Exotica*

R. L. Jaffe†

Center for Theoretical Physics
Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology
Cambridge, MA 02139, USA

The first evidence for Quantum Chromodynamics (QCD), the theory of the strong interactions, came from the systematics of baryon and meson spectroscopy. An important early observation was the apparent absence of exotics, baryons requiring more than three quarks or mesons requiring more than $q\bar{q}$. Years later, QCD is well established, hadron spectroscopy has been relatively inactive, but the absence of exotics remains poorly understood. The recent observation of narrow, prominent exotic baryons has stirred up new interest in hadron spectroscopy. At present the experimental situation is confused; so is theory. The recent discoveries are striking. So too is the complete *absence* of exotic mesons, and, except for the recent discoveries, of exotic baryons as well. Whether or not the new states are confirmed, the way we look at complicated states of confined quarks and gluons has changed. Perhaps the most lasting result, and the one emphasized in these notes, is a new appreciation for the role of diquark correlations in QCD.

I. INTRODUCTION

There is no doubt that Quantum Chromodynamics is the correct theory of the strong interactions. It has been tested quantitatively in hundreds of experiments at high momentum transfer, where asymptotic freedom justifies the use of perturbation theory[1]. Hadrons are clearly bound states of quarks held together by gluon mediated, non-Abelian gauge interactions. After many years, however, a quantitative and predictive theory of confined states of quarks and gluons still eludes us. This is particularly true for the light, u, d, and s, quarks, where non-relativistic approximations fail. Hadron spectroscopy is interesting in its own right, but also because it is a laboratory in which to explore the dynamics of an unbroken gauge interaction with a non-trivial ground state, a model for other unsolved problems in high energy physics. Also, the spectrum of hadrons shows many qualitative regularities that do not follow simply from the symmetries of QCD, and invite a deeper understanding. The one which figures in the present discussion is that all known hadrons can be described as bound states of qqq or $q\bar{q}$. Because QCD conserves the number of quarks of each flavor (u, d, s,...), hadrons can be labeled by their minimum, or *valence*, quark content. Thus, for example, the conserved quantum numbers of the Λ hyperon require that include uds. QCD can augment this with flavor neutral pairs ($u\bar{u}$, $d\bar{d}$, *etc.*) or gluons, but only three quarks are required to account for the conserved quantum numbers of the Λ. Likewise, the K^+ meson must include at least $u\bar{s}$. Hadrons whose quantum numbers require a valence quark content beyond qqq or $q\bar{q}$ are termed "exotics". The classic example is a baryon with positive strangenss, a Z^* as it is known, with valence quark content $uudd\bar{s}$.[1]

The absence of exotics is one of the most obvious features of QCD. In the early years experimenters searched hard for baryons that cannot be made of three quarks or mesons that cannot be made of $q\bar{q}$.[2] Exotic mesons seemed entirely absent. Controversial signals for exotic baryons known as Z^*s came, and usually went, never rising to a level of certainty sufficient for the Particle Data Group's tables[2, 3]. In its 1988 review the Particle Data Group officially

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[1] Accidentally, strangeness was assigned to hadrons in the 1950's in a way such that the s-quark ended up with negative strangeness. A similar choice by B. Franklin assigned the electron negative electric charge.

[2] A small number of mesons whose spin, parity and charge conjugation are forbidden in the *non-relativistic* quark model are also often termed "exotics". These can also be $\bar{q}qg$(gluon) states or even relativistic $\bar{q}q$ bound states. They will not figure in these notes. Also, I will not discuss heavy quark mesons with unexpected masses.

put the subject to sleep[5]:

> "The general prejudice against baryons not made of three quarks and the lack of any experimental activity in this area make it likely that it will be another 15 years before the issue is decided."

After that, the subject of exotic baryons did not receive much attention except from a small band of theorists motivated by the predictions of chiral soliton models[6, 7, 8, 9, 10, 11, 12]. Then, in January of 2003 evidence was reported of a very narrow baryon with strangeness one and charge one, of mass $\approx$ 1540 MeV, now dubbed the Θ^+, with minimum quark content $uudd\bar{s}$[13]. The first experiment was followed by many other sightings[14] and by evidence for other exotics: a strangeness minus two, charge minus two particle now officially named the Φ^{--} by the PDG, with minimum quark content $ddss\bar{u}$[15] at 1860 MeV , and an as-yet nameless charm exotic ($uudd\bar{c}$)[16] at 3099 MeV. Theorists, myself included, descended upon these reports and tried to extract dynamical insight into QCD[17]. Other experimental groups began searches for the Θ^+ and its friends. As time has passed the situation has become more, rather than less, confusing[20]: several experiments have now reported negative results in searches for the Θ^+[22]; no one has confirmed either the $\Phi^{--}(1860)$ or the $uudd\bar{c}(3099)$; and theorists have yet to find a compelling (to me at least) explanation for the low mass or narrow width of the Θ^+.

The existence of the Θ^+ is a question for experimenters. Theorists simply do not know enough about QCD to predict without doubt whether a light, narrow exotic baryon exists. Whether or not the Θ^+ survives, it is clear that exotics are very rare in QCD. Perhaps they are entirely absent. This remarkable feature of QCD is often forgotten when exotic candidates are discussed. The existence of a handful of exotics has to be understood in a framework that also explains their overall rarity. Along the same line, the *aufbau* principle of QCD differs dramatically from that of atoms and nuclei: to make more atoms add electrons, to make more nuclei, add neutrons and protons. However in QCD the spectrum — with the possible exception of a few states like the Θ^+ — seems to stop at qqq and $q\bar{q}$.

Thinking about this problem in light of early work on multiquark correlations in QCD [23], Frank Wilczek and I[25][3] began to re-examine the role of diquark correlations in QCD. Diquarks are not new; they have been championed by a small group of QCD theorists for several decades[28, 29]. We already knew[23, 24] that diquark correlations can naturally explain the general absence of exotics and predict a supernumerary nonet of scalar mesons which seems to exist. We quickly learned that they can rather naturally accommodate exotics like the Θ^+. They also seem to be important in dense quark matter[30], to influence quark distribution[31] and fragmentation functions, and to explain the systematics of non-leptonic weak decays of light quark baryons and mesons[32]. Whether or not the Θ^+ survives, diquarks are here to stay.

In the first part of this paper, after looking briefly at the history of exotics, I assume that the Θ^+ exists, and see how well it fits with other features of light quark spectroscopy. I will take a look at the Θ^+ from several perspectives: general scattering theory, large N_c, chiral soliton models, quark models, and lattice calculations. In general these exercises raise more questions than they answer. In brief: A light, narrow exotic is inconvenient but not impossible for QCD spectroscopy.

Thinking about the Θ^+ in terms of quarks leads one naturally to study quark correlations, and especially diquarks. So the later sections of the paper are devoted to diquarks. I define them carefully and review some of the evidence that they are important in QCD. Next I describe how diquark correlations in hadrons can explain qualitatively most of the puzzles of exotic hadron spectroscopy: first, why exotics are so rare in QCD; next, why there is an extra nonet of scalar mesons; third, why an exotic baryon antidecuplet containing the Θ^+ would be the only prominent baryon exotic; fourth, why non-strange systems of 6, 9, 12, . . . quarks form nuclei not single hadrons; and finally why the H dibaryon ($uuddss$) might not be as bound as simple estimates suggest[33]. "Qualitatively" is an important modifier, however: like all quark model ideas, this one lacks a quantitative foundation. Perhaps lattice QCD studies in the not-too-distant future can confirm some aspects of the analysis, but the need for a systematic and predictive phenomenological framework for QCD spectroscopy has never been greater.

This paper is not a review, but instead an ideosyncratic overview and introduction aimed at readers who may not already be familiar with the subject.[4] I focus principally on quark-based dynamics and on diquark correlations which, I believe, are strongly motivated by other features of hadron phenomenology. Hundreds of theorists have written on the subject of the Θ and other exotic baryons from different perspectives. Presentations of other points of view can be found in Refs. [18, 19]. The paper is not very technical. A few subsections sections are more detailed, and some contain previously unpublished material (see especially, §IV.B and D). Since there is no "Theory of the Spectrum" in QCD, detailed calculations do not seem warranted. I will concentrate on the qualitative features of models that can provide a guide to the study of exotics.

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[3] Closely related ideas have been explored by Nussinov[26] and by Karliner and Lipkin[27].
[4] They has been tested on string theorists, for example.

Here is an outline:

II. HISTORY

A. The absence of exotics

Most talks on the Θ^+ begin by showing the experimental evidence reported in the past two years. I would like to strike a different note by beginning with a brief look at evidence of the absence of exotics. Spectroscopy was at the cutting edge of high energy physics in the '60's and '70's. A great deal of effort and sophisticated analysis was brought to bear on the study of the hadron spectrum, and the conclusions remain important.

The fact that all known hadrons made of light, u, d, and s quarks can be classified in $SU(3)_f$ representations that can be formed from $\bar{q}q$ or qqq played an essential role in the discovery of quarks. Gell-Mann mentions it prominently in his first paper on quarks in 1964[34]:

> "Baryons can be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, *etc*, while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, *etc*. It is assuming [*sic*] that the lowest baryon configuration (qqq) gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\bar{q})$ similarly gives just **1** and **8**."
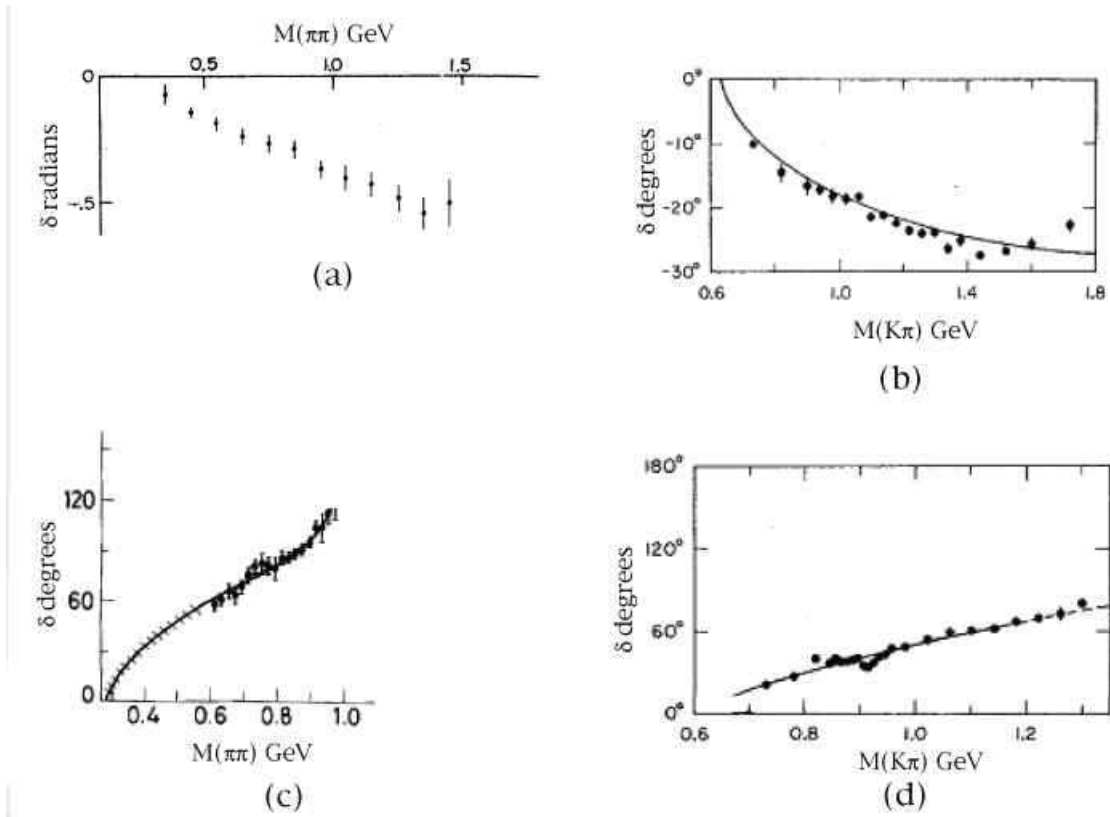
FIG. 1: Meson-meson s-wave scattering phase shifts as they were known in the 1970's. (a) $\pi\pi$ in the exotic $I = 2$ $(\pi^+\pi^+)$ channel[35]; (b) πK in the exotic $I = 3/2$ $(\pi^+ K^+)$ channel[36]; (c) $\pi\pi$ in the non-exotic $I = 0$ channel[35]; (d) πK in the non-exotic $I = 1/2$ channel[36]. The exotic channels show weak repulsion (slowly falling phases). The non-exotic channels show strong attraction (steadily rising phases).

In the decades that followed, many excellent experimental groups studied meson-baryon and meson-meson scattering, and extracted the masses and widths of meson and baryon resonances. Resonances were discovered in nearly all non-exotic meson and baryon channels, but no prominent exotics were found.

Figure 1 shows the $\pi\pi$ and $K\pi$ phase shifts in the s-wave for the exotic ($\pi^+\pi^+$ and $\pi^+ K^+$) and non-exotic channels as they were known in the late 1970's[35, 36]. The non-exotic channels show positive, slowly increasing phases which we now associate with the scalar mesons, $f_0(600)$ and $\kappa(800)$. The exotic channels show small, negative, slowly falling phases characteristic of a weak repulsive interaction. Similar behavior was observed in all exotic channels that were studied.

The situation among the baryons has always been more complicated. More is known because meson-baryon scattering is easier to study than meson-meson. During the 1970's and '80's there were candidates for broad and/or inelastic exotic KN resonances — they were the subjects of the 1982 and 1988 PDG reviews quoted above[2, 5]. The history and status of these states has recently been reviewed by Jennings and Maltman[4]. Figure 2 shows the Argand diagrams for elastic scattering in one non-exotic $\overline{K}N$ channel, and in the exotic KN channels (K^+p with isospin one, and K^0p/K^+n with isospin zero), corresponding to quark content $qqqq\bar{s}$, where q denotes a light, u or d, quark [37]. The non-exotic channel shown for comparison is the $\overline{K}N$ d-wave with $J^\Pi = 3/2^-$, a channel with two well established resonances. In Fig. 2 the Argand amplitude,

$$f_\ell(k) = \frac{i}{2} - \frac{i}{2}\eta_\ell(k)e^{2i\delta_\ell(k)} \tag{II.1}$$

is plotted parametrically as a function of kaon energy in the nucleon rest frame. If the scattering is elastic, then $\eta_\ell = 1$ and $f_\ell(k)$ must lie on the unitarity circle, $|f_\ell(k) - \frac{i}{2}| = \frac{1}{2}$. When $\eta_\ell = 1$ the elastic cross section in each partial wave is given simply by

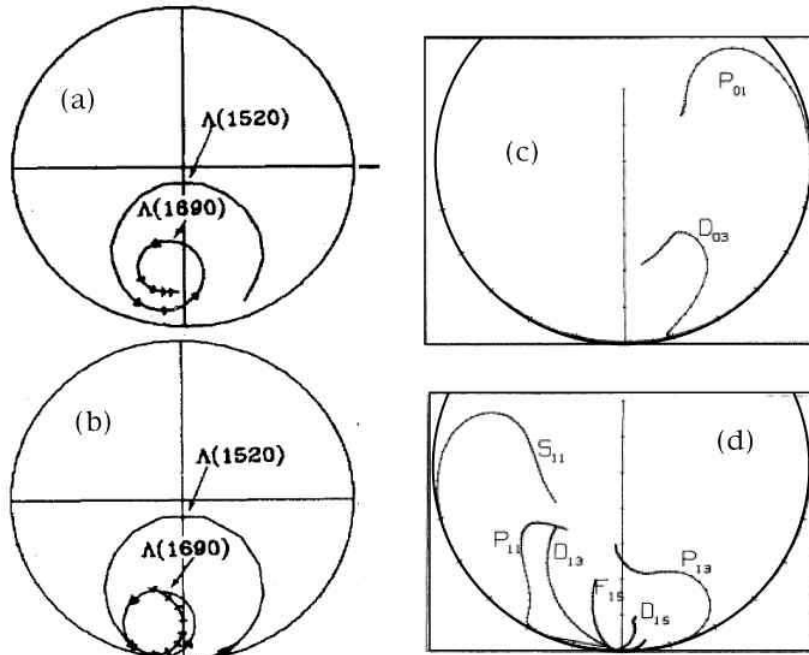$$\sigma_\ell(k) = \frac{4\pi(2\ell+1)}{k^2}\sin^2\delta_\ell(k) \tag{II.2}$$

FIG. 2: Argand diagrams for (a), (b) $I = 0$ $\overline{K}N$ scattering from two different experiments in the non-exotic D_{03} wave, and KN scattering for (c) $I = 0$, and (d) $I = 1$ in various partial waves. The partial wave notation is $L_{I,2J}$. (a) and (b) show evidence for prominent resonances, and the data from two different experiments[38, 39, 40] agree with one another. In the exotic channels ((c) and (d)) there is more and better data on $I = 1$ because the K^+p channel is more easily studied than K^+n. The quality of the figures[37] is not sufficient to see clearly the tick marks on the curves at intervals of 50 MeV in beam energy. The non-exotic resonances are relatively narrow. All the exotic effects change slowly with energy. The $[KN]^{I=0}$ s-wave (S_{01}) is not shown in (c) but is weakly repulsive. The broad counterclockwise motions in the P_{01}, D_{03}, P_{13}, and D_{15} partial waves were interpreted as evidence for broad, inelastic resonances near 1830, 1790, 1810, and 2070 MeV respectively. See Ref. [4] for further discussion. At an elastic resonance, like the Θ^+, the argand amplitude should execute a complete loop on the outer (unitarity) circle.

An elastic resonance gives rise to a rapid increase of $\delta_\ell(k)$ by $\sim 2\pi$ and appears as a counterclockwise circle of radius $1/2$ in $f_\ell(k)$. If there is no background phase, ie if $\delta_\ell \approx 0$ just before the onset of the resonance, then at the peak of the resonance $\delta_\ell = \pi/2$ and $\sigma_\ell = 4\pi(2\ell + 1)/k^2$. A significant background phase can alter the shape of a narrow resonance[41], but because of unitarity it cannot reduce the magnitude of the effect on the cross section. The only way to miss an elastic resonance is if its width is significantly smaller than experimental resolution. At higher energies more channels open, scattering becomes inelastic, and resonances are associated with less pronounced counterclockwise arcs. The non-exotic $\overline{K}N$ channels show many clear resonances at low energy. The exotic channels show none. When the PDG wrote its 1988 review, the closest thing to an exotic was the broad, inelastic counterclockwise motion in the P_{01} partial wave shown in Figure 2(c).

The zeroth order summary prior to January 2003 was simple: no exotic mesons or baryons. In fact the only striking anomaly in low energy scattering was the existence of a supernumerary (ie not expected in the quark model) nonet of scalar, ($J^\Pi = 0^+$) mesons with masses below 1 GeV: the $f_0(600)$, $\kappa(800)$, $f_0(980)$, and $a_0(980)$, about which more later.

When the Θ^+ was first reported, several groups re-examined the old KN scattering data and interpreted the absence of any structure near 1540 MeV as an upper limit on the width of the Θ^+[26, 42, 43, 44]. The limits range from 0.8 MeV through "a few" MeV. It is important to remember that these are not sightings of a narrow Θ^+, rather they are reports of negative results expressed as an upper limit on the width of the Θ^+.

B.   Exotic sightings since January 2003

Space and time do not permit me to present and review all the reports of exotics since January of 2003. Instead I have tried to summarize the situation in two tables. The first, Table I, is derived from one presented by T. Nakano[45],
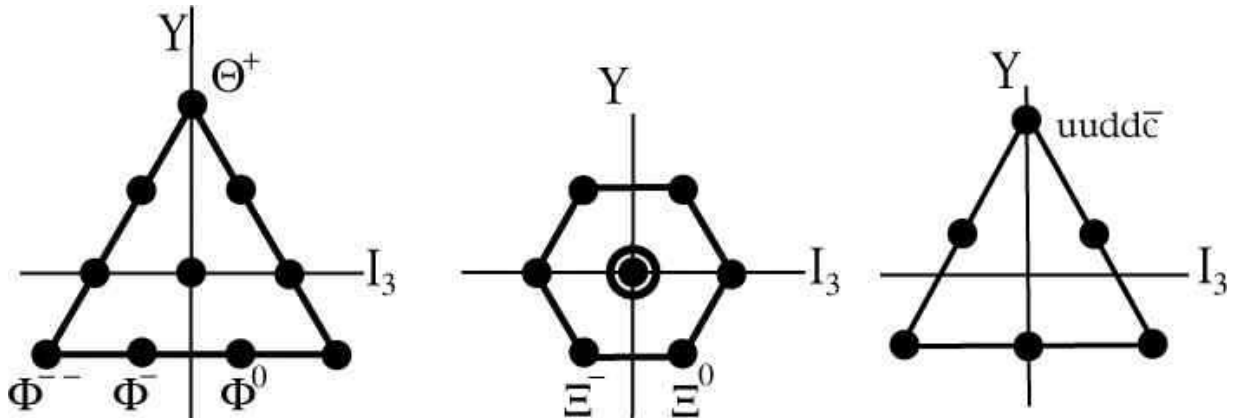
FIG. 3: Simplest possible $SU(3)_f$ representations for the exotic candidate exotic baryons: (a) the $\overline{\mathbf{10}}$; (b) the $\mathbf{8}$; and (c) the $\overline{\mathbf{6}}$.

reporting sightings of the Θ^+. The second, Table II, is a summary of the properties of the reported states. The baryons in Table II can be classified in the $\overline{\mathbf{10}}$ or $\mathbf{8}$ and $\overline{\mathbf{6}}$ representations of $SU(3)_f$ as shown in Fig. 3.[5]

Although they could be in higher representations, the $\overline{\mathbf{10}}$ is the simplest that can accomodate both the Θ^+ and the Φ^{--}. I have not attempted to summarize the searches that fail to see the Θ^+ or the other new exotics. These require a careful discussion, which can be found, for example in Ref. [22].

TABLE I: Summary of sightings of the Θ^+[45]

Experiment	Reaction	Mass	Width	σ_{std}
LEPS	$\gamma C \to K^+ K^-\ X$	$1540\ \pm\ 10$	< 25	4.6
Diana	$K^+ Xe \to K^0 p X$	$1539\ \pm\ 2$	< 9	4.4
CLAS	$\gamma d \to K^+ K^- p(n)$	$1542\ \pm\ 5$	< 21	5.2
SAPHIR	$\gamma p \to K^+ K^0(n)$	$1540\ \pm\ 6$	< 25	4.8
ITEP	$vA \to K^0 p X$	$1533\ \pm 5$	< 20	6.7
CLAS	$\gamma p \to \pi^+ K^- K^+(n)$	$1555\ \pm\ 10$	< 26	7.8
HERMES	$e^+ d \to K^0 p X$	$1528\ \pm\ 3$	$13\ \pm\ 9$	~ 5
ZEUS	$e^+ p \to e' K^0 p X$	$1522\ \pm\ 3$	$8\ \pm\ 4$	~ 5
COSY	$pp \to K^0 p \sum^+$	$1530\ \pm\ 5$	< 18	$4-6$

TABLE II: Properties of reported exotic baryons and related states.

Name	Mass (MeV)	Width (MeV)	Spin$^{\text{parity}}$	Isospin	Decays	Minimal $SU(3)_f$ Irrep
Θ^+	1520—1540	$< 1^{[1]}$, $< 6-10^{[2]}$	$1/2^{?[3]}$	0	$K^+ n$, $K_S p$	$\overline{\mathbf{10}}$
Φ^{--}	1860	< 18	?	$\geq 3/2$	$\Xi^-(1320)\pi^-$	$\overline{\mathbf{10}}$
$\Phi^0/\Xi^{0[4]}$	1860	< 18	?	$\geq 1/2$	$\Xi^-(1320)\pi^+$	$\overline{\mathbf{10}}$ if Φ, $\mathbf{8}$ if Ξ
$\Phi^-/\Xi^{-[4]}$	1855	< 18	?	$\geq 1/2^{[5]}$	$\Xi^{*0}(1530)\pi^-$	$\overline{\mathbf{10}}$ if Φ, $\mathbf{8}$ if Ξ
$\{uudd\bar{c}\}$	3099	< 12	?	≥ 0	pD^{*-} & $\bar{p}D^{*+}$	$\overline{\mathbf{6}}$

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[5] $SU(3)$-flavor representations are denoted by their dimension in boldface. Irreps of other symmetry groups like $SU(3)$-color, or the $SU(6)$ symmetries built from flavor×spin or color×spin are distinguished by appropriate subscripts. Occasionally a subscript "f" is added to an $SU(3)$-flavor representation for clarity.

[1] From analysis of KN scattering;

[2] from direct detection of Θ;

[3] Weak evidence for $J = 1/2$, no information on parity;

[4] Φ if $I = 3/2$, Ξ if $I = 1/2$;

[5] $I = 1/2$ favored if decay to $\Xi^{*0}(1530)$ is correct.

There are several puzzling aspects of the data: the variation of the Θ^+ mass and the claims of HERMES and ZEUS to have measured a non-vanishing width, for example. The interested reader should consult the talks by Nakano[45] and Dzierba[22] and other presentations at QNP2004. Here are some questions and observations about the data...

- Zeus sees the Θ^+ in the current fragmentation region in deep inelastic positron scattering. Particles produced in this region are fragments of the struck quark. Standard factorization arguments imply that the fragmentation function $D_{q/\Theta}(z)$ cannot be zero if the Θ^+ is seen in this experiment. Among those that *have not* seen the Θ^+ are several collider experiments in which particle production also occurs through quark fragmentation. Especially relevant are the e^+e^- colliders (*eg.* Aleph, BABAR) where all hadrons are fragments of high momentum quarks [46]. It would be interesting to know if these negative results can be reconciled with the sighting at Zeus. Initial reports suggest that they are not compatible[47].

- Many unusual hadrons are formed rather abundantly by quark fragmentation in e^+e^- annihilation. Examples taken from the Durham data base[48] include the Ω^- and $f_0(980)$, which is probably dominantly a $q\bar{q}q\bar{q}$ state. For example $\sigma(f_0(980))/\sigma(\rho) \approx 0.1 - 0.2$ over a range of z. It would be very helpful if the collider experiments would phrase their failure to see the Θ^+ as a limit on the $q \to \Theta$ fragmentation function so we could compare it with the fragmentation functions of other hadrons[49].

- The report from COSY/TOF of a sighting of the Θ^+ in the reaction $pp \to \Sigma^+\Theta^+$ is particularly interesting. The Σ^+ is known to couple to K^0p and the Θ^+ has been reported in K_Sp invariant mass plots. Therefore it is possible to estimate the cross section for the reaction $pp \to \Sigma^+\Theta^+$ mediated by K^0 exchange. Theorists have examined the COSY/TOF data and concluded that the cross section is roughly consistent with expectations[50].

There are many predominantly experimental issues that I have not covered here: proposals to measure the parity of the Θ^+, limits on the production of other exotics like a Θ^{++}, and reports of other bumps with the quantum numbers of the Θ^+ at higher mass, to name only a few.

III.   THEORETICAL PERSPECTIVES

A.   Insight from scattering theory

The small width of the Θ^+ poses a challenge for any theoretical interpretation. It is clear that the width is small, but small compared to what? The Θ^+ is unique among hadrons in that its valence quark configuration, $uudd\bar{s}$, already contains all the quarks needed for it to decay into KN. Non-exotic hadrons like the $\rho(770)$ or $\Lambda(1520)$ in their valence quark configuration can only couple to their decay channels ($\pi\pi$ for the former, $\overline{K}N$ for the latter) by creating quark pairs (see Figure 4) The suppression of quark pair creation, known as the Okubo, Zweig, Iizuka (OZI) Rule[51], is often invoked as an explanation for the relative narrowness of hadronic resonances. Some other, as yet unknown mechanism would have to be responsible for the narrowness of the Θ^+[52].

General principles of scattering theory allow one to get at least a qualitative answer to the question: "How unusual is the width of the Θ^+?"[54]. The Θ^+ appears as an elastic resonance in KN scattering at low energy. The center of mass momentum is low enough, $k \approx 270$ MeV, that the motion is arguably non-relativistic, $\beta_p^2 \approx 0.08$ and $\beta_K^2 \approx 0.30$ and the Schrödinger equation can be used to examine the scattering. There is only one open channel (with a definite isospin), simplifying the problem even further.

There are two ways to make a resonance in low energy scattering, either (a) the resonance is generated by the forces between the scattering particles, or (b) it exists in another channel, which is closed (or confined), and couples to the scattering channel by some interaction. The former are the standard resonances of low energy potential scattering, described in any book on quantum mechanics. The latter are "CDD poles" that have to be added by hand into the S-matrix[56, 57]. Potential scattering resonances (case (a)) are generated by the interplay between attraction due to interparticle forces and repulsion, usually due to the angular momentum barrier. These resonances subside into the continuum as the interaction is turned off. The classic example of a CDD pole (case (b)) is the π^--pole in $e^-\overline{\nu}_e$ scattering. It is not generated by the forces between the electron and antineutrino. Rather than disappearing, it decouples, *ie* its width goes to zero, as the interaction is turned off. Another important example of a CDD pole is a bound state in a closed or confined channel that couples to a scattering channel by an interaction. In the case of a qqq
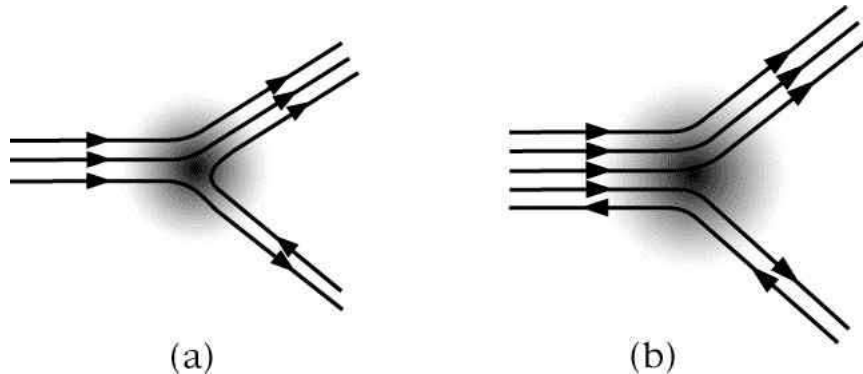
FIG. 4: Quark-line diagrams, (a) for decay of a non-exotic, qqq, resonance into a meson and baryon, requires quark pair creation; (b) for decay of an exotic $qqqq\bar{q}$ resonance is not suppressed by quark pair creation.
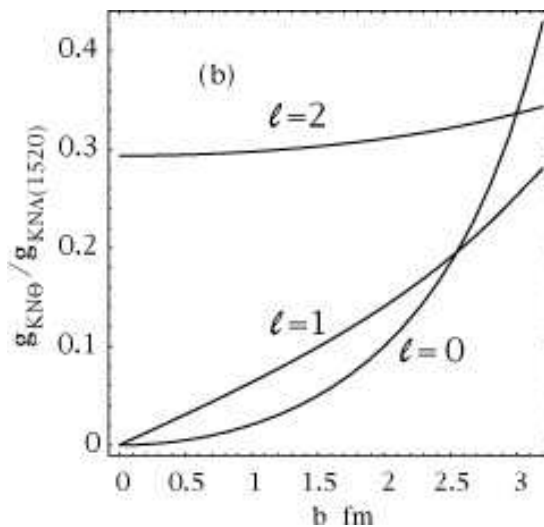

FIG. 5: Ratio of the channel couplings for $KN\Theta$ and $\overline{K}N\Lambda(1520)$, for $\ell = 0, 1, 2$, assuming that the width of the Θ^+ is 1 MeV. The ratio scales like $\sqrt{\Gamma_\Theta}$.

baryon coupling to the meson($\bar{q}q$)-baryons(qqq) continuum, quark pair creation is the interaction. Thus we should expect typical baryon resonances to appear as CDD poles in meson-baryon scattering, not generated by the well known phenomenological meson-baryon potentials. The Θ^+ is unusual: Because its valence quark content, $uudd\bar{s}$, is the same as the valence quark content of KN, the possibility that it arises from the KN potential cannot be excluded *a priori* and has to be analyzed.

For potential scattering we assume an attractive interaction with range, b, and depth, V_0.[6] Keeping the resonance energy fixed at M_Θ=1540 MeV, we obtain a relation between the range and the width, $\Gamma(\Theta)$, for each value of the orbital angular momentum ℓ. $\ell = 0$ can be excluded immediately: there are no s-wave resonances in an attractive potential. The p-wave is excluded because the range would have to be unnaturally short, $b \lesssim 0.05$ fm to obtain $\Gamma(\Theta) \lesssim 5$ MeV. Even the d-wave is marginal. It seems that the Θ^+, narrow as it is, cannot originate in the KN forces, unless it has angular momentum much larger than generally supposed or those forces are bizarre.

It is always possible to introduce a CDD pole at 1540 MeV and couple it weakly enough to give as narrow a width for the Θ^+ as required. However, Nature has given us a "standard" quark model resonance, the $\Lambda(1520)$, with valence quark content uds at nearly the same mass. This enables us to compare the underlying $KN\Theta$ coupling, $g_{KN\Theta}$ to the underlying $\overline{K}N\Lambda(1520)$ coupling, $g_{\overline{K}N\Lambda(1520)}$. The $\Lambda(1520)$ has $J^\Pi = 3/2^-$ and therefore appears in the $\overline{K}N$

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[6] Potentials with repulsion at long distances and attraction at short distances would yield different results, but seem unnatural.

d-wave. Its partial width into $\overline{K}N$ is ≈ 7 MeV. A coupled channel analysis described in Ref. [55] is summarized in Fig. 5, where the ratio of the $g_{KN\Theta}/g_{\overline{K}N\Lambda(1520)}$ is plotted as a function of the range of the interaction that couples the state to the $KN/\overline{K}N$ channel for $\ell = 0$, 1, and 2. Values of $g_{KN\Theta}/g_{\overline{K}N\Lambda(1520)}$ as small as unity would already be surprising. After all, the $\overline{K}N$ coupling of the $\Lambda(1520)$ is suppressed by the OZI rule and the Θ^+ coupling to KN is not. For $\ell = 0$ this ratio would have to be ≈ 0.02 to obtain $\Gamma(\Theta) \approx 1$ MeV. For $\ell = 1$ a suppression of ≈ 0.06 is necessary. Even for $\ell \geq 2$ the KN coupling of the Θ^+ would still have to be less than the the $\overline{K}N$ coupling of the $\Lambda(1520)$ by a factor of order the square root of the ratio of their widths to KN, ie by ≈ 0.3.

The lesson of this exercise is qualitative: If the Θ^+ appears in a low partial wave ($\ell = 0$ or 1), schemes which hope to produce it from KN forces seem doomed from the start; schemes which introduce confined channels (quark models are an example, where reconfiguration of the quark substructure of the Θ^+ could be required for it to decay) are challenged to find a natural physical mechanism that suppresses the Θ^+ decay more effectively than the OZI rule suppresses the decay of the $\Lambda(1520)$.

B. Large N_c and chiral soliton models

1. Large N_c

The number of colors is the only conceivable parameter in light quark QCD, so it is natural to consider exotic baryon dynamics as an expansion in $1/N_c$. It is known from the work of 't Hooft, Witten, and many others, that as $N_c \to \infty$ QCD reduces to a theory of zero width $\bar{q}q$ mesons with masses $\sim \Lambda_{QCD}$[58] and heavy baryons with masses $\sim N_c\Lambda_{QCD}$ in which quarks move in a mean (Hartree) field[59]. This is far from a complete description even at the heuristic level: Almost nothing is known about the spectrum of $\bar{q}q$ mesons from large N_c except for the pseudoscalar Goldstone bosons required by chiral symmetry. The dynamics of baryons is accessible only through a loose association with the chiral soliton model (CSM). It is important to remember that the CSM has not be derived from large N_c QCD. Its appeal is based on its proper implementation of chiral symmetry and anomalies, and on the *resemblance* of collectively quantized solitons to the lightest positive parity baryons. The connection certainly fails for the simplest case of one flavor, where baryons exist and are described by a mean field theory (they large N_c analogues of the $\Delta^{++} \equiv uuu$, for example), but there are no Goldstone bosons, no topology, and no chiral solitons. On the other hand qualitative connections between the lowest lying states of a collectively quantized chiral soliton and the simplest baryons made of N_c quarks have been established for arbitrary N_f ($\neq 0$) and large N_c[60].

Dashen, Jenkins, and Manohar[61] made the application of large N_c ideas to baryons more precise. They eschew dynamics and focus instead on a new $SU(2N_f)_c$ symmetry that emerges as $N_c \to \infty$. The subscript "c" means contracted and refers to a modification of the usual Lie algebra because certain commutators scale to zero as $N_c \to \infty$. Baryons can be organized into (infinite dimensional) irreducible representations of this symmetry. Jenkins and Manohar show that these irreps can be put into correspondence with the spectrum of the non-relativistic quark model, and at least for the ground-state multiplet, with the chiral soliton model. Known relationships among baryon masses and couplings can be classified by the order in $1/N_c$ at which they are broken. New relations can be derived and are generally quite successful[62]. Like any symmetry, however, the utility of large N_c á la Jenkins and Manohar is limited by ignorance of reduced matrix elements.

Jenkins and Manohar have extended their work to exotic baryons[63]. They cannot predict the mass of the lightest exotic — it depends on the $SU(2N_f)_c$ invariant dynamics — but they can enumerate multiplets that have candidates for the Θ^+. They select a positive parity representation which requires a space wavefunction of mixed symmetry for the $N_c + 1$ quarks. In the language of constituent quarks, the mixed symmetry space wavefunction corresponds to a state in which N_c quarks are in the Hartree ground state and one is excited. From a QCD-inspired quark model viewpoint (see below) this seems like an odd choice for the ground state multiplet. To support their choice, Jenkins and Manohar point out that it is natural in models where quarks interact by Goldstone boson exchange[65]. This representation of $SU(2N_f)_c$ contains an infinite tower of $SU(3)$-flavor irreps of definite spin and positive parity. For $N_c = 3$ the lightest $SU(3)_f$ multiplet in this representation is the $\overline{\mathbf{10}}$, where the Θ^+ is expected to lie (see Table II). It occurs with $J^\Pi = \frac{1}{2}$ or $\frac{3}{2}$. Next is a $\mathbf{27}$ with many exotic candidates[7].

There are also two $SU(2N_f)_c$ irreps in which all the quarks are in the Hartree ground state — also a natural

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[7] Actually the situation is a little more complicated: there are two irreps of $SU(2N_f)_c$ allowed with the mixed symmetry space wavefunction. One has a tower of states, $\overline{\mathbf{10}}^{\frac{1}{2}^+}$, $\mathbf{27}^{\frac{1}{2}^+}$, $\mathbf{27}^{\frac{3}{2}^+}$, ... The other has a tower, $\overline{\mathbf{10}}^{\frac{3}{2}^+}$, $\mathbf{27}^{\frac{1}{2}^+}$, $\mathbf{27}^{\frac{3}{2}^+}$, $\mathbf{27}^{\frac{5}{2}^+}$, ... The two towers need not be degenerate. Jenkins and Manohar only discuss the first[64].

candidates for the $q^{N_c}\bar{q}$ ground state. All these states have *negative* parity. These representations have been studied recently by Pirjol and Schat[66]. They consider both the case in which all quarks in the exotic are light ($qqqq\bar{q}$) and the case where the antiquark is heavy, *ie* c or b ($qqqq\bar{Q}$)[67]. For light quarks the towers begins with non-exotic $SU(3)_f \mathbf{1} \oplus \mathbf{8}$ states, followed by exotic multiplets (including degenerate (as $N_c \to \infty$) $SU(3)_f$ $\overline{\mathbf{10}}^{\frac{1}{2}^-}$ and $\overline{\mathbf{10}}^{\frac{3}{2}^-}$) at higher mass.

This illustrates a general feature of all quark model/QCD treatments of the exotic baryons, which I will revisit in the discussion of quark models (Section III.C): the most natural candidate for the ground state multiplet (all quarks in the lowest Hartree level) has negative parity, corresponding to the KN s-wave.[8] So these approaches share a fundamental difficulty at the outset: If the quark configuration is "natural", it's hard to explain why the Θ^+ should be narrow. To obtain a narrow Θ^+, strong quark forces must make a state of mixed spatial symmetry the lightest. Understanding these strong quark correlations and their consequences then becomes a central issue.

The large N_c methods of Jenkins and Manohar cannot determine whether the Θ^+ is light enough to be narrow and prominent, or even whether it is the lightest $qqqq\bar{q}$ state. More dynamical assumptions are needed for that. However, if the existence of the Θ^+ is fed into their machinery, one can predict its properties and the masses and properties of other states in the $qqqq\bar{q}$ spectrum[63].

2. Chiral Soliton Models

Much of the discussion of exotic baryons over the past decade[6, 7, 8, 9, 12] and, in particular, a remarkable prediction of the mass and width of the Θ^+ by Diakonov, Petrov, and Polyakov [11], has been carried out in the context of chiral soliton models. This is not surprising since CSM's are teeming with exotics. Collective quantization of a classical soliton solution to a chiral field theory of pseudoscalar bosons in $SU(2)_f$ or $SU(3)_f$, consistent with anomaly constraints, yields towers of baryons only the lightest of which *are not exotic*[68, 69]. The simplest example is $SU(2)_f$, where the spectrum of baryons begins with a rotational band of positive parity states with $I = J = 1/2, 3/2, 5/2, \ldots$, with masses,

$$M(I, N_c) = M_0 N_c + \frac{J(J+1)}{I N_c}. \tag{III.1}$$

The parameters M_0 and I are $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ and independent of N_c as $N_c \to \infty$. Other excitations, radial for example, are heavier, separated from the ground state band by $\mathcal{O}(N_c^0)$. The lack of evidence for a $I = 5/2$ baryon resonance led most workers to dismiss the heavier states as artifacts of large N_c.

The generalization of the CSM to three flavors with broken $SU(3)_f$ has always been controversial. Guadagnini's original approach (the "rigid rotor" (RR) approach) was to quantize in the $SU(3)_f$ limit and introduce $SU(3)_f$ violation perturbatively[69]. Alternatively, Callan and Klebanov quantized the $SU(2)_f$ soliton and constructed strange baryons as kaon bound states (the "bound state (BS) approach)[70]. Although different in principle, the two approaches give roughly the same spectrum for the octet and decuplet. When generalized to three flavors the rotational band of the RR approach becomes[6, 7, 8],

$$\mathbf{R}^{J^\Pi} = \mathbf{8}^{\frac{1}{2}^+}, \ \mathbf{10}^{\frac{3}{2}^+}, \ \overline{\mathbf{10}}^{\frac{1}{2}^+}, \ \mathbf{27}^{\frac{1}{2}^+, \frac{3}{2}^+}, \ldots \tag{III.2}$$

where $\mathbf{R}$ is the $SU(3)_f$ representation.

Diakonov, Petrov, and Polyakov[11] took the first exotic multiplet in this tower, $\overline{\mathbf{10}}^{1/2^+}$, seriously. They estimated its mass and width and found that it should be light and narrow[71]. Their work stimulated the experimenters who found the first evidence for the Θ^+[13]. Soon after the initial paper by Diakonov *et al*, Weigel examined the spectrum of exotics in the three flavor CSM more closely[12]. He showed that it is inconsistent in the RR approach to ignore the mixing between the $\overline{\mathbf{10}}$ and radial excitations excited by $\mathcal{O}(N_c^0)$ above the ground state. After the first reports of the Θ^+, other groups looked even more closely at the pedigree of the $\overline{\mathbf{10}}$ in the CSM[73, 74]. As Weigel's analysis had implied, they found the mass splitting between the ground state and the $\overline{\mathbf{10}}$ to be $\mathcal{O}(N_c^0)$. Cohen pointed out that the width of the Θ^+ does not vanish as $N_c \to \infty$ in contrast to non-exotic states like the Δ[73], making it hard to understand the very small value of $\Gamma(\Theta)/\Gamma(\Delta)$ obtained in Ref. [11]. Already in 1998 Weigel had pointed out that the Θ^+ does not exist in the BS approach unless the mass of the kaon is of the order of 1 GeV[12], a result confirmed by Itzhaki *et al*, who go on to show that the force between the collectively quantized two-flavor soliton and the kaon

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[8] Although the d-wave is possible in principle, a d-wave Θ^+ does not occur in the lightest quark model multiplet (see section III.D below).

is repulsive for physical kaon masses. So either the RR and BS approaches are inconsistent with one another for the $\overline{\mathbf{10}}$, or the $\overline{\mathbf{10}}$ cannot be included in the ground state rotational band as in Ref. [11].

Chiral soliton models describe at best a piece of QCD: Their picture of the nucleon and Δ (or $\mathbf{8}^{\frac{1}{2}^+}$ and $\mathbf{10}^{\frac{3}{2}^+}$ in $SU(3)_{\mathrm{f}}$) is internally consistent and predictive. Some progress has been made in the description of baryon resonances[75]. However the incorporation of strangeness is not satisfactory and still controversial[74], and the CSM gives no insight at all into the meson spectrum. As for exotics, the candidate for the Θ^+ is controversial and there is no insight into the striking *absence* of exotic mesons and baryons in general. The prediction of a narrow width for the Θ^+ is very controversial.

C. Quark models

The quark model in its many variations has been by far the most successful tool for the classification and interpretation of light hadrons. It predicts the principal features and many of the subtleties of the spectrum of both mesons and baryons, and it matches naturally onto the partonic description of deep inelastic phenomena. Perhaps it receives less recognition than it ought to because it predates QCD. Largely developed during the 1960's by Dalitz and his students, the quark model was already in place when QCD came on the scene to legitimized it.

The limitations of the quark model are, however, as obvious as its successes. It has never been formulated in a way that is fully consistent with confinement and relativity. Of course quarks can move relativistically, governed by the Dirac equation, in first quantized models like the MIT bag[76], but there is no fully relativistic, second quantized version of the quark model. Furthermore, quark models are not the first term in a systematic expansion. No one knows how to improve on them.

Nevertheless all hadrons can be classified as relatively simple configurations of a few confined quarks, and there is no reason to be expect the Θ^+ to be an exception. So looking for a natural quark description of the Θ^+ is a high priority, and if there were none, it would be most surprising.

1. Generic features of an uncorrelated quark model

Although quark models (non-relativistic, bag, flux tube, ...) differ in their details, the qualitative aspects of their spectra are determined by features that they share in common. These important ingredients can be abstracted from the specific models and used to project expectations for a new sector like $qqqq\bar{q}$. They need not be correct — probably they cannot explain the Θ^+ — but they form the context in which other proposals have to be considered. Certainly they do a good job for mesons, baryons, and even tetraquark (*ie.* $\bar{q}\bar{q}qq$) spectroscopy. Here is a summary of the basic ingredients[77, 81], none of which can be "derived" from QCD, with a few words of explanation:

1. The spectrum can be decomposed into sectors in which the numbers of quarks and antiquarks, n_q and $n_{\bar{q}}$, are good quantum numbers — the OZI rule[51].

2. Hadrons are made by filling quark and antiquark orbitals in a hypothetical mean field — a non-relativistic potential or a confining bag, for example. For baryons this might be the Hartree mean field suggested by large N_c, for mesons its origins are less clear. In dual superconductor versions of confinement, like the MIT bag model, it is the normal region where colored fields are confined.

3. The ground state multiplet is constructed by putting all the quarks and antiquarks in the lowest orbital — the "single mode configuration". A natural assumption, but one which must fail for $qqqq\bar{q}$ if the Θ^+ has positive parity (see below).

4. The total angular momentum of the (relativistic) quark in the lowest orbital is $1/2$. Its parity is *even* (relative to the proton). Again a natural assumption. Remember, spin and orbital angular momentum are not separately conserved. Typically the first excited orbitals have *negative* parity and total angular momentum $1/2$ and $3/2$ because orbital excitations are invariably less costly than radial.

5. The lightest multiplet in any sector, $\bar{q}^{n_{\bar{q}}} q^{n_q}$, can be classified using an $SU(6)$ symmetry built from flavor $SU(3)$ and the $SU(2)$ generated by the unitary transformations of the $j_z = \pm 1/2$ eigenstates connecting the lowest $j = 1/2$ quark mode. This generalizes the old $SU(6)$ of flavor×spin to relativistic quarks. For economy of notation I will refer to this $SU(2)$ symmetry as "spin" and the $SU(6)_{\mathrm{fs}}$ as "flavorspin". However it is not spin and it cannot be used for excited states which include both $j = 3/2$ and $1/2$ orbitals without further work[78].
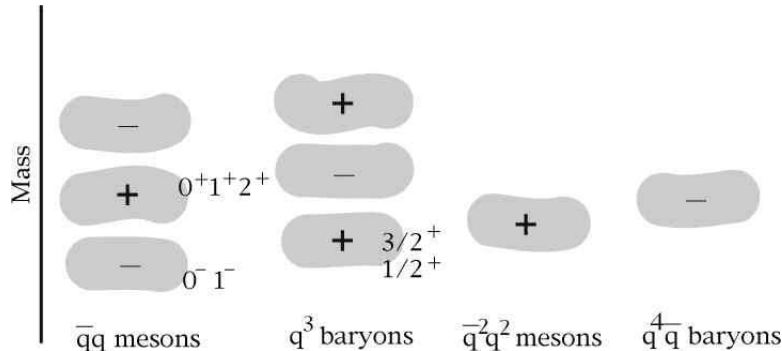
FIG. 6: Pattern of $\bar{q}q$ meson and qqq baryon parities in the quark model and in Nature. The shaded areas represent broad bands of states, the $\pm$ signs label parity. When looked at more closely, the bands overlap, but the general pattern is as shown. Uncorrelated quark model predictions for the parity of the lightest $\bar{q}^2q^2$ "tetraquarks" and $qqqq\bar{q}$ pentaquarks are shown schematically. In other quark models[25, 27, 65] dynamics may alter the pattern.

6. These ideas can be applied to the analysis of local, gauge invariant operators. Suppose $\mathcal{O}$ is a such an operator, built from quark and gluon fields. $\mathcal{O}$ can create states of various spins and parities from the vacuum. Generically, the lower the dimension of $\mathcal{O}$ the lighter the states it creates [77] . If the operator vanishes in the "single mode configuration", then the states it creates are heavier than those created by an operator of the same dimension that does not vanish. Here an example will help: The dimension three operators, $\bar{q}q$, $\bar{q}\gamma_5 q$, $\bar{q}\gamma_\mu q$, $\bar{q}\gamma_\mu\gamma_5 q$, and $\bar{q}\sigma_{\mu\nu}q$, can create mesons with $J^{\Pi C} = 0^{++}$, 0^{-+}, 1^{--}, 1^{+-}, and 1^{++}. When the quark fields are replaced by the lowest mode, the 0^{++}, 1^{++}, and 1^{+-} operators vanish, leaving 0^{-+} and 1^{--}, which are indeed the lightest meson quantum numbers[77].

These are the ingredients in an uncorrelated quark model. Even though the quarks can be relativistic, the classification of states and operators proceeds as if they were non-relativistic.

2. Quark model "states" and scattering

As the number of quarks and antiquarks grows, the number of $\bar{q}^{n_{\bar{q}}}q^{n_q}$ eigenstates proliferates wildly. Even in the ground state multiplet (see 3. above) there are 36 $\bar{q}q$ states, 56 qqq states, 666 $\bar{q}\bar{q}qq$ states and 1260 $qqqq\bar{q}$ states (counting each flavor and spin state separately). The $\bar{q}q$ and qqq states are candidates for the lightest mesons and baryons. Although the $\bar{q}\bar{q}qq$ and $qqqq\bar{q}$ states are stationary states in a potential or bag, they do not in general correspond to stable hadrons or even resonances. Far from it, most, perhaps even all of them fall apart into $\bar{q}q$ mesons and qqq baryons without leaving more than a ripple on the meson-meson or meson-baryon scattering amplitude. A $\bar{q}\bar{q}qq$ state has the same quantum numbers and the same quark content as a $\bar{q}q$-$\bar{q}q$ meson scattering state. In a fairly precise way the $\bar{q}\bar{q}qq$ state can be considered a piece of the meson-meson continuum that has been artificially confined by a confining boundary condition or potential that is inappropriate in the meson-meson channel[79, 80]. If the multiquark state is unusually light or sequestered (by the spin, color and/or flavor structure of the wavefunction) from the scattering channel, it may be prominent. If not, it is just an artifact of a silly way of enumerating the states in the continuum.

3. Pentaquarks in the uncorrelated quark model

The spectrum of an uncorrelated quark model begins with all quarks and antiquarks in the same orbital. Next, one quark or antiquark is excited, and so forth. At zeroth order, the spectrum consists of families of states of alternating parities as shown schematically in Fig. 6. The parity of the ground state of n_q quarks and $n_{\bar{q}}$ antiquarks is $(-1)^{n_{\bar{q}}}$. When a quark or antiquark is excited, the parity flips. Light meson and baryon multiplets *do* (roughly) alternate in parity — one of the remarkably simple and successful predictions of the quark model. The pseudoscalar and vector ($\bar{q}q$) mesons (negative parity) are followed by $J^{PC} = 0^{++}$, 1^{++}, 1^{+-}, and 2^{++} multiplets; the nucleon octet and decuplet (qqq) baryons are followed by many negative parity multiplets. There are a few famous exceptions: for example, the "Roper" resonance, with $J^{\Pi} = 1/2^+$, is the lightest excited nucleon and the 0^{++} $f_0(600)$ is the lightest excited meson. (Interestingly, both these exceptional states are candidates for multiquark states: $qqqq\bar{q}$ for the Roper
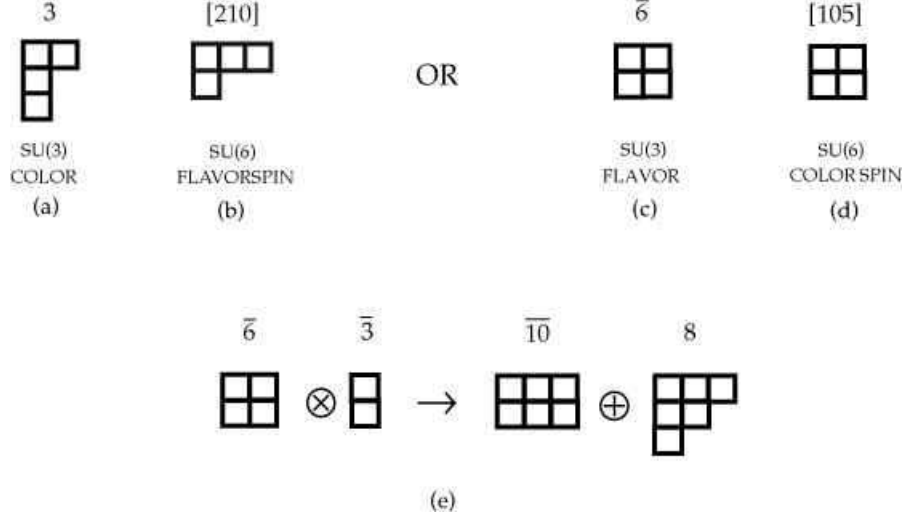
FIG. 7: Young diagrams for quark model exotics. (a) and (b) show all the negative parity $qqqq\bar{q}$ states in the space symmetric uncorrelated quark ground state. The color must be **3** to make a singlet with an antiquark. The flavor-spin $SU(6)$ representation that goes with the color **3** is the [210]. It contains only one flavor $SU(3)$ $\overline{\mathbf{6}}$ which has spin 1. (c) and (d) shows the flavor (c) and $SU(6)$ color-spin (d) diagrams when the q^4 flavor is $\overline{\mathbf{6}}$. As shown in (e), the $\overline{\mathbf{6}}$ combines with an antiquark, $\overline{\mathbf{3}}$ to make an exotic $\overline{\mathbf{10}}$ (and a non-exotic **8**). The color spin representation (d) that goes with the flavor $\overline{\mathbf{6}}$ is the [105]. It contains only one color **3**, which has spin 1. So either way you look at it, there is a unique operator that creates the negative parity $\overline{\mathbf{10}}$ with all quarks in the same mode.

and $\bar{q}\bar{q}qq$ for the $f_0(600)$.) But it is broadly successful, and so far, it has always got the parity of the ground state multiplets right.

I will return to the quark model predictions for the lightest tetraquark states later. The uncorrelated quark model predicts that the pentaquark ground state has *negative* parity.[9] This makes the existence of the Θ^+ embarrassing for this model for many reasons:

1. The $\overline{\mathbf{10}}$ is characteristically accompanied by a nearby **8** in quark models (see later). The $\overline{\mathbf{10}}$ and **8** mix to produce a non-strange Θ^+ analogue, $uudd(\bar{u}, \bar{d})$ which should be lighter. There is no candidate for a *negative parity* nucleon resonance below the Θ^+.

2. In the $\bar{q}q$ (36 states of spin and flavor) and qqq (56 states) sectors the ground state multiplets are complete. In the $qqqq\bar{q}$ sector the ground state multiplet contains 1260 states. The Θ^+ and its $SU(3)_{\rm f}$ brethren account only for a few — 36 to be precise. Of course most will be heavy enough to disappear into the continuum. Still, there are many states, and it is hard to imagine that only the antidecuplet should be seen.

   The counting of states is relatively simple. Going through it will be useful for later purposes: the q^4 configuration is symmetric in space and therefore antisymmetric in color×flavor×spin.[10] In color the q^4 must couple to the $\mathbf{3}_{\rm c}$ in order make a singlet with the antiquark. The Young diagram for $[q^4]^{\mathbf{3}_{\rm c}}$ is shown in Fig. 7 along with the (conjugate) diagram that determines the $SU(6)_{\rm fs}$ flavor×spin state.

   The flavor-spin representation of Fig. 7(b) is 210-dimensional and can be decomposed into $SU(3)_{\rm f} \otimes SU(2)_s$ multiplets (labelled by the dimension of the $SU(3)_{\rm f}$ representation and their spin$^{\rm parity}$) as follows:

$$[\mathbf{210}]_{\rm fs} = \{\mathbf{3}, 0^+ \oplus 1^+\} \oplus \{\overline{\mathbf{6}}, 1^+\} \oplus \{\mathbf{15}, 0^+ \oplus 1^+ \oplus 2^+\} \oplus \{\mathbf{15}', 1^+\} \tag{III.3}$$

   These, in turn, must be combined with the antiquark in the $(\overline{\mathbf{3}}, 1/2^-)$ representation, and yield an array of both exotic and non-exotic negative parity baryons in $SU(3)_{\rm f}$ representations including $\mathbf{1}, \mathbf{8}, \mathbf{10}, \overline{\mathbf{10}}, \mathbf{27}$ and $\mathbf{35}$. Note

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[9] The "flavor exchange" quark model which emphasizes quark-quark forces mediated by pseudoscalar meson exchange is an exception, one example of a *correlated* quark model[83]. It agrees with generic quark model predictions of the parity of $\bar{q}q$ and qqq states, but appears to prefer *positive parity* for the lightest $qqqq\bar{q}$ states.

[10] Remember, by "spin" I really mean the $\uparrow$ and $\downarrow$ states of total angular momentum of a $j = 1/2$ state.

that the all-important $\overline{\mathbf{10}}$ comes from

$$(\overline{\mathbf{6}}, 1^+) \otimes (\overline{\mathbf{3}}, \ 1/2^-) = (\overline{\mathbf{10}} \oplus \mathbf{8}, 1/2^- \oplus 3/2^-). \tag{III.4}$$

as illustrated in Fig. 7(e). Not only is the antidecuplet accompanied by an octet (though their approximate degeneracy will not be explained until Section V.D.1), but also the candidate for the Θ^+ occurs with both $J^\Pi = 1/2^-$ and $3/2^-$. This "spin-orbit doubling" will turn out to be a robust, and problematic, prediction of quark models in general[84].

3. A negative parity Θ^+ would have appear in the s-wave of KN scattering. Odd parity requires $\ell = 0, 2, 4, \ldots$. The $1/2^-$ could only couple to the s-wave. Therefore the most promising uncorrelated quark model candidate for the Θ^+ would have to appear in the KN s-wave, where its narrow width would be very hard to accomodate.

4. The problem of the width of a negative parity $\overline{\mathbf{10}}$ is even more severe than it seems. However, I would like to defer this discussion to the next section, where lattice calculations are discussed.

All in all, the uncorrelated quark model gives little reason to expect a light, narrow, exotic baryon with the quantum numbers of the Θ^+. Later I will describe the somewhat more positive perspective of *correlated* quark models.

D. Early lattice results

Exotic baryons present an excellent target for lattice QCD: the existence of the Θ^+ remains in doubt, and its spin and parity are unknown. Modern lattice calculations should be able to estimate the mass of the lightest state in various $qqqq\bar{q}$ channels, shedding light on the reality of the Θ^+ and its quantum numbers, and predicting other exotics, if they exist.

The first lattice studies of the Θ^+ appeared soon after the first experimental reports[85, 86]. After some initial confusion about the parity, both groups agreed that there is evidence for the KN threshold and for a state in the $uudd\bar{s}$ $J^\Pi = 1/2^-$ channel. They also reported evidence for a state in the $1/2^+$ channel, but at a higher mass. Subsequent work by the Kentucky group[87] does not find a state in either parity channel.[11]

These results are troubling: lattice calculations have, in the past, got the quantum numbers of the ground state correct in each sector of QCD. In this case the calculations support negative parity (Ref. [87] excepted), which, as we have seen, is hard to reconcile with the narrow width of the Θ^+. Several comments are in order:

1. Both Refs. [85] and [86] use single, local $qqqq\bar{q}$ sources. For the negative parity $\overline{\mathbf{10}}$ channel the local source is unique in a certain sense(see below), however for positive parity there are eight local sources[90] and it is far from clear that the chosen source optimizes the overlap with a possible positive parity state. Calculations of the full correlation matrix with the eight local sources are underway[91].

2. There is reason to believe, on the basis of diquark ideas, that the better positive parity sources may contain explicit derivatives, making it "non-local" in lattice parlance. An example of such an operator can be found in eq. (6) of Ref. [92].

3. The calculations are done rather far from the chiral limit. Chiral symmetry is at the heart of the chiral soliton model, where the light $1/2^+$ Θ^+ was first predicted. It is also important in diquark models, because diquark correlations disappear as quark masses increase. It is not clear whether the lattice calculations are close enough to the chiral limit to capture the effects that bind the Θ^+. The N-Δ mass difference, which should be a good measure of the importance of diquark correlations, is already quite well developed at the quark masses used in Refs. [85] and [86], so perhaps they are trustworthy on this score.

Lattice theorists classify sources according to their properties in the "non-relativistic" limit, in which four component Dirac quark fields are replaced by two component Pauli fields. This sounds like a rather uninteresting limit for nearly massless quarks. However it applies just as well to the more general quark model outlined in III.C.1 where the quarks can be massless. So it can shed light on the nature of the negative parity Θ^+ that seems to be cropping up in lattice calculations. In the "single mode configuration" the classification of operators used as lattice sources reduces to the

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[11] There is another published study[88], which claims a $1/2^+$ state and general agreement with the diquark model of Ref. [25], but this work has been criticized for several technical reasons[89].

problem we solved in the previous section for the uncorrelated quark model. The important result, summarized by eq. (III.4), is that there is a *single* $\overline{\mathbf{10}}$ with either $J^\Pi = 1/2^-$ or $3/2^-$. [In comparison, it is easy to work out that there are *four* octets with $J^\Pi = 3/2^+$.]

Now to the point: We already know a very simple operator with $qqqq\bar{q}$ content and the quantum numbers $(\overline{\mathbf{10}}, 1/2^-)$, namely the local operator that creates a kaon and a nucleon in the s-wave! No matter what they may look like — and because of the complexities of Fierz transformations, they can look completely different — any operator that creates a $1/2^-$ Θ^+ and survives the "single mode approximation" is just a kaon and an nucleon on top of one another. This analysis applies to the operators used in Refs. [85, 86, 87] to create the negative parity Θ^+. There is no way to decouple a *negative parity* Θ^+ source from the KN channel by adroitly superposing superficially distinct operators, unless one abandons the single mode approximation, which would presumably add considerable energy to the state. On this basis one would expect that the negative parity state observed in Refs. [85, 86, 87] will turn out to have an enormous width[93]. The same argument has implications for the coupling of a negative parity Θ^+ constructed in quark models: the state has exactly the same spin, color, and flavor wavefunction as KN in the s-wave and therefore should be very broad.

In summary: lattice studies of $qqqq\bar{q}$ are only beginning. Initial results seem to add to the confusion surrounding the Θ^+. They suggest negative parity, but once again the narrow width of the Θ^+ looks like a major problem. Advocates of a positive parity Θ^+ can hope that better sources and better approximations to the chiral limit will reverse the order of states.

IV. DIQUARKS

It seems that an uncorrelated quark model leads to a negative parity ground state multiplet, which contains $1/2^-$ and $3/2^-$ candidates for the Θ^+. However the very narrow width of the Θ^+ seems to be an insuperable difficulty. So a quark description of the Θ^+ must look to some correlation to invert the naive ordering of parity supermultiplets. This is where diquarks enter[25, 26].

The rest of this paper is devoted to diquarks and their role in understanding exotics in QCD. As mentioned in the Introduction, diquarks are not new. They are almost as old as QCD[28] and have been the subject of intense study by many theorists. The 1993 review by Anselmino *et al.* gives references to earlier work[29]. Their roles in baryon spectroscopy, in deep inelastic structure functions, and in dynamics at the confinement scale[94] have been especially emphasized. Early work on multiquark states in QCD hinted at the importance of diquarks in suppressing exotics[23], but their wide-ranging importance in the study of exotics and the *aufbau* principle of QCD does not seem to have been recognized previously.

Diquark correlations in hadrons suggest qualitative explanations for many of the puzzles of exotic hadron spectroscopy: first and foremost, why exotics are so rare in QCD; next, why the most striking supernumerary hadrons are a nonet of scalar mesons; third, why an exotic baryon antidecuplet would be the only prominent baryon exotic; fourth, why non-strange systems of 6, 9, 12, ... quarks form nuclei not single hadrons; and finally why the H dibaryon ($uuddss$) might not be as bound as simple estimates suggest.

A. Introducing diquarks

QCD phenomena are dominated by two well known quark correlations: confinement and chiral symmetry breaking. Confinement hardly need be mentioned: color forces only allow quarks and antiquarks correlated into color singlets. Chiral symmetry breaking can be viewed as the consequence of a very strong quark-antiquark correlation in the color, spin, and flavor singlet channel: $[\bar{q}q]^{\mathbf{1}_c \mathbf{1}_f 0}$. The attractive forces in this channel are so strong that $[\bar{q}q]^{\mathbf{1}_c \mathbf{1}_f 0}$ condenses in the vacuum, breaking $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry.

The "next most attractive channel" in QCD seems to be the color antitriplet, flavor antisymmetric (which is the $\overline{\mathbf{3}}_f$ for three light flavors), spin singlet with even parity: $[qq]^{\overline{\mathbf{3}}_c \overline{\mathbf{3}}_f 0^+}$. This channel is favored by one gluon exchange[95, 96] and by instanton interactions[97, 98]. It will play the central role in the exotic drama to follow.

The classification of diquarks is not entirely trivial. Several of the ideas introduced in Section III.C help us determine which diquark configurations are likely to be most attractive and therefore most important spectroscopically.

Operators that will create a diquark of any (integer) spin and parity can be constructed from two quark fields and insertions of the covariant derivative. We are interested in potentially low energy configurations, so we omit the derivatives. There are eight distinct diquark multiplets (in color×flavor×spin) that can be created from the vacuum by operators bilinear in the quark field, which can be enumerated as follows. Since each quark is a color triplet, the pair can form a color $\overline{\mathbf{3}}_c$, which is antisymmetric, or $\mathbf{6}_c$, which is symmetric. The same is true in $SU(3)$-flavor. The spin couplings are more complicated. Consider $q_\alpha q_\beta$, where α and β are Dirac indices[99]. The constructions look more

familiar if we represent one of the quarks by the charge conjugate field: $q_\alpha q_\beta \to \overline{q}_{C\alpha} q_\beta$, where $\overline{q}_C = -iq^T\sigma^2\gamma_5$. Then the classification of diquark bilinears is analogous to the classification of $\bar{q}q$ bilinears: It is easy to show (remembering that the Dirac fields are in the $(0, 1/2) \oplus (1/2, 0)$ representation of the Lorentz group) that $\overline{q}_{C\alpha}q_\beta$ can have spin zero and one with either even or odd parity: $0^\pm$ and $1^\pm$.. For example, $\bar{q}_C\gamma_5 q$ creates 0^+ and $\bar{q}_C\vec{\gamma}\gamma_5 q$ creates 1^-. The parity is opposite from the more familiar classification of Dirac currents composed of quark and antiquark. Furthermore 0^+ and 1^- even under quark exchange and 0^- and 1^+ are odd under quark exchange. The eight candidate diquarks are

TABLE III: Properties of diquarks.

Spins and Parities	(Flavor, Color)	Operators	Single mode survival
0^+	$(\overline{\mathbf{3}}, \overline{\mathbf{3}})$ $(\mathbf{6}, \mathbf{6})$	$\overline{q}_C\gamma_5 q,\ \overline{q}_C\gamma^0\gamma_5 q$	yes
1^+	$(\overline{\mathbf{3}}, \mathbf{6})$ $(\mathbf{6}, \overline{\mathbf{3}})$	$\overline{q}_C\vec{\gamma}q,\ \overline{q}_C\sigma^{0i}q$	yes
0^-	$(\overline{\mathbf{3}}, \mathbf{6})$ $(\mathbf{6}, \overline{\mathbf{3}})$	$\overline{q}_C q,\ \overline{q}_C\gamma^0 q$	no
1^-	$(\overline{\mathbf{3}}, \overline{\mathbf{3}})$ $(\mathbf{6}, \mathbf{6})$	$\overline{q}_C\vec{\gamma}\gamma_5 q,\ \overline{q}_C\sigma^{ij}q$	no

$\mathcal{A}\{(\overline{\mathbf{3}}_c \oplus \mathbf{6}_c) \otimes (\overline{\mathbf{3}}_c \oplus \mathbf{6}_c) \otimes (0^\pm \oplus 1^\pm)\}$, where $\mathcal{A}\{\dots\}$ denotes the antismmetrization required by fermi statistics. Their properties are summarized in Table III. The candidates can be pared down quickly:

- Color $\mathbf{6}_c$ diquarks have much larger color electrostatic field energy. All models agree that this is not a favored configuration.

- Odd parity diquark operators vanish identically in the single mode configuration. The reasons for using this criterion were described in the previous section: it corresponds to quarks that are unexcited relative to one another.

This leaves only two diquarks,

$$\left|\{qq\}\ \overline{\mathbf{3}}_c(A)\ \overline{\mathbf{3}}_f(A)\ 0^+(A)\right\rangle$$
$$\left|\{qq\}\ \overline{\mathbf{3}}_c(A)\ \mathbf{6}_f(S)\ 1^+(S)\right\rangle \tag{IV.1}$$

where A or S denotes the exchange symmetry of the preceding representation. Both of these configurations are important in spectroscopy. In what follows I will refer to them sometimes as the "scalar" and "vector" diquarks, or more suggestively, as the "good" and "bad" diquarks. Remember, though, that there are many "worse" diquarks that we are ignoring entirely. As an example of the process by which operators are constructed, here is the good diquark operator,

$$\mathcal{Q}_{ia} = \epsilon_{ijk}\epsilon_{abc}(i\sigma_2)_{\alpha\beta}\ q_\alpha^{jb}q_\beta^{kc} = \epsilon_{ijk}\epsilon_{abc}\ \overline{q}_c^{jb}\gamma_5 q^{kc} \tag{IV.2}$$

Models universally suggest that the scalar diquark is lighter than the vector. For example, one gluon exchange evaluated in a quark model gives rise to a color and spin[12] dependent interaction,

$$\mathcal{H}_{\text{color spin}} = -\alpha_s \sum_{i\neq j} M_{ij}\vec{\sigma}_i \cdot \vec{\sigma}_j \tilde{\lambda}_i \cdot \tilde{\lambda}_j \tag{IV.3}$$

where $\vec{\sigma}_i$ and $\tilde{\lambda}_i$ are the Pauli and Gell-Mann matrices that operate in the spin and color space of the ith quark. M_{ij} is a model-dependent matrix element that depends on the mass of the quarks but not on their spin and color. M_{ij} is largest for massless quarks (M_{oo}), decreases if one (M_{os}) or both (M_{ss}) of the quarks are strange, and decreases like $1/m_i m_j$ for heavy quarks. Ignoring quark mass effects the matrix elements of this operator in the "good" and "bad" diquark states are $-8M_{oo}$ and $+8/3M_{oo}$ respectively. To set the scale, the Δ–nucleon mass difference is $16M_{oo}$, so the energy difference between good and bad diquarks is $\sim \frac{2}{3}(M_\Delta - M_N) \sim 200$ MeV. Not a huge effect, but large enough to make a significant difference in spectroscopy. After all, the nucleon is stable and the Δ is 300 MeV heavier and has a width of 120 MeV!

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[12] Here again "spin" refers to the total angular momentum of a $1/2^+$ quark, which coincides with spin only in the non-relativistic limit.
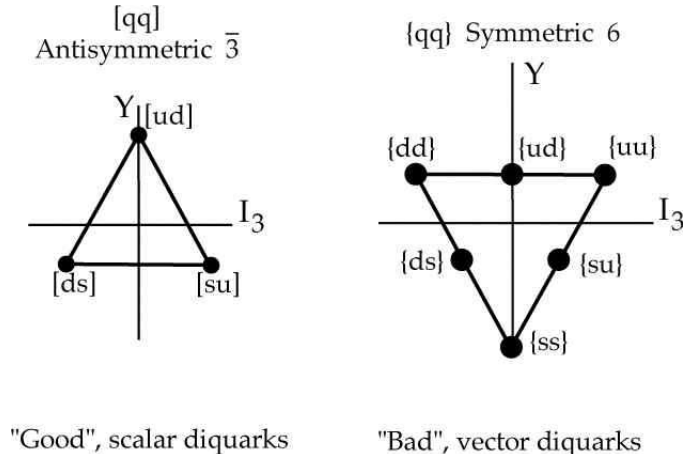
FIG. 8: $SU(3)$ weight diagrams and quark content of the "good" scalar diquarks and the "bad" vector diquarks. Braces denote symmetrization and brackets denote antisymmetrization in flavor.

B. Characterizing diquarks

The good scalar and bad vector diquarks are our principal subjects. Their quark content and flavor quantum numbers are summarized in Fig. 8. Since the good diquarks are antisymmetric in flavor, we will denote them by $[q_1, q_2] : \{[u, d] \, [d, s] \, [s, u]\}$ when flavor is important and by $\mathcal{Q}$ when it is not. It is particularly easy to construct flavor wavefunctions for baryons, pentaquarks, *etc*, made of good diquarks by using the correspondence

$$
\begin{aligned}
[u, d] &\leftrightarrow \bar{s} \\
[d, s] &\leftrightarrow \bar{u} \\
[s, u] &\leftrightarrow \bar{d}
\end{aligned}
\tag{IV.4}
$$

which is obvious from Fig. 8[13]. The bad diquarks are symmetric in flavor, suggesting the notation $\{q_1, q_2\}$: $\{\{u, u\} \, \{u, d\} \, \{d, d\} \, \{d, s\} \, \{s, s\} \, \{s, u\}\}$.

Diquarks are, of course, colored states, and therefore not physical. However, their properties can be studied in a formally correct, color gauge invariant way on the lattice. To define the non-strange diquarks, introduce an infinitely heavy quark, Q, *ie* a Polyakov line. Then study the qqQ correlator with the qq quarks either antisymmetric ($[u, d]Q$) or symmetric ($\{u, d\}Q$) in flavor. The results, $M[u, d]$ and $M\{u, d\}$ — labelled unambiguously — are meaningful in comparison, for example, with the mass of the lightest $\bar{q}Q$ meson, $M(u) = M(d)$[14]. $M\{u, d\} - M[u, d]$ is the good-bad diquark mass difference for massless quarks. It a is measure of the strength of the diquark correlation. The diquark-quark mass difference, $M[u, d] - M(u)$, is another. The same analysis can be applied to diquarks made from one light and one strange quark giving $M[u, s]$ and $M\{u, s\}$. The mass of the doubly strange vector diquark, $M\{s, s\}$ can be measured similarly. These mass differences are *fundamental* characteristics of QCD, which should be measured carefully on the lattice.

Of course, a moment's thought reveals that $M[u, d]$ is the mass of the particle usually called the Λ_Q. $M\{u, d\}$ corresponds to the Σ_Q ($J = 1/2$) or Σ_Q^* ($J = 3/2$), the two being degenerate in the infinite quark mass limit. $M[u, s]$ and $M\{u, s\}$ are related to masses of Ξ_Qs and Ξ_Q^*s; and $M\{s, s\}$ to the masses of the Ω_Qs[101].

Lattice calculations of these quantities may take a while, and will be subject to debates about how close they are to the chiral and continuum limits. In the meantime we already know enough about the masses of charm mesons and baryons to extract an estimate of these mass differences, although we are handicapped by the fact that the spin interactions between the light quarks and the charm quark are not negligible. It would be even better if we knew the masses of enough bottom baryons to perform the analysis with the b-quark as the heavy spectator. So far only enough is known about b-baryon masses to extract the diquark-quark mass difference. Of course the scalar diquark

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[13] Note, however, that the signs in eqs. (IV.4) are important. They are determined by cyclic permutation. If you use $[u, s]$ instead of $[s, u]$ you will get into sign trouble!

[14] I ignore small isospin violating effects throughout.

has no spin interaction with the spectator heavy quark (Q), but the vector diquark does,

$$\mathcal{H}(Q, \{q_1, q_2\}) = K(Q, \{q_1, q_2\})2\vec{S}_{\{q_1, q_2\}} \cdot \vec{S}_Q \tag{IV.5}$$

where $\vec{S}_{\{q_1, q_2\}}$ is the spin of the vector diquark, and the coefficient $K(Q, \{q_1, q_2\})$ depends on the quark masses. The light antiquark and heavy quark in a $\bar{q}Q$ meson has a similar interaction,

$$\mathcal{H}(Q, \bar{q}) = K(Q, \bar{q})2\vec{s}_{\bar{q}} \cdot \vec{S}_Q \tag{IV.6}$$

This interaction splits D_Q^* from the D_Q,[15] the Σ_Q^* from the Σ_Q and the Ξ_Q^* from the Ξ_Q and Ξ_Q'. Other spin dependent interactions mix the $\Xi_Q = [[u, s]Q]^{J=1/2}$ with the $\Xi_Q' = [\{u, s\}Q]^{J=1/2}$.

In order to obtain estimates of diquark mass differences, it is necessary to take linear combinations of baryon and meson masses that eliminate these spin interactions. Among the non-strange quarks, we obtain

$$M\{u, d\}|_Q - M[u, d]|_Q = \frac{1}{3}\left(2M(\Sigma_Q^*) + M(\Sigma_Q)\right) - M(\Lambda_Q)$$

$$M[u, d]|_Q - M(u)|_Q = M(\Lambda_Q) - \frac{1}{4}\left(M(D_Q) + 3M(D_Q^*)\right)$$

$$K(Q, \{u, d\}) = \frac{1}{3}\left(M(\Sigma_Q^*) - M(\Sigma_Q)\right) \tag{IV.7}$$

To obtain useful information from the Ξ_Q and Ω_Q $(\Omega = (Qss)^{J=1/2})$ states, it is necessary to assume that both the bad diquark mass and the spin interaction are linear functions of the strange quark mass,

$$M\{s, s\}|_Q + M\{u, d\}|_Q = 2M\{u, s\}|_Q$$
$$K(Q, \{s, s\}) + K(Q, \{u, d\}) = 2K(Q, \{u, s\}) , \tag{IV.8}$$

amounting to first order perturbation theory in m_s. With this we can deduce,

$$M\{u, s\}|_Q - M[u, s]|_Q = \frac{2}{3}\left(M(\Xi_Q^*) + M(\Sigma_Q) + M(\Omega_Q)\right) - M(\Xi_Q) - M(\Xi_Q')$$

$$M[u, s]|_Q - M(s)|_Q = M(\Xi_Q) + M(\Xi_Q') - \frac{1}{2}(M(\Sigma_Q) + M(\Omega_Q)) - \frac{1}{4}(M(D_{sQ}) + 3M(D_{sQ}^*))$$

$$K(Q, \{u, s\}) = \frac{1}{6}\left(2M(\Xi_Q^*) - M(\Omega_Q) - M(\Sigma_Q)\right) . \tag{IV.9}$$

This analysis can be applied directly in the charm sector, where all the required hadron masses are known. Only the middle of eqs. (IV.7) can be applied in the bottom sector due to lack of information about bottom baryons. Finally, if we are daring, we can apply the first part of this analysis, eqs. (IV.7), to the strange baryons. This is not as ill-founded as it might seem, since we are not ignoring the spin-spin interactions between the light (u and d) quarks and the s-quark. However, there is no useful analogue of eqs. (IV.9) in the strange sector — as the absence of a strange state with the symmetry structure of the Ω_Q should make clear — without making a more detailed model[16].

When we substitute numbers into eqs. (IV.7)–(IV.9), quite a consistent picture of diquark mass differences and diquark-spectator interactions emerges: First,

$$M\{u, d\}|_s - M[u, d]|_s = 205 \text{ MeV}$$
$$M\{u, d\}|_c - M[u, d]|_c = 212 \text{ MeV}$$

$$M[u, d]|_s - M(u)|_s = 321 \text{ MeV}$$
$$M[u, d]|_c - M(u)|_c = 312 \text{ MeV}$$
$$M[u, d]|_b - M(u)|_b = 310 \text{ MeV} \tag{IV.10}$$

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[15] I denote the $(Q\bar{u}/\bar{d})$ pseudoscalar and vector mesons as D_Q and D_Q^* respectively, and the $(Q\bar{s})$ mesons as D_{sQ} and D_{sQ}^*.

[16] In fact, quark models suggest a more microscopic model in which all residual quark interactions are described by a spin-spin interaction, $\mathcal{H} = \sum_{i \neq j} K_{ij}\vec{s}_i \cdot \vec{s}_j$. The reader is invited to work out the diquark masses in this model.

shows that the properties of hypothetical non-strange diquarks are the pretty much the same when extracted from the charm and bottom, and even strange, baryon sectors. Second,

$$M\{u,s\}|_c - M[u,s]|_c = 152 \text{ MeV}$$
$$M[u,s]|_c - M(s)|_c = 498 \text{ MeV} \tag{IV.11}$$

shows that the diquark correlation decreases when one of the light quarks is strange. This is certainly to be expected, since it originates in spin dependent forces. As the correlation decreases the mass difference between the scalar and vector diquarks decreases ($\sim 210 \rightarrow \sim 150$ MeV) and the mass difference between the scalar diquark and the antiquark increases ($\sim 310 \rightarrow \sim 500$ MeV). Finally,

$$K(s,\{u,d\}) = 64 \text{ MeV}$$
$$K(c,\{u,d\}) = 21 \text{ MeV}$$
$$K(c,\{u,s\}) = 24 \text{ MeV} \tag{IV.12}$$

shows that the non-strange vector diquark interaction with the spectator charm quark is significantly weaker than with a spectator strange quark, as expected from heavy quark theory. The only mildly surprising result is that the $\{u,s\}$ and $\{u,d\}$ vector diquarks have roughly the same interaction with the charm spectator. It will be very interesting to compare these results with further measurements in the b-quark sector and, of course, with the results of lattice calculations.

C. Phenomenological evidence for diquarks

Once you start looking, there is evidence for diquarks everywhere. What follows is hardly more than a list to whet the appetite, with occasional explanations.

- Baryon spectroscopy.

  Diquarks were born in the regularities of the baryon spectrum[28], which seem be described qualitatively by viewing baryons as quark-diquark bound states. In addition to the spin splittings that were described in the previous subsection, another famous piece of evidence is the apparent absence of an $[\mathbf{20}]_{\text{fs}}$ in the second "band" of excited baryons in the traditional quark model $O(3) \times SU(6)_{\text{fs}}$ classification of baryons[102]. In the zeroth and first bands ($L = 0$ $[\mathbf{56}]_{\text{fs}}$ or $L = 1$ $[\mathbf{70}]_{\text{fs}}$) the qqq space wavefunction is either symmetric or of mixed symmetry, allowing pairs of quarks to form correlated diquarks. Only in the second band of excited baryons is it possible to have a totally antisymmetric space wavefunction, which cannot be made of a quark-diquark pair. Since the qqq state is by hypothesis antisymmetric in space and already antisymmetric in color, it must be antisymmetric in flavor × spin, which is the $[\mathbf{20}]_{\text{fs}}$. The $[\mathbf{20}]_{\text{fs}}$ contains a $(\mathbf{8}_{\text{f}}, \frac{1}{2})$ and a $(\mathbf{1}_{\text{f}}, \frac{3}{2})$. This is the only spin-3/2, flavor singlet in any qqq $SU(6)_{\text{fs}}$ multiplet. Its absence is noted in the PDG review [102] of the quark model classification of baryon resonances.[17]

- The $\Delta I = 1/2$ rule in weak non-leptonic decays.

  The four-quark ($\bar{q}\Gamma q \, \bar{q}\Gamma q$) operators in the effective Lagrangian for weak non-leptonic decays transform with either $I = 1/2$ or $I = 3/2$ (or, in the case of three flavors $\mathbf{8}$ or $\mathbf{27}$). These operators mediate $K \rightarrow 2\pi$, $K \rightarrow 3\pi$ decays as well as hyperon decays like $\Lambda \rightarrow N\pi$, $\Sigma \rightarrow N\pi$, etc. The $I = 1/2$ operator appears to be enhanced over the $I = 3/2$ operator by over an order of magnitude. A small part of that enhancement can be attributed to the perturbative evolution of the operator from the weak (M_Z) scale down to the QCD scale[103]. The rest of the enhancement is presumably non-perturbative in origin. The $I = 1/2$ operator is built from good, scalar diquarks, the $I = 3/2$ involves bad, vector diquarks. In the 1980's Neubert, Stech, and their collaborators showed how a systematic enhancement of the scalar diquark over the vector would explain the $\Delta I = 1/2$ rule in both meson and baryon non-leptonic weak decays[32].

- Regularities in parton distribution functions.

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[17] Amsler and Wohl suggest that members of the $[\mathbf{20}]_{\text{fs}}$ would be hard to produce "since a coupling to the ground state would require a two-quark excitation", although this is not a well tested dynamical principle.

The famous 4:1 ratio of proton and neutron deep inelastic structure functions as $x \to 1$

$$\lim_{x \to 1} \frac{F_2^{\text{en}}(x, Q^2)}{F_2^{\text{ep}}(x, Q^2)} = \frac{1}{4} \tag{IV.13}$$

follows from the dominance of the scalar diquark[31, 104]. Data are not available all the way to $x = 1$, but the tendency for the data to decrease toward the positivity bound of $1/4$ is clear. As the Bjorken-x of the struck quark approaches one, the two spectator quarks are forced to their most tightly bound configuration. If the scalar diquark dominates then only the u quark in the proton and the d quark in the neutron can survive as $x \to 1$. The $1/4$ is the ratio of their squared charges. Similar regularities are predicted for spin dependent structure functions[31],

$$\lim_{x \to 1} \frac{\Delta d}{d} = -\frac{1}{3}$$
$$\lim_{x \to 1} \frac{\Delta u}{u} = 1 \tag{IV.14}$$

New data coming out of low-Q^2 inelastic electron scattering experiments at JLab seem to support these predictions[105].

- Quark condensation in dense matter.

  The phenomenon of color superconductivity in dense quark matter has attracted widespread attention over the past few years[30]. It does not qualify as phenomenological support for diquarks because no one has figured out how to observe cold quark matter at high baryon density. However, the fundamental Cooper pair of color superconductivity is the good scalar diquark. In dense enough matter one can *prove* that this correlated scalar diquark is so tightly bound that it condenses, breaking color×flavor down to a subgroup with many interesting, if hard to observe, consequences.

- $\Lambda(1116)$ and $\Lambda(1520)$ fragmentation functions.

  The $\Lambda(1116)$ is special among stable baryons. Because it is an isosinglet, the ud pair is 100% in the good, scalar diquark configuration. The $\Sigma(1192)$ resembles the Λ except for a less favorable diquark content. It is therefore interesting to compare their production in a clean environment, like fragmentation in e^+e^- annihilation at LEP, where baryons are seen as fragments of the quarks produced in $e^+e^- \to q\bar{q}$. A summary of the production cross sections for various particles can be found in the Ref. [21]. Typically cross sections fall roughly exponentially with the mass of the produced hadron (about a factor of e for every 100 MeV[106]). The $\Lambda(1116)$ is the only exception among stable baryons: it is produced about 2-3 times more copiously than one would expect, as shown in Fig. 9(a). Interestingly, the $\Lambda(1520)$ resonance is also anomalously abundant. This provides even further support for the dominance of the good diquark: The $\Lambda(1520)$ can be described as a good $[u, d]$ diquark and an s quark in a p-wave[122].

  The contrast between the $\Lambda(1116)$ and the $\Sigma(1192)$ is even more striking when one compares the cross sections as a function of z, the fractional energy of the baryon. At large z where the valence component of hadron wavefunctions dominate, the Λ/Σ ratio, as measured at LEP[107] is more than an order of magnitude (see Fig. 9(b)).

D. Diquarks and higher twist

Diquarks need not be pointlike. As we have seen, the energy difference between the good and bad diquarks is only ~ 200 MeV, enough to be quite important in spectroscopy, but corresponding only to a correlation length of 1 fermi, the same as every other mass scale in QCD. It is interesting, nevertheless to ask whether other hadronic phenomena can constrain the correlation. Although many nucleon properties, like form factors, are often discussed in terms of quark correlations, as far as I know, the correspondence can only be made exact for deep inelastic scattering (DIS).

Any kind of quasi-pointlike (*ie* characterized by a mass scale $\Lambda_{\boldsymbol{Q}} \gg \Lambda_{\text{QCD}}$) correlation in the nucleon is certainly excluded for $\Lambda_{\boldsymbol{Q}}$ ranging from ~ 1 GeV up to the highest scales where deep inelastic data exist (~ 100 GeV). Diquarks would be especially obvious because as bosons they would generate an anomalously large longitudinal/transverse inelastic cross section ratio in DIS at scales below $\Lambda_{\boldsymbol{Q}}$, which would disappear above $\Lambda_{\boldsymbol{Q}}$. Such an effect is certainly ruled out by the early, and apparently permanent, onset of scaling seen in a multitude of experiments.
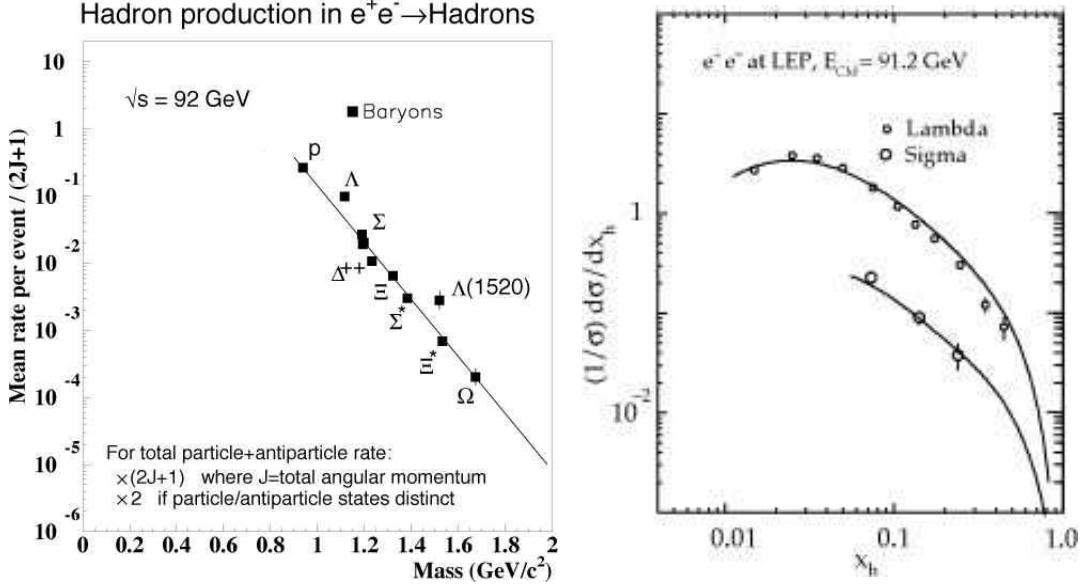
FIG. 9: (a) Total inclusive baryon production in e^+e^- production at LEP energies[106]. The exponential line is only to guide the eye. (b) A comparison of the Λ and Σ fragmentation functions measured at DELPHI[107].
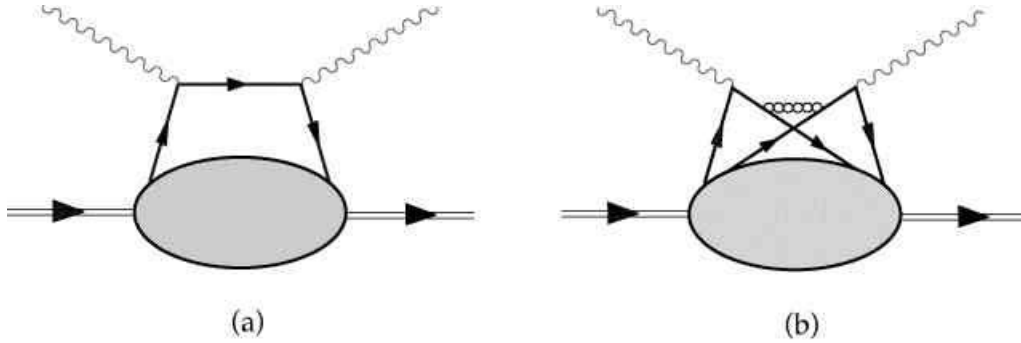

FIG. 10: (a) Leading twist, single quark contribution to DIS (b) Twist-4, diquark contribution to DIS.

On the other hand one might think that the absence of large higher twist effects in DIS could be used to place an uncomfortably *low* limit on the mass scale of diquark correlations. This is not the case[108]. In fact measurements of $1/Q^2$ corrections to DIS *place no limits whatsoever* on scalar diquark correlations in the nucleon. To understand this it is necessary to review some of the basics of the twist analysis of deep inelastic scattering. "Twist" refers to the dimension (d) minus the spin (n) of the operators that contribute to DIS, $t = d - n$. The smaller the twist, the more important the contribution to DIS: A given operator contributes like $1/Q^{t-2}$. The leading operators are twist-2 and act on a single quark[18]. They have the generic structure

$$\mathcal{O}_\mu^{(2)} \sim \bar{q}\gamma_\mu \mathcal{D}\mathcal{D}\ldots q \qquad (IV.15)$$

The covariant derivatives, their Lorentz indices suppressed, denoted schematically by $\mathcal{D}$, have $d(\mathcal{D}) - n(\mathcal{D}) = 0$, so they are irrelevant for counting twist. The quark fields have $d(q) = 3/2$ and the γ-matrix contributes $n(\gamma) = 1$, so in all, $t = 2(3/2) - 1 = 2$, and these operators' contributions to DIS are independent of Q (modulo logarithmic corrections from perturbative QCD). The $\bar{q}\gamma q$ operators sum up to give the "handbag" diagram shown in Fig. 10(a).

It is easy to write down operators with twist greater than two[109]. The most important are twist-four (twist-three does not contribute to spin average DIS for light quarks), which contribute corrections of order $1/Q^2$ to deep inelastic

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[18] I am ignoring gluon operators, which do not figure in the argument.

structure functions. The factor of $1/Q^2$ is accompanied by some squared mass-scale, M_4^2, in the numerator. Twist-four effects have been studied for years, and the qualitative conclusion is that M_4 is small. How small need not concern us, for we are about to see that it anyway places no limit on the good diquark that interests us.

Twist four operators invariably involve products of more than two quark and gluon fields (again ignoring pure-gluon operators). Examples include quark-gluon operators, $\bar{q}\mathcal{F}q$ and $\bar{q}\mathcal{F}\mathcal{F}q$, and four-quark operators, $\bar{q}q\bar{q}q$. The matrix elements of these operators in the target nucleon determine the magnitude of higher twist effects. The four quark operators are the culprits: they can be Fierz-transformed into diquark-antidiquark operators, $\bar{q}\bar{q}\ldots qq$ and therefore measure the scale of diquark correlations in the nucleon. They can be summed (in a well-defined way) to give diagrams like Fig. 10(b), where two quarks are removed from the nucleon, scattered at high momentum, and then returned[110]. The generic structure of four quark operators is (there are others, but the results are the same),

$$\mathcal{O}_{\mu\nu}^{(4)} \sim \bar{q}\gamma_\mu \mathcal{D}\mathcal{D}\ldots q \; \bar{q}\gamma_\nu \mathcal{D}\mathcal{D}\ldots q. \tag{IV.16}$$

The γ-matrices are necessary. With $d(q) = 3/2$ and $d(\mathcal{D}) - n(\mathcal{D}) = 0$ it is easy to see that the twist of $\mathcal{O}^{(4)}$ would be six if it were not for the two factors of γ, each of which corresponds to a unit of spin. In other words: when Fierzed, the two diquarks in $\mathcal{O}^{(4)}$ must be coupled to spin-2. So *only the vector diquark contributes at twist-four*. Bounds on twist four in DIS tell us that the bad, vector diquark cannot be tightly bound, but they do not constrain the good, scalar diquark at all. It contributes only to twist-six and beyond, where it cannot be separated from the flood of non-perturbative effects that emerge at low Q^2.

We can proceed without concern that correlations of the extent necessary to influence the spectrum are ruled out by deep inelastic phenomena.

V. DIQUARKS AND EXOTICS

A. An overview

Let us consider exotic spectroscopy with diquark correlations in mind. I will assume little more than that two quarks prefer to form the good, scalar diquark when possible. States dominated by that configuration should be systematically lighter, more stable, and therefore more prominent, than states formed from other types of diquarks. This qualitative rule leads to qualitative predictions — all of which seem to be supported by the present state of experiment. This is clearly an idealization — a starting place for describing exotic spectroscopy. Important effects are ignored, for example residual QCD interactions can turn a scalar diquark into a vector diquark[19]. A more sophisticated treatment would have to consider these effects quantitatively. In fact, the scheme I am describing here is a step back in complexity — though it may capture the underlying physics better — from the first work on multiquark spectroscopy in QCD[23]. There the spectrum of $\bar{q}\bar{q}qq$ mesons was obtained by diagonalizing the one-gluon exchange interaction. The light states turned out to be predominantly $\overline{\mathcal{Q}}\mathcal{Q}$, but other diquark types mixed in as well. So the pure diquark model being described here is more radical and more elementary than the one proposed there. To learn the real extent of $\mathcal{Q}$ dominance will require more models and more information from experiment. The qualitative ideas explored here are not powerful enough to fix the overall mass scale of any given sector in QCD. So we cannot *predict* the existence of (nearly) stable exotic pentaquarks, or determine whether the H-dibaryon is stable. As was the case of the large N_c-dynamics of Jenkins and Manohar, once a particle like the Θ^+ is found, it sets the scale, and leads to many interesting predictions.

The predictions that follow from $\mathcal{Q}$-dominance are simple, and striking. They capture all the important features of exotic spectroscopy and provide the conceptual basis of a unified description of this sector of QCD.

- There should be no (light, prominent) exotic mesons.

  The good diquark, $\mathcal{Q}$, is a flavor $\overline{\mathbf{3}}$, just like the antiquark. Tetraquarks, $\bar{q}\bar{q}qq$, potentially include exotics in $\mathbf{27}$, $\mathbf{10}$, and $\overline{\mathbf{10}}$ representations of flavor $SU(3)$. However $\overline{\mathcal{Q}} \otimes \mathcal{Q}$ contains *only non-exotic* representations, $\mathbf{1}$ and $\mathbf{8}$, just like $\bar{q} \otimes q$:

$$
\begin{aligned}
q^{\mathbf{3}} \otimes \bar{q}^{\overline{\mathbf{3}}} &= (\bar{q}q)^{\mathbf{1}} \oplus (\bar{q}q)^{\mathbf{8}} \\
\overline{\mathcal{Q}}^{\mathbf{3}} \otimes \mathcal{Q}^{\overline{\mathbf{3}}} &= (\overline{\mathcal{Q}}\mathcal{Q})^{\mathbf{1}} \oplus (\overline{\mathcal{Q}}\mathcal{Q})^{\mathbf{8}}
\end{aligned}
\tag{V.1}
$$

---

[19] Because gluons are flavor singlets they cannot transform a good, $\overline{\mathbf{3}}_{\mathrm{f}}$-diquark into a bad, $\mathbf{6}_{\mathrm{f}}$-diquark. Instead quark exchange between the diquarks is required as well.

Other diquark-antidiquark mesons are heavier, where they would be buried in the meson-meson continuum. As described in Section III.C probably they are not just "broad", but in fact absent[79].

- The only prominent tetraquark mesons should be an $SU(3)$ nonet with $J^\Pi = 0^+$.

  This prediction — a simple corollary of the one just above — dates back to the late 1970's[23]. Since the diquarks in eq. (V.1) are spinless bosons, the spin$^{\text{parity}}$ of the lightest nonet is $J^\Pi = 0^+$. Over the years evidence has accumulated that the nine 0^+-mesons with masses below 1 GeV (the $f_0(600)$, $\kappa(800)$, $f_0(980)$, and $a_0(980)$) have important tetraquark components[111, 112, 113]. Details will follow.

- If there are any exotic pentaquark baryons, they lie in a positive parity $\overline{10}$ of $SU(3)_\text{f}$.

  This is also a simple consequence of combining good diquarks. To make pentaquarks it is necessary to combine two diquarks and an antiquark. The result is

  $$\boldsymbol{Q}^{\overline{\boldsymbol{3}}} \otimes \boldsymbol{Q}^{\overline{\boldsymbol{3}}} \otimes \bar{q}^{\overline{\boldsymbol{3}}} = (\boldsymbol{Q}\boldsymbol{Q}\bar{q})^{\boldsymbol{1},\frac{1}{2}^-} \oplus (\boldsymbol{Q}\boldsymbol{Q}\bar{q})^{\boldsymbol{8},\frac{1}{2}^-} \oplus (\boldsymbol{Q}\boldsymbol{Q}\bar{q})^{\boldsymbol{8},(\frac{1}{2}^+,\frac{3}{2}^+)} \oplus (\boldsymbol{Q}\boldsymbol{Q}\bar{q})^{\overline{\boldsymbol{10}},(\frac{1}{2}^+,\frac{3}{2}^+)} \qquad \text{(V.2)}$$

  The only exotic in $\overline{\boldsymbol{3}} \otimes \overline{\boldsymbol{3}} \otimes \overline{\boldsymbol{3}}$ is the $\overline{\boldsymbol{10}}$. Other exotic flavor multiplets, like the $\boldsymbol{27}$ and $\boldsymbol{35}$, which occur in the uncorrelated quark picture and/or the chiral soliton models, should be heavier and most likely lost in the meson-baryon continuum. The spin and parity assignments in eq. (V.2) and the many properties of these pentaquark baryons are discussed below.

- Nuclei will be made of nucleons.

  To a good approximation, nuclei are made of nucleons — a fact which QCD should explain. If diquark correlations dominate, systems of $3A$ quarks should prefer to form individual nucleons, not a single hadron.

  The argument is based on statistics: Good diquarks are spinless color anti-triplet bosons. Only one, $[u, d]$, is non-strange. A six-quark system made of three of these, antisymmetrized in color to make a color singlet, would have to have fully antisymmetric space-wavefunction to satisfy Bose statistics. The simplest would be a triple-scalar product, $\vec{p}_1 \cdot \vec{p}_2 \times \vec{p}_3$, which should be much more energetic than two separate, color-singlet nucleons in an s-wave (*eg.* the deuteron). The argument generalizes to heavy nuclei. Of course it does not explain nuclear binding or the rich phenomena of nuclear physics.

- The H-dibaryon looks less attractive

  Long ago I argued that perturbative color-spin interactions were maximally attractive in the *uuddss* system and might bind a spinless, doubly strange dihyperon[33]. Searches for the H have so far come up empty, restricting its binding energy to be less than $\sim$ a few MeV[115].

  From a diquark perspective, the H is special: The only way to put three diquarks in a totally symmetric (low energy) space state requires one diquark of each flavor, $[u, d][d, s][s, u]$ — just the quantum numbers of the H. There is, hidden in this description, a hint of repulsion in the H-system coming from Pauli blocking. Although good diquarks are bosons, they are composed of fermions, the quarks, and each diquark has quark components identical in flavor, color, and spin with quarks in the other diquarks. Therefore the exclusion principle will generate a repulsion between good diquarks, or equivalently mix other, less favorable diquarks into the ground state. Of course the ground state with respect to one-gluon exchange, which was constructed in Ref. [33] obeys the proper statistics and therefore does not consist solely of good diquarks. If multiquark hadron stability is driven mainly by the good diquark correlation, there is reason for the H to be less bound. Our tools are too blunt to settle the question, which will be decided either by more accurate theoretical methods or by experiment[116, 117].

B.   The scalar mesons

Nearly all known mesons made of u, d, and s quarks fit neatly into the multiplets expected in generic constituent quark models. The single, striking exception are the scalar (*ie* $J^{\Pi C} = 0^{++}$) mesons with masses below 1 GeV. The classification of these mesons has been a bone of contention for more than 30 years. The history of the σ-meson, the broad, isosinglet $\pi\pi$ s-wave resonance near 600 MeV, now known as the $f_0(600)$ is a good case in point: For years listed by the PDG, it was exiled to the gulag of particle physics in the 1980's, but now has been rehabilitated and lives comfortably in the pages of the latest edition of the PDG. The $f_0(980)$ and $a_0(980)$ — twin scalar resonances just at $\overline{K}K$ threshold, the first an isoscalar, the second an isovector — are well established, but their shapes and interpretation are complicated by their proximity to and strong interaction with the $\overline{K}K$ threshold. They prefer to couple to $\overline{K}K$, a channel with little phase space, instead of $\pi\pi$ or $\pi\eta$. Finally, the four light $K\pi$ isospin-1/2 resonances
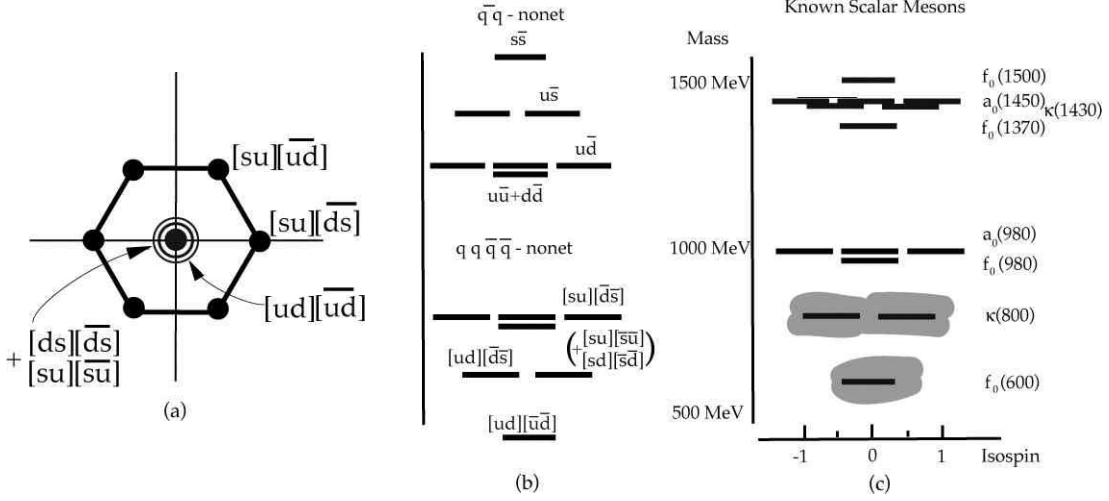
FIG. 11: A $\overline{\boldsymbol{Q}}\boldsymbol{Q}$ nonet: (a) Quark content of a nonet composed of $\overline{\boldsymbol{Q}} \otimes \boldsymbol{Q}$. (b) Comparison of the mass spectrum of an ideally mixed $\bar{q}q$ nonet (like the vector mesons) and an ideally mixed $\overline{\boldsymbol{Q}}\boldsymbol{Q}$ nonet. Note the inversion. (c) The masses of the light scalar mesons listed in the Particle Data Tables[21].

near 800 MeV, known as the $\kappa(800)$ or $K_0^*(800)$, remain too controversial for inclusion in the latest PDG Summary Table, although they make an appearance in the "Particle Listings". Whether or not they are recognized as "states" by the PDG, all these s-wave enhancements are obvious in the data, where they correspond to strong attractive interactions in meson-meson scattering at low energies.

Altogether these nine states form an anomalously light nonet of scalar mesons. $J^{\Pi C} = 0^{++}$ $\bar{q}q$ states are expected near the other positive parity mesons ($1^{+\pm}$ & 2^{++}) between 1200 and 1500 MeV. And over the years a nonet and more have been found in this region. Since scalar glueballs are also expected at these mass scales, this domain has important issues of its own. The existence of a scalar nonet above 1 GeV renders the light scalars supernumerary in a $\bar{q}q$ classification scheme. The $\overline{\boldsymbol{Q}}\boldsymbol{Q}$ interpretation and other possibilities have been discussed in two recent, thorough reviews[111, 112].

The diquark model of the scalar mesons is quite straightforward. More detailed descriptions can be found in Refs. [114, 118] in addition to the reviews, Refs. [111, 112]. Briefly: the simplest hadrons made of a scalar diquark and antidiquark are shown schematically in Fig. 11. Fig. 11(a) shows the quark content of mesons composed of the diquark $\overline{\mathbf{3}}_{\rm f}$ and the antidiquark $\mathbf{3}_{\rm f}$ assuming that the strange quark mass effects are treated to first order. At this order, the two isoscalars mix ideally, so one is $\bar{u}\bar{d}ud$ and the other is $\bar{s}s(\bar{u}u + \bar{d}d)$, naturally degenerate with the isovector, $\bar{s}s\bar{d}u$, $\bar{s}s(\bar{u}u - \bar{d}d)$, and $\bar{s}s\bar{u}d$. Fig. 11(b) compares the spectrum of $\overline{\boldsymbol{Q}}\boldsymbol{Q}$ mesons with a traditional $\bar{q}q$ nonet like the vector mesons. The $\overline{\boldsymbol{Q}}\boldsymbol{Q}$ spectrum is inverted. The lightest state is the non-strange isosinglet ($\bar{u}\bar{d}ud$). The heaviest are the degenerate isosinglet and isovector which contain "hidden" $\bar{s}s$ pairs. The four strange states lie in between. In contrast, the ρ and ω are light and degenerate and the predominantly $\bar{s}s$ ϕ meson is heavy. The spectrum of known light-quark scalar mesons is shown in Fig. 11(c), taken from the PDG Tables[21] (the smudges denote the very wide $f_0(600)$ and $\kappa(800)$). The similarity between the pattern of the known light mesons and the $\overline{\boldsymbol{Q}}\boldsymbol{Q}$ states speaks for itself. There is much more to be considered: widths, branching ratios, photon decays, production in $\gamma\gamma$ collisions, etc, all of which are discussed in Refs. [111] and [112].

C. Pentaquarks from diquarks I: The general idea

The diquark picture of pentaquarks follows the same general principles as the description of tetraquark mesons. We assume that the scalar diquark dominates the spectrum. The rest follows from rather simple considerations of the symmetry of the $\boldsymbol{Q}\boldsymbol{Q}$ wavefunction in color, flavor, and space (the spin wavefunction is trivial)[25, 26]. The good diquark is a spinless boson so the $\boldsymbol{Q}\boldsymbol{Q}$ wavefunction must be symmetric under interchange of the two diquarks. The two diquarks must couple to a color $\mathbf{3}_{\rm c}$:

$$\left[\boldsymbol{Q}^{\overline{\mathbf{3}}_{\rm c}}\boldsymbol{Q}^{\overline{\mathbf{3}}_{\rm c}}\right]^{\mathbf{3}_{\rm c}} \tag{V.3}$$

so the $\boldsymbol{Q}\boldsymbol{Q}$ wavefunction is *antisymmetric* in color. Two choices remain: It can be (a) antisymmetric in flavor
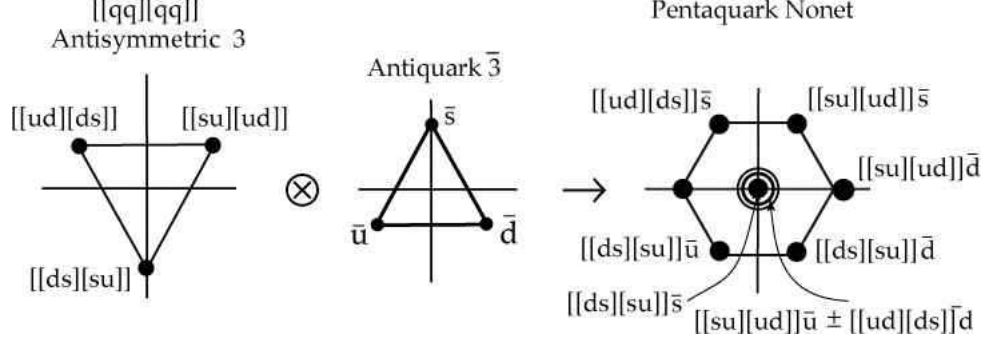
FIG. 12: Odd parity pentaquark nonet: Flavor antisymmetric diquarks, $\mathcal{Q}$, in the $\overline{\mathbf{3}}_{\mathrm{f}}$ representation, are combined antisymmetrically, hence the notation $[[qq][qq]]$, and then combined with the antiquark, $\overline{\mathbf{3}}_{\mathrm{f}}$. The quark structure of most of the nonet states is shown assuming ideal mixing.

and symmetric in space; or (b) symmetric in flavor and antisymmetric in space. Symmetric in flavor means $\mathbf{6}$ and antisymmetric means $\overline{\mathbf{3}}$: $\left[\overline{\mathbf{3}} \otimes \overline{\mathbf{3}}\right]_{\mathcal{S}} = \overline{\mathbf{6}}$, $\left[\overline{\mathbf{3}} \otimes \overline{\mathbf{3}}\right]_{\mathcal{A}} = \mathbf{3}$. Symmetry in space means even parity and a tower of states presumably beginning with $\ell = 0$. Antisymmetry in space means odd parity and a tower beginning with $\ell = 1$.

So the candidates for light pentaquarks in the diquark scheme fall into two categories,

(a) A negative parity nonet with $\mathbf{J^{\Pi} = 1/2^{-}}$

The space, color, and flavor structure of the state is summarized by,

$$\left|[\mathcal{Q}\mathcal{Q}]^{\ell=0,\mathbf{3}_{\mathrm{c}},\mathbf{3}_{\mathrm{f}}} \;\; \bar{q} \;\; ^{j=\frac{1}{2},\overline{\mathbf{3}}_{\mathrm{c}},\overline{\mathbf{3}}_{\mathrm{f}}}\right\rangle^{J^{\Pi}=\frac{1}{2}^{-},\mathbf{1}_{\mathrm{c}},(\mathbf{1}_{\mathrm{f}}\oplus\, \mathbf{8}_{\mathrm{f}})} \tag{V.4}$$

and the quark content of the nine states is summarized in Fig. 12. In the figure, I assume ideal mixing (ie diagonalizing the number of strange quarks), the motivation for which is discussed below.

(b) A positive parity 18-plet (an octet and antidecuplet) with $\mathbf{J^{\Pi} = 1/2^{+}}$ & $\mathbf{3/2^{+}}$

Here the space, color, and flavor structure is summarized by,

$$\left|[\mathcal{Q}\mathcal{Q}]^{\ell=1,\mathbf{3}_{\mathrm{c}},\overline{\mathbf{6}}_{\mathrm{f}}} \;\; \bar{q} \;\; ^{j=\frac{1}{2},\overline{\mathbf{3}}_{\mathrm{c}},\overline{\mathbf{3}}_{\mathrm{f}}}\right\rangle^{J^{\Pi}=(\frac{1}{2}^{+}\oplus\frac{3}{2}^{+}),\mathbf{1}_{\mathrm{c}},(\mathbf{8}_{\mathrm{f}}\oplus\, \overline{\mathbf{10}}_{\mathrm{f}})} \tag{V.5}$$

and the quark content of the eighteen states is summarized in Fig. 13, where, as in the previous case, ideal mixing can be assumed. The figure deserves careful study: the $SU(3)_{\mathrm{f}}$ weight diagrams of the unmixed octet and antidecuplet are shown on the left. The results after ideal mixing are shown on the right. The exotics in the antidecuplet do not mix with the octet. Isospin symmetry precludes mixing between the Λ and the Σ^{0}s or between the $\Xi^{0,-}$ and the $\Phi^{0,-}$. The other states, the Ns and the Σs, mix to diagonalize the number of $\bar{s}s$ pairs. One set has hidden strangeness, the other does not.

It is straightforward to construct the explicit wavefunctions for all these states using the Clebsch-Gordan coefficients for angular momentum and for the symmetric and antisymmetric combinations of color and flavor triplets. There is a small subtlety concerning the phases of the diquark antitriplet states, which is correctly implemented in eq. (IV.4)[119]. The wavefunctions can be found written out in more detail, for example, in Ref. [120].

Which multiplet, the odd parity nonet or the even parity 18-plet, is lighter depends on the quark model dynamics. This is exactly the same question we encountered in the large N_{c} classification of Jenkins and Manohar. Color-spin interactions modeled after one gluon exchange (see eq. (IV.3)) favor the s-wave, ie the odd parity nonet, but there may be Pauli blocking in this state as in the H. This effect would elevate the mass of the negative parity nonet. Flavor-spin interactions, modeled after pseudoscalar meson exchange[83], apparently favor the p-wave (in contrast to the $\bar{q}q$, qqq, and $\bar{q}\bar{q}qq$ sectors where the ground state is always the s-wave), making the even-parity 18-plet the lightest. Whichever way, the diquark picture leads to clear predictions for the light pentaquarks:

- The only potentially light, prominent exotic multiplet is the antidecuplet, which contains candidates for the Θ^{+}, the Φ^{--}, and an as yet unreported Φ^{+}.

- The exotics are accompanied by an non-exotic octet, which mixes with the antidecuplet to give several non-exotic (or "cryptoexotic") analogue states, for example a $[u,d][u,d]\bar{u}$ and $[u,d][u,d]\bar{d}$ pair, which should be lighter than the Θ^{+}.
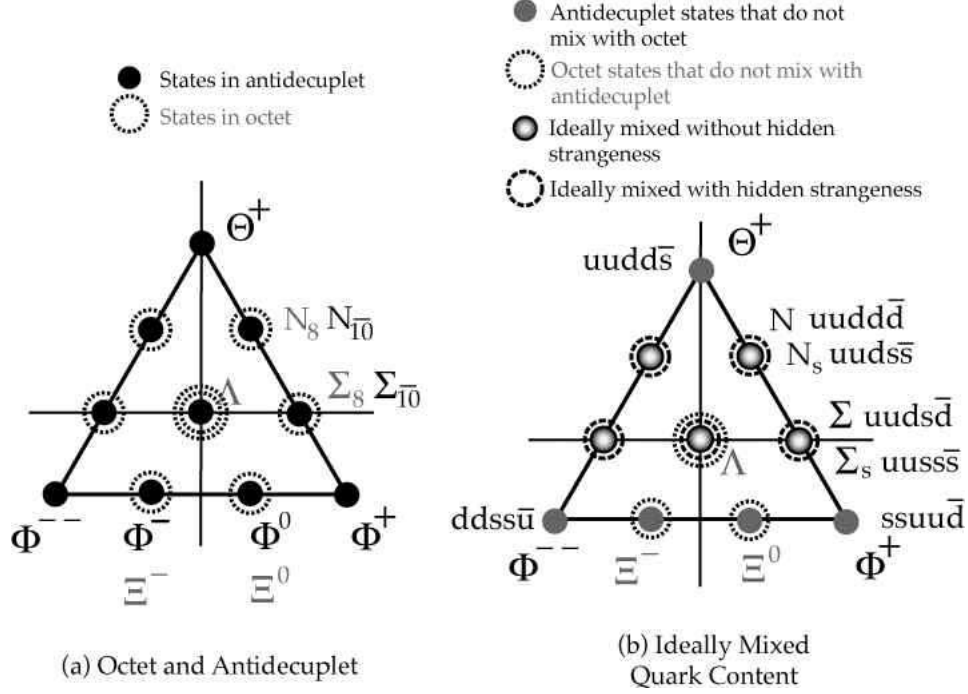
FIG. 13: Even parity pentaquark 18-plet: diquark pairs in the $\overline{\mathbf{6}}_{\mathrm{f}}$ combine with an antiquark in the $\overline{\mathbf{3}}_{\mathrm{f}}$ to make a $\mathbf{8}_{\mathrm{f}}$ and $\overline{\mathbf{10}}_{\mathrm{f}}$. The $SU(3)$ weight diagram for the $\mathbf{8}_{\mathrm{f}}$ and $\overline{\mathbf{10}}_{\mathrm{f}}$ is shown at left, where the unmixed states are named (the decuplet in black, the octet in grey). The ideally mixed states, some with their valence quark content, are shown at right. The exotics (Θ^+, Φ^{--}, and Φ^+) and certain octet states (Λ, Ξ^0, Ξ^-) do not mix if isospin is a good symmetry.

- There are no other light, prominent exotics, like the **27** that figures prominently in the chiral soliton model.

- The Θ^+ should have positive parity.

- The exotics should come in spin-orbit pairs with $J^\Pi = \frac{1}{2}$ and $\frac{3}{2}$.

More predictions include $SU(3)_{\mathrm{f}}$ mass splittings and the existence of charm and bottom analogue states discussed below.

I will have little further to say about the negative parity nonet. These states couple strongly to the meson-nucleon s-wave. The non-strange members of the multiplet contain an $\bar{s}s$ pair and should therefore couple to $N\eta$ and ΛK, not to $N\pi$. Unless these states were below fall apart decay threshold they would be lost in the meson-nucleon continuum. The absence of candidates in the PDG tables should not be surprising.

A word about complications that I have ignored in this presentation: First are the states constructed from the other diquarks: Residual interactions will certainly mix them into the $\mathcal{QQ}\bar{q}$ states, but at zeroth order the good×bad and bad×bad states are ∼200 and ∼400 MeV heavier than the good×good states. If the lightest states in each family are the s-waves — as QCD based interactions prefer — then these states are all well above threshold to fall apart into meson and baryon, and disappear into the continuum. Among them are many exotics, but only one candidate for a negative parity antidecuplet, consistent with our earlier discussion (see eq. (III.4)). Good×bad states lie in the $\overline{\mathbf{3}} \otimes \mathbf{6} \otimes \overline{\mathbf{3}}$ of $SU(3)_{\mathrm{f}}$ which includes the exotic **27**. Bad×bad states lie in the $\mathbf{6} \otimes \mathbf{6} \otimes \overline{\mathbf{3}}$ and include a negative parity antidecuplet (as well as the **35**). So the first candidate for a *negative parity* Θ^+ lies in the "bad-bad" sector and furthermore is created by the same operator that creates KN in an s-wave. So the diquark picture is quite firm that a *negative parity* Θ^+ is much heavier and strongly coupled to the KN s-wave continuum.

Second is mixing between $qqqq\bar{q}$ states and ordinary qqq baryons. Mixing is possible when the $qqqq\bar{q}$ states are not exotic, especially if there are qqq states with the same quantum numbers nearby. Mixing will alter both the spectrum and the decay widths that would otherwise be determined by $SU(3)$ flavor symmetry.

D. Pentaquarks from diquarks II:

A more detailed look at the positive parity octet and antidecuplet

If the Θ^+ and its brethren are confirmed, and if they have positive parity, then the diquark based pentaquark picture seems like a strong candidate for a quark description of the structure. This section and the next are devoted to describing the predictions of the diquark picture in some detail, as presented in Refs. [25, 92, 121]

The states of the positive parity 18-plet are labeled in Fig. 13, with names assigned according to the new PDG conventions: $Y = 2$, $I = 0$ $\Rightarrow \Theta$, $Y = -1$, $I = 3/2$ $\Rightarrow \Phi$; and in the case of residual ambiguity, by appending a subscript "s" to states with hidden $\bar{s}s$ pairs.

1. Flavor SU(3) violation and mass relations

The standard approach to incorporating $SU(3)$ violation in quark spectroscopy is to include the effects of the strange quark mass to lowest order in perturbation theory, which has been perfectly adequate for all qqq baryons and $\bar{q}q$ mesons in the past. The perturbing hamiltonian, $\mathcal{H}'$, is therefore proportional to the $SU(3)_\mathrm{f}$ hypercharge. There is no *a priori* reason to expect it to fail for pentaquarks. At this point there is no reason to assume ideal mixing, so I refer to the $Y = 1$ mass eigenstates as N and N' and the $Y = 0$, $I = 1$ mass eigenstates as Σ and Σ'. This gives eight masses to be fit (in order of decreasing strangeness): Θ^+, N, N', Λ, Σ, Σ', Ξ, and Φ (assuming no isospin violation). The parameters of the fit include: the unperturbed octet and antidecuplet masses, $M_\mathbf{8}$, and $M_{\overline{\mathbf{10}}}$, and the reduced matrix elements of the perturbing Hamiltonian:

$$\langle \overline{\mathbf{10}} \parallel \mathcal{H}' \parallel \overline{\mathbf{10}} \rangle, \quad \langle \overline{\mathbf{10}} \parallel \mathcal{H}' \parallel \mathbf{8} \rangle, \quad \langle \mathbf{8} \parallel \mathcal{H}' \parallel \mathbf{8} \rangle_D, \quad \langle \mathbf{8} \parallel \mathcal{H}' \parallel \mathbf{8} \rangle_F \tag{V.6}$$

where the subscripts F and D refer to the usual duplication of $\mathbf{8}$ in $\mathbf{8} \otimes \mathbf{8}$. Six parameters and eight masses leave two mass relations. One is hopelessly non-linear, the other,

$$2(N + N' + \Xi) = \Sigma + \Sigma' + 3\Lambda + \Theta \tag{V.7}$$

was, to my knowledge, first written down by Diakonov and Petrov[123]. It can be violated if the pentaquark states mix with other, *eg qqq*, multiplets nearby. For example the pentaquark "Roper" can mix with a radially excited nucleon.

It is possible to make more progress by imposing some of the structure suggested by the one-gluon exchange motivated interaction, eq. (IV.3). The Hamiltonian of eq. (IV.3) is color and spin dependent, and depends on the flavor of the quarks explicitly through M_{ij} and implicitly via fermi statistics which correlates flavor with color and spin in states of definite space symmetry. However, $\mathcal{H}_\mathrm{color\ spin}$ does not distinguish between the $\mathbf{8}_\mathrm{f}$ and $\overline{\mathbf{10}}_\mathrm{f}$ in the "final" flavor coupling of $\boldsymbol{QQ}^{\overline{\mathbf{6}}_\mathrm{f}}$ to $\bar{q}^{\overline{\mathbf{3}}_\mathrm{f}}$. Furthermore the $SU(3)$ breaking in the kinetic and confining pieces of the Hamiltonian simply counts the number of s-quarks.[20] As a result, a) the octet and antidecuplet are degenerate in the absence of $SU(3)_\mathrm{f}$ symmetry violation, and b) the symmetry breaking Hamiltonian, $\mathcal{H}'$ acts in the $\boldsymbol{QQ}$ and $\bar{q}$ sectors independently. This leaves only three parameters,

$$M_\mathbf{8} = M_{\overline{\mathbf{10}}}, \quad \langle \{\boldsymbol{QQ}\}\overline{\mathbf{6}}_\mathrm{f} \parallel \mathcal{H}' \parallel \{\boldsymbol{QQ}\}\overline{\mathbf{6}}_\mathrm{f} \rangle, \quad \langle \{\bar{q}\}\overline{\mathbf{3}}_\mathrm{f} \parallel \mathcal{H}' \parallel \{\bar{q}\}\overline{\mathbf{3}}_\mathrm{f} \rangle \tag{V.8}$$

and considerably more predictive power. In Ref. [25] Wilczek and I chose three different parameters which are linear combinations of these,

$$
\begin{aligned}
M_0 &= M_\mathbf{8} + \frac{4}{3}\langle \{\boldsymbol{QQ}\}\overline{\mathbf{6}}_\mathrm{f} \parallel \mathcal{H}' \parallel \{\boldsymbol{QQ}\}\overline{\mathbf{6}}_\mathrm{f} \rangle - \frac{1}{3}\langle \{\bar{q}\}\overline{\mathbf{3}}_\mathrm{f} \parallel \mathcal{H}' \parallel \{\bar{q}\}\overline{\mathbf{3}}_\mathrm{f} \rangle \\
\mu &= \langle \{\bar{q}\}\overline{\mathbf{3}}_\mathrm{f} \parallel \mathcal{H}' \parallel \{\bar{q}\}\overline{\mathbf{3}}_\mathrm{f} \rangle \\
\alpha &= -\langle \{\bar{q}\}\overline{\mathbf{3}}_\mathrm{f} \parallel \mathcal{H}' \parallel \{\bar{q}\}\overline{\mathbf{3}}_\mathrm{f} \rangle - \langle \{\boldsymbol{QQ}\}\overline{\mathbf{6}}_\mathrm{f} \parallel \mathcal{H}' \parallel \{\boldsymbol{QQ}\}\overline{\mathbf{6}}_\mathrm{f} \rangle
\end{aligned} \tag{V.9}
$$

where μ is the matrix element of $m_s\bar{s}s$ and α is $M[u, s] - M[u, d]$.[21] In terms of these,

$$M(N) = M_0$$

---

[20] These observations do not apply to other pictures of the residual quark-quark interactions, like the flavor-spin picture of Refs. [83], where different patterns of $SU(3)_\mathrm{f}$ symmetry violation arise.

[21] In Ref. [25] μ was denoted as m_s engendering some confusion.

$$M(\Theta) = M_0 + \mu$$
$$M(\Lambda) = M(\Sigma) = M_0 + \mu + \alpha$$
$$M(N_s) = M_0 + 2\mu + \alpha$$
$$M(\Phi) = M(\Xi) = M_0 + 2\mu + 2\alpha$$
$$M(\Sigma_s) = M_0 + 3\mu + 2\alpha \tag{V.10}$$

To pin down the spectrum it is necessary to identify some of these $QQ\bar{q}$ states with physical hadrons or import values of the parameters from elsewhere in the hadron spectrum. Before the report of the Φ^{--}, Wilczek and I used the Θ^+ with mass 1540 MeV and the Roper — identified with the N — at 1440 MeV, to extract $\mu \approx 100$ MeV. We took the parameter $\alpha \approx 60$ MeV from the quark model analysis of the Λ-Σ mass difference. The resulting predictions for the masses of the 18-plet — together with possible candidates — are listed in the fourth column (labelled "Mass I") of Table IV.

TABLE IV: Masses and possible assignments of 18-plet pentaquark states in the diquark picture.

Name	Quark content	Mass	Mass I (MeV)[25]	Mass II (MeV)[121]	Possible Assignment	Comments
N	$[ud][ud]\bar{u}, \dots$	M_0	1440	1450	$P_{11}(1440)$ Roper	Width is problematic Input for II
Θ^+	$[ud][ud]\bar{s}$	$M_0 + \mu$	1540	1550	$P_{01}(1540)\ \Theta^+$	Input for II
Λ, Σ	$[ud][su]\bar{d}, \dots$	$M_0 + \mu + \alpha$	1600	1650	$P_{13}(1660)\ \Sigma$	Input for II
N_s	$[ud][su]\bar{s}, \dots$	$M_0 + 2\mu + \alpha$	1700	1750	$P_{11}(1710)$	
Φ, Ξ	$[su][su]\bar{d}, \dots$	$M_0 + 2\mu + 2\alpha$	1760	1850	$\Phi^{--}(1860)$ $\Xi^-(1860)$	
Σ_s	$[su][ds]\bar{s}, \dots$	$M_0 + 3\mu + 2\alpha$	1860	1950	?	

At the time the prediction of a relatively light Φ was rather daring. Now that the Φ^{--} has reported at 1860 MeV, it is appropriate to re-examine these predictions and assignments. In retrospect taking α from the Λ-Σ system may not have been particularly appropriate. In the notation of Section IV, $\alpha = M[u,s] - M[u,d]$, which cannot be extracted reliably from any measured baryon mass differences. The value extracted in Ref. [25] assumed the full color-spin Hamiltonian of eq. (IV.3). Instead in Ref. [121] Wilczek and I propose to identify $\Sigma(1660)$, a $1/2^+$ resonance given three stars by the PDG[21], with the Σ states in the 18-plet. This choice is motivated by a global fit to baryon resonances[122]. We are also mindful that the $N(1710)$ is a candidate for the N_s. Given the widths of these states (the Roper alone has a width of 350 MeV), it makes no sense to quote masses to an accuracy greater than, say, 50 MeV. The resulting alternative mass spectrum is shown in the fifth column of Table IV.

There is an important, qualitative difference between the diquark picture with its 18-plet and other models of the Θ^+ with an antidecuplet alone, which will help sort out the correct physical picture of the exotics. $SU(3)$-flavor splittings within the antidecuplet obey an equal spacing rule, just like the better known decuplet (Δ, Σ^*, Ξ^*, Ω^-) where it is very successful. In an antidecuplet-only picture the Θ^+, $N_{\overline{10}}$, $\Sigma_{\overline{10}}$, and Φ must be spaced at equal mass intervals. In their original paper[11] Diakonov, Petrov, and Polyakov identify the $N_{\overline{10}}$ with the $N(1710)$, which puts the Φ at 2070 MeV, much higher than the quark content ($uudd\bar{s} \rightarrow ddss\bar{u}$) would suggest[25]. If the Φ lies at 1860, the $N_{\overline{10}}$ and $\Sigma_{\overline{10}}$ must lie near 1650 MeV and 1750 MeV respectively. The $N(1710)$ is an imperfect candidate for the first, but there is no candidate for the second excepting a dubious (one-star) P_{11} at 1770. In their revised discussion of the spectrum, Diakonov and Petrov, fit the $\Phi(1860)$ and predict $1/2^+$ N and Σ resonances in the intervals 1650-1690 and 1760-1810 respectively[123]. Weigel has considered mixing of the $\overline{\mathbf{10}}$ with (non-degenerate) radially excited octets and other exotics like $\mathbf{27}$ and $\mathbf{35}$[12, 124], and Diakonov and Polyakov mix the $\overline{\mathbf{10}}$ with the ground state $\mathbf{8}$[123]. Others have proposed non-linear $SU(3)_{\rm f}$ violation, but this seems *ad hoc* given the lack of similar effects elsewhere in the spectrum[125].

In contrast with the $\overline{\mathbf{10}}$-only, the 18-plet picture suggested by diquark arguments allows the Θ^+ and Φ to be interior to the multiplet, with the $\Sigma_s(uuss\bar{s})$ and $N(uudd\bar{u})$ at the top and bottom respectively. The spectrum of the diquark 18-plet picture is contrasted with the antidecuplet-only spectrum in Fig. 14

As explained above there are possible candidates for all the 18-plet states.[22] Time will tell which of these qualitatively different spectra are closest to Nature — provided, of course, the exotics survive the next round of experiments.

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[22] Although the *width* of the Roper presents a problem[126], suggesting that non-exotic $qqqq\bar{q}$ states may mix significantly with qqq states.
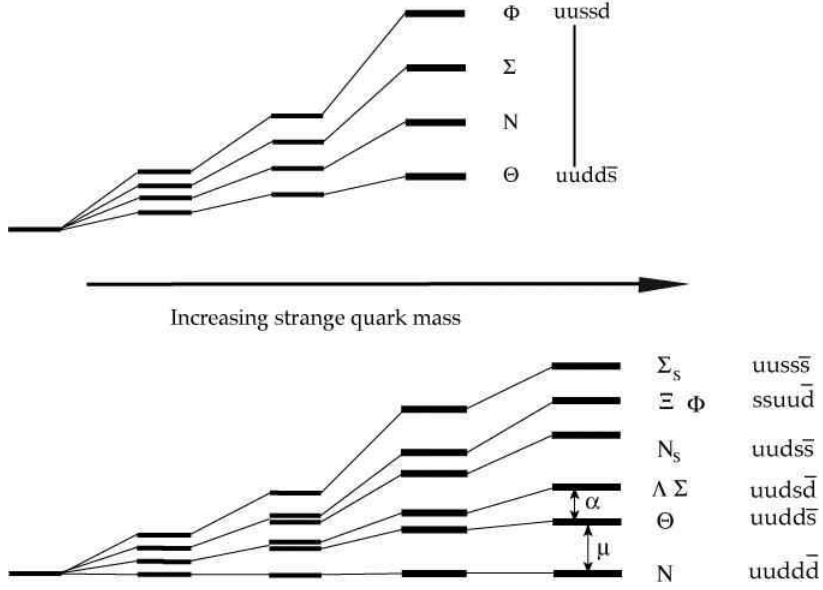
FIG. 14: The $SU(3)_f$ splittings among light $qqqq\bar{q}$ states as a function of m_s. The antidecuplet -only case on top. The 18-plet case below.

2. Isospin and $SU(3)$ selection rules

The existence of *two* nearly degenerate isomultiplets of strangeness minus two pentaquarks (the Ξ and the Φ) is one of the most robust and striking predictions of diquark picture. The degeneracy is exact in the ideal mixing limit described above. Isospin conservation prevents the $\Xi^{-,0}$ and $\Phi^{-,0}$ from mixing, unless they are degenerate within a few MeV (the scale of typical hadronic isospin violation). There are interesting predictions for the decays of these states based on $SU(3)$-flavor selection rules[121, 127]. For example, $SU(3)_f$ forbids the decay of the Φ into a pseudoscalar meson and a member of the baryon decuplet ($\overline{\mathbf{10}} \nrightarrow \mathbf{10} \otimes \mathbf{8}$). The NA49 Group, which has observed the Φ^{--}, can only detect charged particles. They have no evidence for the exotic Φ^+, even though they would be sensitive to $\Phi^+ \rightarrow \Xi^{*0}(1530)\pi^+$. This is consistent with the selection rule because the Φ^+ must be in the $\overline{\mathbf{10}}$ and the $\Xi^{*0}(1530)$ lies in the $\mathbf{10}$. So $SU(3)_f$ seems to be working here. On the other hand, they do have (weak) evidence for a $S = -2$, $Q = -1$ state at 1860 MeV decaying into $\Xi^{*0}(1530)\pi^-$. If this stands up, it identifies this state as the Ξ^-, not the Φ^- (since $\mathbf{8} \rightarrow \mathbf{10} \otimes \mathbf{8}$ is allowed) and supports the existence of degenerate Ξ and Φ pentaquarks. For a more detailed tour of the decay selection rules, see Refs. [121] and [127].

E. Pentaquark from diquarks III: Charm and bottom analogues

Charm and bottom analogues of the Θ^+ can be obtained by substituting the heavy $\bar{c}$ or $\bar{b}$ quark for the $\bar{s}$ in the Θ^+,

$$\Theta_c^0 = |[u,d][u,d]\bar{c}\rangle \qquad \Theta_b^+ = |[u,d][u,d]\bar{b}\rangle \qquad (V.11)$$

Charm pentaquark exotics[23]were predicted many years ago in quark models with flavor dependent interactions, but were not taken very seriously at the time[128]. The existence of the $\Theta_s^+ \equiv \Theta^+(1540)$ fixes the mass scale for exotics and leads to rather robust predictions of the masses of the Θ_c^0 and Θ_b^+. The simplest, though not necessarily the least accurate approach, is to find an analogy among qqQ baryons and apply it to the $\mathcal{Q}\mathcal{Q}\overline{Q}$ system. The obvious choice is the Λ_Q, which has the quark content $\mathcal{Q}\mathcal{Q}$. The heavy anti-quark in the Θ_Q sits in the background of an isosinglet, color $\mathbf{3}$, spin singlet pair of diquarks. The heavy quark in the $\mathcal{Q}\mathcal{Q}$ sits in a background *identical* in isospin, color, and

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[23] The states predicted by Lipkin and Gignoux *et al* were not analogues of the Θ^+. They have the quantum numbers of the states discussed by Stewart, Wessling, and Wise[129].

spin. The only difference is that the spin of the $\overline{Q}$ in the Θ_Q can interact with the orbital angular momentum ($\ell = 1$) in the Θ_Q and this interaction is not present in the Λ_Q. Were it not for this, one would expect the relations

$$M(\mathbf{QQ}\bar{c}) - M(\mathbf{QQ}\bar{s}) = M(\mathbf{Q}c) - M(\mathbf{Q}s) \quad \text{that is} \quad M(\Theta_c^0) - M(\Theta_s^+) = M(\Lambda_c) - M(\Lambda)$$
$$M(\mathbf{QQ}\bar{b}) - M(\mathbf{QQ}\bar{c}) = M(\mathbf{Q}b) - M(\mathbf{Q}c) \quad \text{that is} \quad M(\Theta_b^+) - M(\Theta_c^0) = M(\Lambda_b) - M(\Lambda_c)$$

$$\text{(V.12)}$$

to be nearly exact. QCD spin-orbit interactions are not strong, so these rules should not be badly violated. Because these interactions vanish as $m_Q \to \infty$, the second relation should be quite accurate. The differences among the predictions of various QCD based quark models reflect the different ways that the residual interactions are treated. Taking eqs. (V.12) as is, Wilczek and I estimated,

$$M(\Theta_c^0) = 2710 \text{ MeV} \quad \text{and} \quad M(\Theta_b^+) = 6050 \text{ MeV} \tag{V.13}$$

If these estimates are correct, the Θ_c^0 and Θ_b^+ will be stable against strong decay. The lightest strong decay channel for the Θ_c^0 is ND with a threshold at 2805 MeV, and for the Θ_b^+, it is NB with a threshold at 6220 MeV. They would have to decay weakly with lifetimes of order 10^{-12} sec.

How did this happen? The Θ_s^+ is light, but it is not stable. The reason lies not in the linear scaling of the masses of the heavy pentaquarks with the heavy quark mass, but rather in the *non-linear* scaling of the pseudoscalar meson masses, which determine the strong decay thresholds. Consider the four analogue states, $[u,d][u,d]\overline{Q}$, with $Q = u, s, c, b$, and identify the Θ_u^0 with the Roper as I advocated earlier. Then

$$\begin{aligned}
\Theta_u^0 &\to N\pi &&\text{has decay } \mathcal{Q}\text{-value} &&\mathcal{Q} \approx 350 \text{ MeV} \\
\Theta_s^+ &\to NK &&\text{has decay } \mathcal{Q}\text{-value} &&\mathcal{Q} \approx 100 \text{ MeV} \\
\Theta_c^0 &\to ND &&\text{has decay } \mathcal{Q}\text{-value} &&\mathcal{Q} \approx -100 \text{ MeV} \\
\Theta_b^0 &\to NB &&\text{has decay } \mathcal{Q}\text{-value} &&\mathcal{Q} \approx -150 \text{ MeV}
\end{aligned} \tag{V.14}$$

The Θ_u^0, *ie* the Roper, is unstable because the pion is anomalously light, a consequence of approximate chiral symmetry. The effect is still significant enough for the kaon to make the Θ_s^+ unstable. The D and B-meson masses are not significantly lowered by chiral symmetry, the thresholds are proportionately higher, and the Θ_c^0 and Θ_b^+ are stable. The details are model dependent. Other model estimates are generally higher than the simple scaling law described here[130], some predict stable c and b-exotics, others predict light and narrow, but not stable states.

For the record, here is a list of some of the more obvious (Cabibbo allowed) weak decay modes,

$$\begin{aligned}
\Theta_c^0 &\to pK^+\pi^-, \ pK_S\pi^-, \ \Theta_s^+\pi^-, \ldots \\
\Theta_b^+ &\to p\overline{D}^0, \ p\pi^+D^-, \ \Theta_c^0\pi^+, \ pJ/\psi K_S, \ pJ/\psi K^+\pi^-, \ \Theta_s^+ J/\psi \ (!)\ldots
\end{aligned} \tag{V.15}$$

and many more.

When the light antiquark is replaced by a charm or bottom, the previously *non-exotic*, negative parity baryons of eq. (V.4) and Fig. 12 become exotic[128, 129]. They form a $\overline{\mathbf{3}}$ of $SU(3)_f$ with valence quark content $\{[u,d][d,s]\overline{Q},$ $[d,s][s,u]\overline{Q}, [s,u][u,d]\overline{Q}\}$. If the s-wave pentaquarks are lighter than the p-wave, then these charm and bottom exotics would be even more tightly bound than the Θ^+-analogues. With such strong attraction that the ground states are stable, it is not surprising that models predict bound or at least very narrow excited states as well [130].

The exotic charm baryon reported by H1 is not bound. With a mass of 3099 MeV, it is much too heavy to be the Θ_c^0 as I have described it. The width is reported to be less than 12 MeV. It has been observed through its strong decay into $D^{*-}p$, into which it has a $\mathcal{Q}$ value of 150 MeV. If it were the Θ_c^0, and if it has $J^\Pi = 1/2^+$, it would have a significant decay into D^-p (which would not have been seen at H1), with a partial width that can be related to the width of the Θ_s^+ by scaling p-wave phase space. The result is $\Gamma(3099)/\Gamma(\Theta_s^+) \approx 15$, barely consistent with the H1 limit if the width of the Θ_s^+ is 1 MeV. An interesting possibility — if the 3099 state should be confirmed — is that it is an $L = 2$ Regge excitation of the Θ_c^0 with $J^\Pi = 3/2-$. This object can decay into $D^{*-}p$ in the s-wave, but D^-p in the d-wave, accounting perhaps for its surprisingly narrow width. Should this assignment prove correct, there must be many other excited charm exotic baryons awaiting discovery.

Clearly, if the initial reports are confirmed, there is a fascinating spectroscopy of heavy exotic baryons awaiting us. But it is a big "if"!

F. A paradigm for spectroscopy

If diquarks are as important in exotic spectroscopy as I have suggested, their role in ordinary meson and baryon spectroscopy should be re-examined. For decades it has been traditional to classify hadron resonances according to

the rules of the non-relativistic quark model: essentially $SU(N_{\rm f}) \times O(3)$. The latest checklist can be found in the 2004 Particle Data Tables[21]. To the extent that diquarks dominate the structure of hadrons, baryons are more like mesons, $q^{\mathbf{3}_{\rm c}} - \{qq\}^{\overline{\mathbf{3}}_{\rm c}}$ in analogy to $q^{\mathbf{3}_{\rm c}} - \bar{q}^{\overline{\mathbf{3}}_{\rm c}}$, than the three-body bound states implicit in the quark model classification. Meson quantum numbers *and masses* can be understood using a mix of ideas from QCD and Regge theory: quarks and antiquarks on the ends of flux-tubes (or strings) with a spectrum determined by the Chew-Frautschi formula, $M^2 = \sigma L + \alpha$, where σ is the universal slope of Regge-trajectories (the "string tension"). In a diquark paradigm baryons should lie on trajectories with the *same slope*, and furthermore group into families depending on the nature of the diquark ("good" versus "bad") and the coupling of its spin to the orbital excitation. This classification program has been carried out by Selem and Wilczek[122], and leads to a compact and predictive unified picture of mesons, baryons, and tetraquarks. One of their observations is that the diquark correlation appears to become stronger in Regge-extended hadrons. In particular, the diquark-quark mass difference, which we called $M[u,d] - M[u]$, becomes smaller — evidence that the diquark correlation is medium dependent. This is a rich subject, but well beyond the scope of this review.

VI. CONCLUSIONS

There are two distinct, but related issues at the core of this discussion: first, a question: are there light, prominent exotic baryons, and if so, what is the best dynamical framework in which to study them? and second, a proposal: diquark correlations are important in QCD spectroscopy, especially in multiquark systems, where they account naturally for the principal features.

I believe the case for diquarks is already quite compelling. There are many projects ahead: re-evaluating the qqq spectrum[122]; systematically exploring the role of diquarks in deep inelastic distribution and fragmentation functions, and in scaling violation; seeing if diquarks can help in other areas of hadron phenomenology like form-factors, low p_T particle production, and polarization phenomena; developing a more sophisticated treatment of quark correlations, recognizing that diquarks are far from pointlike inside hadrons; establishing diquark parameters and looking for diquark structure in hadrons using lattice QCD; and — the holy grail of this subject — seeking a more fundamental and quantitative phenomenological paradigm for light quark dynamics at the confinement scale. Diquark advocates have considered many of these issues in the past[29]. No doubt many other important contributions, like the diquark analysis of the $\Delta I = 1/2$-rule[32], have already been accomplished. We can hope eventually to have as sophisticated an understanding of diquark correlations as we have of $\bar{q}q$ correlations, as expressed in chiral dynamics.

The situation with the Θ^+ is less clear. Of course it will eventually be clarified by experiment — a virtue of working on QCD as opposed to string theory! However, theorists' attempts to understand the Θ^+ have raised more questions than they have answered. To wit:

- A negative parity (KN s-wave) Θ^+ is intolerable to theorists, but that is what lattice studies find, if they find anything at all.

- No one has come up with a simple, qualitative explanation for the exceptionally narrow width of the Θ^+.

- The original prediction of a narrow, light Θ^+ in the chiral soliton model does not appear to be robust.

- Quark models can accomodate the Θ^+, but only by reversing the naive, and heretofore universal, parity of the $q^{n_q}\bar{q}^{n_{\bar{q}}}$ ground state. It is necessary to excite the quarks in order to capture the correlation energy of the good diquarks. This does not sound like a way to make an exceptionally light and stable pentaquark.

- When models are adjusted to accomodate the Θ^+, they predict the existence of other states that should have been observed by now: The diquark picture wants both a $\Theta^{\frac{1}{2}^+}$ and a $\Theta^{\frac{3}{2}^+}$; the CSM and large $N_{\rm c}$ want a relatively light $\mathbf{27}$, which includes an $I = 1$ triplet: $\Theta^{*0}, \Theta^{*+}, \Theta^{*++}$.

None of these problems seems insuperable. Indeed, there are papers appearing every day that propose an interesting solution to one or another. Taken together, however, they are an impressive set. They leave us in limbo: Either the Θ^+ will go away, or it will force us to rewrite several chapters of the book on QCD.

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