

Coupled channel effects in $\pi\text{-}\pi$ S -wave interaction

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- Coupled channel effects in $l=2$ pi-pi
S-wave interaction
- Coupled channel effects in $l=0$ pi-pi
S-wave interaction
- Summary and Outlook

1. Motivation

$I=0$ $\pi\pi$ S-wave interaction



$I=0$ $J^{PC}=0^{++}$ particles: σ , glueball

However, to really understand the $I=0$ $\pi\pi$ S-wave interaction, one must first understand the $I=2$ $\pi\pi$ S-wave interaction

Because: a, **simpler**. There are no known s-channel resonances and less coupled channels in $I=2$ $\pi\pi$ system, so it is much simpler than the $I=0$ $\pi\pi$ S-wave interaction;

b, **necessary input for extracting $I=0$ phase shifts from data.** To extract $I=0$ $\pi\pi$ S-wave phase shifts from experimental data obtained by $\pi + N \rightarrow \pi + \pi + N$ reactions, one needs an input of the $I=2$ $\pi\pi$ S-wave interaction.

However, in the previous analysis of $l=2$ amplitude, the feature of inelasticities η_0^2 which start to deviate from 1 for energies above 1.1 GeV often was **overlooked**.

In this report, we show in a K-matrix formalism that the features can be well reproduced by $\pi\pi - \rho\rho - \pi\pi$ coupled-channel effect. The same coupled channel effects in $l=0$ will also be discussed.

2. Coupled channel effects in $l=2$ pi-pi S-wave interaction

Coupled channel K-matrix formalism:

$$K = \begin{pmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{pmatrix}, \quad \rho(s) = \begin{pmatrix} \rho_1(s) & 0 \\ 0 & \rho_2(s) \end{pmatrix},$$

$$K_{11} : \pi\pi \rightarrow \pi\pi,$$

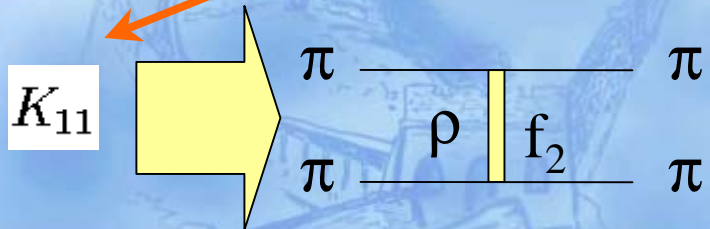
$$K_{12} : \pi\pi \rightarrow \rho\rho,$$

$$K_{22} : \rho\rho \rightarrow \rho\rho$$

$$T = \frac{K}{1 - i\rho K}$$

$$T_{11} = \frac{K_{11} - i\rho_2(K_{11}K_{22} - K_{12}K_{21})}{1 - i\rho_1K_{11} - i\rho_2K_{22} - \rho_1\rho_2(K_{11}K_{22} - K_{12}K_{21})},$$

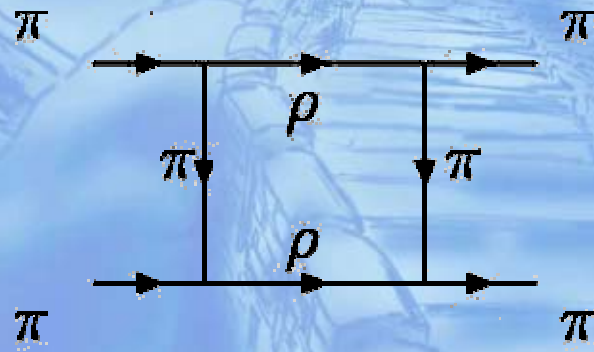
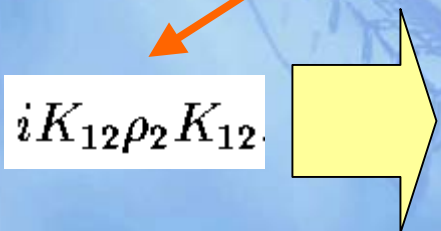
If $K_{22} = 0$, $T_{11} = \frac{K_{11} + iK_{12}\rho_2 K_{21}}{1 - i\rho_1(K_{11} + iK_{12}\rho_2 K_{21})}$,



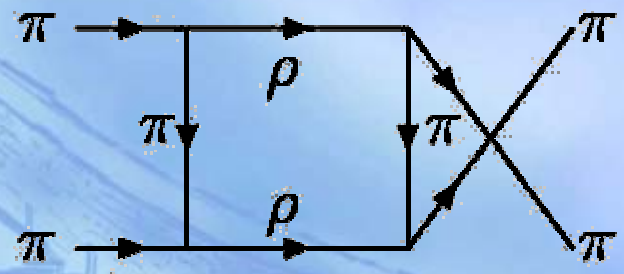
$$V_{\rho\pi\pi} = g_{\rho\pi\pi} \epsilon_{\mu\nu\alpha\beta} q^\mu \mathbf{I}_{\rho} \cdot (\mathbf{I}_{\pi_1} * \mathbf{I}_{\pi_2})$$

$$V_{f_2\pi\pi} = g_{f_2\pi\pi} F_{\mu\nu} q^\mu q^\nu (\mathbf{I}_{\pi_1} \cdot \mathbf{I}_{\pi_2})$$

In order to obtain K_{11} , we incorporate the t-channel $f_2(1270)$ contribution into the ρ exchange term by the **Dalitz-Tuan method**.



(a)



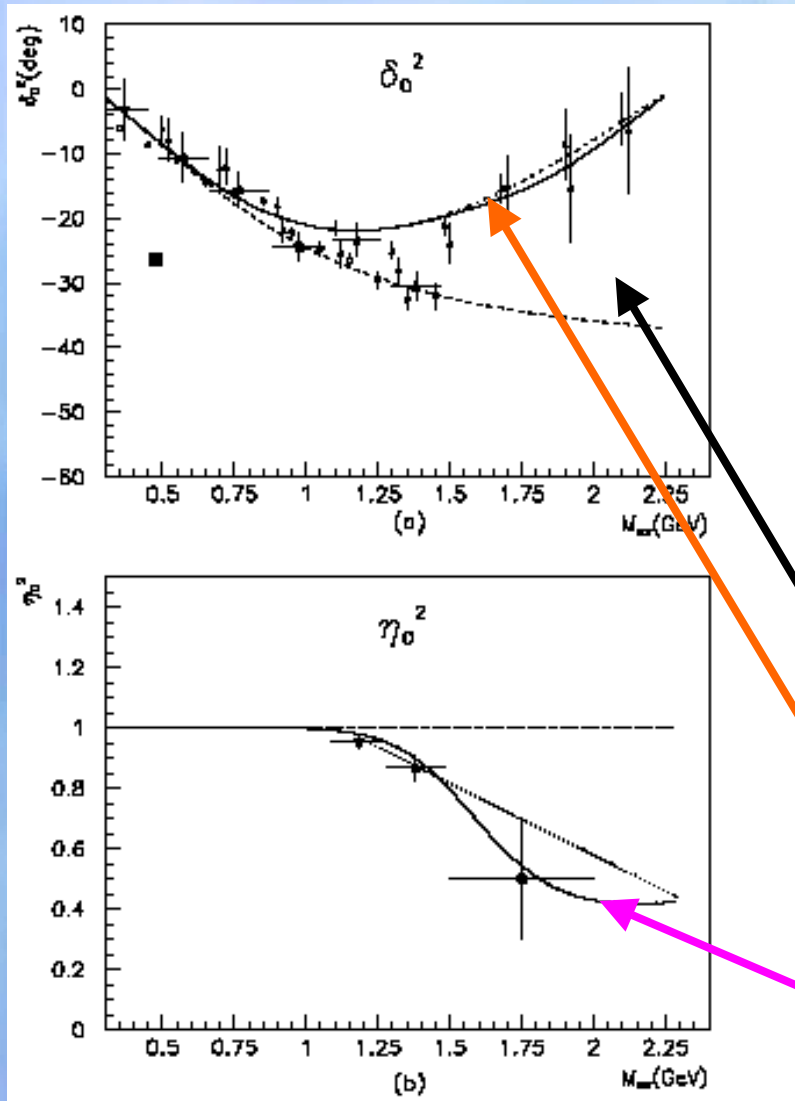
(b)

Off-shell form factor: $F(q^2) = \frac{\Delta^2 + m^2}{\Delta^2 + s^2}$ $F(q^2) = \frac{\Delta^2 - m^2}{\Delta^2 - q^2}$

Only two parameter in our theory: $\Delta_{\rho\pi\pi} = 1.5\text{Gev}$ $\Delta_{f_2\pi\pi} = 1.7\text{Gev}$

With t-channel ρ , $f_2(1270)$ exchange, we reproduce the pion pion isotensor S-wave and D-wave scattering phase shifts up to 2.2 GeV quite well, but inelasticity parameter $\eta=1$. **In order to reproduce this inelasticity, it is necessary to consider the pi pi \rightarrow rho rho coupling channel effect.**

I=2 $\pi\pi$ S-wave phase shift δ and inelastic parameter η



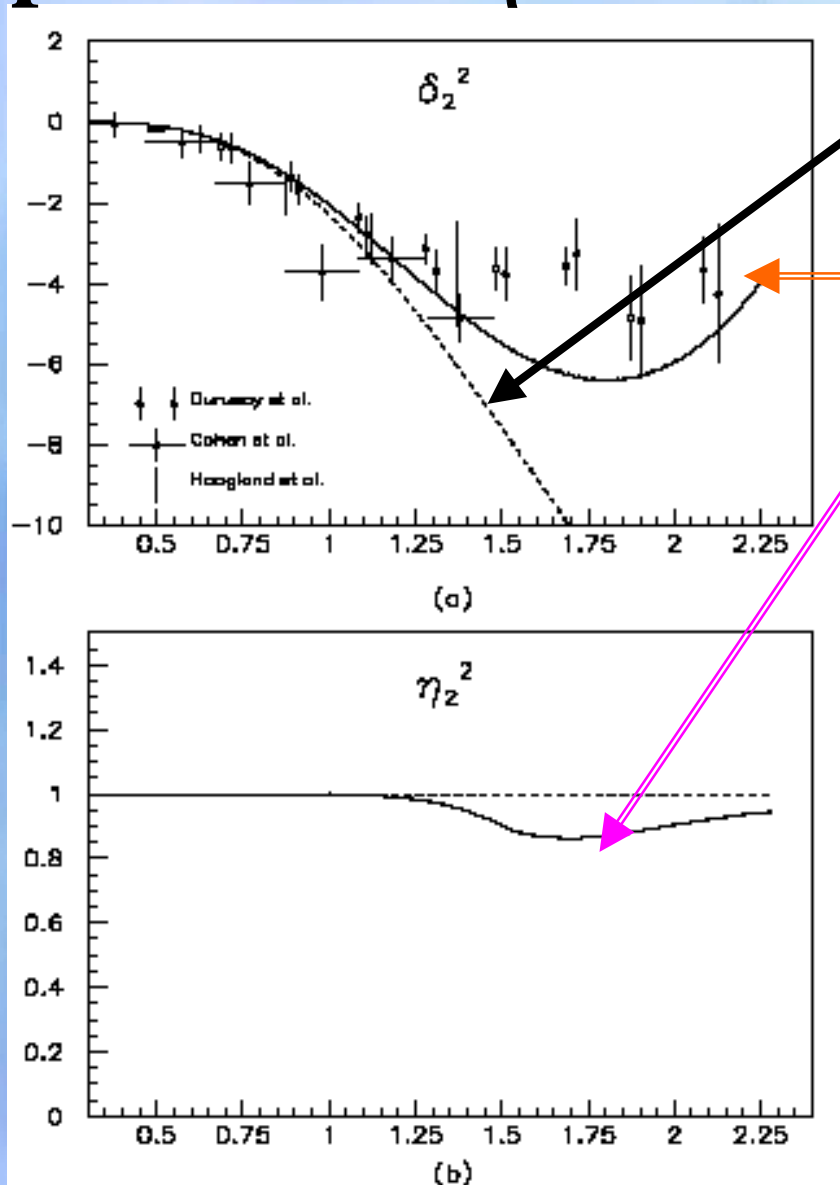
Main features :

(1) the δ_0^2 goes down more and more negative as the $\pi\pi$ invariant mass increases from pion pion threshold up to 1.1 GeV; (2) the δ_0^2 starts to increase for energies above about 1.1 GeV; (3) the η_0^2 starts to deviate from 1 for energies above 1.1 GeV.

Explain :

- 1, repulsive force by t-channel ρ
- 2, attractive force by f_2 (1270)
- 3, inelasticity by $\pi\pi$ - $\rho\rho$ - $\pi\pi$ box

$I=2$ $\pi\pi$ D-wave phase shift δ and inelastic parameter η



Explain :

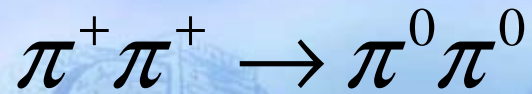
1, repulsive force by t-channel ρ

2, attractive force by f_2 (1270)

3, inelasticity by $\pi\pi$ - $\rho\rho$ - $\pi\pi$ box

the δ_2^2 data around 1.7 GeV has the largest discrepancy with our theoretical result. **Main reason** may be they assume $\eta_2^2 = 1$, when they disposal the experimental data .

$\pi\pi$ S-wave full amplitude squared



The same $T^{I=0}$ plus various input
Of $T^{I=2}$

various $T^{I=2}$:

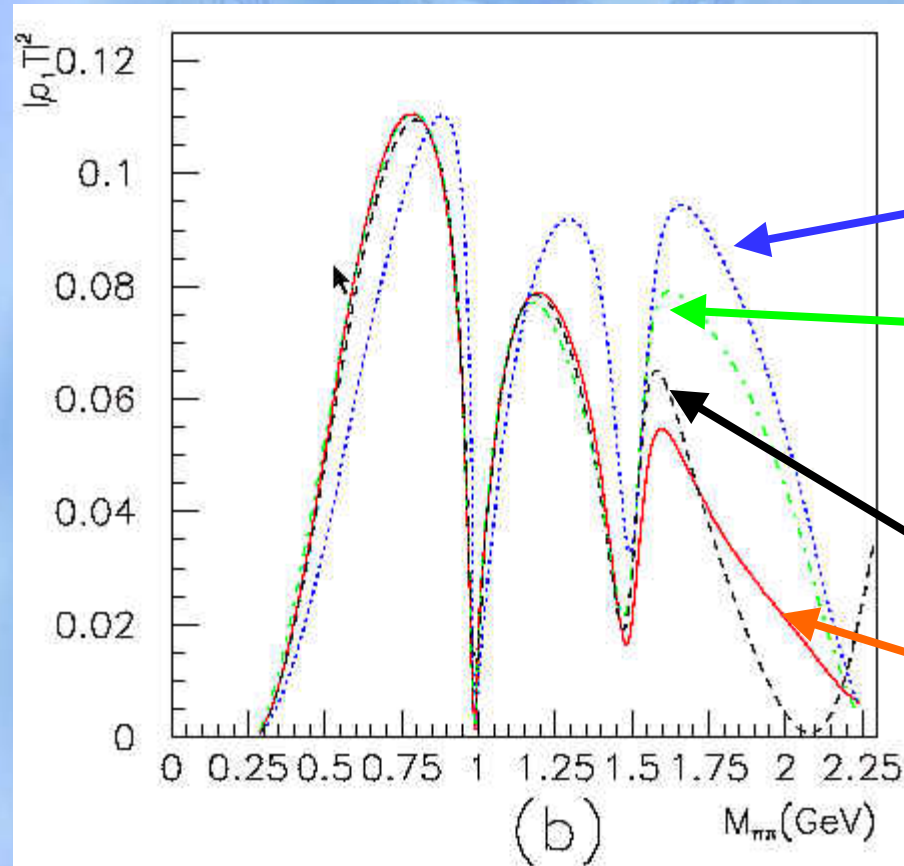
1, $T^{I=2} = 0$

2, $T^{I=2} = \frac{a_0 q}{1 - ia_0 q}$

t-channel ρ exchange

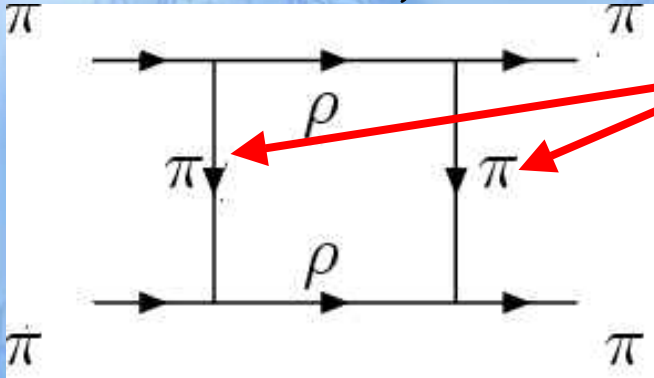
3, include ρ , f_2 (1270)
exchange

4, include ρ , f_2 , $\pi\pi$ - $\rho\rho$ - $\pi\pi$
box diagram.



Discussion:

A, In our calculation of the $\pi\pi$ - $\rho\rho$ - $\pi\pi$ box diagram, besides π , we also consider



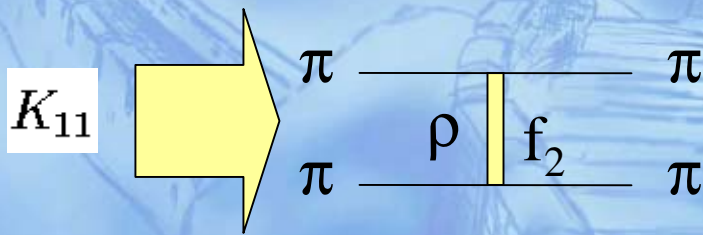
$\omega(783), a_1(1260), \pi(1300), a_2(1320)$

Which have large coupling to $\rho\pi$, The result show these contribution is about a quarter of what we get from the pi exchange and will possible cancel each other.

B, About $K_{22} = 0$ assumption. The interaction of $\rho\rho \rightarrow \rho\rho$ channel is not clear up to now, by assuming $K_{22} = xK_{11}$, with x a constant. It is found that for $x \leq 5$ the results are still within the experimental error bars. For $x \leq 10$ by adjusting off-shell cutoff parameter, the data can still be well reproduced

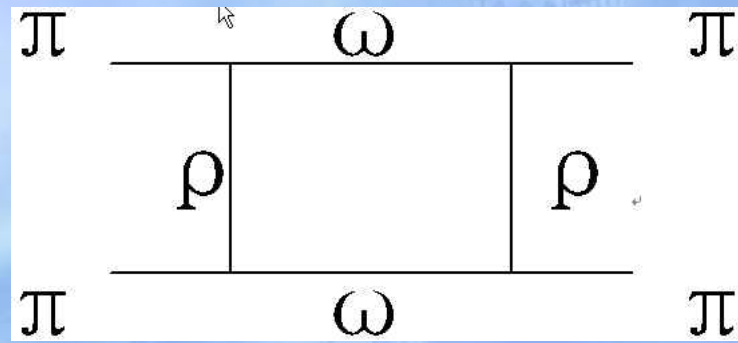
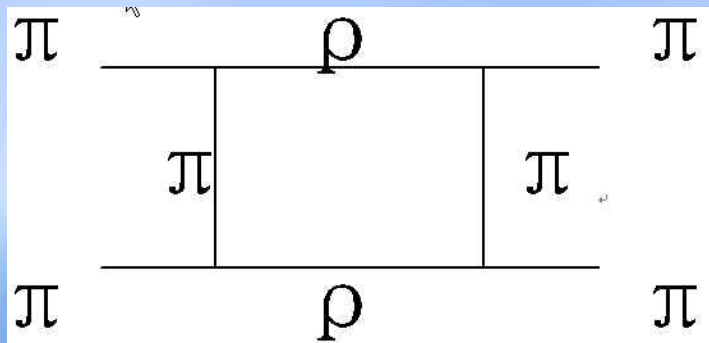
3. Coupled channel effects in pi-pi l=0 S-wave interaction

Without including s-channel resonances, we only consider the effect of ρ , f_2 exchanges and various coupled channel box diagrams upon pi pi inelastic parameter.

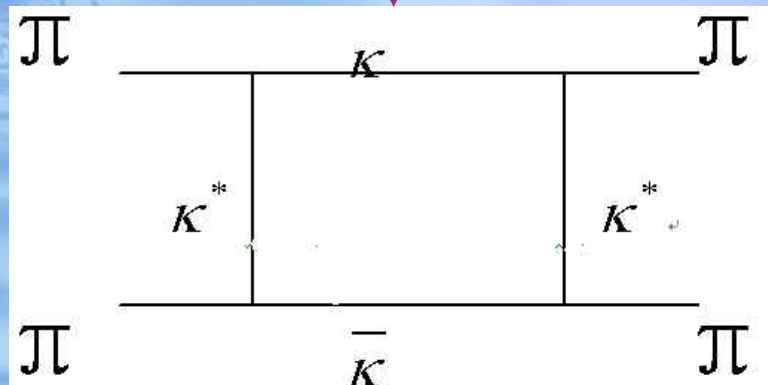
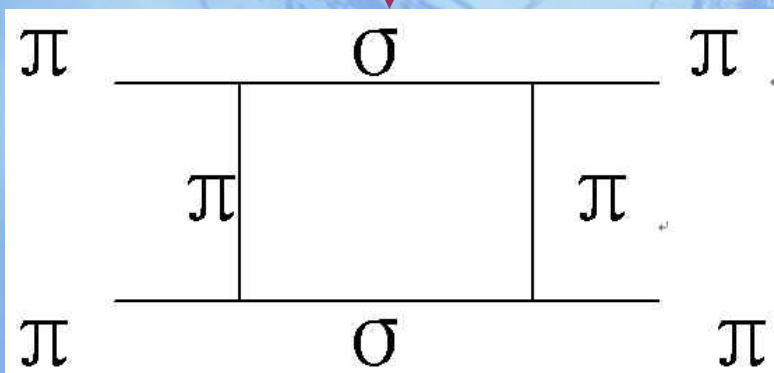


$$\pi\pi \rightarrow \rho\rho \rightarrow \pi\pi$$

$$\pi\pi \rightarrow \omega\omega \rightarrow \pi\pi$$



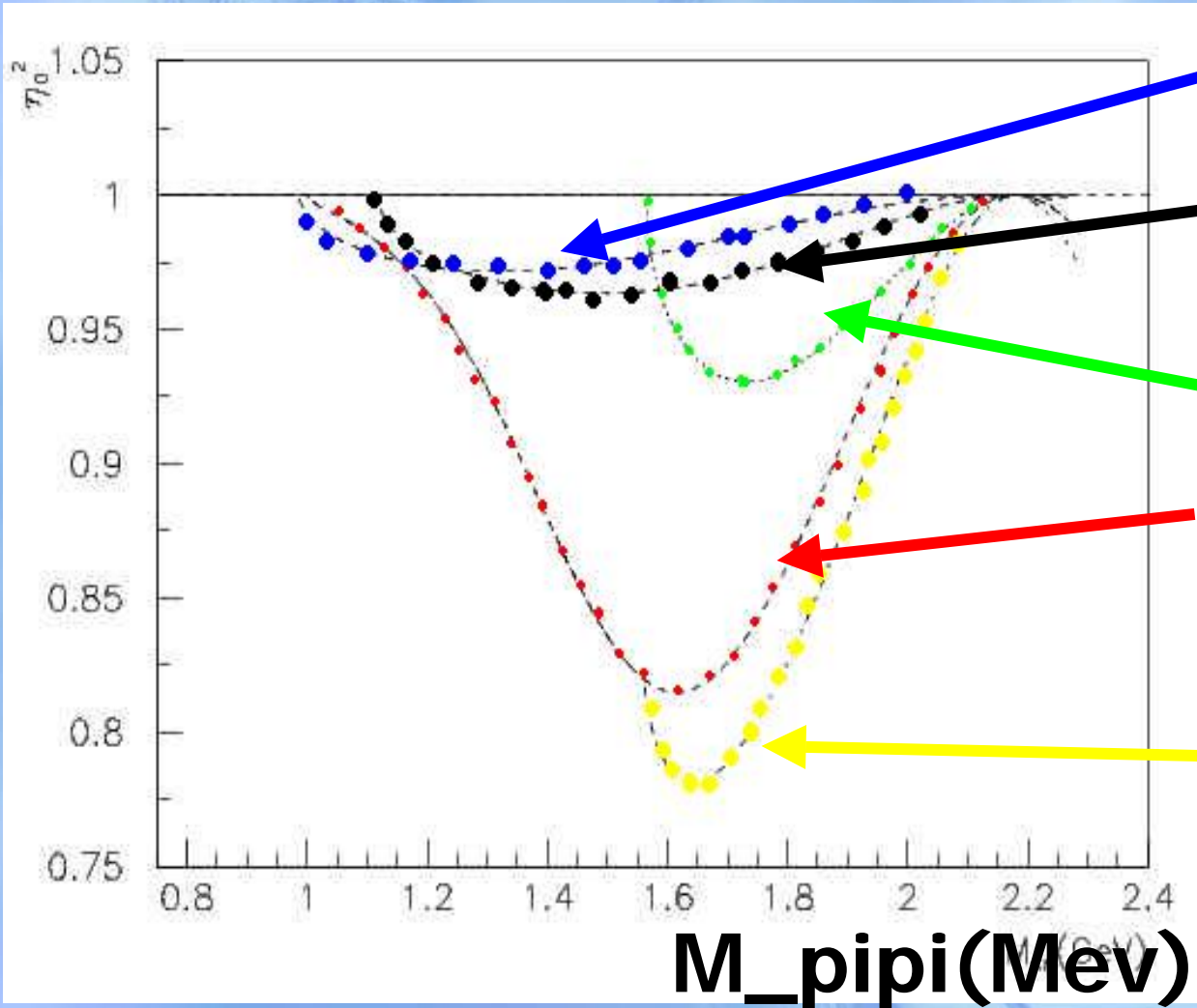
$$K_{ab}\rho_{bb}K_{ba}$$



$$\pi\pi \rightarrow \sigma\sigma \rightarrow \pi\pi$$

$$\pi\pi \rightarrow KK \rightarrow \pi\pi$$


$I = 0 \pi \pi$ S-wave inelastic parameter (without s-channel resonances)



$\pi\pi \rightarrow KK \rightarrow \pi\pi$

$\pi\pi \rightarrow \sigma\sigma \rightarrow \pi\pi$

$\pi\pi \rightarrow \omega\omega \rightarrow \pi\pi$

 $\pi\pi \rightarrow \rho\rho \rightarrow \pi\pi$

three channel $\left\{ \begin{array}{l} \pi\pi \rightarrow \omega\omega \\ \pi\pi \rightarrow \rho\rho \end{array} \right.$

4. Summary and Outlook

- 1) Three basic features of $I=2$ $\pi\pi$ scattering phase shifts and inelasticities can be well reproduced by the t-channel (ρ, f_2) meson exchange and the $\pi\pi - \rho\rho$ coupled-channel effect.
- 2) A correct description of $I=2$ $\pi\pi$ scattering has significant impact on the extraction of $I=0$ scattering amplitudes from $\pi^+\pi^- \rightarrow \pi^+\pi^-$ and $\pi^+\pi^- \rightarrow \pi^0\pi^0$ data, especially for energies above 1.2 GeV.

3) The t-channel (ρ, f_2) meson exchange and the $\pi\pi-\rho\rho$ coupled-channel effect should also be important in $I=0$ $\pi\pi$ scattering.

4) Further study of other channels like $K\bar{K} \rightarrow K\bar{K}$ will make more clear of $\pi\pi$ interaction.



谢谢!

Thanks!