CHIRAL PERTURBATION THEORY

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Lecture I

- Classification of effective field theories
- Spontaneous symmetry breaking and nonlinear realization
- Chiral symmetry breaking
- Chiral Lagrangians and low-energy expansion

Lecture II

- Renormalization and chiral logs
- Matching CHPT with QCD
- Survey of CHPT applications
- Pion-pion scattering

Lecture III

- Isospin violation and precision physics
- CKM matrix elements V_{ud}, V_{us}
- $K \to 2\pi$ decays and CP violation
- Hadronic vacuum polarization and the muon magnetic moment

Literature on CHPT

Classics

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Classification of effective field theories

field theoretic formulation of the

quantum ladder

- relevant degrees of freedom depend on energy
- energy scales well separated

steps of quantum ladder characterized by energy Λ:

$E < \Lambda$		$E > \Lambda$
long distances		short distances
effective	QFT	"fundamental"

criterion for classification :

transition from fundamental to effective theory

A. Decoupling of heavy states

$E < \Lambda$: heavy fields "integrated" out

no additional light fields generated

EFT Lagrangian contains only the remaining light fields ($M < \Lambda$)

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{n\leq 4} + \sum_{n>4} \frac{1}{\Lambda^{n-4}} \sum_{i_n} g_{i_n} O_{i_n}$$

n : operator dimension (in energy units)

O_{i_n} : monomials in light fields with dim=n

g_{i_n} : dimensionless constants

$\mathcal{L}_{n\leq 4}$: potentially renormalizable part

("relevant" and "marginal" operators)

$\mathcal{L}_{n>4}$: "irrelevant" operators example of EFT of type A:

Standard Model

from experiment :

$\Lambda > 100$ GeV $\longrightarrow$ $\mathcal{L}_{n\leq 4}$ sufficient at present

remarks :

- $\mathcal{L}_{n\leq 4}$ "naturally" renormalizable (power counting)
- perturbative expansion of $\mathcal{L}_{n\leq 4}$ vs. $\mathcal{L}_{n>4}$ at lowest order

B. Spontaneous symmetry breaking

phase transition $\rightarrow$ additional light fields :

(pseudo) Goldstone bosons

symmetry acts nonlinearly on GBs $\longrightarrow$

- separation between $\mathcal{L}_{n\leq 4}$ and $\mathcal{L}_{n>4}$ meaningless
- EFTs generically nonrenormalizable : perturbative treatment ?

Goldstone theorem

- existence of GBs
- GBs decouple for $E_{GB} \to 0$
  (independent of underlying interaction !)

$\longrightarrow$ **systematic low-energy expansion**

$\mathcal{L}_{\text{eff}}$: derivatives instead of operator dimension

phase transition $\to$ no perturbative matching in general

Examples

- condensed matter : magnons, phonons, superconductivity
- electroweak gauge symmetry breaking:
  $\Lambda = G_F^{-1/2}$, $GBs \to W_{\text{long}}, Z_{\text{long}}$
- chiral symmetry of QCD : $\Lambda \simeq 1$ GeV

Spontaneous symmetry breaking

realization of symmetries in relativistic quantum field theories

continuous (internal) symmetry:

Noether current $J^\mu(x)$: $\partial_\mu J^\mu = 0$
conserved charge $Q = \int d^3x J^0(x)$
translation invariance : $[Q, P_\mu] = 0$

ground state : $P_\mu|0\rangle = 0$

$$\begin{aligned} ||Q|0\rangle||^2 &= \langle 0|QQ|0\rangle = \int d^3x \langle 0|QJ^0(x)|0\rangle \\ &= \int d^3x \langle 0|Qe^{iPx}J^0(0)e^{-iPx}|0\rangle = \int d^3x \underbrace{\langle 0|QJ^0(0)|0\rangle}_{\text{x}-\text{independent}} \end{aligned}$$

$\rightarrow$ only 2 possibilities

Goldstone alternative

$Q\|0\rangle = 0$	$\|\|Q\|0\rangle\|\| = \infty$
Wigner–Weyl	Nambu–Goldstone
linear representation	nonlinear realization
degenerate multiplets	massless GBs
exact symmetry	spont. broken symm.

Def.: $Q^V(x^0) = \int_V d^3x J^0(x)$ (finite volume V)

$$\lim_{V\to\infty} [H, Q^V] = 0 \text{ and } \lim_{V\to\infty} ||Q^V|0\rangle|| = \infty$$

Ass.: $\exists$ operator A with $\underbrace{\lim_{V\to\infty} \langle 0|[Q^V(x^0), A]|0\rangle}_{\text{order parameter of SSB}} \neq 0$

only possible for $Q|0\rangle \neq 0$

example: scalar fields $\varphi_i(x)$ with $[Q, \varphi_i] = c_{ij}\varphi_j$ and $\langle 0|\varphi_j|0\rangle \neq 0$

Goldstone theorem $\longrightarrow$

$\exists$ massless state $|GB\rangle$ with $\langle 0|J^0(0)|GB\rangle\langle GB|A|0\rangle \neq 0$

necessary and sufficient condition for SSB :

Goldstone matrix element $\langle 0|J^0(0)|GB\rangle \neq 0$

$\longrightarrow$ $|GB\rangle$ same quantum numbers as $J^0(0)|0\rangle$

remarks :

- $|GB\rangle$ need not correspond to physical particle (gauge symm.)
- $J^0(0)$ (usually) rot. inv. bosonic operator $\Rightarrow$ $|GB\rangle$ spin 0
- discrete SSB does not produce GBs
- in general (inequality only possible in nonrelativ. systems):

  $$N_{\text{broken symm.}} \geq N_{\text{GB}}$$

Goldstone model

simplest QFT example for SSB with complex scalar field $\phi(x)$

$$\mathcal{L}_{\text{Goldstone}} = \partial_\mu \phi \partial^\mu \phi^\dagger - \lambda(\phi\phi^\dagger - \frac{v^2}{2})^2 \quad (\lambda > 0, \quad v \text{ real, positive})$$

$U(1)$ symmetry : $\qquad \phi(x) \to e^{i\alpha}\phi(x)$

minimum of (Mexican hat) potential at $\quad \phi\phi^\dagger = \dfrac{v^2}{2}$

$\longrightarrow \qquad$ SSB with

$$\phi(x) = (R(x) + iG(x))/\sqrt{2}\ , \qquad R(x), G(x) \ \text{hermitian}$$

$$\langle 0|R(x)|0\rangle = v, \qquad \langle 0|G(x)|0\rangle = 0$$

spectrum (tree level)

GB field $G(x)$ $\qquad\qquad M_G = 0$

massive field $H(x) = R(x) - v$ $\qquad\qquad M_H = \sqrt{2\lambda}v$

scattering amplitudes at tree level:

$$A(GG \to GG) = O(p_G^4), \qquad A(GH \to GH) = O(p_G^2)$$

p_G generic momentum of GBs, p_H arbitrary

Why do GBs decouple for $E_{\text{GB}} \to 0$?

different choice of fields (polar decomposition)

$$\phi(x) = \frac{1}{\sqrt{2}}[h(x) + v]e^{ig(x)/v}$$

in terms of fields $g(x), h(x)$ $\longrightarrow$

$$\begin{aligned}\mathcal{L}_{\text{Goldstone}} &= \frac{1}{2}(\partial_\mu g)^2 + \frac{1}{2v^2}(h^2 + 2vh)(\partial_\mu g)^2 \\ &+ \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 - \frac{\lambda}{4}(h^4 + 4vh^3)\end{aligned}$$

Haag–Borchers: fields G, H and g, h lead in general to different Green functions, but S-matrix elements are identical

Observations

- spectrum unchanged

  GB field $g(x)$ $\qquad M_g = 0$

  massive field $h(x)$ $\qquad M_h = M_H = \sqrt{2\lambda}v$

- only derivative couplings for Goldstone field g

$$\longrightarrow \qquad \lim_{p_{GB}\to 0} A = 0$$

anticipating spontaneously broken chiral symmetry

general property for processes with massless pions
(and with at most 2 nucleons):

S-matrix elements vanish if all $p_{GB} \to 0$

Nonlinear realization

$\phi^a(x)$: Goldstone fields (zero curvature fluctuations)

general case of spontaneous symmetry breaking:

$$G \longrightarrow H$$

$G,\ H$: (compact) Lie groups (dim $= N_G, N_H$)

H: subgroup of G, invariance group of the vacuum

$g \in H$: linear representation on GB fields ϕ^a

$g \notin H$: nonlinear realization on ϕ^a

$N_G - N_H$ Goldstone fields "parametrize" coset space G/H

transformation of non-Goldstone fields

$$\psi \xrightarrow{g \in G} \psi' = h(g,\phi)\psi$$

$h(g,\phi)$: depends on ϕ for $g \notin H$

$\Rightarrow$ standard construction of **G-invariant $\mathcal{L}_{\rm eff}(\phi,\psi)$**

Chiral symmetry breaking

starting point: **QCD in chiral limit**

$N_f = 2$ [or 3] massless quarks u, d [, s]

$$\mathcal{L}^0_{\rm QCD} = \overline{q_L} i \not{D} q_L + \overline{q_R} i \not{D} q_R + \mathcal{L}_{\text{heavy quarks}} + \mathcal{L}_{\text{gauge}}$$

$$q_{R,L} = \frac{1}{2}(1 \pm \gamma_5) q \; , \qquad q = \begin{pmatrix} u \\ d \\ [\mathrm{s}] \end{pmatrix}$$

exhibits global symmetry $\underbrace{SU(N_f)_L \times SU(N_f)_R}_{\text{chiral group } G} \times U(1)_V \times U(1)_A$

$U(1)_V$: baryon number

$U(1)_A$: Abelian anomaly $\longrightarrow$ $M_{\eta'} \neq 0 \quad (N_f = 3)$

strong evidence (phenomenology + theory) for

spontaneous chiral symmetry breaking

$$G \longrightarrow H = SU(N_f)_V$$

with H = isospin ($N_f = 2$) or flavour $SU(3)$ ($N_f = 3$)

1. absence of parity doublets
2. pseudoscalar mesons lightest hadrons
3. V and A spectral functions very different (ρ vs. a_1)
4. anomaly matching conditions (t'Hooft) + confinement
   $\rightarrow$ G spontaneously broken for $N_f \geq 3$
5. for $\theta_{\rm QCD} = 0$, $SU(N_f)_V$ remains unbroken (Vafa, Witten)
6. lattice QCD: quark condensate non-zero

Mechanism for SCSB ?

order parameter of SSB: $\lim_{V\to\infty}\langle 0|[Q^V(x^0), A]|0\rangle \neq 0$

QCD : axial charges $Q_A^a = Q_R^a - Q_L^a$ $(a = 1, \ldots, N_f^2 - 1)$

A : colour singlet, pseudoscalar operators

unique possibility for operator with lowest dimension (dim =3):

$$A_b = \overline{q}\gamma_5\lambda_b q \quad \rightarrow \quad [Q_A^a, A_b] = -\frac{1}{2}\overline{q}\{\lambda_a, \lambda_b\}q$$

$SU(N_f)_V$ invariance of the vacuum : $\langle 0|\overline{u}u|0\rangle = \langle 0|\overline{d}d|0\rangle$ $[= \langle 0|\overline{s}s|0\rangle]$

$\longrightarrow$ sufficient condition for SCSB :

quark condensate $\langle 0|\overline{q}q|0\rangle \neq 0$

certainly not necessary: $\exists$ condensates with dim > 4

important question :

$\langle 0|\bar{q}q|0\rangle$ dominant order parameter of SCSB ?

answer could depend on N_f: $\langle 0|\bar{q}q|0\rangle$ probably decreases with N_f

Moussallam; Descotes, Girlanda, Stern

$N_f = 2$: positive answer from $\pi\pi$ scattering

Implementation of SCSB

dim $G/H = N_f^2 - 1$ Goldstone boson fields ϕ^a

$$L(\phi) = (u_L(\phi), u_R(\phi)) \in G/H$$

chiral transformation $g = (g_L, g_R) \in G$: nonlinear realization

$$u_A(\phi) \xrightarrow{g} g_A u_A(\phi) h(g,\phi)^{-1} \qquad (A = L, R)$$

Def.: matrix field $U(\phi)$ (transforming linearly)

$$U(\phi) := u_R(\phi) u_L(\phi)^\dagger \xrightarrow{g} g_R U(\phi) g_L^{-1}$$

nonlinear realization $\longrightarrow$ $U(\phi)$ cannot be a polynomial in ϕ

standard choice of coset coordinates (exponential parametrization)

$$u_R(\phi) = u_L(\phi)^\dagger := u(\phi) = \exp\left(i\lambda_a\phi^a/2F\right) \quad \longrightarrow \quad U(\phi) = u(\phi)^2$$

Goldstone matrix element defines F (meson decay constant) :

$$\langle 0|\overline{q(x)}\gamma^\mu\gamma_5\frac{\lambda_a}{2}q(x)|\phi_b(p)\rangle = i\delta_{ab}Fp^\mu e^{-ipx}$$

non-Goldstone fields (meson resonances, baryons , . . .)

$$\psi \stackrel{g\in G}{\longrightarrow} \underbrace{h_\psi(g,\phi)}_{SU(N_f)_V \text{ repr. of } \psi} \psi$$

back to the real world: **no chiral symmetry in nature !**

- explicit breaking : $m_q \neq 0$

$$\begin{array}{lcll} m_u, m_d & \ll & \Lambda \simeq M_\rho & N_f = 2 : \pi \\ m_s & < & M_\rho & N_f = 3 : \pi, K, \eta \end{array}$$

- electroweak interactions :
  included perturbatively in α, G_F

Main assumption of CHPT

expansion around chiral limit meaningful

therefore (even for $\alpha = G_F = 0$) :

2-fold expansion of chiral Lagrangians

i. derivatives $\sim$ momenta

ii. quark masses

$$\mathcal{L}_{\rm eff} = \sum_{i,j} \mathcal{L}_{ij} \;, \qquad \mathcal{L}_{ij} = O(\partial^i m_q^j)$$

relation through meson masses :

$$M_M^2 \sim B m_q + O(m_q^2), \qquad B = -\langle 0|\overline{u}u|0\rangle / F^2$$

Standard CHPT

Weinberg, Gasser, Leutwyler

Ass.: $M_M^2 \sim B m_q$ dominant contribution

corresponds to $B(\nu = 1\ \text{GeV}) \simeq 1.4\ \text{GeV}$

$\rightarrow$ $3M_{\eta_8}^2 = 4M_K^2 - M_\pi^2$ Gell-Mann, Okubo

$\rightarrow$ chiral counting : $m_q = O(M^2) = O(p^2)$

$$\mathcal{L}_{\text{eff}} = \sum_n \mathcal{L}_n \;, \qquad \mathcal{L}_n = \sum_{i+2j=n} \mathcal{L}_{ij}$$

mesons $n = 2, 4, 6, \ldots$

mesons + baryons $n = 1, 2, 3, \ldots$

standard CHPT:

clearly established for $N_f = 2$ ($\rightarrow$ pion-pion scattering)

Chiral Lagrangians and low-energy expansion

couple external matrix fields v_μ, a_μ, s, p

$$\mathcal{L}^0_{\rm QCD} \to \mathcal{L}^0_{\rm QCD} + \overline{q}\gamma^\mu(v_\mu + a_\mu\gamma_5)q - \overline{q}(s - ip\gamma_5)q$$

$\to$ $\mathcal{L}_{\rm eff}$ inherits local chiral symmetry

Advantage

- $\mathcal{L}_{\rm eff}$ and Green functions (of quark currents) manifestly chiral invariant
- phys. amplitudes : $v_\mu = a_\mu = p = 0, \quad s = {\rm diag}(m_u, m_d[, m_s])$

$\mathcal{L}_{\rm eff}$ for the strong interactions of mesons

$$\mathcal{L}_{\rm eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots$$

$$\mathcal{L}_2 = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle$$

gauge-covariant derivative

$$D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)$$

$$\chi = 2B(s + ip) \ , \qquad \chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u$$

2 parameters :

$$F_\pi = F\,[1 + O(m_q)], \qquad \langle 0|\overline{u}u|0\rangle = -F^2 B\,[1 + O(m_q)]$$

CHPT at $O(p^2)$

lowest-order amplitudes: $\mathcal{L}_2$ at tree level $\simeq$ current algebra

amplitudes depend only on F_π, M_M^2

e.g.: $\pi\pi \to \pi\pi$

$$A_2(s,t,u) = \frac{s - M_\pi^2}{F_\pi^2} \qquad \text{Weinberg}$$

$\to$ absolute prediction from pure symmetry !

CHPT at $O(p^4)$

chiral Lagrangian $\mathcal{L}_4$ contains 10 (7) measurable low-energy constants (LECs) for $N_f = 3$ (2) T

$\mathcal{L}_{\rm eff}$ hermitian $\rightarrow$ tree amplitudes real

BUT unitarity + analyticity require complex amplitudes

example : $\pi\pi \rightarrow \pi\pi$

$$\mathrm{Im}\ t_l^I(s) \geq (1 - \frac{4M_\pi^2}{s})^{\frac{1}{2}} |t_l^I(s)|^2$$

partial waves $t_l^I(s)$ start at $O(p^2)$ $\rightarrow$ $\mathrm{Im}T \neq 0$ at $O(p^4)$

systematic low-energy expansion

$\rightarrow$ loop expansion for $n > 2$

loop amps. are in general divergent

regularization <u>and</u> renormalization necessary

Chiral Lagrangian of $O(p^4)$ for $N_f = 3$

Gasser, Leutwyler

$$\mathcal{L}_4 = L_1\langle D_\mu U^\dagger D^\mu U\rangle^2 + L_2\langle D_\mu U^\dagger D_\nu U\rangle\langle D^\mu U^\dagger D^\nu U\rangle + L_3\langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U\rangle$$

$$+ L_4\langle D_\mu U^\dagger D^\mu U\rangle\langle \chi^\dagger U + \chi U^\dagger\rangle + L_5\langle D_\mu U^\dagger D^\mu U(\chi^\dagger U + U^\dagger \chi)\rangle + L_6\langle \chi^\dagger U + \chi U^\dagger\rangle^2$$

$$+ L_7\langle \chi^\dagger U - \chi U^\dagger\rangle^2 + L_8\langle \chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger\rangle$$

$$- iL_9\langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U\rangle + L_{10}\langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu}\rangle$$

$$+ 2 \text{ contact terms} = \sum\nolimits_i L_i P_i$$

$$F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu] , \qquad r^\mu = v^\mu + a^\mu$$

$$F_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu] , \qquad l^\mu = v^\mu - a^\mu$$

$$L_i = L_i^r(\mu) + \Gamma_i \Lambda(\mu)$$

$$\Lambda(\mu) = \frac{\mu^{d-4}}{(4\pi)^2}\left\{\frac{1}{d-4} - \frac{1}{2}[\log 4\pi + 1 + \Gamma'(1)]\right\}$$

$$L_i^r(\mu_2) = L_i^r(\mu_1) + \frac{\Gamma_i}{16\pi^2}\ln\frac{\mu_1}{\mu_2}$$

Consequences:

- renormalization $\rightarrow$ no cutoff dependence
- divergences absorbed by coupling constants in $\mathcal{L}_4, \mathcal{L}_6, \ldots$
- renormalized LECs scale dependent:
  contain effects of "heavy" states

LECs in principle dimensionless functions of $\Lambda_{\rm QCD}$, m_c, m_b, m_t

matching (at $\Lambda \simeq M_\rho$) not possible in perturbation theory $\rightarrow$

need to bridge gap between

CHPT	$\longleftrightarrow$	perturbative QCD
$E < M_\rho$		$E > 1.5$ GeV

most successful (or promising) approaches :

phenomenological	: resonance saturation ($\leftarrow$ large N_c)
theoretical	: lattice

Renormalization and chiral logs

loop expansion $\sim$ expansion around classical solution

of which equation ?

chiral Lagrangian (mesons only)

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots$$

to ensure systematic chiral expansion :

expand around solution of

equation of motion of lowest-order Lagrangian $\mathcal{L}_2$

$\rightarrow$ perturbative propagator always $O(p^{-2})$

successive orders in the chiral expansion characterized by

chiral dimension D_L

$=$ degree of homogeneity in external momenta and meson masses general loop diagram (mesons only) with

L : number of loops

N_n : number of vertices from $\mathcal{L}_n$

I : number of internal meson lines

$$\rightarrow \quad D_L = 4L + \sum_{n\geq 2} nN_n - 2I$$

connected diagram : $L = I - \sum_n N_n + 1$

$$\rightarrow \quad D_L = 2L + 2 + \sum_{n\geq 4}(n-2)N_n \geq 2L + 2$$

scale of chiral expansion

D_L increases with L, phys. dim. fixed $\rightarrow$ $\frac{1}{F^2}\frac{1}{(4\pi)^2}$ for each loop

for mesons $\rightarrow$ **expansion in $\frac{p^2}{(4\pi F)^2}$**

$N_f = 2$: $\frac{p^2}{(4\pi F)^2} = 0.014\frac{p^2}{M_\pi^2}$,

$N_f = 3$: $\frac{p^2}{(4\pi F)^2} = 0.18\frac{p^2}{M_K^2}$

Renormalization at $O(p^4)$

$$D_L = 2L + 2 + \sum_{n\geq 4}(n-2)N_n \quad \rightarrow \quad \text{2 possibilities for } D_L = 4$$

$L = 0$: $\quad N_4 = 1 \quad N_2$ arbitrary

$L = 1$: $\quad N_2$ arbitrary

process independent procedure useful (e.g., with heat kernel expansion) $\rightarrow$ generating functional of Green functions $Z[v,a,s,p]$

Advantages

- nontrivial check for explicit loop calculations
- RGE for renormalized, scale dep. LECs
- leading infrared singularities : chiral logs

divergent part (with $\Lambda(\mu) = \dfrac{\mu^{d-4}}{(4\pi)^2}\left\{\dfrac{1}{d-4} - \dfrac{1}{2}[\ln 4\pi + 1 + \Gamma'(1)]\right\}$)

$$Z^{(L=1)}_{4,\text{div}} = \int d^4x \mathcal{L}_{4,\text{div}} = -\Lambda(\mu)\int d^4x \sum_i \Gamma_i P_i$$

renormalization : with $2\ln c := -\ln 4\pi - 1 - \Gamma'(1)$

$$L_i(d) = (c\mu)^{d-4}\left[\frac{\Gamma_i}{(4\pi)^2(d-4)} + \underbrace{L_i^r(\mu)}_{\text{ren. LEC}} + O(d-4)\right]$$

generating functional (up to Wess-Zumino-Witten functional)

$$\begin{aligned} Z_4 &= Z_4^{(L=1)} + \int d^4x\mathcal{L}_4(L_i) = Z_{4,\mathrm{fin}}^{(L=1)}(\mu) + \int d^4x\mathcal{L}_4(L_i^r(\mu)) \\ &= Z_{4,\mathrm{fin}}^{(L=1)}(M) + \int d^4x\mathcal{L}_4(L_i^r(\mu) - \frac{1}{2}\Gamma_i l(\mu)) \end{aligned}$$

→ amplitudes depend on scale-independent combinations

$\boxed{L_i^r(\mu) - \frac{1}{2}\Gamma_i l(\mu)}$ with chiral log $l(\mu) = \frac{1}{(4\pi)^2}\ln M^2/\mu^2$

M : typical scale $\longrightarrow$ $M = M_\pi (M_K)$ for $N_f = 2(3)$

$Z_{4,\mathrm{fin}}^{(L=1)}(M)$: known in closed form for ≤ 3 propagators in loop

Gasser, Leutwyler, Schweizer, Unterdorfer

Renormalization group equations

$$\mu \frac{dL_i^r(\mu)}{d\mu} = -\frac{\Gamma_i}{(4\pi)^2} \quad \rightarrow \quad L_i^r(\mu_2) = L_i^r(\mu_1) + \frac{\Gamma_i}{16\pi^2} \ln \frac{\mu_1}{\mu_2}$$

Phenomenological values of $L_i^r(M_\rho)$

i	$L_i^r(M_\rho) \times 10^3$ 1995	2000 (Amoros et al.)	main source	Γ_i
1	0.4 ± 0.3	0.53 ± 0.23	$K_{e4}, (\pi\pi \to \pi\pi)$	3/32
2	1.35 ± 0.3	0.71 ± 0.24	$K_{e4}, (\pi\pi \to \pi\pi)$	3/16
3	-3.5 ± 1.1	-2.72 ± 0.99	$K_{e4}, (\pi\pi \to \pi\pi)$	0
4	-0.3 ± 0.5		Zweig rule	1/8
5	1.4 ± 0.5	0.93 ± 0.12	F_K/F_π	3/8
6	-0.2 ± 0.3		Zweig rule	11/144
7	-0.4 ± 0.2	-0.32 ± 0.15	Gell-Mann–Okubo, L_5, L_8	0
8	0.9 ± 0.3	0.63 ± 0.18	$M_{K^0} - M_{K^+}, L_5$	5/48
9	6.9 ± 0.7		$\langle r^2 \rangle_V^\pi$	1/4
10	-5.5 ± 0.7		$\pi \to e\nu\gamma$	1/4

Renormalization at $O(p^6)$

T

reducible diagrams c,e,g

sum c+e+g finite and scale independent

irreducible diagrams a,b,d

divergences must be polynomials in masses and momenta of $O(p^6)$

however : diagrams a,b,d separately have **nonlocal divergences**

E.g.: diagrams a,d involve Green function $G(x,x)$

$$G(x,x) = \frac{2\mu^{d-4}}{(4\pi)^2}\left(\frac{1}{d-4} + \frac{1}{2}\ln M^2/\mu^2\right) a_1(x,x) + \underbrace{\overline{G}(x,x)}_{\text{finite,nonlocal}}$$

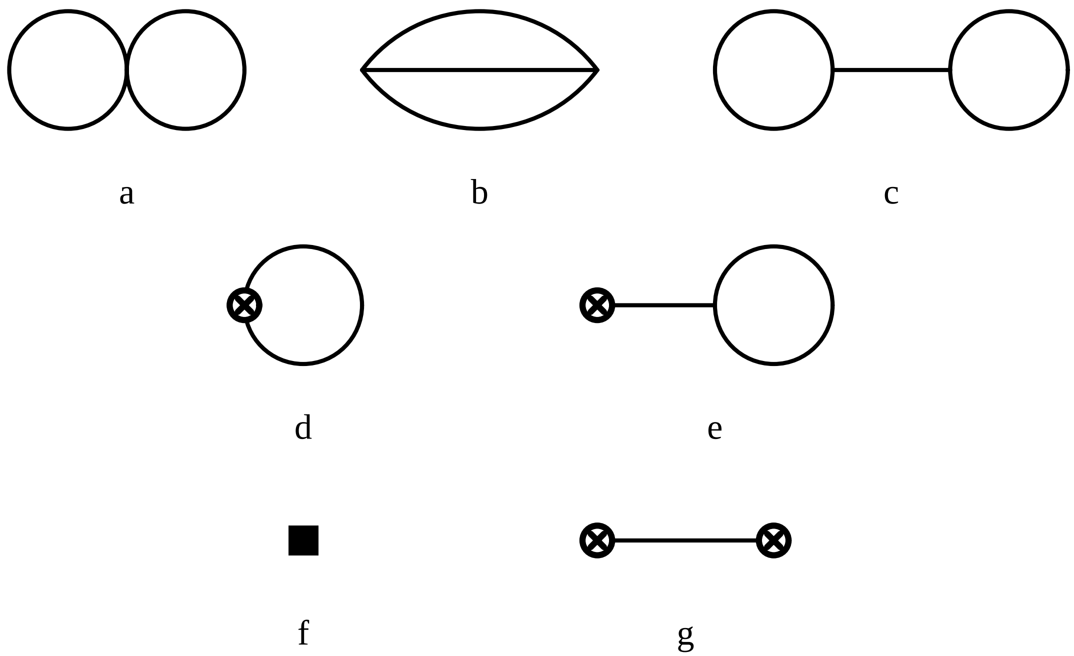
$a_1(x,x)$: (local) Seeley-DeWitt coefficient

$\rightarrow$ nonlocal divergences of the type

$$\frac{1}{d-4}\overline{G}(x,x) \quad \text{and} \quad \frac{1}{d-4}\ln M^2/\mu^2$$

skeleton diagrams for $D_L = 2L + 2 + \sum_{n\geq 4}(n-2)N_n = 6$

		diagrams
$L = 0$:	$N_6 = 1$	f
$L = 0$:	$N_4 = 2$	g
$L = 1$:	$N_4 = 1$	d,e
$L = 2$:		a,b,c



proper renormalization at $O(p^4)$ $\rightarrow$

nonlocal divergences cancel in sum a+b+d

remaining divergences : polynomials of $O(p^6)$

$\rightarrow$ divergent parts of C_i (tree diagram f) chosen
to make the sum a+b+d+f finite $\Rightarrow$

complete functional $Z_6[v, a, s, p]$
finite and scale independent with renormalized LECs $C_i^r(\mu)$

Bijnens, Colangelo, E.

Chiral logs

often numerically important or even dominant
(especially for $N_f = 2$ $\rightarrow$ $\pi\pi$ scattering)

trivial observation : total amplitudes are scale independent
$\rightarrow$ chiral logs vanish for $\mu = M$: $l(\mu) = \frac{1}{(4\pi)^2} \ln M^2/\mu^2 \overset{\mu=M}{=} 0$

However

$\mu = M$ generates "unnaturally" big $L_i^r(M) \quad \longrightarrow$
infrared singularities shifted into LECs

natural size : $\quad L_i^r(\mu \simeq M_\rho)$

full dependence on $l(\mu)^2, \quad l(\mu)L_i^r(\mu), \quad L_i^r(\mu)L_j^r(\mu)$

can be extracted from generating functional of $O(p^6)$

Generalized double-log approximation

Bijnens, Colangelo, E.

remarks :

- chiral logs may give hints for large corrections
- no substitute for full calculation
- leading chiral logs to any order (via recursion relations)
  Büchler, Colangelo

Matching CHPT with QCD

main problem of CHPT: abundance of LECs

2 classes of LECs (exemplified for $\mathcal{L}_4$ with $N_f = 3$)

i. terms surviving in the chiral limit

$$L_1, L_2, L_3, L_9, L_{10}$$

govern momentum dependence of amplitudes $\longrightarrow$ relatively easy to access experimentally

ii. symmetry breaking terms

$$L_4, \ldots, L_8$$

specify quark mass dependence of amplitudes $\longrightarrow$ difficult to extract from exp., but accessible in lattice QCD

Heitger, Sommer, Wittig; Irving et al.;
Fleming, Nelson, Kilcup; Farchioni et al.

More general problem

matching CHPT with QCD (bridging the gap $M_K \lesssim E \lesssim 1.5$ GeV)

observations:

- intermediate region is governed by resonances
- $N_C \to \infty$: Green functions determined by
  exchange of (stable) particles

$\longrightarrow$ large-N_C motivated ansatz for Green functions:

resonance exchange (with minimal number of nonets)

procedure:

a. match with asymptotic behaviour dictated by QCD (OPE, Froissart bound, Brodsky-Lepage rules)

b. match with CHPT at low energies

c. ignore loops that are (formally) suppressed for $N_C \to \infty$

Results

- establish minimal number of resonance multiplets for matching
  $\longrightarrow$ M(inimal) H(adronic) A(pproximation) de Rafael et al.
- relate LECs to resonance parameters
- high-energy constraints $\longrightarrow$ relations between parameters
- results can be encoded in a chiral resonance Lagrangian for nonet fields (E., Gasser, Leutwyler, Pich, de Rafael)

simplest example:

Pion form factor $F_\pi(t)$

Chiral Resonance Lagrangian (EGPR)

$$\mathcal{L}_{\rm CHRL}[V(1^{--}),A(1^{++}),S(0^{++}),P(0^{-+}] = \mathcal{L}_{\rm kin}[V,A,S,P] + \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle + \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu}u^\mu u^\nu\rangle + \ldots$$

$V_{\mu\nu}$ antisymmetric tensor field for $V(1^{--})$ nonet

CHRL

$$F_\pi(t) = 1 + \frac{F_V G_V}{F^2} \frac{t}{M_V^2 - t}$$

CHPT to $O(p^4)$ $(N_f = 3)$

$$F_\pi(t) = 1 + 2L_9 t/F^2 \quad (+\text{ small loop cont.})$$

$$\longrightarrow \qquad L_9 = \frac{F_V G_V}{2M_V^2}$$

asymptotic condition: $\quad \lim_{t\to\infty} F_\pi(t) = 0$

$$\longrightarrow \qquad F_V G_V = F^2, \qquad L_9 = \frac{F^2}{2M_V^2} = 7.2 \cdot 10^{-3}$$

in excellent agreement with $L_9^r(M_\rho) = (6.9 \pm 0.7) \cdot 10^{-3}$
N.B.: scale dependence is higher order in $1/N_C$

EGLPR :

all LECs of $O(p^4)$ can be understood in terms of resonance exchange

Situation at $O(p^6)$

- many more LECs
- experimental information meager $\longrightarrow$ additional theoretical input essential for NNLO analysis
- large-N_C ansatz still compatible with chiral resonance Lagrangian (must contain terms bilinear in resonance fields) ?

no complete solution exists so far $\longrightarrow$ consider special example

$\langle VAP \rangle$ three-point function

$$(\Pi_{VAP})^{abc}_{\mu\nu}(p,q) = \int d^4x \int d^4y \; e^{i(p\cdot x + q\cdot y)} \langle 0|T\{V^a_\mu(x) A^b_\nu(y) P^c(0)\}|0\rangle$$

with $SU(3)$ octet vector, axial-vector and pseudoscalar currents

$$V^a_\mu = \bar{\psi}\gamma_\mu \frac{\lambda^a}{2}\psi\,, \qquad A^a_\mu = \bar{\psi}\gamma_\mu\gamma_5 \frac{\lambda^a}{2}\psi\,, \qquad P^a = \bar{\psi} i\gamma_5 \frac{\lambda^a}{2}\psi$$

special property of $\langle VAP \rangle$: order parameter of SCSB

$\longrightarrow$ vanishes to all orders in QCD perturbation theory

$\longrightarrow$ smooth behaviour at short distances

OPE Moussallam

$$\lim_{\lambda\to\infty} (\Pi_{VAP})^{abc}_{\mu\nu}(\lambda p, \lambda q) = \frac{\langle\overline{\psi}\psi\rangle_0}{\lambda^2} f^{abc} \frac{1}{p^2 q^2 (p+q)^2} \{\ldots\}_{\mu\nu} + O\left(\frac{1}{\lambda^4}\right)$$

Knecht, Nyffeler: additional OPE constraints with p or q $\to \infty$

large-N_C inspired ansatz for invariant function $\mathcal{F}$ in $\{\ldots\}_{\mu\nu}$

$$\mathcal{F}(p^2, q^2, (p+q)^2) = \frac{\langle\overline{\psi}\psi\rangle_0}{(p^2 - M_V^2)(q^2 - M_A^2)} \times \left[a_0 + \frac{b_1 + b_2 p^2 + b_3 q^2}{(p+q)^2} + \frac{c_1 + c_2 p^2 + c_3 q^2}{(p+q)^2 - M_P^2}\right]$$

ass.: single nonet of V, A, P resonances each Cirigliano et al.

include asymptotic behaviour of form factors with pion(s) on-shell:

$$F_\pi(t) \in \langle\pi|V_\mu|\pi\rangle \quad \text{and} \quad G_A(t) \in \langle\gamma|A_\mu|\pi\rangle$$

original approach of Moussallam, Knecht, Nyffeler :

V, A resonances only

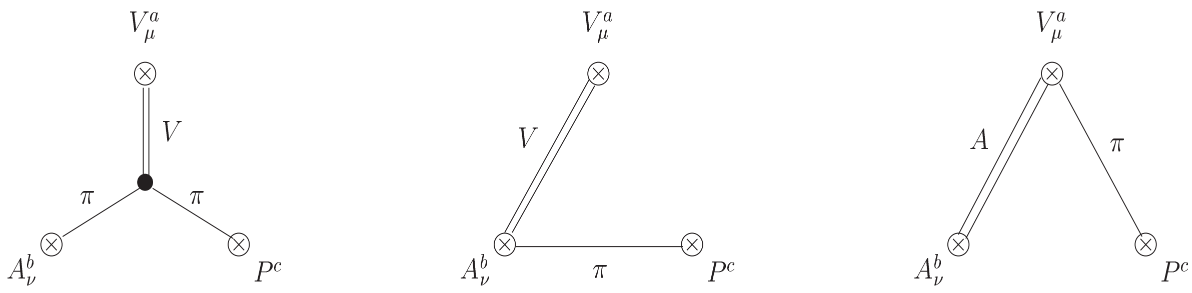
width $\Gamma(a_1 \to \pi\gamma)$ much too small

conflict with (minimal) chiral resonance Lagrangian

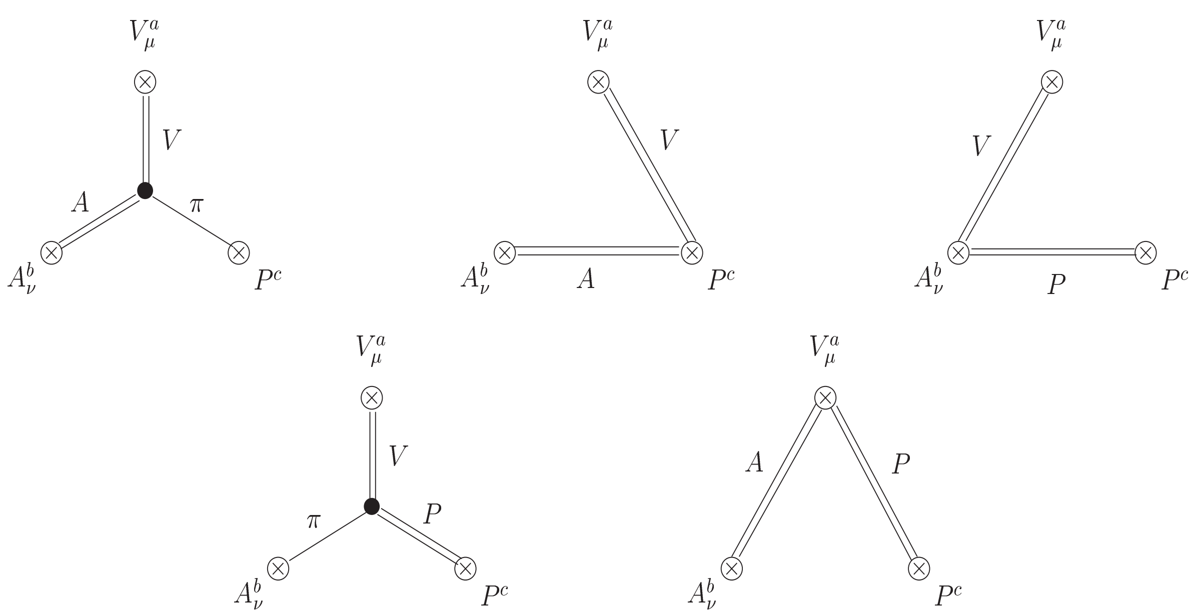
new solution of Cirigliano et al. :

- V, A and P nonets
- $\Gamma(a_1 \to \pi\gamma)$ in agreement with experiment
- other favourable properties compared to previous solution ($G_A(t), \tau \to 3\pi\nu_\tau$)
- previous results for L_9, L_{10} recovered
- six LECs of $O(p^6)$ predicted in terms of F, M_V, M_A, M_P
- completely compatible with a minimal CHRL F
- new resonance couplings fixed in terms of M_V, M_A

single-resonance exchange for $\langle VAP \rangle$



double-resonance exchange for $\langle VAP \rangle$



Survey of CHPT applications

CHPT not restricted to strong interactions of mesons

Inclusion of baryons

straightforward in principle: $\psi \xrightarrow{g \in G} h_\psi(g,\phi)\psi$

BUT

- baryons are not Goldstone particles
  - $\rightarrow$ there are no "soft" baryons
  - $\rightarrow$ interactions less constrained
  - $\rightarrow$ nucleon mass m complicates chiral counting
- $\mathcal{L}_{\rm eff}^{\rm MB} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \ldots$
  $\rightarrow$ more unknown LECs for given accuracy
- baryon resonances close to threshold (Δ, $\Lambda^*(1405)$, ...)

2 versions:

1. Heavy Baryon CHPT (Jenkins, Manohar)

   same procedure as in HQET $\longrightarrow$ expand both in

$$\frac{\vec{p}}{4\pi F_\pi} \qquad \text{and} \qquad \frac{\vec{p}}{m}$$

   properties :

   - procedure frame dependent; Lorentz invariant amplitudes can be recovered uniquely (to any given chiral order)
   - expansion does not converge in some kinematic regions

2. Relativistic Baryon CHPT (Becher, Leutwyler, Ellis, Tang)

   properties :

   - manifest Lorentz invariance at each stage
   - (infrared) regularization more involved than dim. reg.
   - usually fewer diagrams, but calculations more cumbersome (full Dirac algebra)

Dynamical photons

needed for radiative corrections

necessary ingredients:

- photon kinetic term (+ gauge fixing)
- replace external vector field by $v_\mu \to v_\mu - eQA_\mu$
  with quark charge matrix Q, photon field A
- add chiral Lagrangian bilinear in spurion fields $Q_L(x), Q_R(x)$

with transformation properties

$$Q_A(x) \stackrel{g}{\to} g_A Q_A(x) g_A^{-1}, \qquad A = L, R$$

setting $\quad Q_A(x) = Q = \text{ diag}(2/3, -1/3, -1/3)$

$\longrightarrow$ chiral Lagrangian of $O(e^2 p^{2n}) \quad (n \geq 0)$ Urech

for radiative corrections to semileptonic decays :

include leptons as dynamical fields

Knecht, Neufeld, Rupertsberger, Talavera

Nonleptonic weak interactions

integrate out W and heavy quarks $\longrightarrow$ OPE

$$\mathcal{H}_{\text{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i + \text{h.c.}$$

$C_i(\mu)$: Wilson coefficients $\qquad Q_i$: four-quark operators (dim=6)

transformation properties under G (for $\alpha = 0$)

$$\mathcal{H}_{\text{eff}}^{\Delta S=1} \sim (8_L, 1_R) + (27_L, 1_R)$$

NEEDED: **Effective Lagrangian for an effective Hamiltonian**

use again spurion fields, e.g., for octet Hamiltonian :

weak spurion $\lambda(x)$ transforming as octet field

$$\lambda(x) \xrightarrow{g} g_L \lambda(x) g_L^{-1}$$

construct general chiral Lagrangian linear in $\lambda(x)$ (and thus in G_F)

energy scale	fields	effective theory
M_W	W, Z, γ, G leptons, quarks	standard model
	$\Downarrow$ OPE $\Downarrow$	
m_c	$\gamma, G; \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\rm QCD}^{N_f=3}, \mathcal{L}_{\rm eff}^{\Delta S=1,2}$
	$\Downarrow N_C \to \infty \Downarrow$	
M_K	$\gamma, G; \mu, e, \nu_i$ π, K, η	CHPT

project out octet part by setting

$$\lambda(x) = \frac{1}{2}(\lambda_6 - i\lambda_7)$$

to get nonleptonic chiral Lagrangian of $O(G_8 p^{2n}) \quad (n \geq 1)$
and similarly for $O(G_{27} p^{2n})$ Cronin; Kambor, Missimer, Wyler

inclusion of electromagnetic penguin operators

$$\mathcal{H}_{\rm eff}^{\Delta S=1} \sim (8_L, 8_R)$$

$\rightarrow$ electroweak chiral Lagrangian of $O(G_8 e^2 p^{2n}) \quad (n \geq 0)$

Bijnens, Wise; E., Müller, Isidori, Neufeld, Pich

analogous constructions for meson-baryon sector

altogether : T

effective chiral Lagrangian of the standard model

Effective chiral Lagrangian of the SM

(meson and single-baryon sectors)

$\mathcal{L}_{\text{chiral order}}$ (# of LECs)	loop order
$\mathcal{L}_{p^2}(2) + \mathcal{L}^{\text{odd}}_{p^4}(0) + \mathcal{L}^{\Delta S=1}_{G_F p^2}(2) + \mathcal{L}^{\text{em}}_{e^2 p^0}(1) + \mathcal{L}^{\text{emweak}}_{G_8 e^2 p^0}(1)$ $+ \mathcal{L}^{\pi N}_{p}(1) + \mathcal{L}^{\pi N}_{p^2}(7) + \mathcal{L}^{MB,\Delta S=1}_{G_8 p^0}(2) + \mathcal{L}^{MB,\Delta S=1}_{G_8 p}(8)$ $+ \mathcal{L}^{\pi N,\text{em}}_{e^2 p^0}(3)$	$L = 0$
$+ \underline{\mathcal{L}^{\text{even}}_{p^4}(10)} + \underline{\mathcal{L}^{\text{odd}}_{p^6}(32)} + \underline{\mathcal{L}^{\Delta S=1}_{G_8 p^4}(22)} + \underline{\mathcal{L}^{\Delta S=1}_{G_{27} p^4}(28)}$ $+ \underline{\mathcal{L}^{\text{em}}_{e^2 p^2}(14)} + \underline{\mathcal{L}^{\text{emweak}}_{G_8 e^2 p^2}(14)} + \underline{\mathcal{L}^{\text{leptons}}_{e^2 p}(5)}$ $+ \underline{\mathcal{L}^{\pi N}_{p^3}(23)} + \underline{\mathcal{L}^{\pi N}_{p^4}(118)} + \underline{\mathcal{L}^{MB,\Delta S=1}_{G_8 p^2}(?)} + \underline{\mathcal{L}^{\pi N,\text{em}}_{e^2 p}(8)}$	$L = 1$
$+ \underline{\mathcal{L}^{\text{even}}_{p^6}(90)}$	$L = 2$

number of LECs for $N_f = 3$ (except for πN)

even (odd) intrinsic parity: mesonic Lagrangian without (with) ε tensor

<u>underlined Lagrangians:</u> all completely renormalized

Status of CHPT applications

mesons	$O(p^6)$	2 loops
baryons and mesons	$O(p^4)$	1 loop

Mesons

- strong interactions (+ external γ, W)

  meson-meson scattering, $\gamma\gamma \to \pi\pi$, meson masses ($\to m_q$), meson form factors, meson production, η decays, semileptonic (radiative) decays, $P \to l^+l^-$

- virtual photons

  electromagnetic mass differences, electromagnetic corrections (isospin violation) for strong (hadronic vacuum polarization, pionium, πK atoms, ...) and weak processes (CP violation)

- nonleptonic weak interactions
  $K \to 2\pi, 3\pi$, rare K decays, rad. corrections, isospin violation
- leptons and photons
  elm. corrections for semileptonic decays : $\pi_\beta \to V_{ud}$, $K_{l3} \to V_{us}$

Baryons and mesons

- strong interactions (+ ext. γ, W)
  $\pi N \to \pi(\pi)N, KN \to KN$, σ terms, photo-, electro-, ν-production of π, K, η, baryon form factors, Compton scattering, magnetic moments, polarizabilities
- virtual photons
  elm. corrections, isospin violation (π-H, K-H atoms, ...)
- nonleptonic weak interactions
  (radiative) weak baryon decays
- nuclear physics [not included in $\mathcal{L}_{\rm eff}^{\rm SM}$]
  NN interactions, few-nucleon systems

Pion-pion scattering

Motivation

- the fundamental scattering process of CHPT for $N_f = 2$
- sensitive to mechanism of SCSB
- new experimental activity

BNL-E865	K_{e4} $(K \to \pi\pi e \nu_e)$
NA48 (CERN)	"
KLOE (DAΦNE)	"
DIRAC (CERN)	pionium

Scattering amplitude in CHPT

isospin limit : $m_u = m_d \, , \alpha = 0$

kinematics T

Kinematics

$$\pi^a(p_a) + \pi^b(p_b) \to \pi^c(p_c) + \pi^d(p_d)$$

scattering amplitude $(m_u = m_d)$

$$T_{ab,cd}(s,t,u) \;=\; \delta_{ab}\delta_{cd}A(s,t,u) + \delta_{ac}\delta_{bd}A(t,s,u) + \delta_{ad}\delta_{bc}A(u,t,s)$$

Bose symmetry $\quad\to\quad A(s,t,u) = A(s,u,t)$

$$\begin{aligned} s &= (p_a + p_b)^2 = 4(q^2 + M_\pi^2) \\ t &= (p_a - p_c)^2 = -2q^2(1 - \cos\theta) \\ u &= (p_a - p_d)^2 = -2q^2(1 + \cos\theta) \end{aligned}$$

$q\ ,\theta$: CM momentum, scattering angle

$T^I(s,t)$: amplitudes with definite isospin I in s-channel

$$\begin{aligned} T^0(s,t) &= 3A(s,t,u) + A(t,u,s) + A(u,s,t) \\ T^1(s,t) &= A(t,u,s) - A(u,s,t) \\ T^2(s,t) &= A(t,u,s) + A(u,s,t) \end{aligned}$$

partial wave expansion

$$T^I(s,t) = 32\pi \sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)t_l^I(s)$$

$t_l^I(s)$: partial wave amplitudes

elastic region $(4M_\pi^2 \leq s \leq 16M_\pi^2)$:

$$t_l^I(s) = (1 - \frac{4M_\pi^2}{s})^{-1/2} \exp i\delta_l^I(s) \sin\delta_l^I(s)$$

$\delta_l^I(s)$: phase shifts

expansion near threshold :

$$\mathrm{Re}\, t_l^I(s) = q^{2l}\{a_l^I + q^2 b_l^I + O(q^4)\}$$

threshold parameters

a_l^I : scattering lengths

b_l^I : slope parameters ($\sim$ effective ranges)

$O(p^2)$ $(L=0)$

$$A_2(s,t,u) = \frac{s - M_\pi^2}{F_\pi^2}$$ Weinberg

together with $(2\hat{m} := m_u + m_d)$: $\quad r := \frac{m_s}{\hat{m}} = \frac{2M_K^2}{M_\pi^2} - 1 \simeq 26$

S-waves especially sensitive to size of quark condensate:

a_0^0	r	$B(\nu = 1\ \text{GeV})$
0.16	26	1.4 GeV
0.26	10	F_π

$O(p^4)$ $(L \leq 1)$

Gasser, Leutwyler

$$\begin{aligned} F_\pi^4 A_4(s,t,u) &= c_1 M_\pi^4 + c_2 M_\pi^2 s + c_3 s^2 + c_4 (t-u)^2 \\ &+ F_1(s) + G_1(s,t) + G_1(s,u) \end{aligned}$$

F_1, G_1 : 1-loop functions

$c_1, \ldots, c_4$: contain ren. LECs $l_i^r(\mu)$ and chiral logs (first power)

many observables dominated by chiral logs :

	$O(p^2)$	$O(p^4)$
a_0^0	0.16	0.20

most of the difference due to chiral logs ($\mu \simeq M_\rho$)

experimental value (until recently) : $a_0^0 = 0.26 \pm 0.05$

Rosselet et al. (Geneva-Saclay, 1977)

$O(p^6)$ $(L \leq 2)$

Bijnens, Colangelo, E., Gasser, Sainio

- 2-loop amplitude calculated analytically (elementary functions)
- confirm previous dispersive treatment of Knecht et al.

$r_i^r(\mu)$ $(i = 1, \ldots, 6)$: new LECs of $O(p^6)$

Advantages over dispersive amplitude

- full infrared structure (chiral logs)
- dependence on LECs $\rightarrow$ comparison with other processes
- dependence on m_q (via M_π^2) $\rightarrow$ lattice QCD

results: with $r_i^r(\mu)$ estimated from meson resonance exchange, small uncertainties for low partial waves, much more sensitive to $l_i^r(\mu)$ (LECs of $O(p^4)$)

Recent developments

combination with Roy equations (dispersion relations)

Ananthanarayan, Colangelo, Gasser, Leutwyler

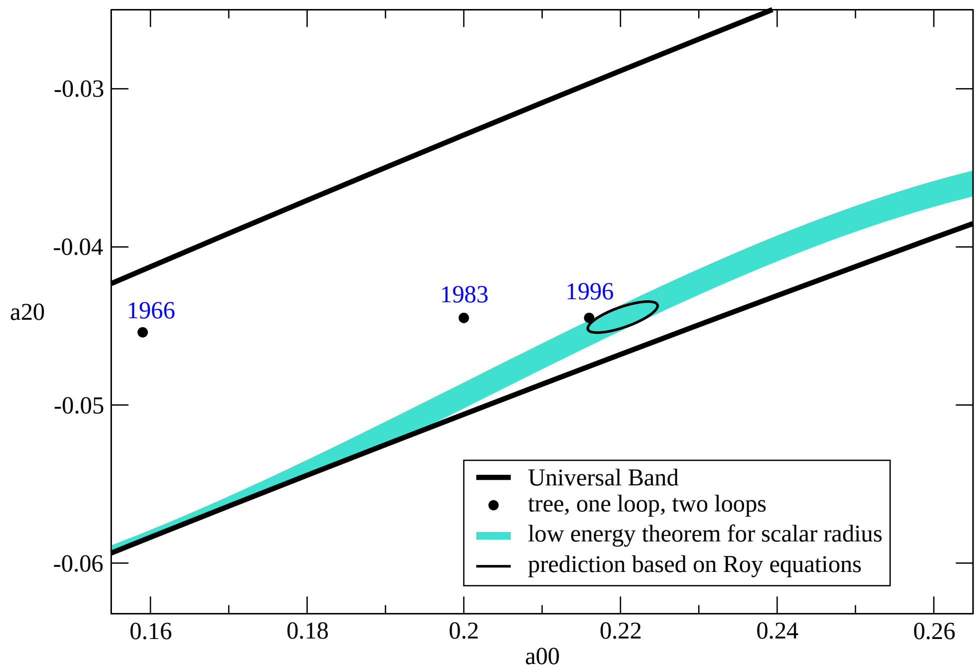
$\Rightarrow$ low partial waves (S, P) from input

i. high-energy data ($E \geq 0.8$ GeV)

ii. scattering lengths a_0^0, a_0^2 (subtraction constants)

→ amazingly precise predictions for phase shifts and threshold parameters in terms of a_0^0, a_0^2

from current algebra to $O(p^6)$



final step: Colangelo, Gasser, Leutwyler

match Roy equations and $O(p^6)$ CHPT

input :

LECs of $O(p^4)$: l_3^r (main sensitivity), l_4^r

LECs of $O(p^6)$: $r_1^r, r_2^r, r_3^r, r_4^r$

output : $a_0^0, a_0^2,$ $l_1^r, l_2^r,$ r_5^r, r_6^r

threshold parameters for $\pi\pi$ scattering

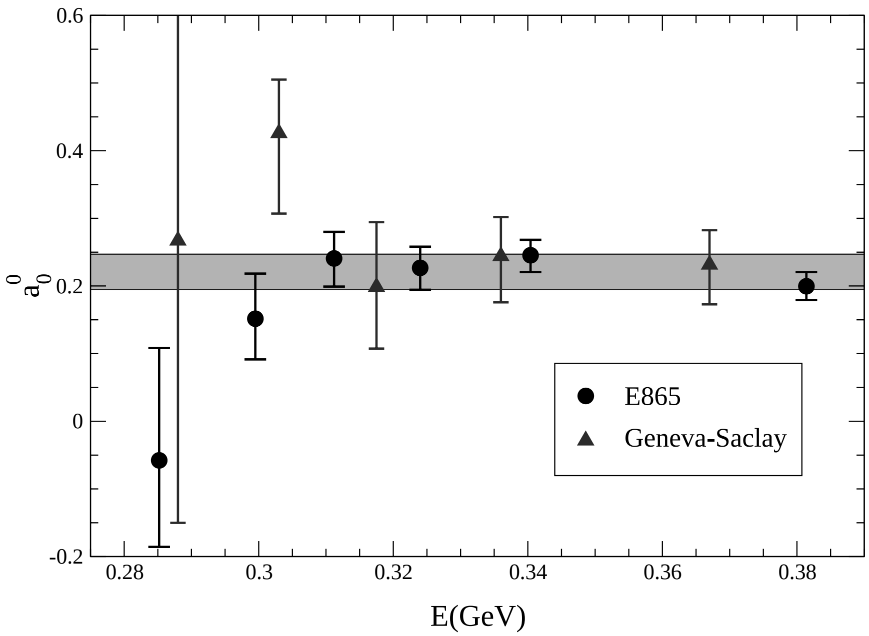
	$O(p^6)$ set ABT	$O(p^6)$ set I	Roy
a_0^0	0.219 ± 0.005	0.222	0.220 ± 0.005
$-10a_0^2$	0.420 ± 0.010	0.420	0.444 ± 0.010
b_0^0	0.279 ± 0.011	0.282	0.276 ± 0.006
$-10b_0^2$	0.756 ± 0.021	0.729	0.803 ± 0.012
$10a_1^1$	0.378 ± 0.021	0.404	0.379 ± 0.005
$10^2 b_1^1$	0.59 ± 0.12	0.83	0.567 ± 0.013
$10^2 a_2^0$	0.22 ± 0.04	0.28	0.175 ± 0.003
$10^3 a_2^2$	0.29 ± 0.10	0.24	0.170 ± 0.013

caveat: large quark condensate assumed (standard CHPT) $\rightarrow$ consistent picture, but is it unambiguous?

Impact of new K_{e4} data (BNL-E865)

Colangelo, Gasser, Leutwyler

use only Roy equation ($\rightarrow$ correlation a_0^0, a_0^2), phase shift difference $\delta_0^0 - \delta_1^1$ considered as function of scattering length a_0^0



fitted value

$$a_0^0 = 0.221 \pm 0.026$$

chiral expansion of pion mass

$$M_\pi^2 = M^2 - \frac{\bar{l}_3}{32\pi^2 F^2} M^4 + O(M^6)$$

$$M^2 = (m_u + m_d)|\langle 0|\overline{u}u|0\rangle|/F^2 \qquad \text{GMOR}$$

Consequence for SCSB: with new K_{e4} data $\longrightarrow$

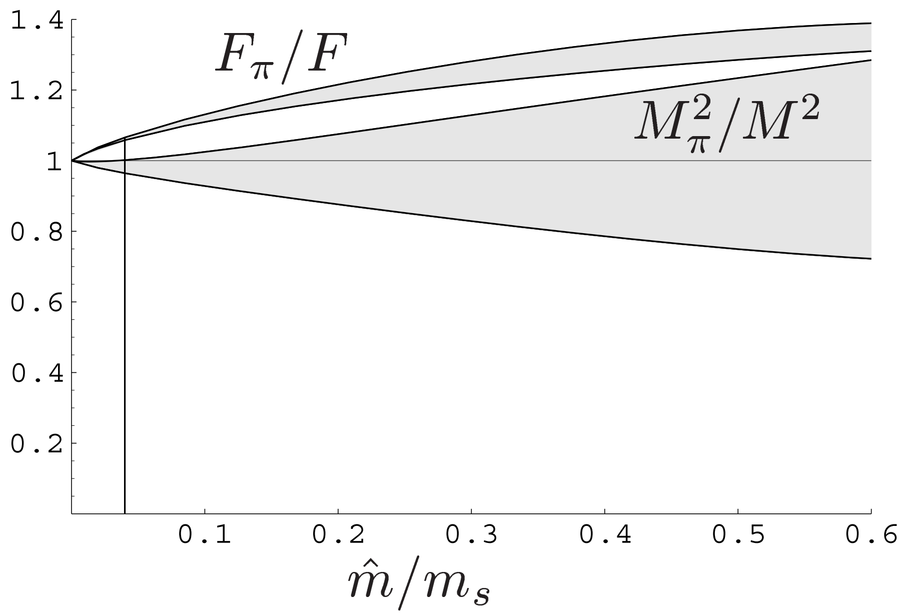
more than 94 % of M_π due to leading term !

Standard mechanism of SCSB confirmed

Quark mass dependence of M_π, F_π to $O(p^6) = O(m_q^3)$

relevant for lattice QCD

Leutwyler



Summary for $\pi\pi$ scattering

- chiral expansion "converges"
- many observables dominated by chiral logs (especially S-waves)
- unambiguous predictions of standard CHPT + Roy equations :

$$a_0^0 = 0.220 \pm 0.005 \qquad a_0^2 = -0.0444 \pm 0.0010$$

$\rightarrow$ S- and P-wave phase shifts precisely predicted
$\rightarrow$ pionium lifetime $\tau = (2.9 \pm 0.1) \cdot 10^{-15}$

- comparison with new K_{e4} data (BNL-E865) confirms standard picture of SCSB with large quark condensate (at least for $N_f = 2$)
- small caveat : at this level of accuracy, isospin violation and electromagnetic corrections must be included available for $\pi\pi \to \pi\pi$ and partially also for K_{e4}

Isospin violation and precision physics

2 sources of isospin violation in SM

i. $m_u \neq m_d$

ii. electromagnetic corrections

order-of-magnitude estimates

$O(m_u - m_d)$	$O(\alpha)$
$\dfrac{M_{K^0}^2 - M_{K^+}^2}{M_\rho^2} \sim 0.7\%$	$\dfrac{M_{\pi^+}^2 - M_{\pi^0}^2}{M_\rho^2} \sim 0.2\%$

observations

- 2 contributions comparable in general
- small effects, can be enhanced in some cases

example : $I = 0$ S-wave $\pi\pi$ scattering length at $O(p^2)$

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2} = \begin{cases} 0.159 & M_{\pi^+} \\ 0.149 & M_{\pi^0} \end{cases} \Rightarrow \quad 6\% \text{ effect}$$

difference comparable to $O(p^6)$ corrections !

Isospin violation in pion-pion scattering

leading $\Delta I \neq 0$ effects ($N_f = 2$)

$$\mathcal{L}_2 + \mathcal{L}_4 = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle + \ldots - \frac{l_7}{16}\langle \chi U^\dagger - \chi^\dagger U\rangle^2 + \ldots$$

l_7 : tiny contribution $\sim (m_u - m_d)^2$ to $M_{\pi^0}^2$
no contribution to $A(\pi\pi \to \pi\pi)$

$L = 1$: vertices and propagators with $m_u = m_d$

$\rightarrow \quad m_u \neq m_d$ does not influence $A(\pi\pi \to \pi\pi)$ to $O(p^4)$

general result in QCD :

$A(\pi\pi \to \pi\pi)$ does not contain terms linear in $m_u - m_d$

proof :

isospin violating mass term has odd G-parity

$$\mathcal{L}_{QCD}^{\Delta I \neq 0} = (m_d - m_u)\left(\overline{u}u - \overline{d}d\right)$$

Leading electromagnetic corrections

$$\mathcal{L}^{\text{em}}_{e^2p^0} = e^2 Z F^4 \langle QUQU^\dagger \rangle$$

$\to \quad \Delta M_\pi^2 = M_{\pi^+}^2 - M_{\pi^0}^2 = 2e^2ZF^2 \qquad\qquad Z \simeq 0.8$

no contribution to scattering amplitude (except via M_π^2)
genuine leading electromagnetic corrections are $O(e^2p^2)$:

Knecht, Urech, Nehme, Meißner, Müller

$\to$ leading isospin violating corrections are

$O(e^2p^2)$ dominant

$O[(m_u - m_d)^2 p^2]$ suppressed

- relevant for isospin violation in pionium

  Gall, Gasser, Lyubovitskij, Rusetsky

- not applicable to K_{e4} ($\rightarrow \pi\pi$ phase shifts) :

  both $O(e^2p^2)$ and $O[(m_u - m_d)p^2]$

many precision measurements at low energies

require systematic account of isospin violating effects

examples to be discussed :

semileptonic decays	CKM mixing
K decays	ϵ'/ϵ
hadronic vacuum polarization	a_μ, $\alpha(M_Z)$

CKM matrix elements V_{ud}, V_{us}

issue: unitarity of the quark mixing matrix

PDG 2004: ($|V_{ub}| = 0.00367(47)$ irrelevant)

$|V_{us}| = 0.2200(26)$ K_{e3}

$|V_{ud}| = 0.9738(5)$ SAFT $0^+ \to 0^+$, neutron β decay

$$\sum_i |V_{ui}|^2 - 1 = -0.0033(15) \qquad 2.2\sigma$$

essential for unitarity test: isospin violating and electromagnetic corrections for V_{ud} and V_{us}

Task for CHPT

isospin violating corrections for P_{l3} decays

$$P(p_P) \to \pi(p_\pi) l^+(p_l) \nu_l(p_\nu) \qquad (P = \pi^+, K^+, K^0)$$

advantage : unified treatment for V_{ud}, V_{us}

invariant amplitude (for $e = 0$, but $m_u \neq m_d$)

$$A \sim f_+^{(0)}(t)(p_P + p_\pi)_\mu + f_-^{(0)}(t)(p_P - p_\pi)_\mu$$

$$(t = (p_P - p_\pi)^2)$$

aim : systematic calculation to $O(p^4, p^2(m_u - m_d), e^2 p^2)$

general procedure for radiative corrections:

- photonic one-loop corrections

$$f_+^{(0)}(t), f_-^{(0)}(t) \rightarrow F_+(t,u), F_-(t,u)$$

  with second Dalitz variable u

$$u = \begin{cases} (p_P - p_l)^2 & P = K^+, \pi^+ \\ (p_\pi + p_l)^2 & P = K^0 \end{cases}$$

- radiative decay: Dalitz plot density modified

$$\rho^{(0)}(t,u) \rightarrow \rho(t,u) = \rho^{(0)}(t,u) + \rho^\gamma(t,u)$$

- only the sum of the rates is infrared finite $\longrightarrow$ radiative rate depends on (exp.) treatment of real photon emission

$P = \pi^+$: pionic β decay

Cirigliano, Knecht, Neufeld, Pichl

main correction comes from $O(e^2p^2)$:

$$f_+(0) = 1.0046 \pm 0.0005$$

N.B.: very small theoretical uncertainty!

PSI experiment PIBETA (hep-ex/0312030)

$$BR(\pi_{e3} + \pi_{e3\gamma}) = (1.034 \pm 0.004 \pm 0.007) \times 10^{-8}$$

PIBETA + CHPT: $|V_{ud}| = 0.9716 \pm 0.0039$

PDG'04: $|V_{ud}| = 0.9738 \pm 0.0005$

expected final accuracy of PIBETA : $\Delta V_{ud} = \pm 0.0025$

$P = K^+, K^0$: K_{l3} decay

Cirigliano, Knecht, Neufeld, Pichl, Rupertsberger, Talavera

combined analysis of K^+_{e3} and K^0_{e3} now available

K_{e3}: again only one form factor $f_+^{K\pi}$ relevant

complication : isospin conserving corrections of $O(p^6)$ not negligible

$\rightarrow$ main source of theoretical uncertainty

Bijnens, Talavera, Post, Schilcher

$$f_+^{K\pi}(t=0)|_{p^6} = -8\frac{(M_K^2 - M_\pi^2)^2}{F_\pi^4}\left[C_{12}^r(\mu) + C_{34}^r(\mu)\right] + \Delta_{\text{loops}}(\mu)$$

$\Delta_{\text{loops}}(M_\rho) = 0.0146(64)$ error due to L_i in loops

$$f_+^{K\pi}(t=0)|_{p^6}^{\text{local}} = \begin{cases} -0.016(8) & \text{Leutwyler, Roos} \\ -0.012 & \text{resonance saturation} \end{cases}$$

destructive interference between loop and local contributions final result (non-negligible scale dependence):

$$f_+^{K\pi}(t=0)|_{p^6} = -0.001 \pm 0.010$$

vector form factors at $t = 0$ Cirigliano, Neufeld, Pichl

$$f_+^{K^0\pi^-}(t=0) = 0.981 \pm 0.010$$
$$f_+^{K^+\pi^0}(t=0) = 1.002 \pm 0.010$$

new experimental results in 2004 T

$\longrightarrow$ small discrepancy between K^0 and $K^\pm$ data

Recent theoretical developments

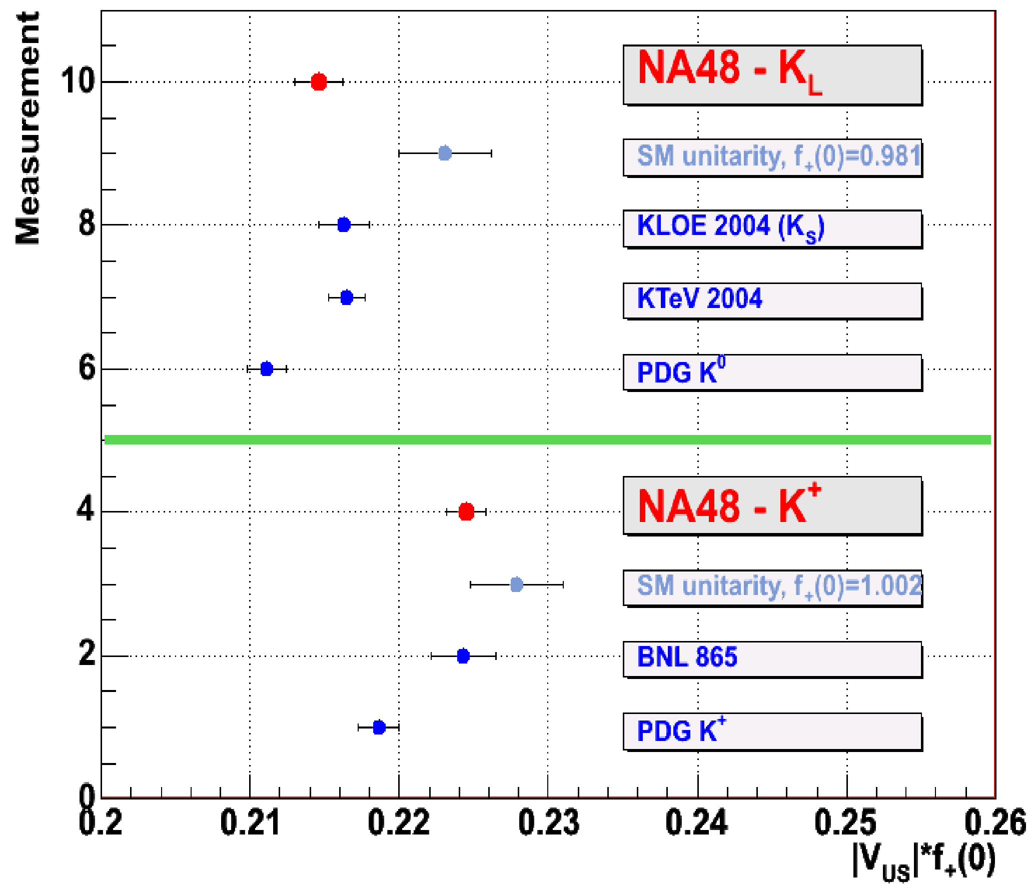
- lattice calculation of $f_+(0)$ Becirevic et al.

  extrapolation to $t = 0$ using $f_0(t_{\max})$ and $|V_{us}|f_+(0)$ (KTeV)

  $$f_+(0) = 0.960 \pm 0.005(\text{stat}) \pm 0.007(\text{syst})$$

  quenching error ?

Determination of $V_{us}f_+(0)$



- lattice calculation of F_K/F_π MILC-Collaboration

$$F_K/F_\pi = 1.210(4)(13)$$

extract $|V_{us}|$ using (ancient) exp. result on $K_{\mu 2}$ (room for improvement); partially quenched QCD

- hadronic τ decays Gamiz et al.

	$\lvert V_{us}\rvert$
Becirevic et al.	0.2259(21)
MILC	0.2219(26)
Gamiz et al.	0.2208(34)
unitarity	0.2265(20)
PDG'04	0.2200(26)

Ultimate CHPT method

CHPT relates slope and curvature of $f_0(t)$ to $f_+(0) = f_0(0)$

$$f_0(t) = 1 - \frac{8\left(M_K^2 - M_\pi^2\right)^2}{F_\pi^4}(C_{12}^r + C_{34}^r) + \frac{8t\left(M_K^2 + M_\pi^2\right)}{F_\pi^4}(2C_{12}^r + C_{34}^r)$$
$$+ \frac{t}{M_K^2 - M_\pi^2}(F_K/F_\pi - 1) - \frac{8t^2}{F_\pi^4}C_{12}^r + \text{ loop conts.}$$

advantage of chiral analysis:

$C_{12}^r(M_\rho) + C_{34}^r(M_\rho)$ accessible from slope and curvature of scalar form factor $f_0^{K\pi}(t)$

$\longrightarrow$ more precise $K_{\mu 3}$ data needed

Summary for $|V_{us}|$

- unitarity problem solved by joint effort of theory and experiment
- precise value for $|V_{us}|$ yet to come

$K \to \pi\pi$ decays and CP violation

$K \to \pi\pi$: isospin decomposition (standard model)

$$
\begin{aligned}
A(K^0 \to \pi^+\pi^-) &= A_0 e^{i\delta_0^0} + \frac{1}{\sqrt{2}} A_2 e^{i\delta_0^2} \\
A(K^0 \to \pi^0\pi^0) &= A_0 e^{i\delta_0^0} - \sqrt{2} A_2 e^{i\delta_0^2} \\
A(K^+ \to \pi^+\pi^0) &= \frac{3}{2} A_2 e^{i\delta_0^2}
\end{aligned}
$$

$\delta^I_{l=0}(M_K^2)$: $\pi\pi$ phase shifts at $s = M_K^2$

from $K \to \pi\pi$ branching ratios (isospin limit) :

$$
\begin{aligned}
A_0 &= (2.715 \pm 0.005) \times 10^{-7}\ \text{GeV} \\
A_2 &= (1.225 \pm 0.004) \times 10^{-8}\ \text{GeV} \\
\delta_0^0 - \delta_0^2 &= (48.5 \pm 2.6)^o
\end{aligned}
$$

remark :

new KLOE measurement has changed phase difference substantially (instead of 58^o during past decades) recall effective Lagrangian for $\Delta S = 1$

$$\mathcal{L}_{\rm eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i + \text{h.c.}$$

chiral realization at $O(G_F p^2)$ for $m_u = m_d, \alpha = 0$ (lowest order)

$$\mathcal{L}_2^{\Delta S=1} = G_8 F^4 \langle \lambda L_\mu L^\mu \rangle + G_{27} F^4 \left(L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right) + \text{h.c.}$$

$$\lambda = (\lambda_6 - i\lambda_7)/2 \ , \qquad L_\mu = i U^\dagger D_\mu U$$

$K \to \pi\pi$ amplitudes

$$A_0 = F(M_K^2 - M_\pi^2)\sqrt{2}\,(G_8 + G_{27}/9)$$

$$A_2 = F(M_K^2 - M_\pi^2) 10\ G_{27}/9$$

branching ratios $\longrightarrow$ $G_{27}^{(0)}/G_8^{(0)} \simeq 1/18$

$\Delta I = 1/2$ rule (only qualitative understanding)

less ambitious project (G_8, G_{27} free parameters):

isospin violation in $K \to 2\pi$

Cirigliano, E., Neufeld, Pich

first complete NLO calculation $[O(G_F p^4)]$ including
isospin violation (in octet amplitudes)

strong	$O(G_8(m_u - m_d)p^2)$
electromagnetic	$O(e^2 G_8 p^2)$

main motivation

- $\Delta I = 1/2$ rule
- ϵ'/ϵ

isospin violation at leading order

$m_u - m_d$	$\pi^0 - \eta$ mixing
electromagnetic	mass differences
"	elm. penguin

Lowest-order weak chiral Lagrangian for $m_u \neq m_d, \alpha \neq 0$

$$\mathcal{L}_2^{\Delta S=1} = G_8 F^4 \langle \lambda L_\mu L^\mu \rangle + G_{27} F^4 \left(L_{\mu 23} L^\mu_{11} + \frac{2}{3} L_{\mu 21} L^\mu_{13} \right) + e^2 F^6 G_8 g_{\rm ewk} \langle \lambda U^\dagger Q U \rangle + \text{h.c.}$$

$g_{\rm ewk} \simeq -1$ accounts for electroweak penguin contribution general amplitude decomposition in presence of isospin violation

$$
\begin{aligned}
A(K^0 \to \pi^+\pi^-) &= A_0 e^{i\chi_0} + \tfrac{1}{\sqrt{2}} A_2 e^{i\chi_2} & A_0 e^{i\chi_0} &= \mathcal{A}_{1/2} \\
A(K^0 \to \pi^0\pi^0) &= A_0 e^{i\chi_0} - \sqrt{2} A_2 e^{i\chi_2} & A_2 e^{i\chi_2} &= \mathcal{A}_{3/2} + \mathcal{A}_{5/2} \\
A(K^+ \to \pi^+\pi^0) &= \tfrac{3}{2} A_2^+ e^{i\chi_2^+} & A_2^+ e^{i\chi_2^+} &= \mathcal{A}_{3/2} - \tfrac{2}{3}\mathcal{A}_{5/2}
\end{aligned}
$$

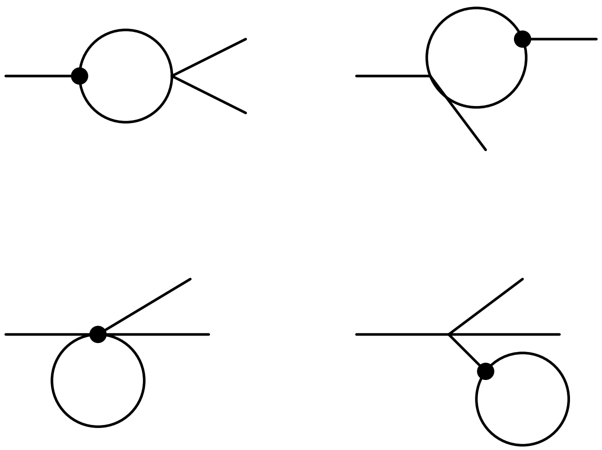
$$
\begin{aligned}
\mathcal{A}_{1/2} &= \sqrt{2} F_\pi \left\{ G_8 \left[(M_K^2 - M_\pi^2)\left(1 - \frac{2}{3\sqrt{3}}\,\varepsilon^{(2)}\right) - \frac{2}{3}\, e^2 F_\pi^2\,(g_{\rm ewk} + 2Z) \right] \right. \\
&\quad \left. + \frac{1}{9} G_{27} (M_K^2 - M_\pi^2) \right\} \qquad \mathcal{A}_{5/2} = 0 \\
\mathcal{A}_{3/2} &= \frac{2}{3} F_\pi \left\{ \left(\frac{5}{3} G_{27} + \frac{2}{\sqrt{3}} \varepsilon^{(2)} G_8 \right) (M_K^2 - M_\pi^2) - e^2 F_\pi^2 G_8 (g_{\rm ewk} + 2Z) \right\}
\end{aligned}
$$

$Z \simeq (M_{\pi^+}^2 - M_{\pi^0}^2)/(2e^2F_\pi^2) \simeq 0.8\,;\ \varepsilon^{(2)} = \frac{\sqrt{3}}{4}(m_d - m_u)/(m_s - \hat{m}) \simeq 0.011$

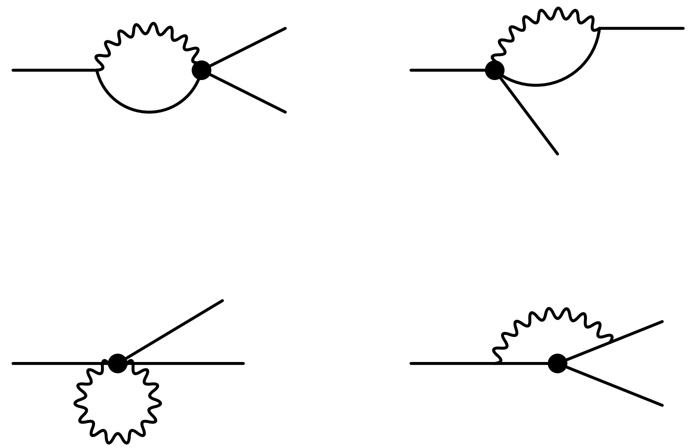
much more involved at NLO: T

i. loop amplitudes

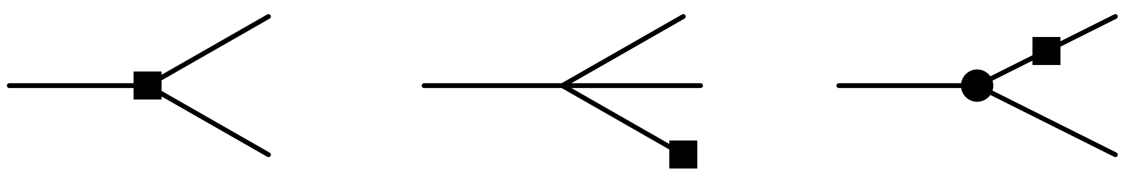
ii. local amplitudes contain many LECs



purely mesonic loop diagrams



meson-photon loop diagrams



local amplitudes (square: NLO vertex)

local amplitudes $\longrightarrow$ host of LECs involved

Lagrangians

order	LECs
$O(p^4)$	L_i
$O(G_F p^2)$	G_8, G_{27}
$O(G_F p^4)$	N_i, D_i
$O(e^2 p^0)$	Z
$O(e^2 p^2)$	K_i
$O(e^2 G_8 p^0)$	$g_{\rm ewk}$
$O(e^2 G_8 p^2)$	Z_i
[$O(p^6)$	X_i]

determination of LECs

with few exceptions
(mainly G_8, G_{27}):

leading large-N_c
(factorization)

primary output	errors
$G_8, G_{27}, \chi_0 - \chi_2$	exp., SD (Wilson coeff.), LD (chiral scale)

fitted phase difference $\chi_0 - \chi_2$

IC	IB
$(48.6 \pm 2.6)^\circ$	$(54.6 \pm 2.4)^\circ$

isospin limit: Watson theorem $\rightarrow$

$\pi\pi$ S-wave phase shifts $\delta_I(M_K)$

$\chi_0 - \chi_2 = \delta_0(M_K) - \delta_2(M_K)$

$= (47.7 \pm 1.5)^\circ$

Colangelo, Gasser, Leutwyler

general parametrization of phases in presence of isospin violation

$$\chi_I = \delta_I(M_K) + \gamma_I \qquad (I = 0, 2)$$

γ_I : purely electromagnetic origin

CHPT

$$\gamma_0 = -0.2^\circ$$
$$\gamma_2 = 3.0^\circ$$

optical theorem

$$\gamma_0 = -0.2^\circ$$
$$\gamma_2 = (6 \pm 3)^\circ$$

$\gamma_2 \gg |\gamma_0|$ due to $\Delta I = 1/2$ rule

Status of phase difference

$(\delta_0 - \delta_2)_{\rm th}$	$(47.7 \pm 1.5)^\circ$	Colangelo, Gasser, Leutwyler
$(\delta_0 - \delta_2)_{K\to\pi\pi}$	$(60.2 \pm 6.3)^\circ$	CENP

most of difference due to $\Delta I = 5/2$ loop amplitude $\longrightarrow$ difference not understood

$\Delta I = 1/2$ rule

	IC	IB
$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}$	22.2 ± 0.1	20.3 ± 0.5
$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2^+}$	22.2 ± 0.1	22.1 ± 0.1

consequence:

IB accounts for

only about 10 % of

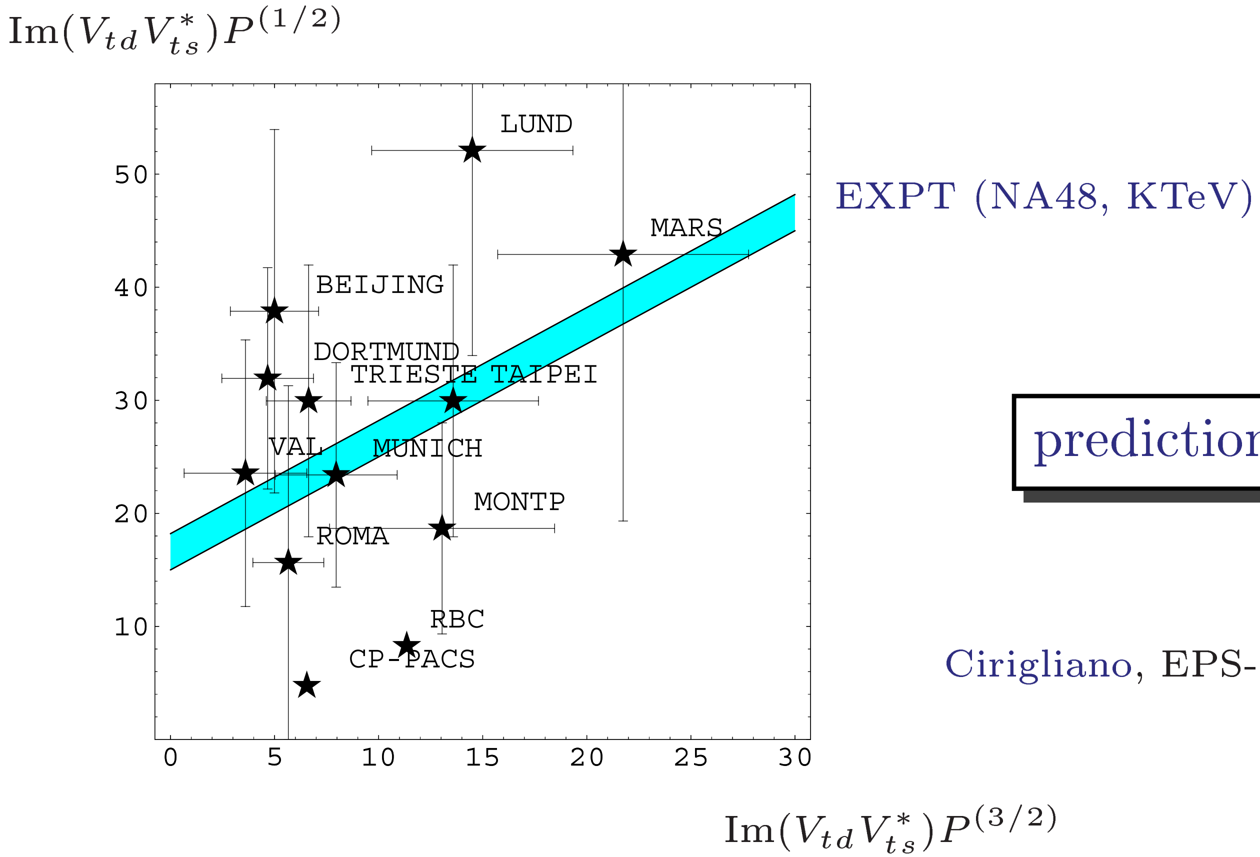
the $\Delta I = 1/2$ rule

Direct CP violation in $K^0 \to 2\pi$

$$\eta_{+-} = \frac{A[K_L \to \pi^+\pi^-]}{A[K_S \to \pi^+\pi^-]} \simeq \epsilon + \epsilon'$$

$$\eta_{00} = \frac{A[K_L \to \pi^0\pi^0]}{A[K_S \to \pi^0\pi^0]} \simeq \epsilon - 2\epsilon'$$

$$\mathrm{Re}\,\frac{\epsilon'}{\epsilon} \sim \mathrm{Im}(V_{td}V_{ts}^*)\cdot\left[P^{(1/2)} - P^{(3/2)}\right]$$



predictions for ϵ'/ϵ

Cirigliano, EPS-HEP2003 (Aachen)

rather than calculate weak matrix elements $P^{(1/2)}$, $P^{(3/2)}$

CHPT project: calculate all effects in ϵ' of first order in $m_u - m_d$ or α

3 sources of isospin violation

i. Q_6 effect in $\mathrm{Im}A_2$ $[O(\alpha, m_u - m_d)]$

$$\Omega_{\mathrm{IB}} = \frac{\mathrm{Re}A_0^{(0)}}{\mathrm{Re}A_2^{(0)}} \cdot \frac{\mathrm{Im}A_2^{\mathrm{non-emp}}}{\mathrm{Im}A_0^{(0)}}$$

ii. IB in A_0 [mainly $O(\alpha)$]

$$\Delta_0 = \frac{\mathrm{Im}A_0}{\mathrm{Im}A_0^{(0)}} \cdot \frac{\mathrm{Re}A_0^{(0)}}{\mathrm{Re}A_0} - 1$$

iii. $\Delta I = 5/2$ amplitude [purely $O(\alpha)$]

$$\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} = \frac{\mathrm{Re}A_2^{+}}{\mathrm{Re}A_0} \left(1 + f_{5/2}\right)$$

notation: A_0^0, A_2^0 amplitudes in isospin limit

$$\epsilon' = -\frac{i}{\sqrt{2}} e^{i(\chi_2-\chi_0)} \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} - \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} \right]$$

$$= -\frac{i}{\sqrt{2}} e^{i(\chi_2-\chi_0)} \frac{\mathrm{Re}A_2^+}{\mathrm{Re}A_0} \left[\frac{\mathrm{Im}A_0^{(0)}}{\mathrm{Re}A_0^{(0)}} (1-\Omega_{\mathrm{eff}}) - \frac{\mathrm{Im}A_2^{\mathrm{emp}}}{\mathrm{Re}A_2^{(0)}} \right]$$

$$\boxed{\Omega_{\mathrm{eff}} = \Omega_{\mathrm{IB}} - f_{5/2} - \Delta_0}$$

	$\alpha = 0$		$\alpha \neq 0$	
	LO	LO+NLO	LO	LO+NLO
Ω_{IB}	11.7	15.9 ± 4.5	18.0 ± 6.5	22.7 ± 7.6
Δ_0	$-\ 0.004$	$-\ 0.41 \pm 0.05$	8.7 ± 3.0	8.4 ± 3.6
$f_{5/2}$	0	0	0	8.3 ± 2.4
Ω_{eff}	11.7	16.3 ± 4.5	9.3 ± 5.8	6.0 ± 7.7

isospin violating corrections for ϵ' in units of 10^{-2}

- strong IB $\sim m_u - m_d$ dominated by $\pi^0 - \eta$ mixing
- destructive interference between strong and elm. contributions

Summary for $K \to \pi\pi$

i. NLO corrections in $K \to 2\pi$ sizable $\longrightarrow$ well-known example: G_8 decreases by $\sim 30\%$ (Kambor, Missimer, Wyler)

ii. IB accounts for about 10% of the $\Delta I = 1/2$ rule

iii. destructive interference of IB effects for both G_{27} and ϵ'

iv. IB seems to go in the "wrong" direction for $\chi_0 - \chi_2$

v. separate IB contributions to ϵ' big, overall effect smaller

IB	correction factor $\Omega_{\rm eff}$
strong	0.16 ± 0.05
strong + elm.	0.06 ± 0.08

Hadronic vacuum polarization and the muon magnetic moment

status of $a_\mu = (g_\mu - 2)/2$

Bennett et al. (BNL Muon g-2 Coll., Feb. '04)

$$a_\mu^{\text{exp}} = (11659208 \pm 6) \times 10^{-10}$$

muon anomalous magnetic moment in the SM

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{had}}$$

from now on: a_μ in units 10^{-10}

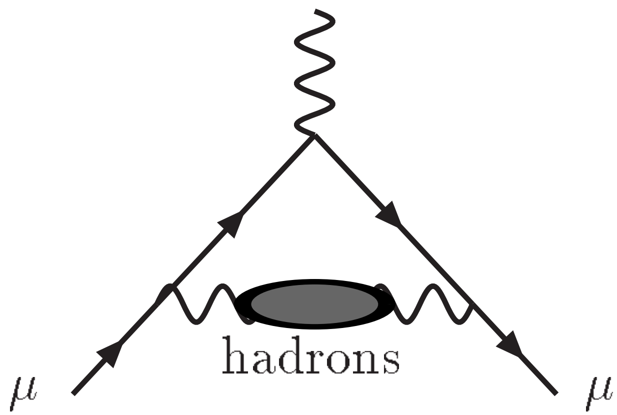
precision needed for hadronic contribution:

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED+weak}} = 721 \pm 6$$

conventional decomposition of hadronic contribution

$$a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,HO} + a_\mu^{\rm had,LBL}$$

$$a_\mu^{\rm had,LO} = a_\mu^{\rm vac.pol.} = \int_{4M_\pi^2}^\infty dt K(t) \sigma_0(e^+e^- \to \text{ hadrons})(t)$$



most important contribution:

73 % due to $\pi\pi$ state

(70 % for $t \leq 0.8$ GeV^2)

N.B.: $a_\mu^{\rm had,LO}$ contains many contributions of $O(\alpha^3)$

$$a_\mu^{\pi\pi} = \int_{4M_\pi^2}^\infty dt K(t) \quad \{\sigma_{\rm rad}(e^+e^- \to \pi^+\pi^-[\gamma])(t)$$
$$+ \quad \sigma_{\rm non-photonic}(e^+e^- \to \pi^+\pi^-)(t)\}$$

CVC relation (isospin limit: $m_u = m_d, \alpha = 0$)

$$\sigma_0(e^+e^- \to \pi^+\pi^-)(t) = h(t)\,\frac{d\Gamma(\tau^- \to \pi^0\pi^-\nu_\tau)}{dt}$$

Modified CVC relation

$S_{\rm EW}$: SD correction factor

$$\sigma_{\rm non-phot}(t) = h(t)\,\frac{d\Gamma(\tau^- \to \pi^0\pi^-\nu_\tau[\gamma])}{dt}\,\frac{R_{IB}(t)}{S_{EW}}$$

$$R_{IB}(t) = \frac{1}{G_{EM}(t)}\,\frac{\beta^3_{\pi^+\pi^-}(t)}{\beta^3_{\pi^0\pi^-}(t)}\left|\frac{F_V(t)}{f_+(t)}\right|^2$$

$\beta^3_{\pi^+\pi^-}(t)/\beta^3_{\pi^0\pi^-}(t)$
phase space correction factor

ratio of form factors:
mainly sensitive to
$\rho - \omega$ mixing

Electromagnetic corrections $\to G_{\rm EM}(t)$

must be subtracted from τ data (ALEPH, CLEO, OPAL) $\longrightarrow$
photon-inclusive rate $\Gamma(\tau^- \to \pi^0\pi^-\nu_\tau[\gamma])$ Cirigliano, E., Neufeld

Isospin violating corrections for a_μ

isospin violating corrections for $a_\mu^{\pi\pi}$ from τ data ($t_{\rm max} \leq m_\tau^2$):

$$\Delta a_\mu^{\pi\pi} = \int_{4M_\pi^2}^{t_{\rm max}} dt K(t) \left[h(t) \frac{d\Gamma_{\pi\pi[\gamma]}}{dt} \right] \times \left(\frac{R_{\rm IB}(t)}{S_{\rm EW}} - 1 \right)$$

Contributions to $\Delta a_\mu^{\pi\pi}$

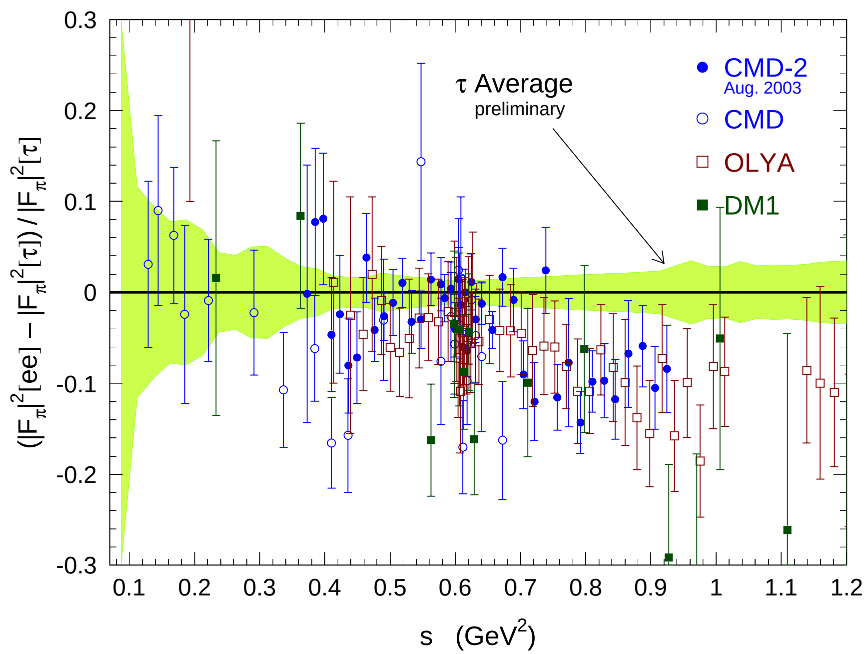
$t_{\rm max}$(GeV^2)	$S_{\rm EW}$	KIN	EM	FF	total
1	- 9.6	- 7.5	- 1.1	6.1 ± 2.6	- 11.9 ± 2.6
3	- 9.7	- 7.5	- 1.0	6.1 ± 2.6	- 12.0 ± 2.6

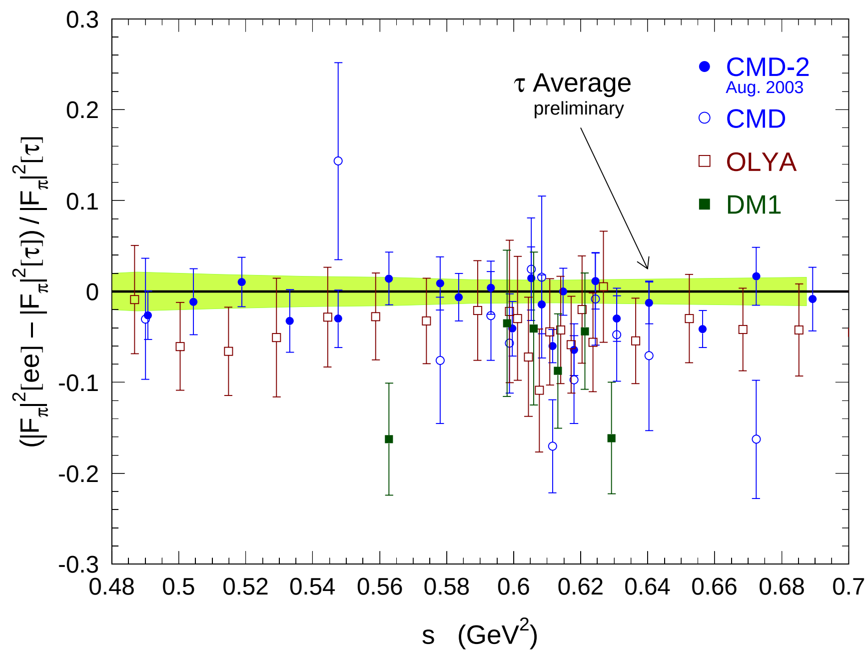
- shift insensitive to $t \geq 1$ GeV^2
- electromagnetic shift relatively small (“destructive interference” between loop and radiative contributions)

final results (Teubert, Davier et al. ICHEP Beijing)

$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = \begin{cases} 25.2 \pm 9.0 \quad (2.7\ \sigma) \quad [e^+e^-] \\ 11.0 \pm 9.0 \quad (1.2\ \sigma) \quad [\ \tau\ ] \end{cases}$$

other recent evaluations: Ghozzi, Jegerlehner
de Troconiz, Yndurain





Conclusions

- CHPT : effective field theory of the standard model
  at low energies
- systematic expansion in $|\vec{p}|/4\pi F_\pi$ (not a Taylor expansion)
- mechanism of spontaneous chiral symmetry breaking
  experimentally confirmed (at least for $N_f = 2$):
  quark condensate accounts for more than 94 % of M_π
- renormalization of a non-renormalizable QFT
  $\rightarrow$ RGE $\rightarrow$ chiral logs
- pion-pion scattering: combination of CHPT and dispersion relations $\rightarrow$ very precise predictions for $E_{\rm cms} < 0.8$ GeV
- isospin violation : generically small, but enhanced in some cases $\rightarrow$ important for precision physics

- CKM matrix: unitarity problem solved due to recent experimental and theoretical improvements
  precise value for $|V_{us}|$ yet to come

- isospin violation in $K \to \pi\pi$:
  i. accounts for about 10 % of the $\Delta I = 1/2$ rule
  ii. separate IB contributions to ϵ' big
  overall effect relatively small
  iii. puzzle with phase difference $\chi_0 - \chi_2$ remains

- a_μ: discrepancy between e^+e^- and τ data (isospin violation too small?) must be understood for comparison with BNL measurement