

Covariant L-S Scheme for the effective N^*NM couplings

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Abstract

For excited nucleon states N^* of arbitrary spin coupling to nucleon (N) and meson (M), we propose a Lorentz covariant orbital-spin (L-S) scheme for the effective N^*NM couplings. To be used for the partial wave analysis of various N^* production and decay processes, it combines merits of two conventional schemes, *i.e.*, covariant effective Lagrangian approach and multipole analysis with amplitudes expanded according to angular momentum L. As examples, explicit formulae are given for $N^* \rightarrow N\pi$, $N^* \rightarrow N\omega$ and $\psi \rightarrow N^*\bar{N}$ processes which are under current experimental studies.

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1 Introduction

The study of the nucleon and its excited states N^* can provide us with critical insights into the nature of QCD in the confinement domain [1]. They are the simplest system in which the three colors of QCD neutralize into colorless objects and the essential nonabelian character of QCD is manifest. However our present knowledge on the N^* spectroscopy is still very poor, with information coming almost entirely from the old generation of πN experiments of more than twenty years ago [2] and with many fundamental issues not well understood[3]. Considering its importance for the understanding of the nonperturbative QCD, much effort has been devoted to the study of the N^* spectrum. A series of new experiments on N^* physics with electromagnetic probes have been started at modern facilities such as TJNAF[4], ELSA[5], GRAAL[6], SPRING8[7] and BEPC[8].

Abundant data have been accumulated for various N^* production and decay channels at these facilities in last few years. Now an important task facing us is to perform partial wave amplitude analysis (PWA) of these data to extract properties of N^* resonances, such as their spin-parity, mass, width, decay branching ratios,

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and so on. For πN or γN to meson-nucleon final states, the most commonly used PWA formalism is the multipole analysis with amplitudes expanded according to angular momentum L of meson-nucleon system[9, 10, 11, 12, 13]. This formalism is usually written in the meson-nucleon CM system, not in a covariant form, hence not very convenient to be used for multi-step chain processes, such as $J/\psi \rightarrow N^* \bar{N}$ with N^* further decaying to meson-nucleon. For a multi-step chain process, the covariant effective Lagrangian approach[14, 15, 16, 17, 8] is more convenient. In this approach, the effective N^*NM couplings are constructed by Rarita-Schwinger wave functions for particles of arbitrary spin[18], 4-momenta of involved particles, Dirac γ matrices, etc., with constraint of general symmetries required by the strong interaction. A problem for this approach is that the amplitude is usually a mixture of various orbital angular momenta L . Hence the usual centrifugal barrier (Blatt-Weisskopf) factor[12, 19], commonly used in multipole analysis and mesonic decays, cannot be used here since the barrier factor is L -dependent. Instead vertex form factors with exponential form or other forms are used in the effective Lagrangian approach. This makes comparison to results from usual multipole approach very difficult.

In this paper we propose a covariant L-S Scheme for the effective N^*NM couplings to be used for the partial wave analysis of N^* data. In this scheme, the amplitudes are expanded according to the orbital angular momentum L of two decay products, meanwhile Lorentz invariant. Hence it combines the merits of multipole analysis and the effective Lagrangian approach.

2 General Formalism

In our construction of the covariant L-S Scheme for the effective N^*NM couplings, we need to combine some knowledge from the covariant tensor formalism for meson decays[19] and covariant wave functions for hadrons of arbitrary spin[20].

For a given hadronic decay process $A \rightarrow BC$, in the L-S scheme on hadronic level, the initial state is described by its 4-momentum P_μ and its spin state $\mathbf{S}_A$; the final state is described by the relative orbital angular momentum state of BC system $\mathbf{L}_{BC}$ and their spin states ($\mathbf{S}_B, \mathbf{S}_C$).

The spin states ($\mathbf{S}_A, \mathbf{S}_B, \mathbf{S}_C$) can be well represented by the relativistic Rarita-Schwinger spin wave functions for particles of arbitrary spin[18, 19, 21, 17]. The spin- $\frac{1}{2}$ wavefunction is the standard Dirac spinor $u(p,s)$ or $v(p,s)$ and the spin-1 wave function is the standard spin-1 polarization four-vector $\varepsilon^\mu(p,s)$ for particle with momentum p and spin projection s .

$$\sum_{s=0,\pm 1} \varepsilon_\mu(p,s) \varepsilon_\nu^*(p,s) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \equiv \tilde{g}_{\mu\nu}(p). \quad (1)$$

Spin wave functions for particles of higher spins are constructed from these two basic spin wave functions with C-G coefficients $(j_1, j_{1z}; j_2, j_{2z} | j, j_z)$ as the following:

$$\varepsilon_{\mu_1 \mu_2 \dots \mu_n}(p, n, s) = \sum_{s_{n-1}, s_n} (n-1, s_{n-1}; 1, s_n | n, s) \varepsilon_{\mu_1 \mu_2 \dots \mu_{n-1}}(p, n-1, s_{n-1}) \varepsilon_{\mu_n}(p, s_n) \quad (2)$$

for a particle with integer spin $n \geq 2$, and

$$u_{\mu_1 \mu_2 \dots \mu_n}(p, n + \frac{1}{2}, s) = \sum_{s_n, s_{n+1}} (n, s_n; \frac{1}{2}, s_{n+1} | n + \frac{1}{2}, s) \varepsilon_{\mu_1 \mu_2 \dots \mu_n}(p, n-1, s_n) u(p, s_{n+1}) \quad (3)$$

for a particle with half integer spin $n + \frac{1}{2}$ of $n \geq 1$.

The orbital angular momentum $\mathbf{L}_{\mathbf{BC}}$ state can be represented by covariant tensor wave functions $\tilde{t}_{\mu_1 \dots \mu_L}^{(L)}$ as the same as for meson decay[19]. Define $r = p_B - p_C$, then

$$\tilde{t}^{(0)} = 1, \quad (4)$$

$$\tilde{t}_{\mu}^{(1)} = \tilde{g}_{\mu\nu}(p_A) r^{\nu} \equiv \tilde{r}_{\mu}, \quad (5)$$

$$\tilde{t}_{\mu\nu}^{(2)} = \tilde{r}_{\mu} \tilde{r}_{\nu} - \frac{1}{3}(\tilde{r} \cdot \tilde{r}) \tilde{g}_{\mu\nu}, \quad (6)$$

$$\begin{aligned} \tilde{t}_{\mu\nu\lambda}^{(3)} &= \tilde{r}_{\mu} \tilde{r}_{\nu} \tilde{r}_{\lambda} - \frac{1}{5}(\tilde{r} \cdot \tilde{r})(\tilde{g}_{\mu\nu} \tilde{r}_{\lambda} + \tilde{g}_{\nu\lambda} \tilde{r}_{\mu} + \tilde{g}_{\lambda\mu} \tilde{r}_{\nu}), \\ &\dots \end{aligned} \quad (7)$$

In the L-S scheme, we need to use the conservation relation of total angular momentum:

$$\mathbf{S}_{\mathbf{A}} = \mathbf{S}_{\mathbf{B}} + \mathbf{S}_{\mathbf{C}} + \mathbf{L}_{\mathbf{BC}} \quad \text{or} \quad -\mathbf{S}_{\mathbf{A}} + \mathbf{S}_{\mathbf{B}} + \mathbf{S}_{\mathbf{C}} + \mathbf{L}_{\mathbf{BC}} = 0. \quad (8)$$

Comparing with the pure meson case[19], here for N^*NM couplings we need to introduce the concept of relativistic total spin of two fermions.

For the case of A as a meson, B as N^* with spin $n + \frac{1}{2}$ and C as $\bar{N}$ with spin- $\frac{1}{2}$, the total spin of BC ($\mathbf{S}_{\mathbf{BC}}$) can be either n or $n + 1$. The two $\mathbf{S}_{\mathbf{BC}}$ states can be represented as

$$\psi_{\mu_1 \dots \mu_n}^{(n)} = \bar{u}_{\mu_1 \dots \mu_n}(p_B, s_B) \gamma_5 v(p_C, s_C), \quad (9)$$

$$\begin{aligned} \Psi_{\mu_1 \dots \mu_{n+1}}^{(n+1)} &= \bar{u}_{\mu_1 \dots \mu_n}(p_B, s_B) \left(\gamma_{\mu_{n+1}} - \frac{r_{\mu_{n+1}}}{m_A + m_B + m_C} \right) v(p_C, s_C) \\ &\quad + (\mu_1 \leftrightarrow \mu_{n+1}) + \dots + (\mu_n \leftrightarrow \mu_{n+1}) \end{aligned} \quad (10)$$

for $\mathbf{S}_{\mathbf{BC}}$ of n and $n + 1$, respectively. As a special case of $n = 0$, we have

$$\psi^{(0)} = \bar{u}(p_B, s_B) \gamma_5 v(p_C, s_C), \quad (11)$$

$$\Psi_{\mu}^{(1)} = \bar{u}(p_B, s_B) \left(\gamma_{\mu} - \frac{r_{\mu}}{m_A + m_B + m_C} \right) v(p_C, s_C). \quad (12)$$

Here r_μ term is necessary to cancel out the $\hat{\mathbf{p}}$ -dependent component in the simple $\bar{u}\gamma_\mu v$ expression. In the A at-rest system, we have

$$\psi^{(0)} = C_\psi(-1)^{\frac{1}{2}-s_C}\delta_{s_B(-s_C)}, \quad (13)$$

$$\Psi_i^{(1)} = C_\Psi(-1)^{\frac{1}{2}-s_C}\chi_{s_B}^\dagger\sigma_i\chi_{-s_C} \quad (14)$$

with two-component Pauli spinors $\chi_{1/2}^\dagger = (1, 0)$ and $\chi_{-1/2}^\dagger = (0, 1)$, and

$$C_\psi = \frac{(E_B + m_B)(E_C + m_C) + \mathbf{p}_\mathbf{C}^2}{\sqrt{2m_B 2m_C(E_B + m_B)(E_C + m_C)}}, \quad (15)$$

$$C_\Psi = \sqrt{\frac{(E_B + m_B)(E_C + m_C)}{2m_B 2m_C}} \left(1 + \frac{\mathbf{p}_\mathbf{C}^2}{(E_B + m_B)(E_C + m_C)} \right). \quad (16)$$

In the non-relativistic limit, both C_ψ and C_Ψ are equal to 1. Generally both of them have some smooth dependence on the magnitude of momentum. But both $\psi^{(0)}$ and $\Psi_\mu^{(1)}$ have no dependence on the direction of the momentum $\hat{\mathbf{p}}$, hence correspond to pure spin states with the total spin of 0 and 1, respectively.

For the case of A as N^* with spin $n + \frac{1}{2}$, B as N and C as a meson, one needs to couple $-\mathbf{S}_\mathbf{A}$ and $\mathbf{S}_\mathbf{B}$ first to get $\mathbf{S}_{\mathbf{AB}} \equiv -\mathbf{S}_\mathbf{A} + \mathbf{S}_\mathbf{B}$ states, which are

$$\phi_{\mu_1 \dots \mu_n}^{(n)} = \bar{u}(p_B, s_B) u_{\mu_1 \dots \mu_n}(p_A, s_A), \quad (17)$$

$$\Phi_{\mu_1 \dots \mu_{n+1}}^{(n+1)} = \bar{u}(p_B, s_B) \gamma_5 \tilde{\gamma}_{\mu_{n+1}} u_{\mu_1 \dots \mu_n}(p_A, s_A) + (\mu_1 \leftrightarrow \mu_{n+1}) + \dots + (\mu_n \leftrightarrow \mu_{n+1}) \quad (18)$$

for $\mathbf{S}_{\mathbf{AB}}$ of n and $n + 1$, respectively.

$$\phi^{(0)} = \bar{u}(p_B, s_B) u(p_A, s_A), \quad (19)$$

$$\Phi_\mu^{(1)} = \bar{u}(p_B, s_B) \gamma_5 \tilde{\gamma}_\mu u(p_A, s_A) \quad (20)$$

with $\tilde{\gamma}_\mu = \tilde{g}_{\mu\nu}(p_A)\gamma^\nu$. In the A (N^*) at-rest system, we have

$$\phi^{(0)} = \sqrt{\frac{(E_A + m_A)(E_B + m_B)}{2m_A 2m_B}} \delta_{s_A s_B}, \quad (21)$$

$$\Phi_i^{(1)} = -\sqrt{\frac{(E_A + m_A)(E_B + m_B)}{2m_A 2m_B}} \chi_{s_B}^\dagger \sigma_i \chi_{s_A}. \quad (22)$$

Both have no dependence on the direction of the momentum $\hat{\mathbf{p}}$.

In effective Lagrangian approaches, the effective N^*NM couplings are constructed by p_A , r , $g^{\mu\nu}$, γ^μ , u or v , and may be mixture of various orbital angular momentum states. In our proposed covariant L-S scheme, the effective N^*NM couplings should be composed of p_A , $\tilde{t}^{(L)}$, $g^{\mu\nu}$, $\epsilon_{\alpha\beta\gamma\delta}$ (the full anti-symmetric tensor), ψ (Ψ) or ϕ (Φ), corresponding to a pure orbital angular momentum L state. Then the procedure

for constructing the effective N^*NM couplings is very similar to the case for pure mesons[19]. First the parity should be conserved, which means

$$\eta_A = \eta_B \eta_C (-1)^L \quad (23)$$

where η_A , η_B and η_C are the intrinsic parities of particles A, B and C, respectively. From this relation, one knows whether L should be even or odd. Then from Eq.(8) one can figure out how many different L-S combinations, which determine the number of independent couplings. For a final state with orbital angular momentum of L, $\tilde{t}^{(L)}$ should appear once in the effective coupling without any other $\tilde{t}$ or r . This will guarantee a pure L final state. Then one can easily put into the Blatt-Weisskopf centrifugal barrier factor for each effective coupling with L final state if one wishes. We shall show the concrete procedure by examples in the following section.

3 Examples

We shall start with the simplest case for $N^* \rightarrow N\pi$ process, then for $N^* \rightarrow N\omega$ and $\psi \rightarrow N^* \bar{N}$ where ψ can be J/ψ or ψ' or any other heavy vector mesons.

3.1 $N^* \rightarrow N\pi$

For $N^* \rightarrow N\pi$, it is well known that only one possible L-S coupling for the $N\pi$ final state of each N^* decay. Since the nucleon has spin-parity $\frac{1}{2}^+$ and pion has spin-parity 0^- , $N^*(\frac{1}{2}^+)$ can only decay to $N\pi$ in P-wave with $S_{AB} = 1$ to make $-\mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C + \mathbf{L}_{BC} = \mathbf{S}_{AB} + \mathbf{L}_{BC} = 0$ meanwhile satisfying parity conservation relation Eq.(23). Similarly we have $N^*(\frac{1}{2}^-) \rightarrow N\pi$ in S-wave with $S_{AB} = 0$; $N^*(\frac{3}{2}^+) \rightarrow N\pi$ in P-wave with $S_{AB} = 1$; $N^*(\frac{3}{2}^-) \rightarrow N\pi$ in D-wave; $N^*(\frac{5}{2}^+) \rightarrow N\pi$ in F-wave with $S_{AB} = 3$; $N^*(\frac{5}{2}^-) \rightarrow N\pi$ in D-wave with $S_{AB} = 2$; $N^*(\frac{7}{2}^+) \rightarrow N\pi$ in F-wave with $S_{AB} = 3$; $N^*(\frac{7}{2}^-) \rightarrow N\pi$ in G-wave with $S_{AB} = 4$; and so on. Then the effective $N^*N\pi$ couplings in the covariant L-S scheme are

$$N^*(\frac{1}{2}^+) \rightarrow N\pi : \quad \Phi_\mu^{(1)} \tilde{t}^{(1)\mu}, \quad (24)$$

$$N^*(\frac{1}{2}^-) \rightarrow N\pi : \quad \phi^{(0)} \tilde{t}^{(0)}, \quad (25)$$

$$N^*(\frac{3}{2}^+) \rightarrow N\pi : \quad \phi_\mu^{(1)} \tilde{t}^{(1)\mu}, \quad (26)$$

$$N^*(\frac{3}{2}^-) \rightarrow N\pi : \quad \Phi_{\mu\nu}^{(2)} \tilde{t}^{(2)\mu\nu}, \quad (27)$$

$$N^*(\frac{5}{2}^+) \rightarrow N\pi : \quad \Phi_{\mu\nu\lambda}^{(3)} \tilde{t}^{(3)\mu\nu\lambda}, \quad (28)$$

$$N^*(\frac{5^-}{2}) \rightarrow N\pi : \quad \phi_{\mu\nu}^{(2)} \tilde{t}^{(2)\mu\nu}, \quad (29)$$

$$N^*(\frac{7^+}{2}) \rightarrow N\pi : \quad \phi_{\mu\nu\lambda}^{(3)} \tilde{t}^{(3)\mu\nu\lambda}, \quad (30)$$

$$N^*(\frac{7^-}{2}) \rightarrow N\pi : \quad \Phi_{\mu\nu\lambda\delta}^{(4)} \tilde{t}^{(4)\mu\nu\lambda\delta}. \quad (31)$$

Here for simplicity we omit the vertex form factors. With properties of Rarita-Schwinger wave functions

$$\gamma^{\mu_i} u_{\dots\mu_i\dots} = 0 \quad \text{and} \quad p^{\mu_i} u_{\dots\mu_i\dots}(p, s) = 0 \quad (32)$$

one can easily get the relation between the covariant L-S couplings and the usual effective Lagrangian ones

$$N^*(\frac{1^+}{2}) \rightarrow N\pi : \quad \Phi_{\mu}^{(1)} \tilde{t}^{(1)\mu} = \bar{u}_N \gamma_5 \gamma_{\mu} u_{*} p_{\pi}^{\mu} \cdot C_{\Phi}, \quad (33)$$

$$N^*(\frac{1^-}{2}) \rightarrow N\pi : \quad \phi^{(0)} \tilde{t}^{(0)} = \bar{u}_N u_{*} \cdot 1, \quad (34)$$

$$N^*(\frac{3^+}{2}) \rightarrow N\pi : \quad \phi_{\mu}^{(1)} \tilde{t}^{(1)\mu} = \bar{u}_N u_{*} p_{\pi}^{\mu} \cdot 2, \quad (35)$$

$$N^*(\frac{3^-}{2}) \rightarrow N\pi : \quad \Phi_{\mu\nu}^{(2)} \tilde{t}^{(2)\mu\nu} = \bar{u}_N \gamma_5 \gamma_{\mu} u_{*\nu} p_{\pi}^{\mu} p_{\pi}^{\nu} \cdot 4C_{\Phi}, \quad (36)$$

$$N^*(\frac{5^+}{2}) \rightarrow N\pi : \quad \Phi_{\mu\nu\lambda}^{(3)} \tilde{t}^{(3)\mu\nu\lambda} = \bar{u}_N \gamma_5 \gamma_{\mu} u_{*\nu\lambda} p_{\pi}^{\mu} p_{\pi}^{\nu} p_{\pi}^{\lambda} \cdot 12C_{\Phi}, \quad (37)$$

$$N^*(\frac{5^-}{2}) \rightarrow N\pi : \quad \phi_{\mu\nu}^{(2)} \tilde{t}^{(2)\mu\nu} = \bar{u}_N u_{*\mu\nu} p_{\pi}^{\mu} p_{\pi}^{\nu} \cdot 4, \quad (38)$$

$$N^*(\frac{7^+}{2}) \rightarrow N\pi : \quad \phi_{\mu\nu\lambda}^{(3)} \tilde{t}^{(3)\mu\nu\lambda} = \bar{u}_N u_{*\mu\nu\lambda} p_{\pi}^{\mu} p_{\pi}^{\nu} p_{\pi}^{\lambda} \cdot 8, \quad (39)$$

$$N^*(\frac{7^-}{2}) \rightarrow N\pi : \quad \Phi_{\mu\nu\lambda\delta}^{(4)} \tilde{t}^{(4)\mu\nu\lambda\delta} = \bar{u}_N \gamma_5 \gamma_{\mu} u_{*\nu\lambda\delta} p_{\pi}^{\mu} p_{\pi}^{\nu} p_{\pi}^{\lambda} p_{\pi}^{\delta} \cdot 48C_{\Phi} \quad (40)$$

with

$$C_{\Phi} = (1 + \frac{m_N}{m_*} - \frac{m_{\pi}^2}{m_*^2 + m_* m_N}), \quad (41)$$

u_N, u_* the Rarita-Schwinger wave functions of N and N^* , respectively; m_N, m_* the mass of N and N^* , respectively; p_{π} the four-momentum of the pion. We see the two approaches are equivalent here up to some constants or a smooth m_* dependent factor C_{Φ} . This is because for any $N^* \rightarrow N\pi$ process there is only one possible L-S coupling and hence only one independent coupling.

3.2 $N^* \rightarrow N\omega$

Unlike pion with spin 0, here ω has spin 1. For N^* with spin $\frac{1}{2}$ there are two independent L-S couplings conserving parity (23) and total angular momentum (8);

for N^* with spin larger than $\frac{1}{2}$, there are three independent L-S couplings. Here we list them for N^* with spin up to $\frac{7}{2}$.

$$(S_C, S_{AB}, L_{BC}) : \mathbf{S}_{AB} + \mathbf{S}_C + \mathbf{L}_{BC} = 0$$

$$N^*(\frac{1}{2}^+) \rightarrow N\omega \quad (1, 0, 1) : \phi^{(0)} \varepsilon_\mu^* \tilde{t}^{(1)\mu}, \quad (42)$$

$$(1, 1, 1) : i\Phi_\mu^{(1)} \epsilon^{\mu\nu\lambda\sigma} \varepsilon_\nu^* \tilde{t}_\lambda^{(1)} \hat{p}_{*\sigma}, \quad (43)$$

$$N^*(\frac{1}{2}^-) \rightarrow N\omega \quad (1, 1, 0) : \Phi_\mu^{(1)} \varepsilon^{*\mu}, \quad (44)$$

$$(1, 1, 2) : \Phi_\mu^{(1)} \varepsilon_\nu^* \tilde{t}^{(2)\mu\nu}, \quad (45)$$

$$N^*(\frac{3}{2}^+) \rightarrow N\omega \quad (1, 1, 1) : i\phi_\mu^{(1)} \epsilon^{\mu\nu\lambda\sigma} \varepsilon_\nu^* \tilde{t}_\lambda^{(1)} \hat{p}_{*\sigma}, \quad (46)$$

$$(1, 2, 1) : \Phi_{\mu\nu}^{(2)} \varepsilon^{*\mu} \tilde{t}^{(1)\nu}, \quad (47)$$

$$(1, 2, 3) : \Phi_{\mu\nu}^{(2)} \varepsilon_\lambda^* \tilde{t}^{(3)\mu\nu\lambda}, \quad (48)$$

$$N^*(\frac{3}{2}^-) \rightarrow N\omega \quad (1, 1, 0) : \phi_\mu^{(1)} \varepsilon^{*\mu}, \quad (49)$$

$$(1, 1, 2) : \phi_\mu^{(1)} \varepsilon_\nu^* \tilde{t}^{(2)\mu\nu}, \quad (50)$$

$$(1, 2, 2) : i\Phi_{\mu\alpha}^{(2)} \epsilon^{\mu\nu\lambda\sigma} \varepsilon_\nu^* \tilde{t}_\lambda^{(2)\alpha} \hat{p}_{*\sigma}, \quad (51)$$

$$N^*(\frac{5}{2}^+) \rightarrow N\omega \quad (1, 2, 1) : \phi_{\mu\nu}^{(2)} \varepsilon^{*\mu} \tilde{t}^{(1)\nu}, \quad (52)$$

$$(1, 2, 3) : \phi_{\mu\nu}^{(2)} \varepsilon_\lambda^* \tilde{t}^{(3)\mu\nu\lambda}, \quad (53)$$

$$(1, 3, 3) : i\Phi_{\mu\alpha\beta}^{(3)} \epsilon^{\mu\nu\lambda\sigma} \varepsilon_\nu^* \tilde{t}_\lambda^{(3)\alpha\beta} \hat{p}_{*\sigma}, \quad (54)$$

$$N^*(\frac{5}{2}^-) \rightarrow N\omega \quad (1, 2, 2) : i\phi_{\mu\alpha}^{(2)} \epsilon^{\mu\nu\lambda\sigma} \varepsilon_\nu^* \tilde{t}_\lambda^{(2)\alpha} \hat{p}_{*\sigma}, \quad (55)$$

$$(1, 3, 2) : \Phi_{\mu\nu\lambda}^{(3)} \varepsilon^{*\mu} \tilde{t}^{(2)\nu\lambda}, \quad (56)$$

$$(1, 3, 4) : \Phi_{\mu\nu\lambda}^{(3)} \varepsilon_\sigma^* \tilde{t}^{(4)\mu\nu\lambda\sigma}, \quad (57)$$

$$N^*(\frac{7}{2}^+) \rightarrow N\omega \quad (1, 3, 3) : i\phi_{\mu\alpha\beta}^{(3)} \epsilon^{\mu\nu\lambda\sigma} \varepsilon_\nu^* \tilde{t}_\lambda^{(3)\alpha\beta} \hat{p}_{*\sigma}, \quad (58)$$

$$(1, 4, 3) : \Phi_{\mu\nu\lambda\sigma}^{(4)} \varepsilon^{*\mu} \tilde{t}^{(3)\nu\lambda\sigma}, \quad (59)$$

$$(1, 4, 5) : \Phi_{\mu\nu\lambda\sigma}^{(4)} \varepsilon_\delta^* \tilde{t}^{(5)\mu\nu\lambda\sigma\delta}, \quad (60)$$

$$N^*(\frac{7}{2}^-) \rightarrow N\omega \quad (1, 3, 2) : \phi_{\mu\nu\lambda}^{(3)} \varepsilon^{*\mu} \tilde{t}^{(2)\nu\lambda}, \quad (61)$$

$$(1, 3, 4) : \phi_{\mu\nu\lambda}^{(3)} \varepsilon_\sigma^* \tilde{t}^{(4)\mu\nu\lambda\sigma}, \quad (62)$$

$$(1, 4, 4) : i\Phi_{\mu\alpha\beta\gamma}^{(4)} \epsilon^{\mu\nu\lambda\sigma} \varepsilon_\nu^* \tilde{t}_\lambda^{(4)\alpha\beta\gamma} \hat{p}_{*\sigma}. \quad (63)$$

where $\hat{p}_{*\sigma} = p_{*\sigma}/m_*$. In the N^* at-rest system, $\hat{p}_* = (1, 0, 0, 0)$; $\epsilon^{\mu\nu\lambda\sigma} S_\mu L_\nu J_\lambda \hat{p}_{*\sigma} = (\mathbf{S} \times \mathbf{L}) \cdot \mathbf{J}$ is the standard form for forming a total angular momentum $|\mathbf{J}| = 1$ from two other angular momenta (S,L) of absolute value 1. In the covariant L-S tensor formalism, for S-L-J coupling, if S+L+J is an odd number, then the

$\epsilon^{\mu\nu\lambda\sigma}\hat{p}_{A\sigma}$ is needed. These are the only possible independent couplings because the fact that $p_{*\sigma}t^{(n)\sigma\mu\dots} = 0$, $p_{*\sigma}\phi^{(n)\sigma\mu\dots} = 0$ and $p_{*\sigma}\Phi^{(n)\sigma\mu\dots} = 0$. The corresponding couplings from the simple effective Lagrangian approach are give in Ref.[17]. They have the same number of independent couplings as here and are linear combinations of couplings here. For example, for $N^{*3/2-} \rightarrow N\omega$, the full amplitude in the covariant L-S scheme is

$$A = g_1\phi_\mu^{(1)}\varepsilon^\mu + g_2\phi_\mu^{(1)}\varepsilon_\nu\tilde{t}^{(2)\mu\nu} + g_3i\Phi_{\mu\alpha}^{(2)}\epsilon^{\mu\nu\lambda\sigma}\varepsilon_\nu\tilde{t}_\lambda^{(2)\alpha}p_{*\sigma} \quad (64)$$

with vertex form factors g_1 , g_2 and g_3 , while in the simple effective Lagrangian approach[17] is

$$A = f_1\bar{u}_Nu_{*\mu}\varepsilon^\mu + f_2\bar{u}_N\gamma_\nu u_{*\mu}p_N^\mu\varepsilon^\nu + f_3\bar{u}_Nu_{*\mu}p_\omega^\mu\varepsilon_\nu p_N^\nu \quad (65)$$

with vertex form factors f_1 , f_2 and f_3 . These Vertex form factors are smooth functions of m_* with practically constant m_N and m_ω ; they have no dependence on angular variable. With some simple algebra and the following identity[22]:

$$\begin{aligned} i\epsilon_{\mu abc} = & \gamma_5(\gamma_\mu\gamma_a\gamma_b\gamma_c - \gamma_\mu\gamma_a g_{bc} + \gamma_\mu\gamma_b g_{ac} - \gamma_\mu\gamma_c g_{ab} \\ & - \gamma_a\gamma_b g_{\mu c} + \gamma_a\gamma_c g_{\mu b} - \gamma_b\gamma_c g_{\mu a} + g_{\mu a}g_{bc} - g_{\mu b}g_{ac} + g_{\mu c}g_{ab}), \end{aligned} \quad (66)$$

we have

$$\phi_\mu^{(1)}\varepsilon^\mu = \bar{u}_Nu_{*\mu}\varepsilon^\mu, \quad (67)$$

$$\phi_\mu^{(1)}\varepsilon_\nu\tilde{t}^{(2)\mu\nu} = 2(-1 + \frac{m_N^2 - m_\omega^2}{m_*^2})\bar{u}_Nu_{*\mu}p_\omega^\mu\varepsilon_\nu p_N^\nu + \frac{1}{3}\mathbf{r}^2\bar{u}_Nu_{*\mu}\varepsilon^\mu, \quad (68)$$

$$\begin{aligned} i\Phi_{\mu\alpha}^{(2)}\epsilon^{\mu\nu\lambda\sigma}\varepsilon_\nu\tilde{t}_\lambda^{(2)\alpha}\hat{p}_{*\sigma} = & 2(-3 + \frac{m_N^2 - m_\omega^2}{m_*^2})\bar{u}_Nu_{*\mu}p_\omega^\mu\varepsilon_\nu p_N^\nu + \mathbf{r}^2\bar{u}_Nu_{*\mu}\varepsilon^\mu \\ & + 4\frac{(m_* + m_N)^2 - m_\omega^2}{m_*}\bar{u}_N\gamma_\nu u_{*\mu}p_N^\mu\varepsilon^\nu, \end{aligned} \quad (69)$$

which give the relation between g_i and f_i vertex form factors:

$$f_1 = g_1 + \frac{1}{3}\mathbf{r}^2g_2 + \mathbf{r}^2g_3, \quad (70)$$

$$f_2 = 4\frac{(m_* + m_N)^2 - m_\omega^2}{m_*}g_3 \quad (71)$$

$$f_3 = 2(-1 + \frac{m_N^2 - m_\omega^2}{m_*^2})g_2 + 2(-3 + \frac{m_N^2 - m_\omega^2}{m_*^2})g_3. \quad (72)$$

The g_i and f_i are related by some smooth m_* dependence factors. For an N^* with very broad width, this may cause some model dependence on the determination of their mass and width.

3.3 $\psi \rightarrow N^* \bar{N}$

Here we give an example of a vector meson decaying into $N^* \bar{N}$ final state.

$$(S_A, S_{BC}, L_{BC}) : -\mathbf{S}_A + \mathbf{S}_{BC} + \mathbf{L}_{BC} = 0$$

$$\psi \rightarrow N^*(\frac{1}{2}^+) \bar{N} \quad (1, 1, 0) : \Psi_\mu^{(1)} \varepsilon^\mu, \quad (73)$$

$$(1, 1, 2) : \Psi_\mu^{(1)} \varepsilon_\nu \tilde{t}^{(2)\mu\nu}, \quad (74)$$

$$\psi \rightarrow N^*(\frac{1}{2}^-) \bar{N} \quad (1, 0, 1) : \psi^{(0)} \varepsilon_\mu \tilde{t}^{(1)\mu}, \quad (75)$$

$$(1, 1, 1) : i\Psi_\mu^{(1)} \epsilon^{\mu\nu\lambda\sigma} \varepsilon_\nu \tilde{t}_\lambda^{(1)} \hat{p}_{(\psi)\sigma}, \quad (76)$$

$$\psi \rightarrow N^*(\frac{3}{2}^+) \bar{N} \quad (1, 1, 0) : \psi_\mu^{(1)} \varepsilon^\mu, \quad (77)$$

$$(1, 1, 2) : \psi_\mu^{(1)} \varepsilon_\nu \tilde{t}^{(2)\mu\nu}, \quad (78)$$

$$(1, 2, 2) : i\Psi_{\mu\alpha}^{(2)} \epsilon^{\mu\nu\lambda\sigma} \varepsilon_\nu \tilde{t}_\lambda^{(2)\alpha} \hat{p}_{(\psi)\sigma}, \quad (79)$$

$$\psi \rightarrow N^*(\frac{3}{2}^-) \bar{N} \quad (1, 1, 1) : i\psi_\mu^{(1)} \epsilon^{\mu\nu\lambda\sigma} \varepsilon_\nu \tilde{t}_\lambda^{(1)} \hat{p}_{(\psi)\sigma}, \quad (80)$$

$$(1, 2, 1) : \Psi_{\mu\nu}^{(2)} \varepsilon^\mu \tilde{t}^{(1)\nu}, \quad (81)$$

$$(1, 2, 3) : \Psi_{\mu\nu}^{(2)} \varepsilon_\lambda \tilde{t}^{(3)\mu\nu\lambda}, \quad (82)$$

$$\psi \rightarrow N^*(\frac{5}{2}^+) \bar{N} \quad (1, 2, 2) : i\psi_{\mu\alpha}^{(2)} \epsilon^{\mu\nu\lambda\sigma} \varepsilon_\nu \tilde{t}_\lambda^{(2)\alpha} \hat{p}_{(\psi)\sigma}, \quad (83)$$

$$(1, 3, 2) : \Psi_{\mu\nu\lambda}^{(3)} \varepsilon^\mu \tilde{t}^{(2)\nu\lambda}, \quad (84)$$

$$(1, 3, 4) : \Psi_{\mu\nu\lambda}^{(3)} \varepsilon_\sigma \tilde{t}^{(4)\mu\nu\lambda\sigma}, \quad (85)$$

$$\psi \rightarrow N^*(\frac{5}{2}^-) \bar{N} \quad (1, 2, 1) : \psi_{\mu\nu}^{(2)} \varepsilon^\mu \tilde{t}^{(1)\nu}, \quad (86)$$

$$(1, 2, 3) : \psi_{\mu\nu}^{(2)} \varepsilon_\lambda \tilde{t}^{(3)\mu\nu\lambda}, \quad (87)$$

$$(1, 3, 3) : i\Psi_{\mu\alpha\beta}^{(3)} \epsilon^{\mu\nu\lambda\sigma} \varepsilon_\nu \tilde{t}_\lambda^{(3)\alpha\beta} \hat{p}_{(\psi)\sigma}, \quad (88)$$

$$\psi \rightarrow N^*(\frac{7}{2}^+) \bar{N} \quad (1, 3, 2) : \psi_{\mu\nu\lambda}^{(3)} \varepsilon^\mu \tilde{t}^{(2)\nu\lambda}, \quad (89)$$

$$(1, 3, 4) : \psi_{\mu\nu\lambda}^{(3)} \varepsilon_\sigma \tilde{t}^{(4)\mu\nu\lambda\sigma}, \quad (90)$$

$$(1, 4, 4) : i\Psi_{\mu\alpha\beta\gamma}^{(4)} \epsilon^{\mu\nu\lambda\sigma} \varepsilon_\nu \tilde{t}_\lambda^{(4)\alpha\beta\gamma} \hat{p}_{(\psi)\sigma}, \quad (91)$$

$$\psi \rightarrow N^*(\frac{7}{2}^-) \bar{N} \quad (1, 3, 3) : i\psi_{\mu\alpha\beta}^{(3)} \epsilon^{\mu\nu\lambda\sigma} \varepsilon_\nu \tilde{t}_\lambda^{(3)\alpha\beta} \hat{p}_{(\psi)\sigma}, \quad (92)$$

$$(1, 4, 3) : \Psi_{\mu\nu\lambda\sigma}^{(4)} \varepsilon^\mu \tilde{t}^{(3)\nu\lambda\sigma}, \quad (93)$$

$$(1, 4, 5) : \Psi_{\mu\nu\lambda\sigma}^{(4)} \varepsilon_\delta \tilde{t}^{(5)\mu\nu\lambda\sigma\delta}. \quad (94)$$

Corresponding couplings in the effective Lagrangian approach are give in Ref.[17]. In the multipole approach, the amplitude for $\psi \rightarrow N^* \bar{N}$ generally takes the form

$$A = \sum_{L, m_L, S, m_S} (L, m_L; S, m_S | 1, m_\psi) (S_B, m_B; S_C, m_C | S, m_S) Y_{L m_L}(\hat{\mathbf{p}}_{\mathbf{N}}) G_{LS} |\mathbf{p}_{\mathbf{N}}|^L f_L(|\mathbf{p}_{\mathbf{N}}|) \quad (95)$$

where G_{LS} is the coupling constant for the final state with orbital angular momentum L and total spin S , $\mathbf{p}_{\mathbf{N}}$ is the momentum of $\bar{N}$ in the rest frame of ψ and $f_L(|\mathbf{p}_{\mathbf{N}}|)$ is the vertex form factor. Taking $\psi \rightarrow N^*(\frac{1}{2}^+) \bar{N}$ as an example, the amplitude is as the following

$$\begin{aligned} A &= (\frac{1}{2}, m_B; \frac{1}{2} m_C | 1, m_\psi) Y_{00}(\hat{\mathbf{p}}_{\mathbf{N}}) G_{01} f_0(|\mathbf{p}_{\mathbf{N}}|) \\ &+ (2, m_L; 1, m_S | 1, m_\psi) (\frac{1}{2}, m_B; \frac{1}{2} m_C | 1, m_S) Y_{2 m_L}(\hat{\mathbf{p}}_{\mathbf{N}}) G_{21} |\mathbf{p}_{\mathbf{N}}|^2 f_2(|\mathbf{p}_{\mathbf{N}}|) \end{aligned} \quad (96)$$

with $m_S = m_B + m_C$ and $m_L = m_\psi - m_S$. With some simple algebra, the corresponding amplitude in the covariant L-S scheme can be reduced to the similar form:

$$\begin{aligned} A &= g_0 \Psi_\mu^{(1)} \varepsilon^\mu f_0(|\mathbf{p}_{\mathbf{N}}|) + g_2 \Psi_\mu^{(1)} \varepsilon_\nu \tilde{t}^{(2)\mu\nu} f_2(|\mathbf{p}_{\mathbf{N}}|) \\ &= (\frac{1}{2}, m_B; \frac{1}{2} m_C | 1, m_\psi) Y_{00}(\hat{\mathbf{p}}_{\mathbf{N}}) g_0 \sqrt{8\pi} C_\Psi f_0(|\mathbf{p}_{\mathbf{N}}|) \\ &+ (2, m_L; 1, m_S | 1, m_\psi) (\frac{1}{2}, m_B; \frac{1}{2} m_C | 1, m_S) Y_{2 m_L}(\hat{\mathbf{p}}_{\mathbf{N}}) g_2 \frac{8}{3} \sqrt{4\pi} C_\Psi |\mathbf{p}_{\mathbf{N}}|^2 f_2(|\mathbf{p}_{\mathbf{N}}|). \end{aligned} \quad (97)$$

Comparing Eq.(96) and Eq.(97), we have

$$G_{01} = g_0 \sqrt{8\pi} C_\Psi, \quad (98)$$

$$G_{21} = g_2 \frac{8}{3} \sqrt{4\pi} C_\Psi. \quad (99)$$

In non-relativistic limit, $C_\Psi = 1$, and the covariant L-S scheme gives G_{01} and G_{21} as constants; but generally speaking, the covariant L-S scheme results in G_{01} and G_{21} smoothly dependent on $|\mathbf{p}_{\mathbf{N}}|$.

As a concrete example, here we study the angular distribution and the relative ratio of D-wave and S-wave in the final states of $e^+ e^- \rightarrow J/\psi \rightarrow p \bar{p}$. For this process of positron-electron collision, J/ψ spin projection is limited to be ± 1 along the beam direction. The differential decay rate of the J/ψ is related to the amplitude A as

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2} \overline{|A|^2} \frac{|\mathbf{p}_{\mathbf{N}}|}{M_\psi^2}. \quad (100)$$

With A given by Eq.(97), we have

$$\overline{|A|^2} = \frac{m_\psi^2}{m_p^2} \left(\frac{1}{2} C_S^2 + \frac{20}{9} C_D^2 - \frac{2}{3} C_S C_D \cos\beta \right) (1 + \alpha \cos^2\theta) \quad (101)$$

where

$$\alpha = \frac{2C_S C_D \cos\beta - \frac{4}{3}C_D^2}{\frac{1}{2}C_S^2 + \frac{20}{9}C_D^2 - \frac{2}{3}C_S C_D \cos\beta} \quad (102)$$

with $C_S \equiv |g_0|f_0(|\mathbf{p}_N|)$, $C_D \equiv |g_2|\mathbf{p}_N^2 f_2(|\mathbf{p}_N|)$ and β the relative phase between C_S and C_D . If $C_D = 0$, then $\alpha = 0$ as expected for a pure S-wave decay; if $C_S = 0$, then $\alpha = -\frac{3}{5}$ for the pure D-wave decay.

The relative ratio $R_{D/S}$ of D-wave and S-wave decay rates is

$$R_{D/S} \equiv \frac{\Gamma_D}{\Gamma_S} = \frac{32C_D^2}{C_S^2}. \quad (103)$$

The experimental value of α for the $e^+e^- \rightarrow J/\psi \rightarrow p\bar{p}$ process is about 0.62[23]. This gives the ratio $R_{D/S}$ to be in the range of $0.09 \sim 1.9$. The large uncertainty is due to the unknown relative phase β between S-wave and D-wave amplitudes. For a full determination of the ratio $R_{D/S}$, the polarization information of final state particles is needed.

4 Discussion

Comparing with the simple effective Lagrangian approach, each coupling in the covariant L-S scheme corresponds to a single L final states while a coupling in the simple effective Lagrangian approach may be a mixture of two L final states. The number of independent couplings is same in the two approaches as it should be. In the simple effective Lagrangian approach, the independent couplings are not necessary to be orthogonal to each other; while in the covariant L-S scheme, they are orthogonal and make the partial wave analysis easier. The construction of the full amplitude in the covariant L-S scheme for a multi-step process, e.g., $J/\psi \rightarrow N^*\bar{N} \rightarrow \omega N\bar{N}$, is similar to the simple effective Lagrangian approach[17]. The coupling constants for each couplings are fitted to the data in the procedure of partial wave analysis[8].

For the partial wave analysis, we only demand very basic requirements, i.e., Lorentz, CPT, C and P invariance, for the amplitude and we make formalism more general. Various theories or models or assumptions can bring more constraints to the relations of various couplings, hence reduce the number of independent couplings. For example, a chiral quark model calculation[24] results in a single coupling form for the $N^*(1675)(\frac{5}{2}^-)N\omega$ coupling, which corresponds to our (1,2,2) coupling of Eq.(55), while other quark model[25] gives different prediction. This can be checked in the future by partial wave analysis of processes involving $N^*(1675)(\frac{5}{2}^-) \rightarrow N\omega$. Some people[16] assume $N^*(\frac{3}{2}^\pm)N\omega$ couplings to have the same structure as $N^*(\frac{3}{2}^\pm)N\gamma$ hence only two independent couplings. In our general scheme, we have three independent couplings for $N^*(\frac{3}{2}^\pm)N\omega$ couplings; gauge invariance requirement for the

$N^*(\frac{3}{2}^\pm)N\gamma$ couplings reduces the number of independent couplings to two for the $N^*(\frac{3}{2}^\pm)N\gamma$ couplings.

In this paper we have given explicit formulae for $N^* \rightarrow N\pi$, $N^* \rightarrow N\omega$ and $\psi \rightarrow N^*\bar{N}$ as examples since the relevant processes are understudy by experimental groups. For any baryon resonance decaying to a $\frac{1}{2}^+$ baryon plus a pseudoscalar meson through strong interaction, e.g., $N^* \rightarrow \Lambda K$, $N^* \rightarrow \Sigma K$, $\Lambda^* \rightarrow NK$, $\Lambda^* \rightarrow \Sigma\pi$, *etc.*, the coupling has the same form as for $N^* \rightarrow N\pi$, the only difference is the coupling constants. For any baryon resonance strong decaying to a $\frac{1}{2}^+$ baryon plus a vector meson, the coupling has the same form as for $N^* \rightarrow N\omega$. For any vector meson strong decaying to a baryon resonance plus an anti- $(\frac{1}{2}^+)$ baryon, the coupling has the same form as for $\psi \rightarrow N^*\bar{N}$. Extension to other processes are straightforward by following the basic rules outlined in this work.

In our present L-S scheme for N^* decays, we have added the spin of the incoming nucleon resonance and the final nucleon. This is different with the usual L-S scheme where it is always the spin of the final state particles which are added to make the total spin S. The two schemes are simply related by recoupling various angular momenta involved. With recoupling technique in Ref.[26], we have the relation between the two schemes as the following:

$$[[\mathbf{S}_A \times \mathbf{S}_B]_{S_{AB}} \times \mathbf{S}_C]_{LM} = \sum_{S_{BC}} \sqrt{(2S_{AB}+1)(2S_{BC}+1)} W(S_A S_B L S_C; S_{AB} S_{BC}) [\mathbf{S}_A \times [\mathbf{S}_B \times \mathbf{S}_C]_{S_{BC}}]_{LM} \quad (104)$$

where $W(S_A S_B L S_C; S_{AB} S_{BC})$ is the usual Racah coefficients[26]. From this relation, after we get the coupling constants in our scheme, $g(S_{AB}, L)$, we can easily get the corresponding coupling constants in the usual L-S scheme, $G(S_{BC}, L)$, as

$$G(S_{BC}, L) = \sum_{S_{AB}} g(S_{AB}, L) \sqrt{(2S_{AB}+1)(2S_{BC}+1)} W(S_A S_B L S_C; S_{AB} S_{BC}). \quad (105)$$

Since the covariant L-S scheme combines merits of two conventional schemes, *i.e.*, covariant effective Lagrangian approach and multipole analysis with amplitudes expanded according to angular momentum L, we recommend it to be used in future partial wave analysis.

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