Flavour Dynamics & CP in the SM*: A Tale of Great Successes, Little Understanding -- and Promise for the Future!

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Lecture II (6)

CKM Phenomenology
Recap from Lecture I

The CKM description of flavour transitions:

an amazingly successful piece of `theoretical engineering’
based on central mysteries of the SM -- mass generation
for quark fields.

\[ |V_{\text{CKM}}| \sim \begin{pmatrix}
1 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix} \]

presumably profound message from nature -- in an
encoded form
Menu for Lecture II

I  Phenomenological Landscape → 1999

II  Theoretical Technologies
   Effective Field Theories
   Operator Renormalization & Mixing
   Nonperturbative Tool Chest

III  The CKM Paradigm of Large CP in B Decays
   Prelude '52 - '73
   Growing up '73 - '94
   Completion of a Heroic Era
   Status of CKM Theory end of 2nd Millenium
   CKM Exotica -- EDM’s

IV  Summary of Lecture II
I Phenomenological Landscape → 1999

- 'θ–τ puzzle': θ → 2π, τ → 3π
- production >> decay rates → associated production
- τ(K* → π+π0) >> τ(K → π+π–) → ΔI=1/2 rule
- Cabibbo Universality: |V(us)|^2 << |V(ud)|^2, |V(ud)|^2 + |V(us)|^2 = 1
- K^0-K^0 oscillations → ΔΓ_K, ΔM_K
- K_L → μ^+μ^–, γγ → suppression of FlChNC
- K_L → π^+π^– → ΔL
- charm → |V(cs)|^2 ~ |V(ud)|^2, |V(cd)|^2 ~ |V(us)|^2
- beauty → |V(ub)|^2 << |V(cb)|^2 << 1 → ΔM_B
- top → m_t > M_Z, >> m_b → inferred f. quantum corr.
2.1 Electroweak Dynamics

Can be dealt with perturbatively

consider $\Delta S=1$

\[ L = g_W q_1 \gamma_\mu (1-\gamma_5)q_2 W_\mu \]

with heavy fields -- $W_\mu$ -- `integrated out'
2.1.1 Effective Field Theories

**Wilson prescription:**
- **Define field theory** $L(\Lambda)$ at **UV scale** $\Lambda$ with $\Lambda \gg$ *germane scales* of theory like $M_W, m_Q$ etc.
- for applications characterized by scales $\sim \mu$ integrate out the heavy d.o.f. to arrive at an **effective low energy** field theory using the **Operator Product Expansion (OPE)** as tool

\[ L(\Lambda) \rightarrow L(\mu) = \sum_i c_i(\mu,\Lambda) \, O_i(\mu) \]

**c numbers** providing gateway for **heavy d.o.f.** with frequencies $> \mu$

**local operators** containing dynamical, i.e. active fields with frequencies $\leq \mu$
integrating out heavy d.o.f. induces higher-dimensional (d>4) operators

\[ L(\mu_1) \neq L(\mu_2) \quad \text{for} \quad \mu_1 \neq \mu_2 \]

in principle observables cannot depend on value of \( \mu \) -- it is just a labeling device:

short distances \(<\ \mu^{-1}\ <\ \text{long distances} \)

in practice, however, must be chosen judiciously due to limitations in our computational abilities s.t.

\[ \alpha_s(\mu) < 1 \]

\[ \text{matrix elements can be evaluated} \quad \Rightarrow \mu \sim 1 \text{ GeV} \]
consider $\Delta S=2$ produced by iterating $\Delta S=2$

$\Delta S=2$ produced by iterating $\Delta S=2$

$\Delta M(K)$

$\Delta M(B)$

$m_t > M_B$

$m_t > m_c > M_K$

$L_{\text{eff}}(\Delta S=2) \propto$

$L_{\text{eff}}(\Delta B=2) \propto$
`food for thought' a.k.a. homework assignment

\[ \Delta M(B) \propto \left( \frac{m_t}{M_W} \right)^2 \quad \text{for } m_t \gg M_W \]

😢 "decoupling" ??
radiative QCD corrections affect the strength of these effective weak transition operators -- and create new & novel ones!

$\Delta S=1$

under QCD renormalization the two operators

mix
$I_{\text{init}} = \frac{1}{2}$

$\Delta I = \frac{1}{2}$ & $\frac{3}{2}$

$\Rightarrow I_{\text{final}} = 1$

$\Rightarrow \Delta I = \frac{1}{2}$ & $\frac{3}{2}$

$I_{\text{final}} = 0$

$\Rightarrow \Delta I = \frac{1}{2}$ only
2 multiplicatively renormalized operators

\[ O_{\pm} = \frac{1}{2} \times \begin{cases} s & \text{d} \\ u & \pm s & \text{u} \end{cases} \]

\[ L_{\text{eff}} \propto c_- O_- + c_+ O_+ \]

with \( c_- > 1 > c_+ \) & \( c_- c_+^2 \sim 1 \), \( O_- \) pure \( \Delta I = \frac{1}{2} \)

... the emergence of Penguins!

\( \begin{cases} \text{pure } \Delta I = \frac{1}{2} \\
\text{sensitive to three families in } \\
\Delta S = 1 \\
\text{direct } CP \end{cases} \)
2.2 Nonperturbative Dynamics

To calculate rates need to evaluate on-shell hadronic ME:

\[ T(H \rightarrow f) \propto \langle f | L_{\text{eff}} | H \rangle \propto \Sigma_i c_i(\mu) \langle f | O_i(\mu) | H \rangle, \]

\[ \mu = \text{usual hadronic scale} \]

Challenge of nonperturbative dynamics

-quark models

The `old war horse' -- can still get some mileage out of it, but do not overburden it: excellent for developing intuition, first answers, yet unsatisfactory for final answers

-chiral perturbation theory = QCD at low energies

-heavy quark theory = QCD for heavy flavours (see later)

-LQCD = the perceived Panacea

-QCD Sum Rules
a central example:

\[ \langle K \mid (s_L \gamma_\mu d_L) (s_L \gamma_\mu d_L) \mid K \rangle, \langle B_q \mid (b_L \gamma_\mu q_L) (b_L \gamma_\mu q_L) \mid B_q \rangle \]

control \( K-K, B-B \) oscillations

Vacuum Saturation (VS)/`factorization`

\[ \langle K \mid (s_L \gamma_\mu d_L) (s_L \gamma_\mu d_L) \mid K \rangle \]

\[ 1 = |0><0| + |\text{had}><\text{had}| \]

\[ \langle K \mid (s_L \gamma_\mu d_L) (s_L \gamma_\mu d_L) \mid K \rangle \sim \langle K \mid (s_L \gamma_\mu d_L) \mid 0><0 \mid (s_L \gamma_\mu d_L) \mid K \rangle \]

\[ \text{if } Kp_\mu \quad \text{if } Kp_\mu \]

\[ = B_K f_K^2 M_K^2 , \]

\( B_K^{\text{VS}} = 1, B_K \sim O(1) \)

\[ \not\exists \text{ fact. at } \mu_1 \not\exists \text{ fact. at } \mu_2 \text{ if } \mu_1 \neq \mu_2 \]
III The CKM Paradigm of Large CP in B Decays

- phenomenological distinction between $\Delta F=1$ & $\Delta F=2$ dynamics; yet underlying theory has to yield both!
- same interplay between $\Delta F=1$ & $\Delta F=2$ affects CP

\[
\eta_{+-} = \epsilon + \epsilon', \quad \eta_{00} = \epsilon - 2\epsilon'
\]

as long as CP seen in a single decay of a neutral meson, distinction between direct & indirect CP arbitrary!

\[
\eta_{+-,00} = \frac{A(K_L \rightarrow \pi^{+0} \pi^{-0})}{A(K_S \rightarrow \pi^{+0} \pi^{-0})}
\]
a general comment:

transitions like $K_L \rightarrow \pi\pi$ or $B_d \rightarrow \psi K_S$ involve two phenomenologically distinct dynamics, namely $\Delta F=1$ & $\Delta F=2$;

it is important to deduce from data to which degree both contribute to observed modes;

yet in the end the underlying theory has to explain both.

Lastly:

The `Superweak’ Model is not a theory, not even a model -- it is merely a classification scheme.
3.1 Prelude: ’52 → ’73

- CP vs. \( P \)

Wick, N.D., Wightman, Wigner (1952): the "... disturbing possibility ..." that CP \( \checkmark \), yet \( P \) and \( C \) is "remote at the moment"!

- discovery of \( P \) in ’57 a great shock, yet theorists quickly recovered

“politics and \( R \)”

\[
\begin{align*}
\pi^- &\rightarrow e_L^- \nu & \text{or} & \pi^+ &\rightarrow e_R^+ \nu \\
\text{"L"} &= f ("-")
\end{align*}
\]

If \( CP \checkmark \) ⇒ “L" pure convention!
"the thumb is left on the right hand!"
'64: CP discovered -- caused another even greater shock!
Attempts at evasion:
- $K_L \rightarrow \pi \pi$ implying CP requires Superposition Principle of QM -- give up SuPoPr!

- ∃ invisible CP odd particle $U$  $K_L \rightarrow \pi^+ \pi^- [U]$
a la Pauli's postulate for $\nu$'s in $\beta$ decay $n \rightarrow p e [\nu]$
introduce new invisible particle to save conservation law


\begin{align*}
\text{Commutator} & \quad \nu \\
\text{CP} & \quad \text{energy-momentum} \\
\text{did not work} & \quad \text{did work} \\
\text{`quod licet Jovi, non licet bovi'} & \\
= \text{Pauli} & \quad = \text{non-Pauli}
\end{align*}
- $\text{CP} \Rightarrow \not\exists$

- $\text{CP}$ required to define matter vs. antimatter, $L$ vs. $R$, $+ \text{ vs. } -$ in convention independent way

- smallest observed violation of a symmetry

- $\text{Im } M_{12} \approx 1.1 \times 10^{-8} \text{ eV} \Leftrightarrow \text{Im } M_{12}/m_K \approx 2.2 \times 10^{-17}$

- frustrating -- no `peccate fortiter'  
  $\text{CP}$ invariance as a `near miss' vs. maximal $P$
- '65: Sakharov conditions for baryogenesis
  - $\Delta N_{\text{baryon}} \neq 0$,
  - $\mathbb{C}\mathbb{P}$
  - out-of-thermal equilibrium

- phenomenology of $\mathbb{C}\mathbb{P}$ quickly developed

$$\eta_{+-} = \varepsilon + \varepsilon', \quad \eta_{00} = \varepsilon - 2\varepsilon'$$

- '64 - '72: lack of theory not realized

  even after renormalizibility of $\text{SU}(2)_L \times \text{U}(1)$ recognized

  (except for short remark by Mohapatra in '72)
'64 - '72:

- 3 `quarks`: u, d, s
- 1 mixing angle $\theta_C$
- 2 charged leptons + 2 $\nu$
- charm -- suggested, yet common sentiment: “Nature is smarter than Shelley (Glashow) -- she can do without charm”

☞ asymptotic freedom of QCD discovered & appreciated

'74 `October Revolution'

- quarks viewed as real d.o.f.
- first `heavy' quark -- charm -- found
  open charm hadrons identified in '76
- $\tau$ lepton found -- beginning of 3rd family
'76: discovery of $\Upsilon$

'79: prediction of large CP in $B_d \rightarrow K\pi$

'80: prediction of large CP in $B^0$ decays involving qm state mixing & oscillations, in particular in $B_d \rightarrow \psi K_S$

- before a single B decay channel had been identified!

`beauty prefers charm': $|V(cb)|^2 \gg |V(ub)|^2$

'82: discovery of `long' B lifetime $\sim O(1\ \text{psec})$: $|V(us)|^2 \gg |V(cb)|^2$
the emerging pattern:

- $|V(us)| = \lambda$
- $\tau(B) \sim 1 \text{ psec} \quad \Rightarrow \quad |V(cb)| \sim O(\lambda^2)$
- $|V(ub)|/|V(cb)| \sim O(\lambda)$

Wolfenstein representation

$V_{\text{CKM}} = \begin{pmatrix}
1 - \lambda^2 & \lambda & \Lambda \lambda^3 (\rho - i\eta + \eta \lambda^2/2) \\
-\lambda & 1 - \lambda^2/2 - i\eta \Lambda^2 \lambda^4 & \Lambda \lambda^2 (1 + \eta \lambda^2) \\
\Lambda \lambda^3 (1 - \rho - i\eta) & -\Lambda \lambda^2 & 1
\end{pmatrix}$
3 classes of 2 unitarity triangles each the sides of which have length

- \( \lambda + \lambda^5 \)
  - sd triangle: \( V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = \delta_{sd} = 0 \)
  - cu triangle: \( V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = \delta_{cu} = 0 \)

- \( \lambda^2 + \lambda^2 + \lambda^4 \)
  - bs triangle: \( V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = \delta_{bs} = 0 \)
  - tc triangle: \( V_{td}^* V_{cd} + V_{ts}^* V_{cs} + V_{tb}^* V_{cb} = \delta_{ts} = 0 \)

- \( \lambda^3 + \lambda^3 + \lambda^3 \)
  - bd triangle: \( V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = \delta_{bd} = 0 \)
  - tu triangle: \( V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = \delta_{tu} = 0 \)

all six triangles have equal area!
last 2 triangles have all 3 sides of comparable length
→ all their angles are naturally large

control B transitions

V_{ud} V^*_{ub} \quad \Delta B=1 \quad V_{td} V^*_{tb} \quad \Delta B=2

\phi_1 \quad \phi_2 \quad \phi_3

one more obstacle: due to CPT CP can enter only through complex phases
→ to observe CP need 2 different, yet coherent amplitudes for a process
✿ best & most spectacular realization

B^0 - B^0 oscillations
'86: discovery of $B^0$ oscillations: $x(B_d) = \Delta M(B_d)/\Gamma_B = 0.75$

- indirect bound $m_t > 100$ GeV

[similar, though less precise than later LEP I findings]

$\text{CKM} \rightarrow \text{CP in } B^0 \text{ decays with natural unit 10\%, not 0.1\%}$

Predicted before discovery of top quarks

$$\sin 2\phi_1 \propto \epsilon_K/\Delta M(B) \ [= f(m_t)] \sim 0.6 - 0.7$$

'94: top quarks discovered directly

- all observed CP expressed by 1 number $|\eta_{+-}| \rightarrow \Phi(\Delta S=2) = \arg(M_{12}/\Gamma_{12})$
- intriguing, though not conclusive evidence for direct CP

$$\epsilon'/\epsilon = (2.30 \pm 0.65) \times 10^{-3} \text{ NA31 vs. } (0.74 \pm 0.59) \times 10^{-3} \text{ E731}$$

both launched by theory predictions and done in the '80's
3.3 The Completion of a Heroic Era

- **direct $\bar{CP}$ established by ’99**
- WA ’03: \( \text{Re } \epsilon'/\epsilon = (1.66 \pm 0.16) \times 10^{-3} \)
  \[
  \frac{\Gamma(K^0 \rightarrow \pi^+\pi^-) - \Gamma(K^0 \rightarrow \pi^+\pi^-)}{\Gamma(K^0 \rightarrow \pi^+\pi^-) + \Gamma(K^0 \rightarrow \pi^+\pi^-)} = (5.5 \pm 0.6) \times 10^{-6}
  \]
- a discovery of the first rank -- irrespective of theory
- experimental groups earned our admiration
- **not inconsistent with SM/CKM**
  - CKM is not a superweak theory
  - $\epsilon'/\epsilon$ suppressed by $\Delta I=1/2$ rule, superheavy top mass, being a loop effect
- do not expect quick conclusive reply from theory

standard for CPT tests!
3.4 Status of CKM theory end of 2nd millennium

Yes, indeed ...

large fraction of $\Delta m_K$, $\epsilon_K$, $\Delta m_B$

most of $\epsilon_K'$

or equivalently

data constraints translate into `broad' bands

in unitarity triangle plots

yet such a statement
misses the real point!
broad bands, indeed, it seems ...
... but look at the big picture!

... there must be a good reason!
can be reproduced with

|V(us)| $\sim$ 0.22, |V(ts)| $\sim$ 0.04

|V(td)| $\sim$ 0.004

$m_u \sim 5$ MeV, $m_c \sim 1.2$ GeV

$m_t \approx 180$ GeV, $m_d \sim 10$ MeV

$m_s \sim 0.15$ GeV, $m_b \approx 4.6$ GeV

observables spanning several orders of magnitude accommodated with parameter choices that a priori would seem frivolous!

There could easily have been inconsistencies!

small $|V(td)|$ offset by large $m_t$
hadronization ( & nonperturbative dynamics in general) usually viewed as unwelcome complication (if not outright nuisance);
case in point:

interpretation of observed $\Delta m_K, \epsilon_K, \Delta m_B , \epsilon_K'$ contains sizeable uncertainties
correct -- yet such perspective again misses the deeper truth
without hadronization no formation of bound states

☞ no $K^0$-$\bar{K}^0$ oscillations
  ➞ no indirect CP: $\text{Im } M_{12} \sim O(10^{-8} \text{ eV})$
  ➞ no direct CP à la $\epsilon'$

☞ no $B^0$-$\bar{B}^0$ oscillations
  ➞ no CP in $\Delta B=2$: $\sim O(10^{-4} \text{ eV})$
  ➞ no New Physics in $\Delta B=2$

hadronization

☞ reduces CP $\sqrt{K_L \rightarrow 3\pi}$ by $\sim 500$ due to hadronic PhSp
☞ awards `patience`; i.e. you can `wait' for pure $K_L$ beam
☞ generates CP signal in existence rather than asymmetry

hadronization -- the hero rather than the villain in the tale of CP!
another imminent CKM triumph (?)

at present:

without $\varepsilon_K$ & $\sin 2\phi_1$ `flat' CKM triangle still allowed
[unless accept QCD Fact.'s verdict on BR($B \rightarrow K \pi$)]

if $\Delta M(B_s)$ measured `soon':

$|V(ub)/V(cb)|$ & $\Delta M(B_s)$ require non-trivial CKM triangle
`CP insensitive observables imply CP!'
arrival of the pure Penguins’

\[ B \to \gamma K^* \]
\[ B \to \gamma X_S \]

as 1-loop processes pure quantum effect in SM

already by ’98 CKM dynamics provided a surprisingly successful description of highly diverse phenomena

CKM fits yielded: \( \sin 2\phi_1 [\beta] = 0.72 \pm 0.07 \)

cannot count on New Physics inducing large deviation from the SM

need for precision experimentally & theoretically!
3.5 CKM `Exotica, Outliers, Leftfielders’

**Electric dipole moments**

Energy shift $\Delta \mathcal{E}$ of system inside electric field $\mathbf{E}$:

$$\Delta \mathcal{E} = d_i E_i + d_{ij} E_i E_j + \ldots$$

Linear in $\mathbf{E}$ \quad $d \propto s \Rightarrow d \neq 0 \iff T$ violation!

$d_N < 0.63 \times 10^{-25}$ ecm vs. $d_N^{\text{CKM}} < 10^{-30}$ ecm

from ultracold neutrons

$memento$: Strong CP problem!

$d_e = (0.07 \pm 0.07) \times 10^{-26}$ ecm vs. $d_e^{\text{CKM}} < 10^{-32}$ ecm

from atomic EDM

✍ New Physics scenarios can yield $\sim 10^{-26} - 10^{-28}$ ecm
to visualize the sensitivity achieved

\[ d_N = 6.3 \times 10^{-26} \text{ e cm} \ \Rightarrow \text{radius} \ r_N \sim 10^{-13} \text{ cm} \]

7000 km

search for displacement of \( 10^{-12} R_e \sim 7 \mu \)!

\[ d_e = (-0.3 \pm 0.8) \times 10^{-26} \text{ e cm} \]

\[ \Leftrightarrow \delta[(g-2)/2] \sim 10^{-11} \]

\[ \Leftrightarrow \delta(F_2(0)/2m_e) \approx 2 \times 10^{-22} \text{ e cm}! \]
Pol$_\perp(\mu)$ in $K_{\mu3}$ decays

$K \rightarrow \mu^+\nu\pi$

Pol$_\perp(\mu)=\langle s_\mu \cdot (p_\mu \times p_\pi)/|p_\mu \times p_\pi| \rangle$ -- T odd moment

$K^+ \rightarrow \mu^+\nu\pi^0$

Pol$_\perp(\mu)=(-1.7 \pm 2.3 \pm 1.1) \times 10^{-3}$ vs. Pol$_{\perp}^{SM}(\mu) < 10^{-6}$

● a clean search for CP via Higgs dyn.

$K_L \rightarrow \mu^+\nu\pi^-$

Pol$_{\perp}^{SM}(\mu) \sim 10^{-3} (\sim \alpha/\pi)$ -- Coulomb FSI!

τ decays

most certainly not least!

see lecture VI
from a general perspective ...

take a model with a set of mass related basic quantities -- fermion masses, CKM parameters -- that any sober person would view as frivolous -- were they not forced upon us by data -- in particular since we have no deeper understanding of mass generation in particular for fermions --

you would have no reason to expect success in describing flavour dynamics proceeding on vastly different scales --

yet it does seem to work!
this model has to produce a host of large CP in B decays --

there is no plausible deniability

Some of us concluded that while the seemingly accidental structures in CKM theory strongly suggest a deeper level of dynamics underlying them, one cannot count on New Physics intervening in heavy flavour decays in a numerically massive and thus obvious way