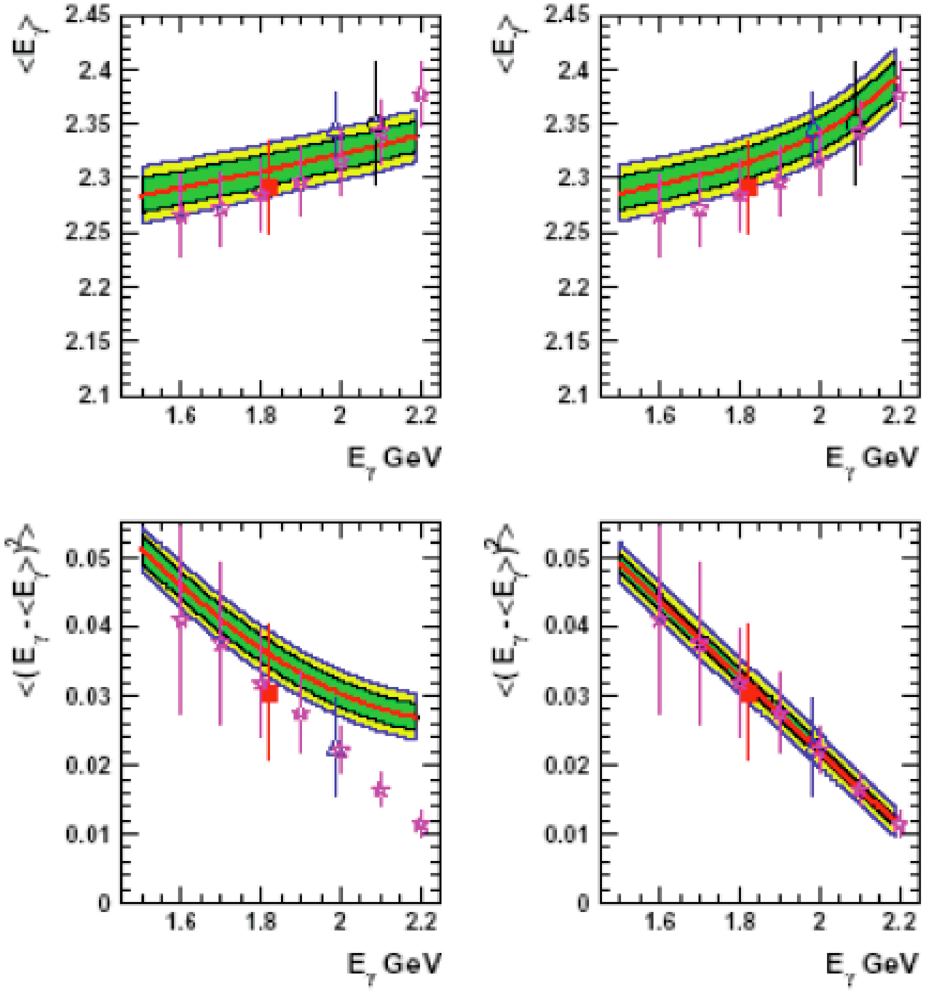
$$\langle(E_\gamma-\langle E_\gamma\rangle)^2\rangle|_{2.0}^{CLEO}=0.0226\pm0.0066\pm0.0020\ GeV^2$$

$$\langle(E_\gamma-\langle E_\gamma\rangle)^2\rangle|_{2.1}^{bias}=0.0271\ GeV^2 \rightarrow \langle(E_\gamma-\langle E_\gamma\rangle)^2\rangle|_{2.1}^{corr}=0.0145\ GeV^2$$

Consistency between $b \to s\gamma$ and $b \to cl\nu$ *

Use extracted HQE parameters from the $cl\nu$ moment fit to predict the moments of the photon energy spectrum.

"HQE only" "bias corrected HQE"



Moment measurements agree well with HQE prediction obtained from the $cl\nu$ moment fit.

Evidence that bias correction is needed for moments above $E_\gamma > 1.8$ GeV

But we can do more …

→Use the shape function parameter that fit the BELLE spectrum to obtain the moments as a function of the cut.

(Test: agrees nicely at $E_\gamma = 1.8$ GeV with the direct measurement from BELLE)

Remarkable agreement with HQE prediction

Strong evidence, especially from the second moment, that bias corrections are needed above $E_\gamma > 1.8$ GeV.

from O. Buchmueller

*

Lessons:

- keep the cuts as low as possible
- bias in the meas. moments induced by cuts
  - ☞ can be corrected for (within a certain range of cut values)
  - ☞ not a pretext for inflating theor. uncert.
- moments meas. as fction of cuts: important cross check!

Personal plea to BELLE/BABAR:

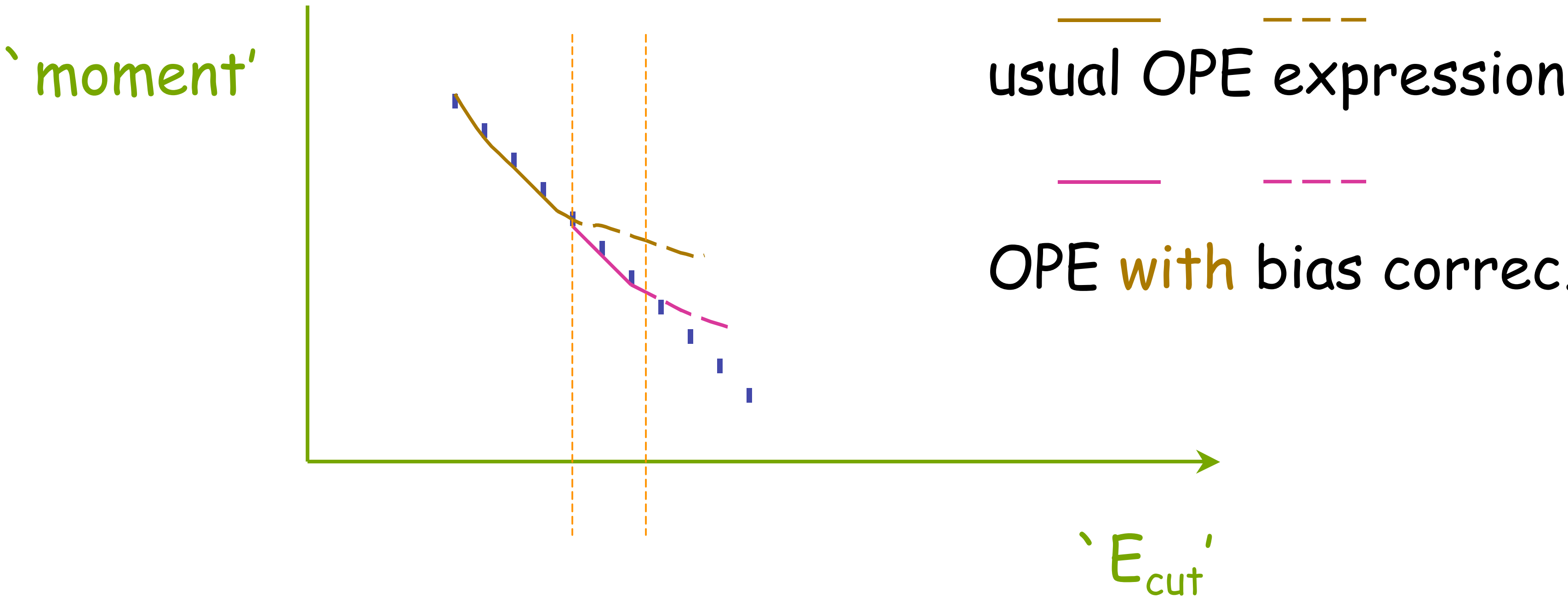
Please tell us what you measure for

- $\langle E_\gamma \rangle$, $\langle E_\gamma^2 - \langle E_\gamma \rangle^2 \rangle$
- with E_{cut} = 1.8, 1.9, 2.0, 2.1, 2.2 GeV!!

done by BaBar

`defensible' ? --

*



4.6 $V(cb)_{excl}$

$B \to l\nu D^*$

measure rate of B -> l ν D*

☹ extrapolate to zero recoil & extract $|V(cb)\, F_{D^*}(0)|$

☺ $F_{D^*}(0) = 1 + O(1/m_Q^2) + O(a_s)$ normalized

☺ holds automatically for $m_b = m_c$

☹ expansion in $1/m_c$!

$F_{D^*}(0) =$
- $0.89 \pm 0.08 [0.05]$ Uraltsev et al.: $O(1/m_Q^2)$
- 0.913 ± 0.042 BaBar Book `par ordre de Mufti'
- $0.913\,^{+0.024}_{-0.017}\,^{+0.017}_{-0.030}$ 2nd *quenched* lattice: H,K et al. $O(1/m_Q^3)$ [~ 0.89 at $O(1/m_Q^2)$]

use: $F_{D^*}(0) = 0.90 \pm 0.05$ for convenience

➲ $|V(cb)|_{excl} = 0.0416 \times (1 \pm 0.022|_{exp} \pm 0.06|_{theor})$

Unorthodoxy: $B \to e/\mu\, \nu\, D$

Uraltsev: BPS expansion

if $\mu_\pi^2 = \mu_G^2$: $\sigma \bullet \pi |B\rangle = 0$, $\rho^2 = 3/4$

in real QCD: $\mu_\pi^2 - \mu_G^2 \ll \mu_\pi^2$, $\rho^2 \approx 3/4$

expansion in $\beta = [3(\rho^2 - 3/4)]^{1/2} = 3\, [\Sigma_n |\tau^{(n)}_{1/2}|^2]^{1/2}$

irreducible $\delta f_+(0) \sim \exp(-2m_c/\mu_{had}) \sim$ few %

Program:

❶ extract |V(cb)| from $B \to e/\mu\, \nu\, D$

❷ compare with `true' |V(cb)| from $\Gamma_{SL}(B)$
to validate BPS expansion
if successful -- see later

4.7 $B \to \tau\nu X$, $B \to \tau\nu D$ and New Physics

- analyze $B \to l\nu D$ & extract $|V(cb)|$
- validate it with $|V(cb)|$ from $B \to l\nu X$
- if successful, measure $B \to \tau\nu D$ - 2nd FF f_- can be measured!
- compare with SM prediction for known $|V(cb)|$

☞ discrepancy could be interpreted in terms of charged Higgs

➻ repeat analysis for $B \to \tau\nu X$

$|V(td)/V(ts)|$

from $B \to \gamma X_d$ vs. $B \to \gamma X_s$?

V Summary of Lecture IV

The new heavy flavour paradigm: add high accuracy to high sensitivity

Extracting CKM parameters with accuracy seemingly

unrealistic less than 10 years ago --

with detailed & defensible error budgets from theorists!

- $\delta V(cb) \sim 2$ % now, ~ 1 % soon
- $\delta V(ub) \sim 5$ % conceivable

without new theoretical breakthrough

Progress based on two key elements:

- robust theory subjected to the challenges of
- high quality data
  - precision, i.e. small defensible uncertainties

overconstraints