Chiral Symmetry, Hadron Physics, and Lattice QCD

- Introduction to Lattice Gauge Theory
- Synopsis of Chiral Symmetry on the Lattice
- Hadron Spectrosopy: Excited States of N, Δ, and Λ ; Nature of Roper, a_0(1450) and σ(600)
- Hadron Structure: πNσ term, Quark Spin, Strangeness Form Factors
- Valence QCD and Quark Model
- Effective Theories for Hadrons: Valence QCD vs Chiral Quark Model

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Introduction to Lattice Gauge Theory

Path Integral Formulation in Discrete Euclidean Space-Time

- Path integral of the partition function of continuum QCD in Minkowski space

$$Z = \int DA_\mu D\Psi D\overline{\Psi}\; e^{iS_M}$$

$$S_M = \int d^4x \{-\frac{1}{4} G^{\mu\nu a} G^a_{\mu\nu} + \overline{\Psi}[i\gamma^\mu(\partial\mu - ig\frac{\lambda^a}{2}A^a_\mu) - m]\Psi\}$$

- Imaginary time with Wick rotation $t \to -it \;\; (x^0 \to -ix_4),$

and $A^0 \to iA_4, \gamma^0 \to \gamma_4, \gamma^i \to i\gamma^i, \{\gamma^\mu, \gamma^\nu\} = g^{\mu\nu} \to \{\gamma_\mu, \gamma_\nu\} = \delta_{\mu\nu}$

then $Z_E = \int DA_\mu D\Psi D\overline{\Psi}\; e^{-S_E}$ $\quad S_E = \int d^4x \{\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \overline{\Psi}[\gamma_\mu(\partial\mu + ig\frac{\lambda^a}{2}A^a_\mu) + m]\Psi\}$

- Integrating Grassmann numbers Ψ and $\overline{\Psi}$ gives Euclidean partition function

$$Z_E = \int DA_\mu \det M\; e^{-S_G},\;\; M = \gamma_\mu D_\mu + m$$

- Note the Grassmann number integration

$$\int D\overline{\Psi} D\Psi\; \overline{\Psi}_i \Psi_j\; e^{-\overline{\Psi}M\Psi} = \det M\; M^{-1}_{ji}, \int D\overline{\Psi} D\Psi\; \overline{\Psi}_i \Psi_j \overline{\Psi}_k \Psi_l e^{-\overline{\Psi}M\Psi} = \det M\; (M^{-1}_{ji} M^{-1}_{lk} - M^{-1}_{li} M^{-1}_{jk})$$

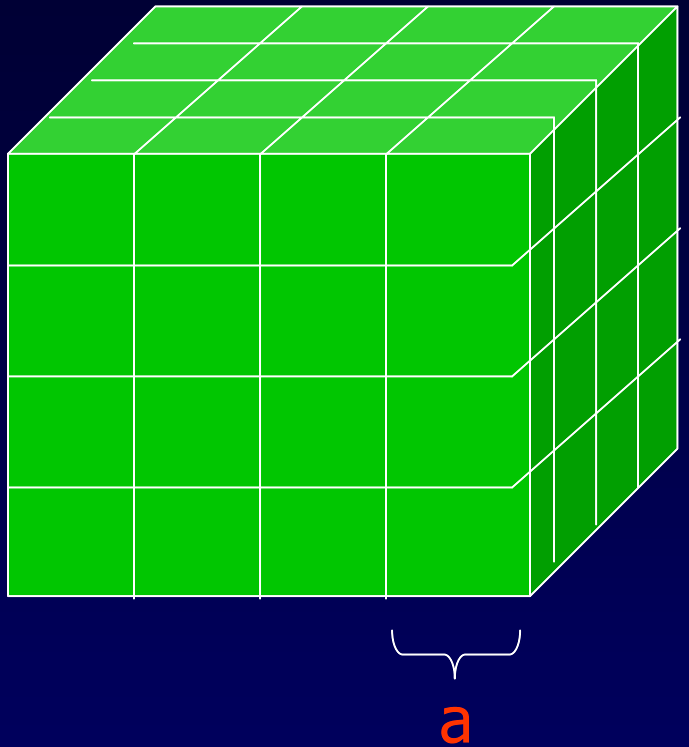
- The Green's function

$$< T[O_1(A,\overline{\Psi},\Psi) O_2(A,\overline{\Psi},\Psi)\ldots] >= \frac{1}{Z}\int DA_\mu\, e^{-S_G} \det M\; Tr\{M^{-1} M^{-1} \ldots A \ldots\}$$

Lattice QCD

Why Lattice?

- Regularization
  - Lattice spacing a
  - Hard cutoff, $p \leq \pi/a$
  - Scale introduced (dimensional transmutation)



- Renormalization
  - Perturbative
  - Non-perturbative

    Regularization independent Scheme
    Schroedinger functional
    Current algebra relations

- Numerical Simulation
  - Quantum field theory → classical statistical mechanics
  - Monte Carlo simulation (important sampling)

$$e^{-S_G} \det M \geq 0$$

Lattice QCD

Correspondence between Euclidean field theory and classical statistical mechanics

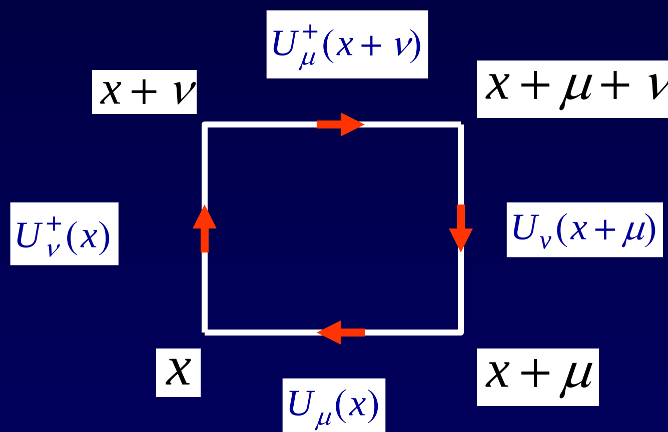
Euclidean field theory	Classical statistical mechanics
Vacuum transition amplitude	Partition function
$A = \int D\varphi\, e^{-S/h}$	$Z = \sum e^{-\beta H}$
Action S	Hamiltonian H
Vacuum Energy	Free Energy
Vacuum Expectation $\langle 0 \vert\, O \,\vert 0 \rangle$	Ensemble Average $\langle O \rangle$
Time-ordered Product	Ordinary Product
Green's Function $\langle 0 \vert T(O_1 O_2 \ldots O_n) \vert 0 \rangle$	Correlation Function $\langle O_1 O_2 \ldots O_n \rangle$
Mass M	Correlation Length $\xi = 1/M$
Regularization with cutoff Λ	Lattice Spacing a
Renormalization: $\Lambda \to \infty$	Continuum Limit : $a \to 0$
Vacuum Change	Phase Transition

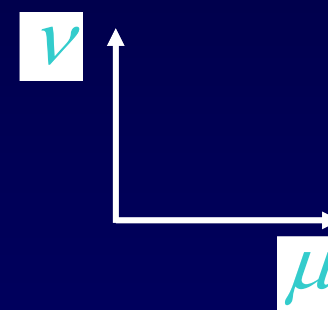
Lattice Formulation of QCD

- Symmetries at Finite Cut-off
  - Gauge Symmetry --- Wilson, 1974 ($A_\mu \rightarrow U_\mu$)

U : Link Variable

$$U_\mu(x) = e^{-iagA_\mu^a \lambda^a / 2}$$





  - Wilson gauge action

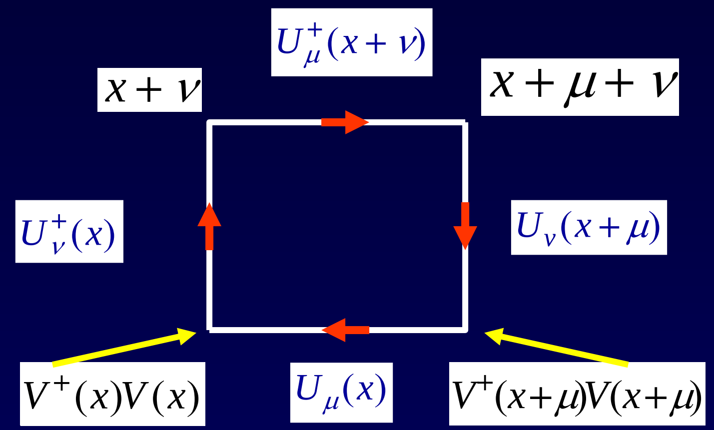
$$S_G = \beta \sum_P \{1 - \frac{1}{N_c} \mathrm{Re}\, TrU_P\}, \quad \beta = \frac{2N_c}{g^2}$$

$$U_P = U_\mu(x)U_\nu(x+\mu)U_{-\mu}(x+\nu)U_{-\nu}(x) = U_\mu(x)U_\nu(x+\mu)U_\mu^+(x+\nu)U_\nu^+(x)$$

Lattice QCD

- Continuum Limit (a → 0)
  - Campbell-Baker-Hausdorff formula $e^A e^B = e^{A+B+(1/2)[A,B]} + \ldots$

$$S_G \rightarrow \frac{1}{4}\sum_x a^4 G^a_{\mu\nu} G^a_{\mu\nu} + \mathrm{O}(a^6)$$



- Gauge Invarance
  - Gauge transformation:

$$U_\mu(x) \rightarrow V(x) U_\mu(x) V^+(x+\mu)$$

$$Tr[U_P] \rightarrow Tr[U_P]$$

  - Integration of link variable U with Haar measure over group manifold satisfies

$$\int DU\ f(U) = \int DU\ f(VU) = \int DU\ f(UW)$$ for arbitrary group elements V and W

  - This proves that $Z_G = \int DU\ e^{-S_G}$ is gauge invariant.
  - A non-perturbative, gauge invariant regularization of pure gauge theory.

- Improvement of gauge action
  - Symanzik improvement to reduce $O(a^2)$ and $O(g^2a^2)$ errors: Lüscher-Weisz, Iwasaki, DBW2
  - Perfect action: Hasenfratz-Niedermayer

Chiral Symmetry on the Lattice

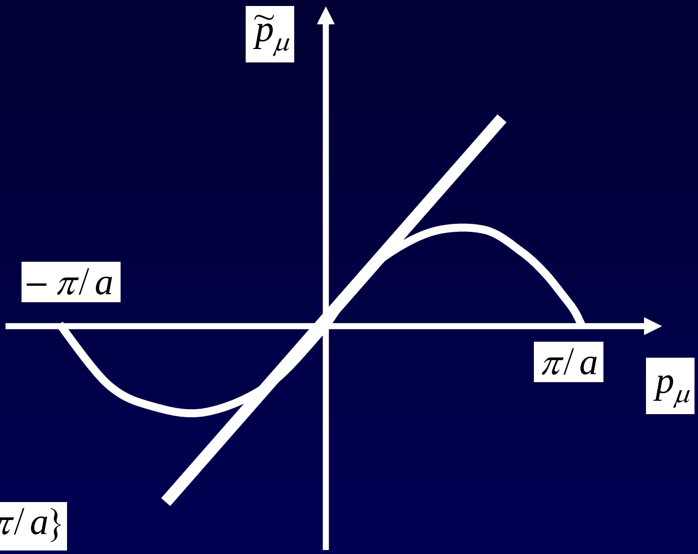
- Fermion species doubling
  - Naïve free fermion with symmetric derivative

$$\partial_\mu \Psi(x) \to \frac{1}{2}[\Psi(x+\hat{\mu}) - \Psi(x-\hat{\mu})]$$

  - Free quark propagator

$$\langle \Psi_\alpha(x)\overline{\Psi}_\beta(y)\rangle = \int_{-\pi/a}^{\pi/a} \frac{d^4p}{(2\pi)^4} \frac{[-i\,\gamma_\mu \tilde{p}_\mu + m]_{\alpha\beta}}{\Sigma_\mu \tilde{p}_\mu^2 + m^2} e^{ip(x-y)}, \quad \tilde{p}_\mu = \frac{1}{a}\sin(p_\mu a)$$

  - 2^d poles at $p = \{0,0,0,0\},\{0,\pi/a,0,0\},\ldots\{\pi/a,\pi/a,\pi/a,\pi/a\}$



- Wilson fermion
  - Introduce an irrelevant term to lift the 15 extra doubles

$$D_W = \frac{a}{2}\gamma_\mu(\nabla_\mu - \nabla_\mu^+) + \frac{a^2}{2} r \nabla_\mu \nabla_\mu^+ + m_0$$

  - Wilson fermion action

$$S_F = a^4 \sum_{x,y} \overline{\Psi}(x)\{\frac{1}{\kappa} - \Sigma_\mu[(r+\gamma_\mu)U_\mu^+(y)\delta_{x-\hat{\mu},y} + (r-\gamma_\mu)U_\mu(x)\delta_{x+\hat{\mu},y}]\}\Psi(y), \quad \frac{1}{\kappa} = 2(m_0 a + 4)$$

  - Breaks chiral symmetry explicitly ! (Hope to recover at the continuum limit.)

- Lattice gauge partition function

$$Z_{lattice} = \int DU D\overline{\Psi} D\Psi \, e^{-S_G - S_F} = \int DU \det D_W \, e^{-S_G}$$

- Chiral Symmetry ? --- Nielsen – Ninomiya no go theorem
- Difficulties of Ultralocal Actions
  - Wilson Fermion
    - Breaking of chiral symmetry at finite a
    - Quark condensate mixes with unity
    - Additive renormalization of quark masses
    - κ_c depends on the gauge configuration
    - Exceptional configurations
    - No unambiguous identification of fermion zero modes with topology
    - Mixing of operators in different chiral sector
    - O(a) error (can be removed with improved action but with fine tuning)
  - Staggered Fermion
    - Breaking of flavor symmetry
    - Topologically blind
    - Dynamical fermion action with $\sqrt{\det D_{stagger}}$ or $\sqrt[1/4]{\det D_{stagger}}$ non-local (?)

Chiral Fermion

Domain wall fermion, overlap fermion, and fixed-point-action fermion

Their Dirac operators satisfy Ginsparg-Wilson relation

$$\{\gamma_5, D\} = D\gamma_5 D \underset{a \to 0}{\Rightarrow} 0, D = aD_a$$

Lattice chiral symmetry (Lüscher)

$$\delta\psi = T\gamma_5 (1 - \tfrac{1}{2} D)\psi, \ \delta\overline{\psi} = \overline{\psi}(1 - \tfrac{1}{2} D)\gamma_5 T, \ \delta(\overline{\psi} D\psi) = 0$$

Chiral SU(N_f) sectors transforms like in the continuum with

$$S = \overline{\Psi}T(1 - \frac{1}{2}D)\Psi \Leftrightarrow P = \overline{\Psi}T\gamma_5(1 - \frac{1}{2}D)\Psi; \ V_\mu = \overline{\Psi}T\gamma_\mu(1 - \frac{1}{2}D)\Psi \Leftrightarrow A_\mu = \overline{\Psi}T\gamma_\mu\gamma_5(1 - \frac{1}{2}D)\Psi$$

Continuum current algebra relations are retained upon the substitution $\psi \to (1 - \tfrac{1}{2}D)\psi$

For example, generalized Gell-Mann-Oakes-Renner relation

$$m\int d^4x \, \langle \pi(x)\pi(0) \rangle = -\langle \overline{\Psi}(1 - \frac{1}{2}D)\Psi \rangle; \ \pi = \overline{\Psi}\gamma_5(1 - \frac{1}{2}D)\Psi$$

is satisfied configuration by configuration, for any source, and any mass.

Synopsis of Overlap Fermion

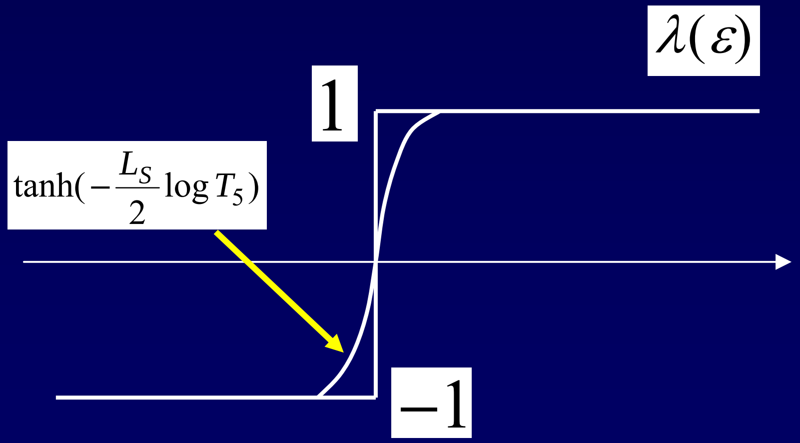
Overlap fermion (Neuberger, 1998)

$$D = 1 + \gamma_5 \varepsilon, \ \varepsilon = H / \sqrt{H^2}$$

$$H = \gamma_5 (D_W(m=0) - m_0), \ 0 < m_0 < 2$$

Free quark: $D(p) \cong ipa$ for small p, no doublers

Eigenvalues of ε are 1 and -1



Discontinuity is crucial for the exact chiral symmetry.

Topology of Overlap Fermion

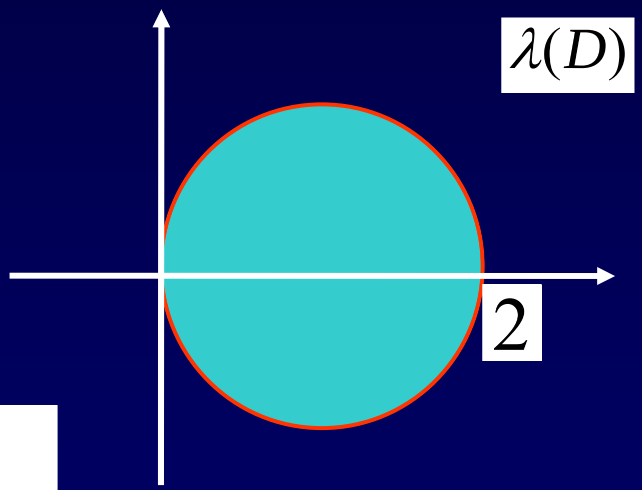
U(1) anomaly and topology is similar to continuum.
Chiral Jacobian *a la* Fujikawa
Atiya-Singer theorem satisfied on the finite lattice

$$Q = n_- - n_+ = -\mathbf{Tr}\gamma_5(1-\tfrac{1}{2}D) \Rightarrow q(x) = -\mathbf{Tr}\gamma_5(1-\tfrac{1}{2}D(x,x))$$

Normality ($D^\dagger D = DD^\dagger$) and GW relation

→ Eigenvalues are on a unit circle and λ = 0, 2 are chiral modes. The rest are complex pairs.

$$D\Psi = \lambda\Psi, \textbf{ then } D(\gamma_5\Psi) = \lambda^*(\gamma_5\Psi)$$



$$\lambda \neq \lambda^* \;\Rightarrow\; (\Psi, \gamma_5\Psi) = 0\,;$$

$$\mathrm{Tr}\,\gamma_5 = 0 \Rightarrow n_+(\lambda=0) - n_- + N_+(\lambda=2) - N_- = 0\,;$$

$$\mathrm{Tr}\,\gamma_5 D = 2(N_+ - N_-)\,;$$

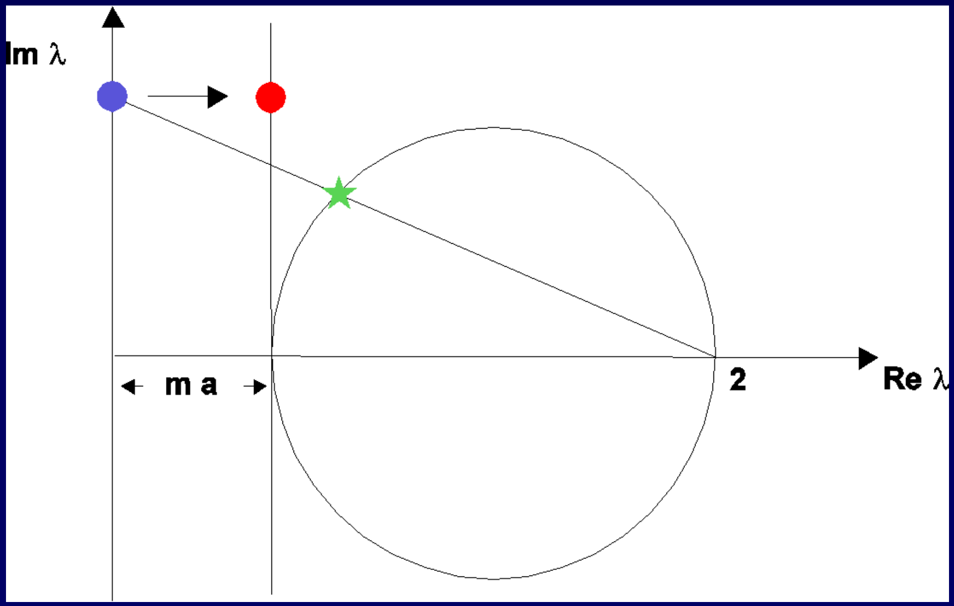
$$\therefore Q = n_- - n_+ = -\,\mathrm{Tr}\,\gamma_5(1-\frac{1}{2}D) \quad \text{can be even or odd.}$$

Effective Quark Propagator on the Lattice

- Consider chirally regulated field $\hat{\psi} = (1 - \tfrac{1}{2} D)\psi,$

  the effective propagator is $(1 - \tfrac{1}{2} D) D(m)^{-1} = (D_c + ma)^{-1}$

- $D_c = D/(1 - D/2)$ is chirally symmetric, i.e. $\{\gamma_5, D_c\} = 0$.
- D is local and normal ($DD^+ = D^+D$), D_c is non-local and anti-hermitian.
- Eigenvalues of $D_c + m$ correspond to the stereographic projection of the circle onto the imaginary axis shifted by m.
- $D_c + m$ is like in the continuum formalism.



Overlap Fermion

- Expected features:
  - Numerically intensive (~100 times of Wilson fermion to invert) Zolotarev approximation of matrix sign function ($< 10^{-9}$ accurancy)
  - No O(a) errors (no dimension 5 chirally symmetric action)
  - No additive quark mass renormalization
  - No exceptional configuration (eigenvalues on a circle)
  - Well defined topology both globally and locally
  - No mixing of operators in different chiral sectors

- Unexpected desirable features:
  - $O(a^2)$ error are small (π and ρ masses on 3 lattice).
  - $O(m^2a^2)$ errors are small (dispersion relation, renormalization constants). This justifies its application on both heavy and light quarks.
  - Critical slowing down is gentle ($m_\pi \sim 160$ MeV).
  - Topological charge density void of large ultra-violet fluctuation (overlap operator is exponentially local)

Lattice QCD Calculation

- Steps in numerical calculation:
  - Monte Carlo simulation to prepare ~100-200 gauge configurations (through Markov chain) according to the distribution

$$[DU]\, e^{-S_G} \det D$$

  - Quenched approximation $\Rightarrow \det D = \textbf{constant}$
  - Calculate quark propagator with a source S by solving the matrix equation

$$DX = S \qquad Dim = L_s^{\,3} \times L_t \times 3(\textbf{color}) \times 4(\textbf{spin})$$

$DX = S$

  - Sew propagators together in each configuration (sum space, color, and spin)
  - Statistical average over gauge configurations

$$< O_1(U, \overline{\Psi}, \Psi) O_2(U, \overline{\Psi}, \Psi)...] >= \frac{1}{N}\sum_{i=1}^{N} Tr\,\{D[U_i]^{-1} D[U_i]^{-1} ... U_i ...\}$$

  - Renormalization of operators and match to $\overline{MS}(2\,\textbf{GeV})$

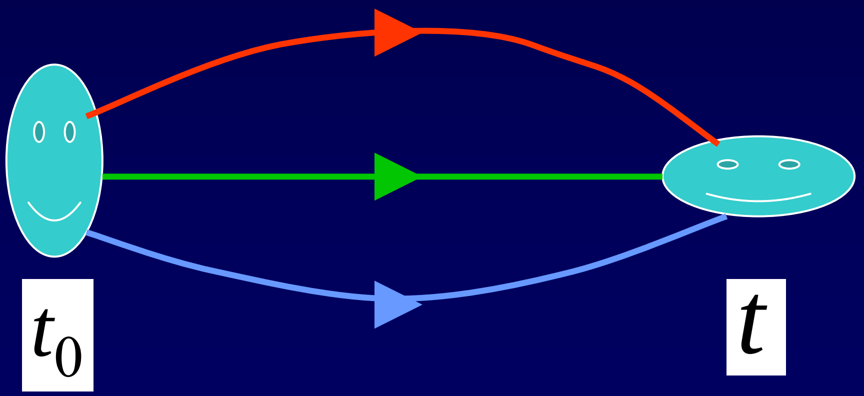
- Systematic errors:
  - Infinite volume limit (V → ∞)
  - Continuum limit (a → 0)
  - Chiral limit (m_π → 137 MeV)
  - Dynamical fermion with u,d, and s flavors

Hadron Mass and Decay Constant

The two-point Green's function decays exponentially at large separation of time

$$G_{NN}^{\alpha\alpha}(t,t_0,\vec{p}) \equiv \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle T(\chi^\alpha(x)\overline{\chi}^\alpha(x_0))\rangle$$

$$\xrightarrow{t-t_0>>1} \langle 0|\chi^\alpha|N^\alpha(\vec{p})\rangle\langle N^\alpha(\vec{p})|\overline{\chi}^\alpha|0\rangle \frac{e^{-E_p(t-t_0)}}{2E_pV_3} \equiv \frac{E_p+m}{E_p}|\phi|^2 e^{-E_p(t-t_0)}$$



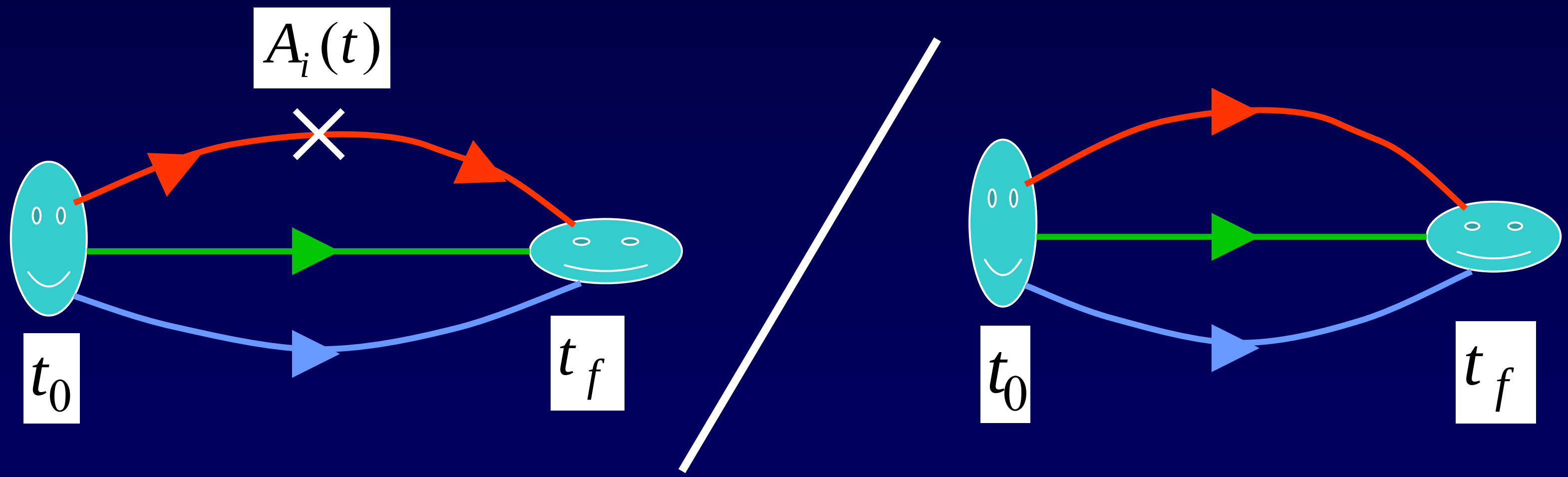
Mass M= $E_p(p=0)$, decay constant ~ Φ

Nucleon Form Factor

The three-point Green's function for the iso-vector axial current is proportional to $\langle N(\vec{p}\,')|A_\mu|N(\vec{p})\rangle$ asymptotically.

$$\Gamma_{\alpha\beta}G_{NN}^{\beta\alpha}(t_f,t,t_0,\vec{q}) \equiv \Gamma_{\alpha\beta}\sum_{\vec{x},\vec{x}_f} e^{i\vec{q}\cdot\vec{x}}\langle T(\chi^\alpha(x_f)A_i\overline{\chi}^\beta(x_0))\rangle$$

$$\xrightarrow{t_f-t,\,t-t_0>>1,} \frac{E_q+m}{E_q}|\phi|^2\, e^{-m(t_f-t)-E_q(t-t_0)}[g_A(q^2)-h_A(q^2)\frac{q_i^2}{E_q+m}];\ \Gamma=\begin{pmatrix}\sigma_i & 0\\ 0 & 0\end{pmatrix}$$

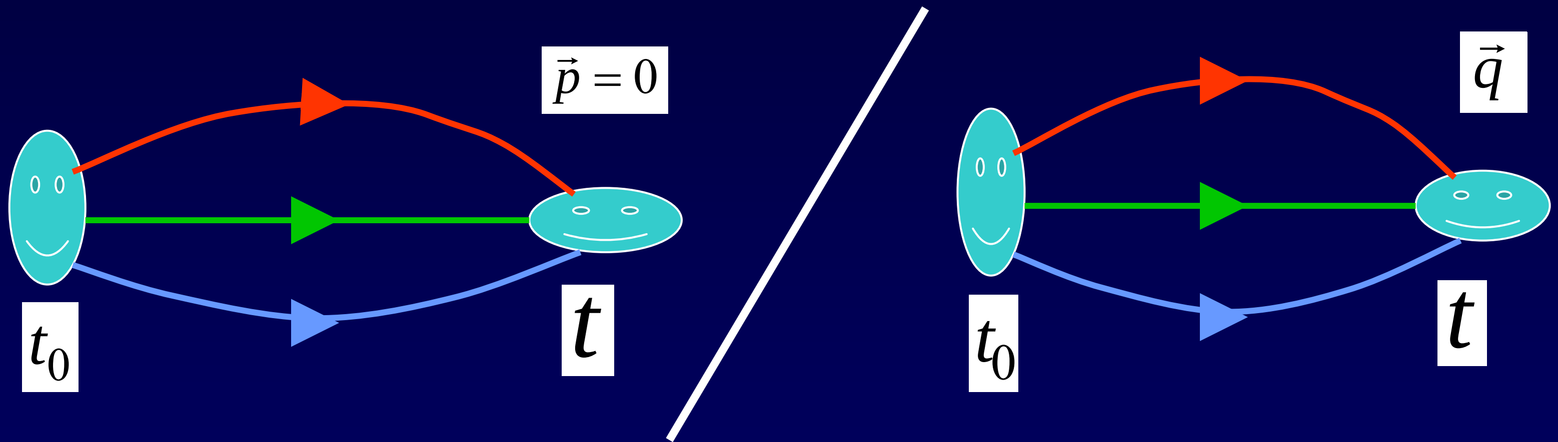


$$\frac{\Gamma_{\alpha\beta}G_{NN}^{\beta\alpha}(t_f,t,t_0,\vec{q})}{G_{NN}^{\alpha\alpha}(t_f,t_0,\vec{p}=0)} \equiv \xrightarrow{t_f-t,\,t-t_0>>1,} \frac{E_q+m}{2E_q}e^{-(E_q-m)(t-t_0)}[g_A(q^2)-h_A(q^2)\frac{q_i^2}{E_q+m}]$$

Nucleon Form Factor

The addition kinematic factor can be removed by the ratio

$$\frac{G_{NN}^{\alpha\alpha}(t,t_0,\vec{p}=0)}{G_{NN}^{\alpha\alpha}(t,t_0,\vec{q})} \xrightarrow{t-t_0>>1} \frac{2E_q}{E_q+m} e^{(E_q-m)(t-t_0)}$$



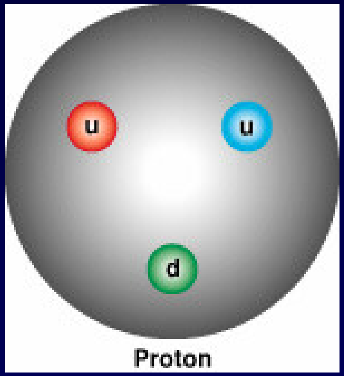
The combined ratios leads to the form factors

$$g_A(q^2)-h_A(q^2)\frac{q_i^2}{E_q+m} \xrightarrow{q_{i=0}} g_A(q^2)$$

How is Spontaneously Broken Chiral Symmetry Realized in Baryon Spectrum?

Valence Quark Model vs Chiral Model of Hadrons

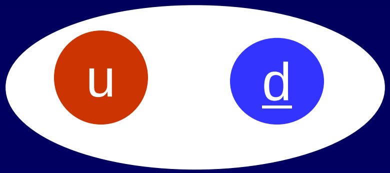
- Attributes of the Valence Quark Model:
  - confined constituent or current quarks; Lowest Fock space, i.e. $q\bar{q}$ or qqq, no meson clouds nor sea quarks; SU(6) symmetry.
- Success of the Valence Quark Model:
  - Classification of Hadrons
  - Spectroscopy -- hyperfine and fine splittings
  - Magnetic moments of baryons, μ_n / μ_p
  - SU(6) relations, e.g. $g_A^8 / g_A^3 = 3/5$
  - OZI rule



- Chiral Symmetry and Chiral Model:

  $SU(2)_L \times SU(2)_R$ and approximate $SU(3)_L \times SU(3)_R$

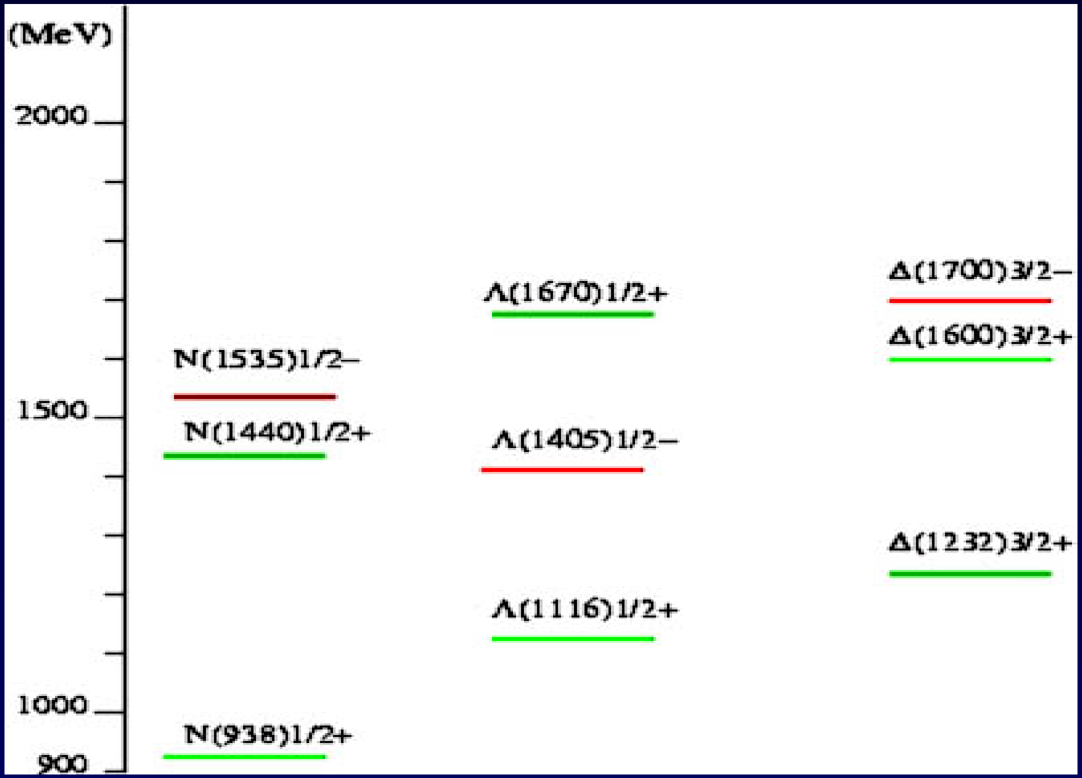
- Success of Current Algebra and Chiral Model:
  - Goldberger-Treiman relaton
  - PCAC
  - π - π scattering
  - Gel-Mann-Oakes-Renner relation
  - Skyrmion



- Suggested resolution:

  Look to lattice QCD calculation for hint and adjudication.

Glozman and Riska Challenge Phys. Rep. 268, 263 (1996)



Hyperfine Interaction of Quarks in Baryons

- Color-spin

$$\lambda_1^C \bullet \lambda_2^C \, \vec{\sigma}_1 \bullet \vec{\sigma}_2$$

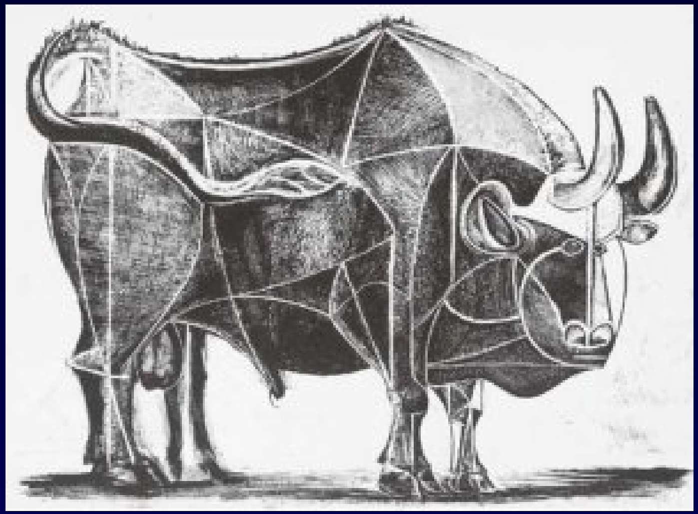
- One-gluon exchange

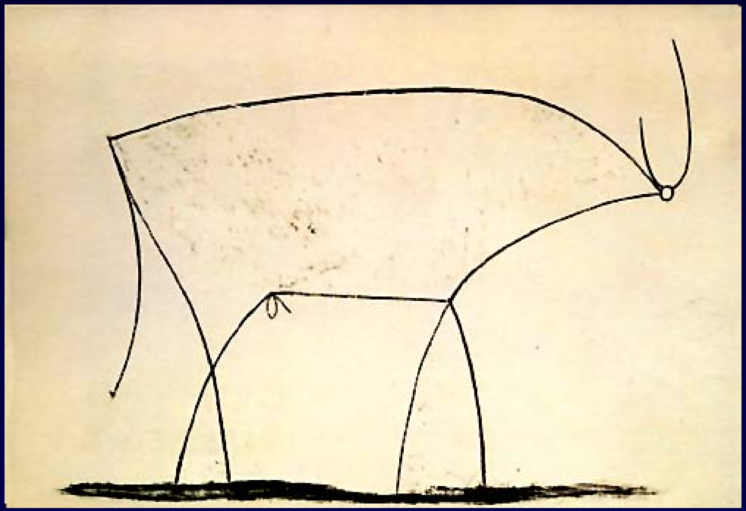
- Flavor-spin

$$\lambda_1^F \bullet \lambda_2^F \, \vec{\sigma}_1 \bullet \vec{\sigma}_2$$

- Goldstone boson exchange

Le Taureau of Pablo Picasso (1945)



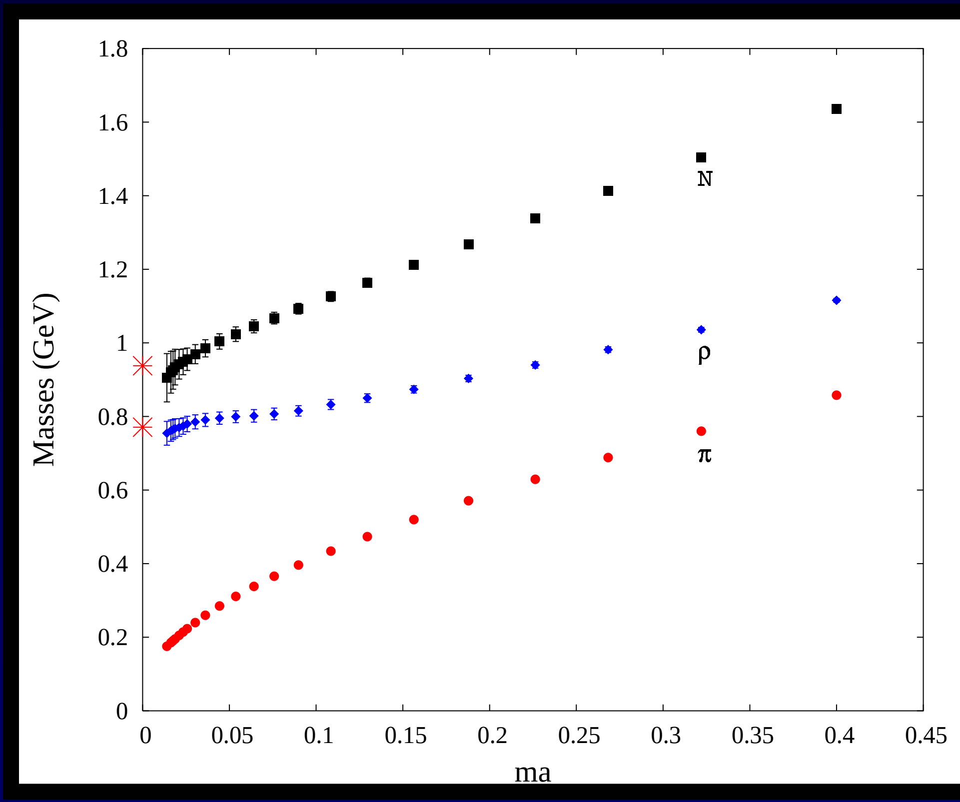


5th stage → 11th stage

Dynamical chiral fermion → Quenched approximation with Chiral symmetry, and light quark masses

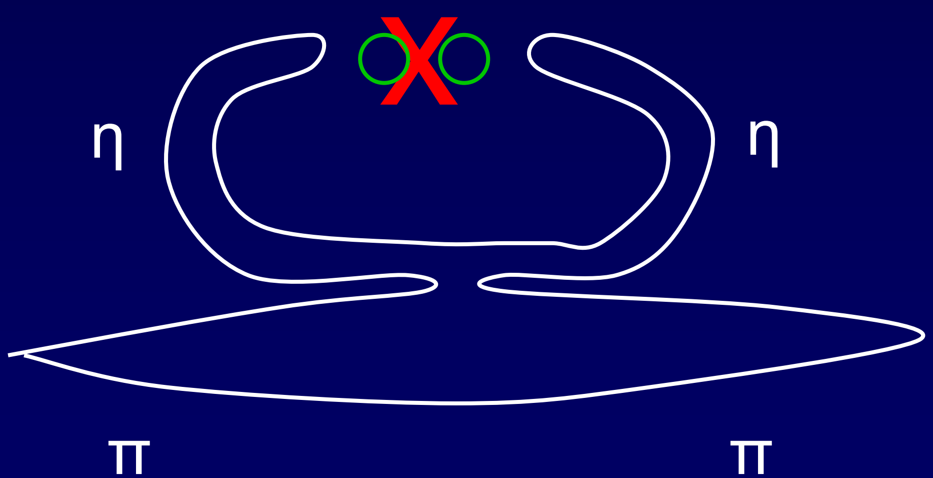
Masses of N, ρ, and π

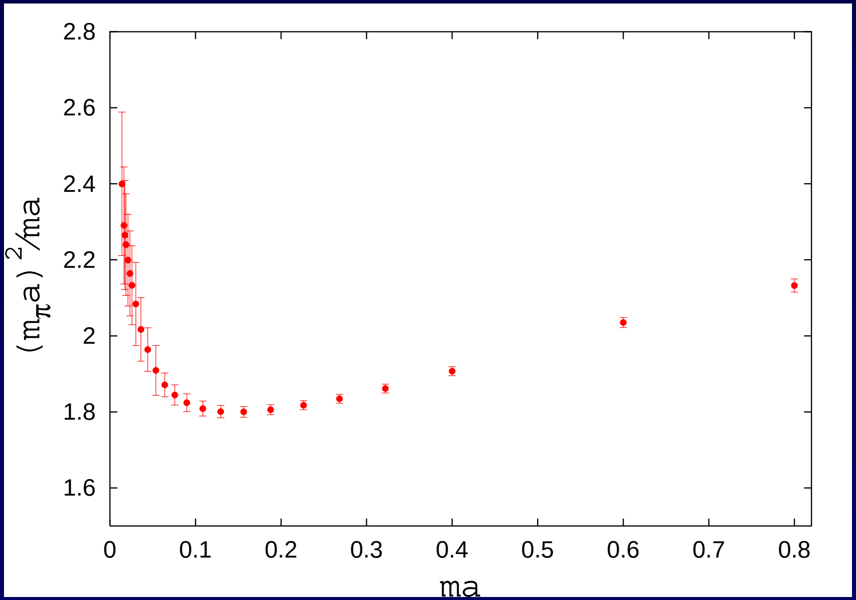
- 16^3 x 28 quenched lattice, Iwasaki action with a = 0.200(3) fm
- Overlap fermion
- Critical slowing down is gentle
- Smallest $m_\pi \sim 180$ MeV
- $m_\pi L > 3$



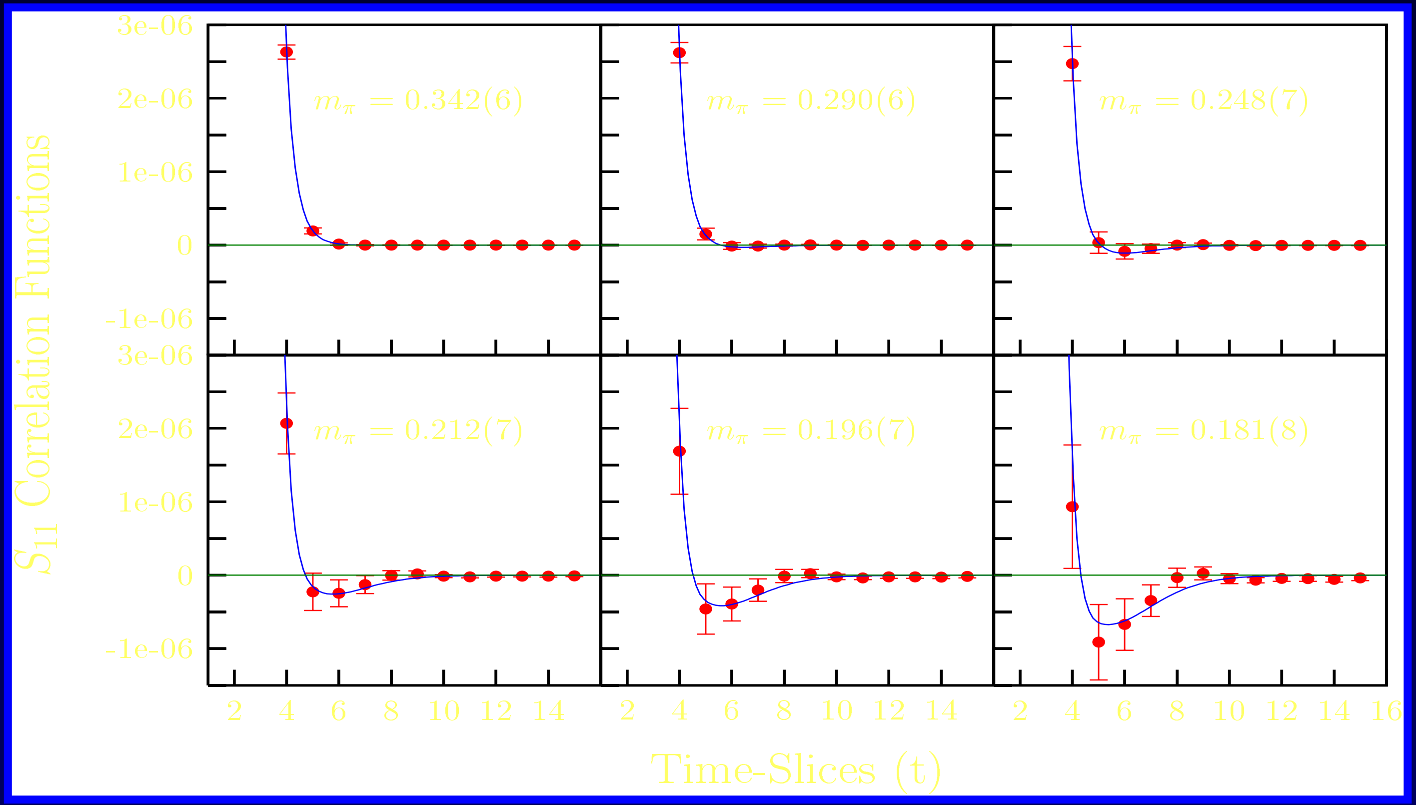
Quenched Artifacts

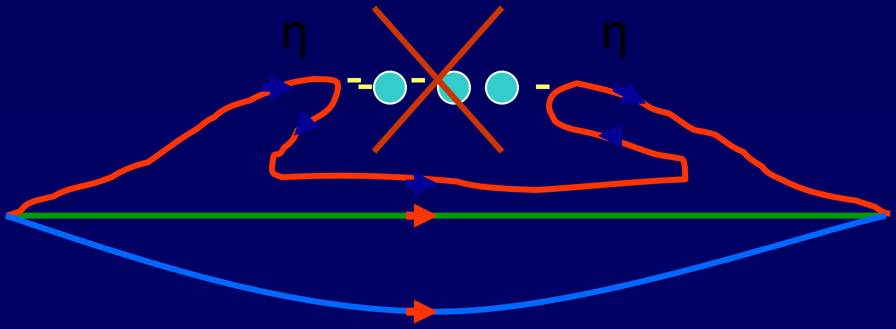
- Chiral log in m_π^2 $$m_\pi^2 = Am\left\{1-\delta\left[\ln(Am/\Lambda_\chi^2)+1\right]\right\}+Bm^2$$

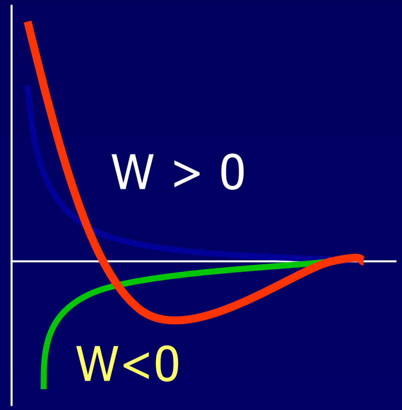




Evidence of η'N GHOST State in S_{11} (1535) Channel







Baryon Two-point Function

Nucleon interpolating field: $\chi_1 = \varepsilon_{abc}(u^{aT} C\gamma_5 d^b)u^c$

$$G(t) = \sum_{\vec{x}} \langle vac \,|\, T\left[\chi_1(x)\overline{\chi_1}(0)\right] |\, vac\rangle$$

$$= (1+\gamma_4)[A_+ e^{-m_+(t-t_0)} + bA_- e^{-m_-(N_t+t_0-t)}] + (1-\gamma_4)[bA_+ e^{-m_+(N_t+t_0-t)} + A_- e^{-m_-(t-t_0)}]$$

Positive-parity channel: N + η' N (p=2π/L) + Roper + …

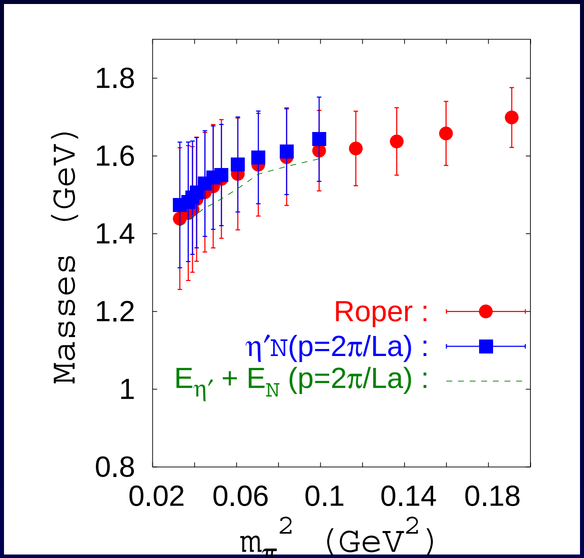
$$G(t) = w_N e^{-m_N t} - w_{\eta' N}(1+E_\pi t)e^{-E_{\eta' N} t} + w_R e^{-m_R t} + \ldots$$

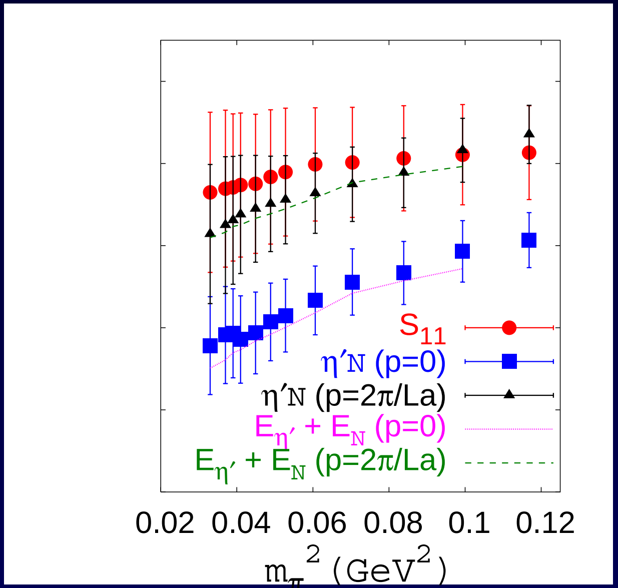
$$E_{\eta' N} \approx \sqrt{m_\pi^2 + p^2} + \sqrt{m_N^2 + p^2}$$

Negative-parity channel: η' N (p=0) + η' N (p=2π/L) + S_{11} +…

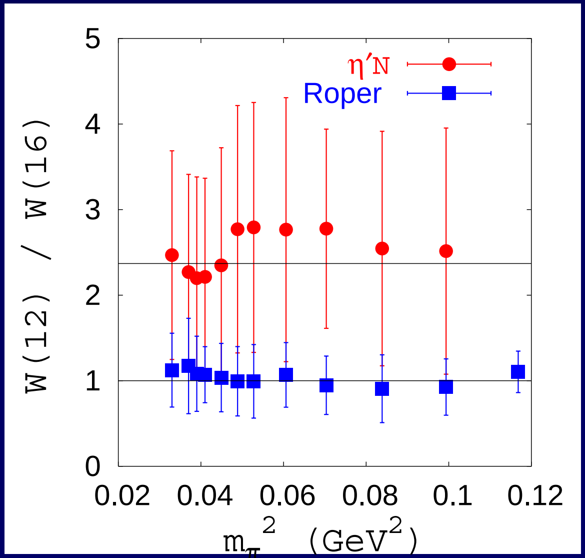
$$G(t) = -w_1(1+m_\pi t)e^{-m_{\eta' N} t} - w_2(1+E_\pi t)e^{-E_{\eta' N} t} + w_{S_{11}} e^{-m_{S_{11}} t} + \ldots$$

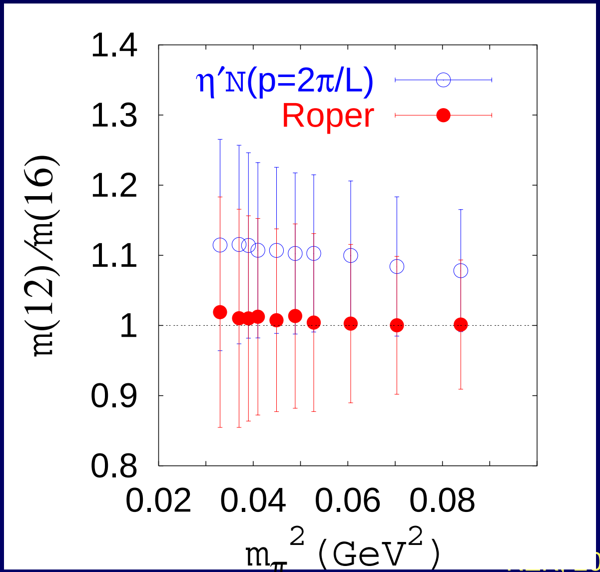
Roper and $S_{11}(1535)$



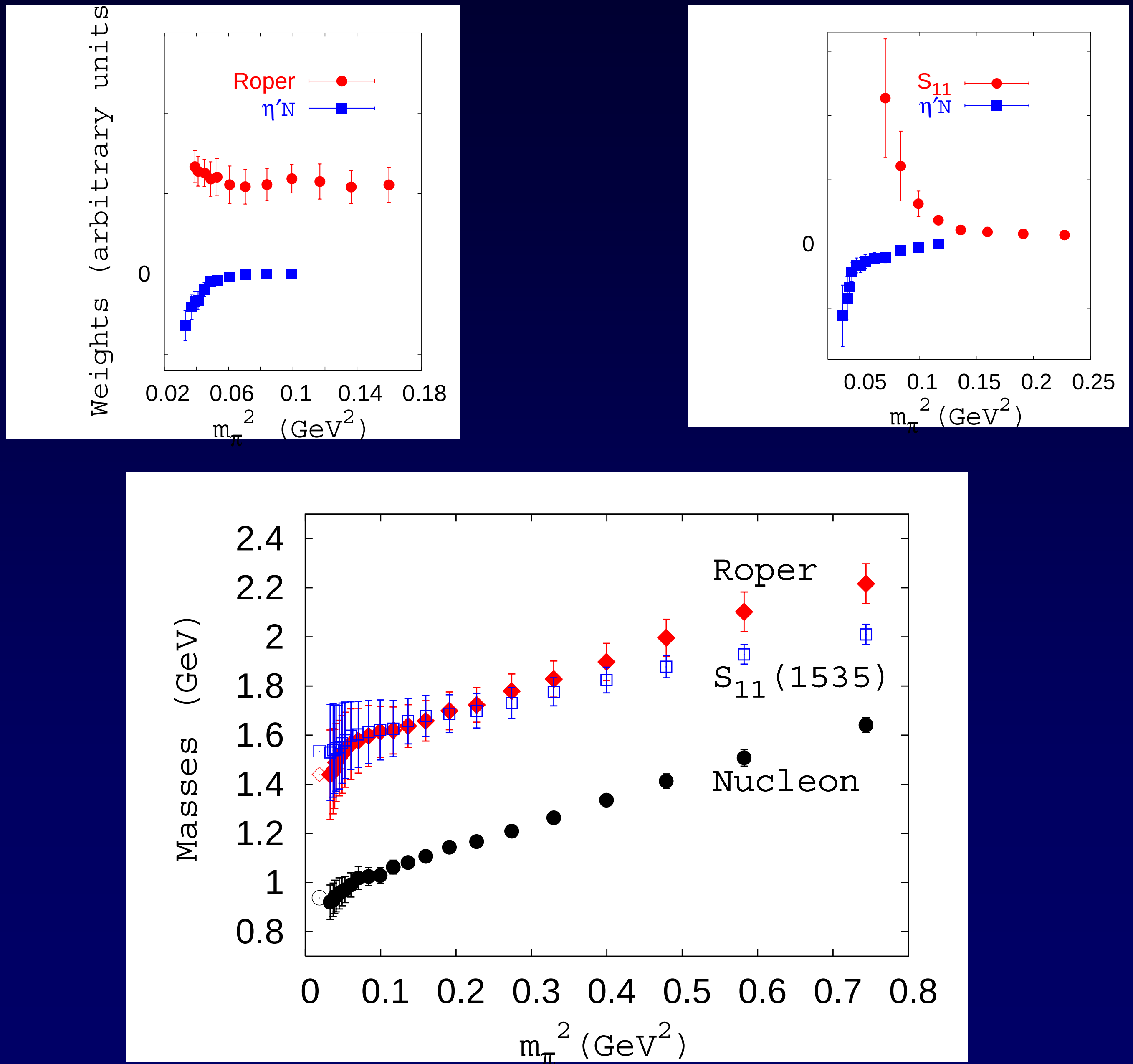


W(12)/W(16) (two-particle) = $16^3/12^3 = 2.37$





Roper and $S_{11}(1535)$

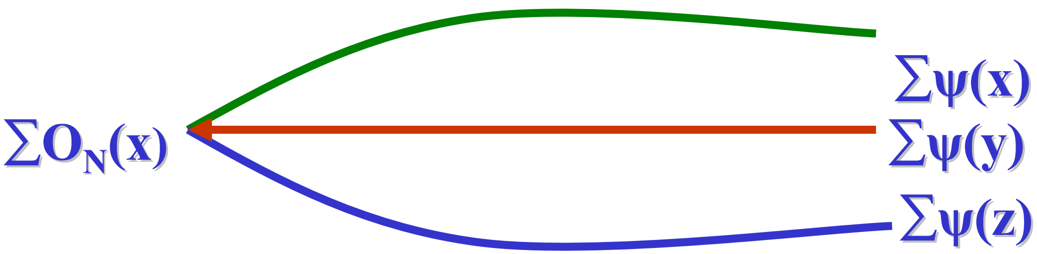


Roper

Radial excitation? $q^4\bar{q}$ State?

- **Roper is seen on the lattice with three-quark interpolation field.**
- **Weight :**

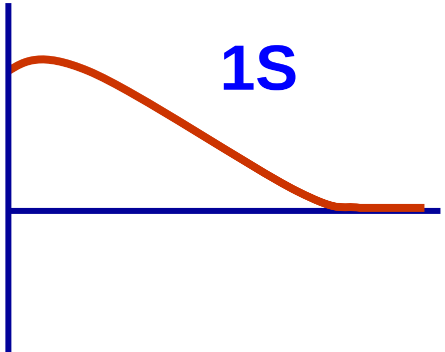
$$|\langle 0|O_N|R\rangle|^2 > |\langle 0|O_N|N\rangle|^2 > 0 \quad \text{(point source, point sink)}$$

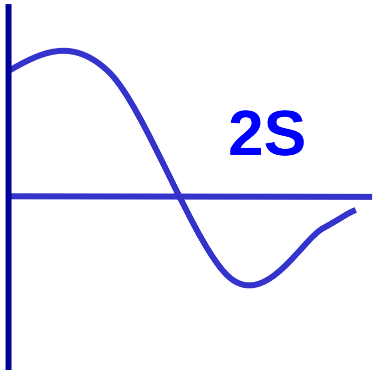


Point sink **Wall source**

$$\langle 0|O_N(0)|N\rangle \langle N|\, \Sigma\psi(x)\, \Sigma\psi(y)\, \Sigma\psi(z)|0\rangle > 0$$

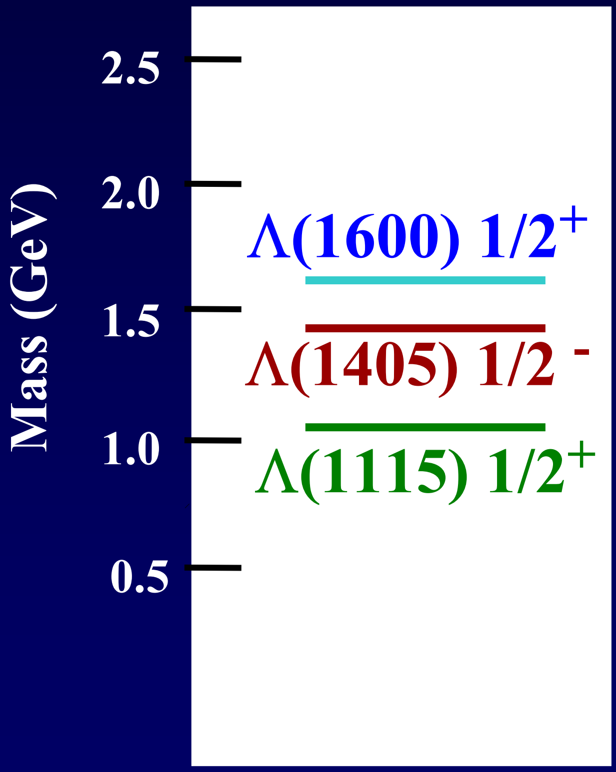
However, $\langle 0|O_N(0)|R\rangle \langle R|\, \Sigma\psi(x)\, \Sigma\psi(y)\, |\, \Sigma\psi(z)|0\rangle < 0$

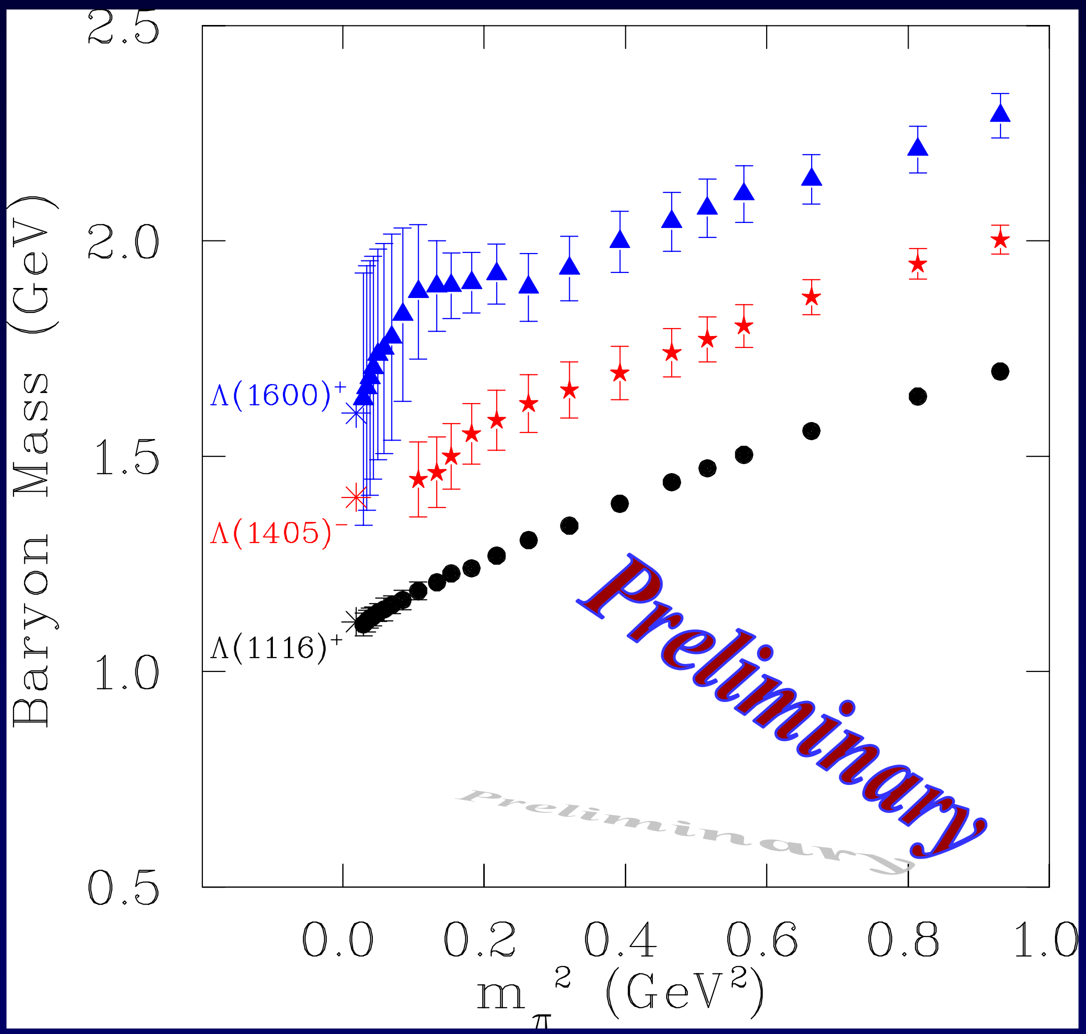




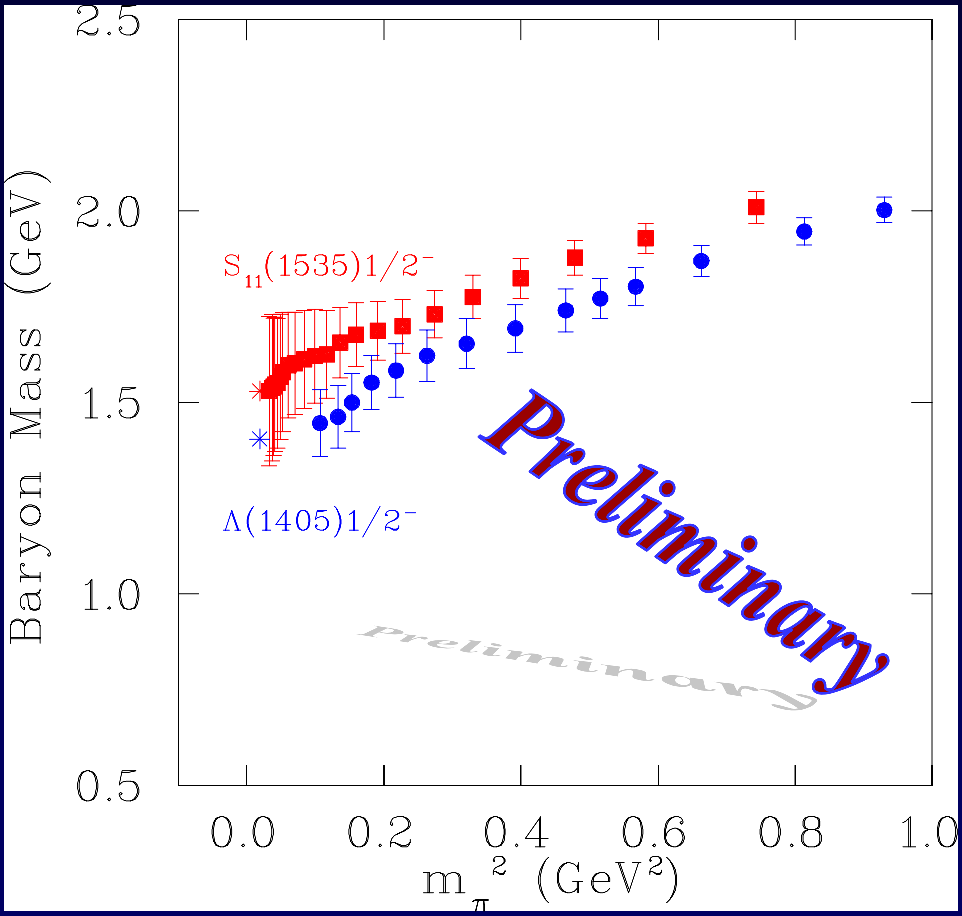
What about Hyperons?
The Λ(1405)?

...*different* story!!



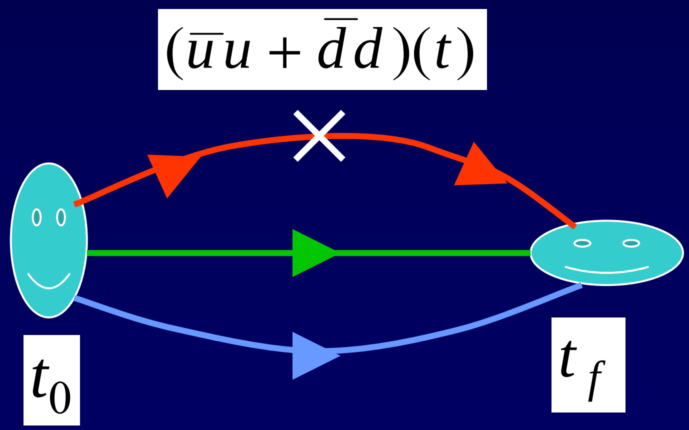


$S_{11}(1535)1/2^-$ and $\Lambda(1405)1/2^-$

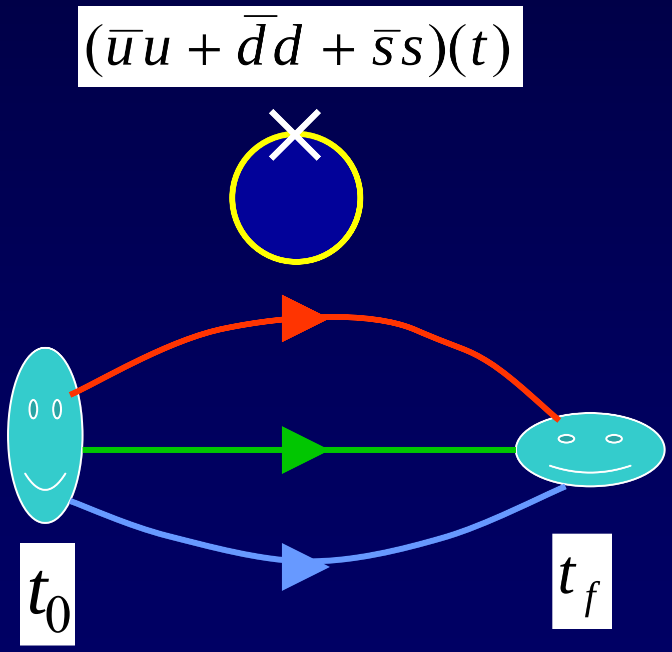


Chiral Symmetry and Nucleon structure

- πNσ Term Puzzle $\sigma_{\pi N} = \frac{m_u + m_d}{2} \langle N | \bar{u}u + \bar{d}d | N \rangle$
  - $\sigma_{\pi N} = \Sigma_{\pi N}$(Cheng-Dashen) - $\Delta\, \sigma_{\pi N}$(Gasser, Leutwyler, Sainio) ~ 45 MeV
  - $\sigma_{\pi N}$ ~ 32 MeV from octect baryon masses, if $\langle N | \bar{s}s | N \rangle = 0$ (T.P. Cheng)
  - $\sigma_{\pi N}$ ~ 15 MeV from valence quark model (average u, d mass ~ 5 MeV)



Connected insertion (CI)



Disconnected insertion (DI)

	Lattice	χ PT	NRQM	RQM
$\langle N \mid \bar{u}u + \bar{d}d \mid N \rangle_{CI}$	3.02(9)		3	< 3
$\langle N \mid \bar{u}u + \bar{d}d \mid N \rangle_{DI}$	5.41(15)		0	0
$\langle N \mid \bar{s}s \mid N \rangle_{DI}$	1.53(7)		0	0
F_S	1.51(12)	1.52-1.81	1	< 1
D_S	-.88(28)	-0.52- -.57	0	0
$\Delta\sigma_{\pi N}$ (MeV)	6.61(59)	15.2(4)		
y	0.36	0.2 – 0.3	0	0
$\sigma_{\pi N}$ (MeV)	49.7(2)	~ 45	~ 15	~ 15
σ_{KN} (MeV)	362(13)	~ 395		

$$F_S = \frac{\langle P \mid \bar{u}u - \bar{s}s \mid P \rangle}{2}; \quad D_S = \frac{\langle P \mid \bar{u}u - 2\bar{d}d + \bar{s}s \mid P \rangle}{2}$$

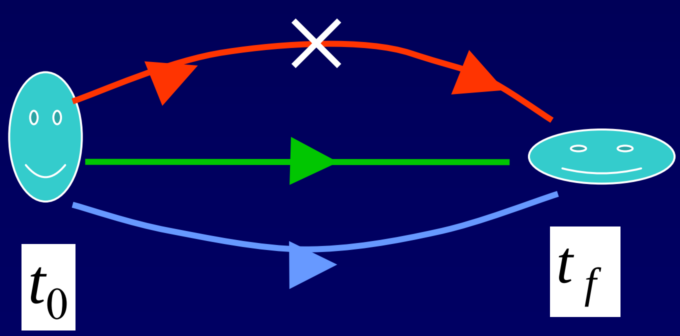
$$y = \frac{2\langle N \mid \bar{s}s \mid N \rangle}{\langle N \mid \bar{u}u + \bar{d}d \mid N \rangle}$$

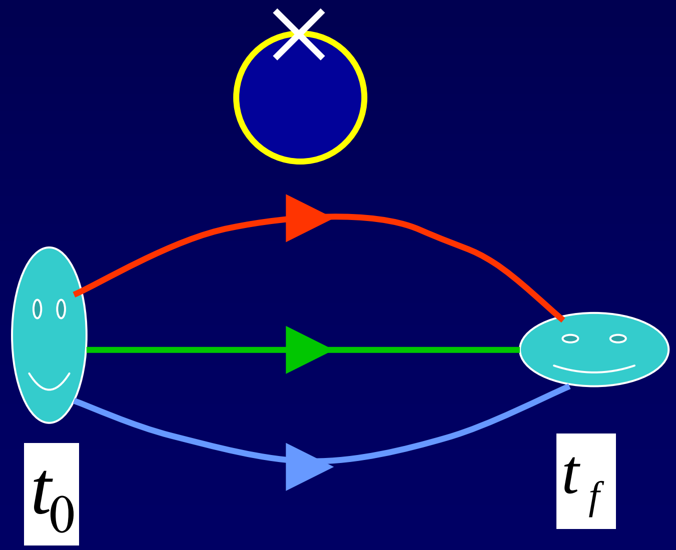
Flavor-singlet g_A

- Quark spin puzzle (dubbed `proton spin crisis')
  - $$g_A^0 = \Delta u + \Delta d + \Delta s = \begin{cases} 1 & \text{NRQM} \\ 0.75 & \text{RQM} \end{cases}$$
  - Experimentally (EMC, SMC, … $\Delta\Sigma = g_A^0 \sim 0.2 - 0.3$

$$\overline{\Psi}\gamma_\mu\gamma_5\Psi(t)(u,d,s)$$

$$(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)(t)$$





$$g^0_{A,con} = (\Delta u + \Delta d)_{con}$$

$$g^0_{A,dis} = (\Delta u + \Delta d + \Delta s)_{dis}$$

Lattice resolution: U(1) anomaly

$$g_A^0 = (\Delta u + \Delta d)_{con} + (\Delta u + \Delta d + \Delta s)_{dis} = 0.62(9) + 3(-0.12(1)) = 0.25(12)$$

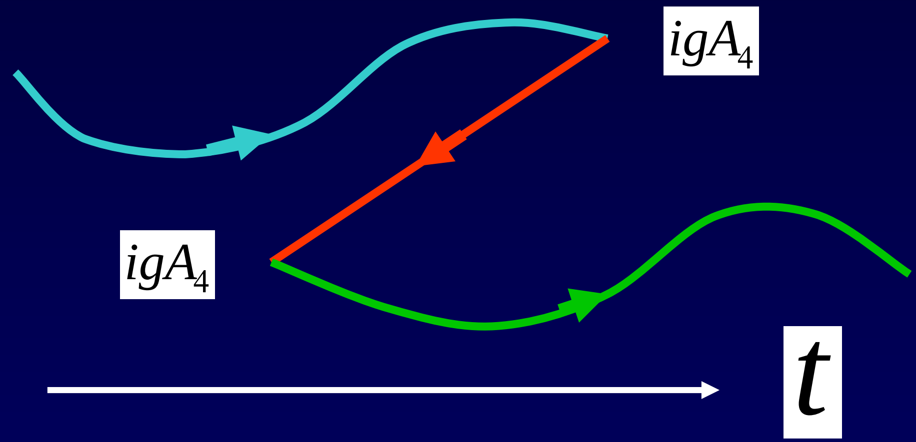
	Lattice	Expt. (SMC)	NRQM	RQM
$g_A^0 = \Delta u + \Delta d + \Delta s$	0.25(12)	0.22(10)	1	0.75
$g_A^3 = \Delta u - \Delta d$	1.20(10)	1.2573(28)	5/3	1.25
$g_A^8 = \Delta u + \Delta d - 2\Delta s$	0.61(13)	0.579(25)	1	0.75
Δu	0.79(11)	0.80(6)	1.33	1
Δd	-.42(11)	-0.46(6)	-0.33	-0.25
Δs	-.12(1)	-0.12(4)	0	0
F_A	0.45(6)	0.459(8)	0.67	0.5
D_A	0.75(11)	0.798(8)	1	0.75
F_A / D_A	0.60(2)	0.575(16)	0.67	0.67

$$F_A = (\Delta u - \Delta s)/2; \quad D_A = (\Delta u - 2\Delta d + \Delta s)/2$$

Valence QCD and Quark Model

- Valence QCD (dropping Z-graphs) to simulate valence quark model

$$L_{VQC} = -\frac{1}{4} G_{\mu\nu} G_{\mu\nu} - \overline{u}[\frac{\gamma_4 + 1}{2} D_4 + \vec{\gamma} \cdot \vec{D} + m]u - \overline{v}[\frac{\gamma_4 - 1}{2} D_4 + \vec{\gamma} \cdot \vec{D} + m]v$$



  - Reduction of Dirac spinor to Pauli spinor

$$L_F(VQCD) = \overline{\chi}(-\gamma_4 D_4 - m + \frac{\vec{D}^2 + \vec{\sigma} \cdot \vec{B}}{m})\chi, \quad \chi = \binom{\chi_1}{\chi_2}$$

  - Particle (χ_1)and antiparticle (χ_2)are decoupled.

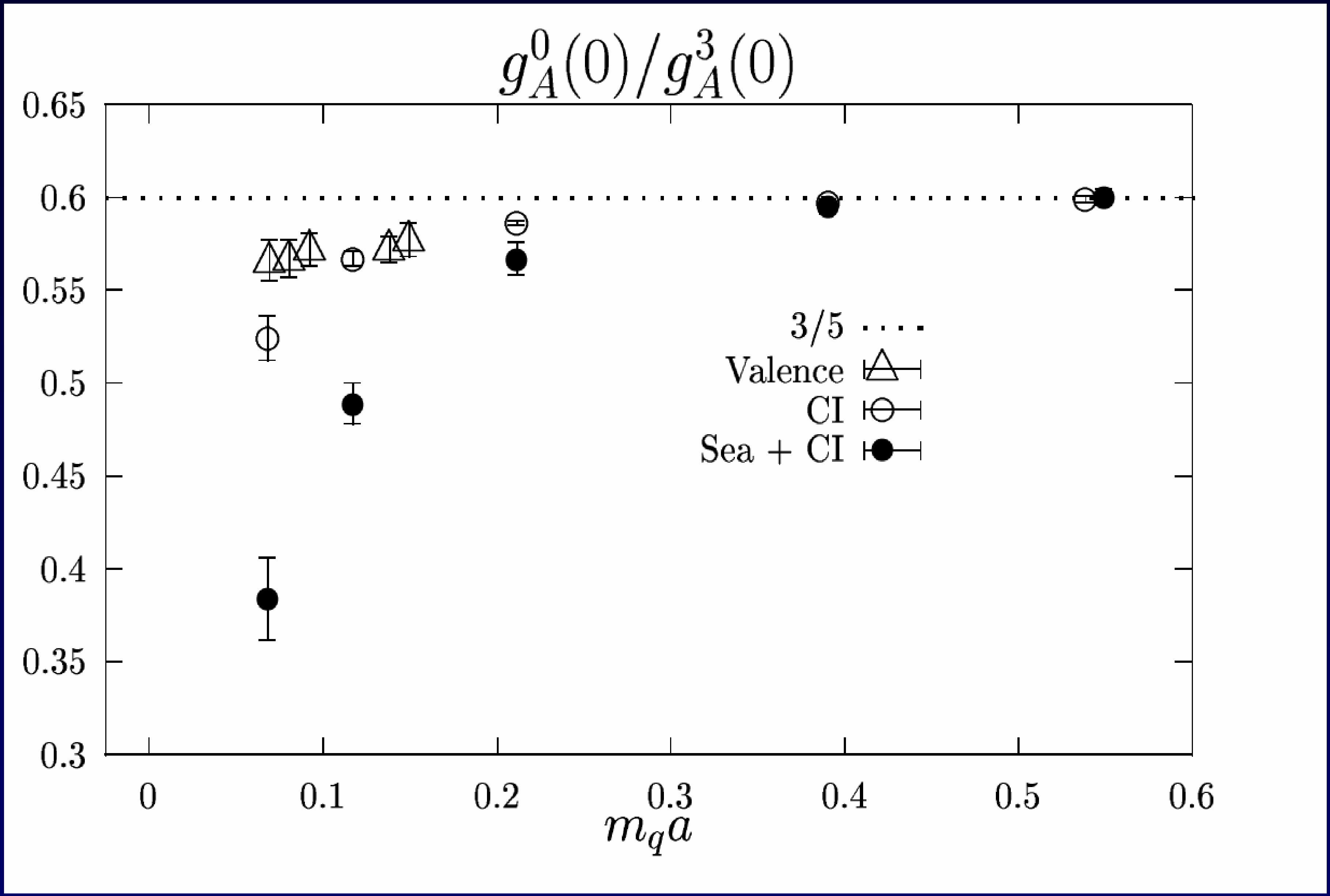
Valence QCD and Quark Model

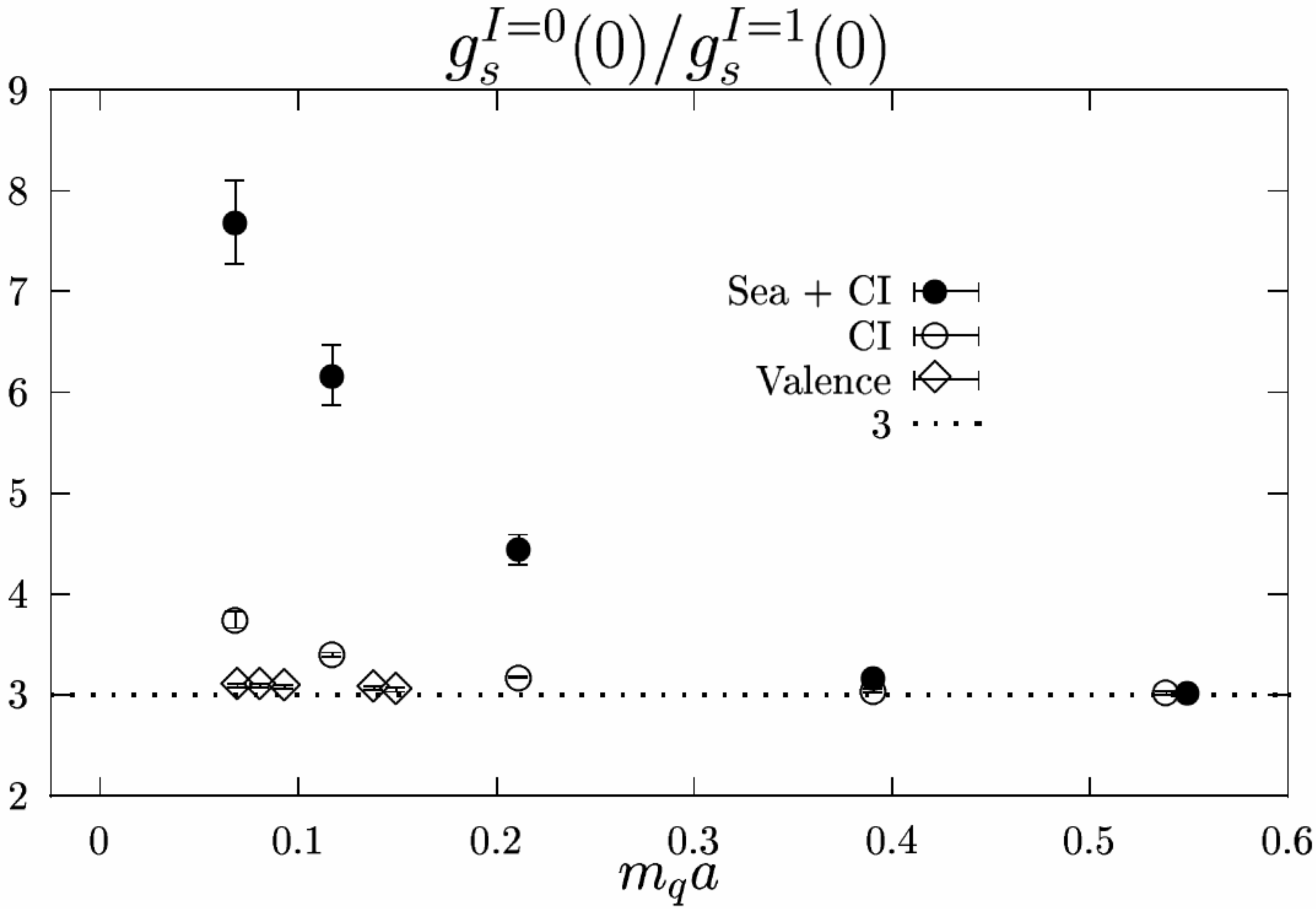
- Lattice simulation reveals:
  - Hyperfine splittings between N-Δ, ρ-π greatly reduced
  - $m_{\pi} \sim m_{q}$
  - Axial, scalar, and EM form factors satisfy SU(6) relations.

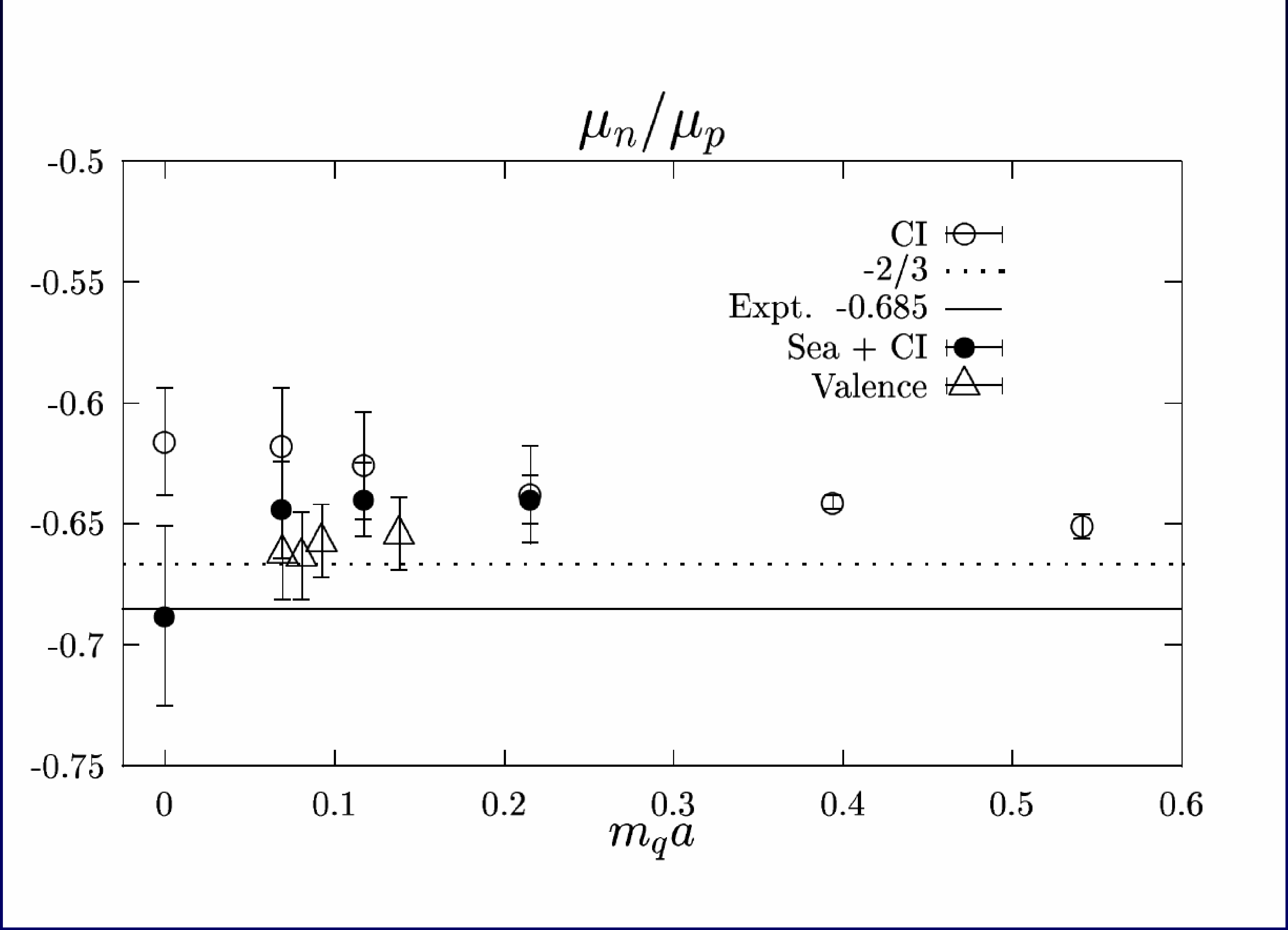
- Symmetry of valence QCD

QCD: $SU_L(N_f) \times SU_R(N_f) \times U_V(1) \rightarrow SU_V(N_f) \times U_V(1)$

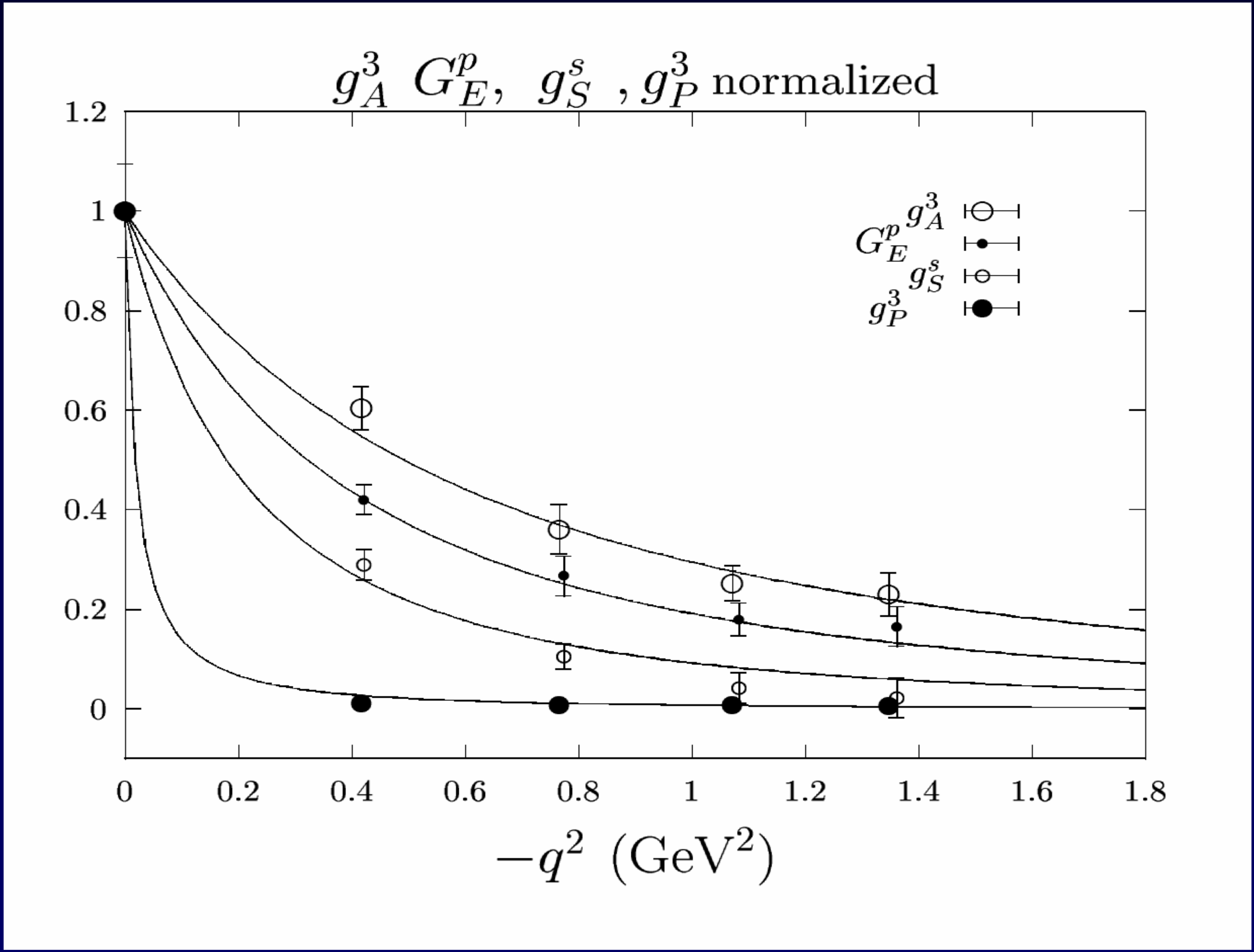
VQCD: $U(2N_f) \rightarrow U^q(N_f) \times U^{\bar{q}}(N_f) \approx U^q(2N_f) \times U^{\bar{q}}(2N_f)$


$g_A^0(0)/g_A^3(0)$
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0
0.1
0.2
0.3
0.4
0.5
0.6
$m_q a$
3/5
Valence
CI
Sea + CI


$g_s^{I=0}(0)/g_s^{I=1}(0)$
9
8
7
6
5
4
3
2
0
0.1
0.2
0.3
0.4
0.5
0.6
$m_q a$
Sea + CI
CI
Valence
3

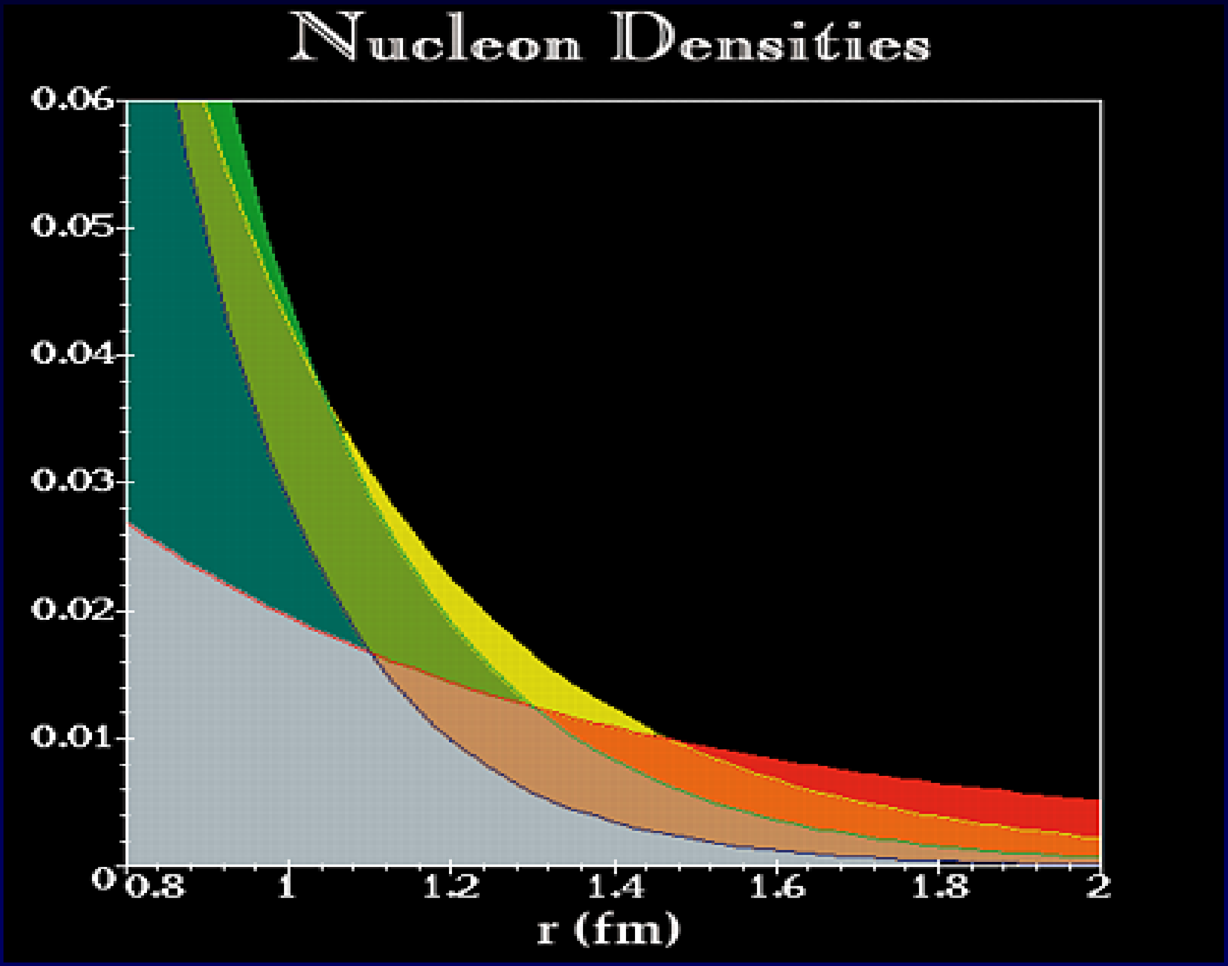


μ_n/μ_p
-0.5
-0.55
-0.6
-0.65
-0.7
-0.75
0
0.1
0.2
0.3
0.4
0.5
0.6
$m_q a$
CI
-2/3
Expt. -0.685
Sea + CI
Valence



g_A^3 G_E^p, g_S^s , g_P^3 normalized
1.2
1
0.8
0.6
0.4
0.2
0
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8
$-q^2$ (GeV^2)
g_A^3
G_E^p
g_S^s
g_P^3

Nucleon Sizes



Blue: axial-vector; Green: proton charge

Yellow: scalar (strangeness); Red: pseudoscalar

Effective Theories

QCD

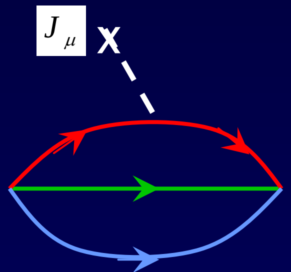
$\Lambda_\chi \sim 4\pi f_\pi \sim 1\,\text{GeV},\ l_M \sim 0.2\,\text{fm}$

0.2 fm

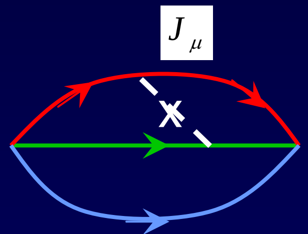
$\psi = \psi_L + \psi_S$ $\quad A_\mu = A_{\mu_L} + A_{\mu_S}$

Chiral Quark Model

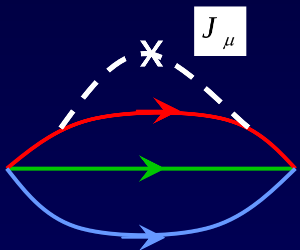
q_L, $g_L(\alpha_s^R)$, π, ρ, ω, a_1, K, ϕ, …



meson dominance
valence + connected sea



meson exchange
connected sea



meson loop
disconnected sea

$\Lambda_B \sim 300\,\text{MeV},\ l_B \sim 0.6\,\text{fm}$

1 fm

Chiral Perturbation Theory