Charmonia transitions in ψ' and its cascade decays

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(Dated: September 22, 2006)

Transitions between the bound $c\bar{c}$ states provide an excellent laboratory for studying heavy quark-antiquark dynamics at short distances. In this note, the radiative and hadronic transitions between charmonia produced in ψ' and its cascade decays are reviewed. All these transitions can be studied using the ψ' data sample which will be collected by BESIII. The theoretical studies of some of the transitions are listed and commented, while for some other transitions, theoretical efforts are called for. This supply as the useful handbook for the analyses using ψ' data in the future at BESIII.

PACS numbers: 14.40.Gx, 14.40.Cs, 13.25.Gv

I. INTRODUCTION

At BESIII, since its peak luminosity is designed as 10^{33} cm^{-2}s^{-1} at the center of mass energy around ψ'' peak, the luminosity at the ψ', which is less than 100 MeV below the ψ'' peak, is about the same. As the beam energy spread of the BEPCII is around 1.4 MeV, the peak cross section of ψ' production is around 600 nb. Assuming the average luminosity is half of the peak luminosity and the effective running time each year around 10^7 s, the expected ψ' events is 3 billion in one year's running [1]. This is a huge data sample compared with the old generation experiments, and the detector performance is also much better than the old generation ones, thus makes a higher precision measurement of the ψ' decays possible, and makes the search for the modes with small decay rates possible.

The charmonium spectrum below the open charm threshold is shown in Fig. 1, since the mass of ψ' is higher than those of all the $n = 1$ charmonia states, all these lower mass charmonia thus can be produced by radiative and/or hadronic decays of ψ'. This has been a fruitful field in charmonium physics [2, 3] both in theoretical and in experimental aspects, however, due to the low statistics of the old generation experiments and the poor detector performance, not all of them were measured, among them, some are crucial in the development of the phenomenological models in charmonium physics.

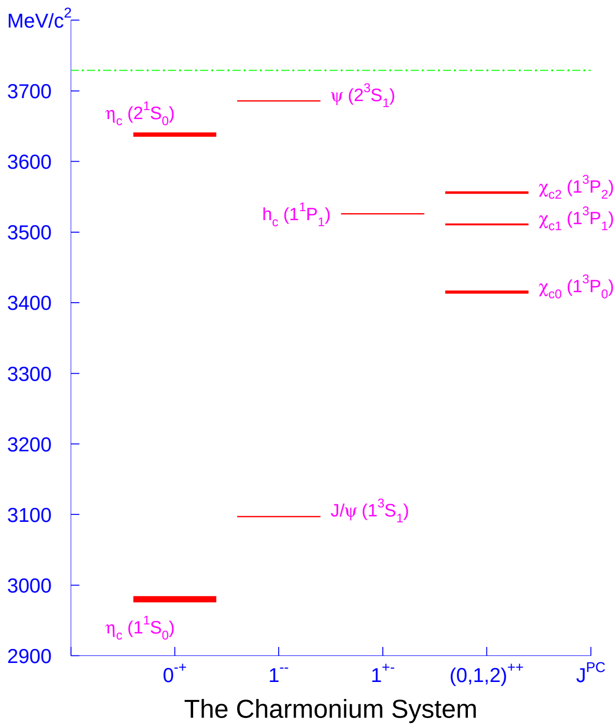

FIG. 1: Charmonium spectroscopy below the open charm threshold.

In this note, we try to list all the allowed radiative transitions and hadronic transitions which can be studied by the large ψ' data sample which will be collected by BESIII, give an overview of the status of the study and point out some topics where theoretical efforts are called for.

II. RADIATIVE TRANSITIONS

Since there is one photon emission between the two charmonia, and the J^{PC} of photon is 1^{--}, the radiative transition only occurs between two C-parity different states. The transitions could be either electric multipole or magnetic multipole transitions depends on the spins and parities of the initial and final states. Assuming the spins of the initial and final states are S_i and S_f, respectively, the total angular momentum carried by the photon (J_γ) can be any integer between $|S_i - S_f|$ and $S_i + S_f$. If the product of the parities of the initial state (π_i) and final state (π_f) equals $(-1)^{J_\gamma}$, the transition is EJ_γ transition; otherwise, if $\pi_i \cdot \pi_f = (-1)^{J_\gamma+1}$, it is an MJ_γ transition. It is obvious that the electric multipole transitions keep the quark spins, while the magnetic multipole transitions are accompanied by quark spin-flip.

In general, when more than one multipole transitions are allowed, only the lowest one is important, however, the contributions of higher multipoles were studied both theoretically and experimentally.

The radiative transition between charmonium states has been studied extensively by many authors both theoretically and experimentally [4–11]. The rates of some of the transitions were also calculated in lattice QCD [12].

A. ψ' decays

- $\psi' \to \gamma\chi_{cJ}$, $J = 0, 1, 2$

  These are the transitions between S-wave spin triplets and the P-wave spin triplets. In $\psi' \to \gamma\chi_{c0}$, there is only $E1$ transition, while in $\psi' \to \gamma\chi_{c1}$, there are $E1$ and $M2$ transitions, and in $\psi' \to \gamma\chi_{c2}$, there could be $E1, M2$, as well as $E3$ transitions.

  In general, it is believed that $\psi' \to \gamma\chi_{cJ}$ is dominated by the $E1$ transition, but with some $M2$ (for χ_{c1} and χ_{c2}) and $E3$ (for χ_{c2}) contributions due to the relativistic correction. These contributions have been used to explain the big differences between the calculated pure $E1$ transition rates and the experimental results [5]. They will also affect the angular distribution of the radiative photon. Thus the measurement of the angular distribution may be used to determine the contributions of the higher multipoles in the transition.

  Furthermore, for $\psi' \to \gamma\chi_{c2}$, the $E3$ amplitude is directly connected with D-state mixing in ψ' which has been regarded as a possible explanation of the large leptonic annihilation rate of ψ'' [9]. Since recent studies [13–15] also suggest the S- and D-wave mixing of ψ' and ψ'' may be the key to solve the longstanding "$\rho\pi$ puzzle" and to explain ψ'' non-$D\bar{D}$ decays, the experimental information on multipole amplitudes gains renewed interest.

  Decay angular distributions were studied in $\psi' \to \gamma\chi_{c2}$ by the Crystal Ball experiment using $\psi' \to \gamma\gamma J/\psi$ [16]; the contribution of the higher multipoles were not found to be significant but the errors were large due to the limited statistics. In a recent analysis at BESII [17], $\psi' \to \gamma\chi_{c2} \to \gamma\pi^+\pi^-$ or γK^+K^- decays were used for a similar study. The analysis gives the magnetic quadrapole amplitude $a'_2 = -0.051^{+0.054}_{-0.036}$ and the electric octupole amplitude $a'_3 = -0.027^{+0.043}_{-0.029}$ [18]. Neither result significantly differs from zero. The results are in good agreement with what is expected for a pure $E1$ transition. As for the D-state mixing of ψ', the results do not contradict with the previous theoretical calculation within one standard deviation [19].

  The contribution of these higher multipoles are interesting theoretically, so further studies at BESIII are expected where a much higher sensitivity for probing the higher multipoles contribution would be possible.

- $\psi' \to \gamma\eta_c$

  This is a hindered $M1$ transition, as it occurs between $n = 2$ and $n = 1$ states.

- $\psi' \to \gamma\eta'_c$

  This is an $M1$ transition, in analogy to the similar transition between J/ψ and η_c, however, the transition rate is very small as the mass difference between the ψ' and η'_c is very small.

  The study of this state in ψ' decays is challenging to the experimentalists and to the detector capability.

B. η_c' decays

The observation of these transitions will be very helpful in understanding the η_c' properties. In experiment aspect, these final states are clean for the observation, however, the rates seem too small. It is a challenge for BESIII.

- $\eta_c' \to \gamma J/\psi$

  This is an $M1$ transition. It has long been calculated [19] to have a partial width at ??

- $\eta_c' \to \gamma h_c(^1P_1)$

  This is an $E1$ transition. It has long been calculated [8] to have a partial width at 16 keV.

C. χ_{cJ} decays

- $\chi_{cJ} \to \gamma J/\psi$

  These are the transitions between P-wave spin triplets and the S-wave spin triplets. In $\chi_{c0} \to \gamma J/\psi$, there is only $E1$ transition, while in $\chi_{c1} \to \gamma J/\psi$, there are $E1$ and $M2$ transitions, and in $\chi_{c2} \to \gamma J/\psi$, there could be $E1, M2$, as well as $E3$ transitions.

  Decay angular distributions were studied in $\psi' \to \gamma\chi_{c2}$ by the Crystal Ball experiment using $\psi' \to \gamma\gamma J/\psi$ [16]; the contribution of the higher multipoles were not found to be significant but the errors were large due to the limited statistics. χ_{c2} decay was also studied by E835 in $p\bar{p}$ annihilation.

- $\chi_{c2} \to \gamma h_c(^1P_1)$

  It could be $M1$, $E2$, $M3$ transitions. There was no calculation in the market.

D. $h_c(^1P_1)$ decays

- $h_c(^1P_1) \to \gamma\eta_c$

  $E1$ transition. This is the discovery mode of the $h_c(^1P_1)$ state at CLEO [20]. The transition branching fraction is expected to be large (more than 50% of the total $h_c(^1P_1)$ decays), this is confirmed by the CLEOc measurement. This should be measured in higher precision at BESIII.

- $h_c(^1P_1) \to \gamma\chi_{c0}$, $\gamma\chi_{c1}$

  $h_c(^1P_1) \to \gamma\chi_{c0}$ is $M1$ transition, and $h_c(^1P_1) \to \gamma\chi_{c1}$ is $M1$, $E2$ transitions. There is no calculation available.

  The measurement of these transitions as well as $\chi_{c2} \to \gamma h_c(^1P_1)$ is challenging, as the rates may be small, and the photons are very soft.

E. J/ψ decays

- $J/\psi \to \gamma\eta_c$

  $M1$ transition, better measurement is necessary to improve the precision, as well as to clarify the difference between measurements from J/ψ decays and from other experiments, such as $\gamma^*\gamma^*$ fusion, B decays and so on.

  This mode can be studied using either ψ' data sample, via $\psi' \to J/\psi\pi^+\pi^-$, or using the J/ψ data sample collected at the J/ψ resonance peak.

All the radiative transitions between the charmonium states listed above are summarized in Fig. 2.

III. HADRONIC TRANSITIONS

There are strong and electromagnetic transitions between two charmonium states if the mass difference is large enough to produce one or more π's, or η. C-parity conservation and Parity conservation may forbid some of the transitions, and they are pointed out in this note. The study of them for a search for the rare decays and the potential signal of new physics is beyond the scope of this note.

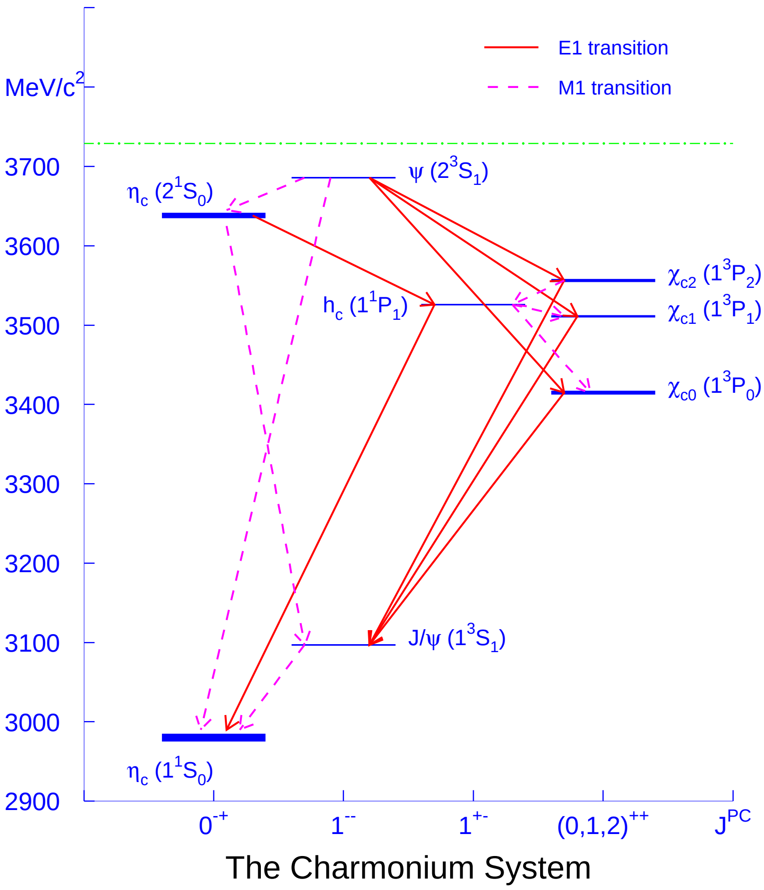

FIG. 2: Radiative transitions between charmonium states below the open charm threshold.

Only the hadronic transitions of the ψ' have been studied experimentally, including $\pi^+\pi^- J/\psi$, $\pi^0\pi^0 J/\psi$, $\eta J/\psi$ and $\pi^0 J/\psi$. The theoretical calculations were also done for these transitions. The other possible transitions were only studied scarcely, we point this out below when available.

It should be noted that since the mass differences between the charmonium states are not large, the light hadrons are generally produced at very low momentum, this may supply a good laboratory for studying the physics in this energy domain.

A. ψ' decays

Since the mass difference between ψ' and many of the charmonium states are much larger than one π mass, thus there are many possible transitions.

All the allowed transitions are summarized in Fig. 3 and explained in detail below.

1. $\psi' \to \eta_c + X$

The mass difference between ψ' and η_c is 706 MeV$/c^2$, greater than $5m_\pi$ and $m_\eta + m_\pi$, all the possible combinations are listed below. No measurement for the study of the channels listed here, only very few theoretical considerations.

- $\psi' \to n\pi^0\eta_c$, $n = 1, 2, 3, 4, 5$: C-violation, not allowed
- $\psi' \to \pi^+\pi^-\eta_c$: G-violation, EM decays, via ρ^*
- $\psi' \to \pi^+\pi^-\pi^0\eta_c$: strong decays, via ω^*

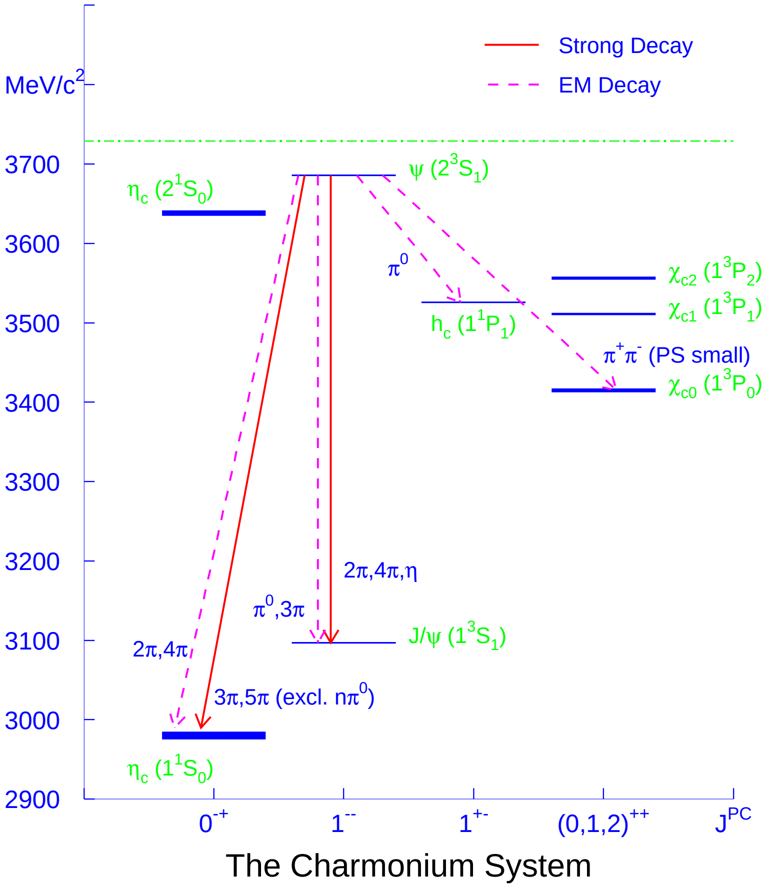

FIG. 3: Hadronic transitions of ψ' to other charmonium states.

It is expected that this mode should be produced at 1% level [21]. The model is developed to explain the "$\rho\pi$ puzzle" between ψ' and J/ψ hadronic decays. Using 14 M ψ' data at BESII, this decay mode is being searched for using three η_c decay modes: $p\bar{p}$, $2(\pi^+\pi^-)$, and $\pi^+\pi^-K^+K^-$. Result is expected in a few months.

- $\psi' \to 2(\pi^+\pi^-)\eta_c$: G-violation, EM decays
- $\psi' \to \pi^+\pi^-2\pi^0\eta_c$: G-violation, EM decays
- $\psi' \to 2(\pi^+\pi^-)\pi^0\eta_c$: strong decays
- $\psi' \to \pi^+\pi^-3\pi^0\eta_c$: strong decays
- $\psi' \to \eta\eta_c$: C-violation, not allowed
- $\psi' \to \eta\pi^0\eta_c$: C-violation, not allowed

2. $\psi' \to J/\psi + X$

The mass difference between ψ' and J/ψ is 589 MeV/c^2, greater than $4m_\pi$ and m_η, all the possible combinations are listed below.

The channels in this category were studied well both experimentally and theoretically, due to the large decay branching fractions and the clear signature of the J/ψ leptonic decays.

- $\psi' \to \pi^0 J/\psi$: G-violation, EM decays

  Observed via $\pi^0 \to \gamma\gamma$ and $J/\psi \to \ell\ell$ by many experiments, the most recent are BESII and CLEO.

  Theoretical calculations are ...

- $\psi' \to \pi^+\pi^- J/\psi$: strong decays

  This is one of the main transition mode of the ψ', the branching fraction is about one third. The $\pi^+\pi^-$ mass shows interesting feature (more events at high $\pi^+\pi^-$ mass) which has been the hot topic of the theoretical effort from the discover of this decay mode until now.

  $\pi\pi$ are produced mainly in S-wave, with the same quantum number as the σ, the D-wave component is found to be small from the analysis of BESI using 4 M produced ψ' events.

  The process has been analyzed in various models by many authors [22–26], all the models fit the data well.

- $\psi' \to \pi^0\pi^0 J/\psi$: strong decays

  The same as $\pi^+\pi^- J/\psi$ mode. Isospin symmetry predicts its production rate is half of that of $\pi^+\pi^- J/\psi$, this is tested in high precision with the CLEO detector using 3 M produced ψ' events.

  The Isospin violation effect may exist, but should be small if compare with the $\pi^0 J/\psi$ and $\eta J/\psi$ rates difference. This may be tested with more data with better precision.

- $\psi' \to \pi^+\pi^-\pi^0 J/\psi$: G-violation, EM decays

  The rate can be roughly estimated by the $\pi^0 J/\psi$ mode.

- $\psi' \to 3\pi^0 J/\psi$: G-violation, EM decays

- $\psi' \to 2(\pi^+\pi^-)J/\psi$: strong decays

  The phase space is small, the rate may not be small since it is a strong decay. The detection in experiment is hard, since the momentum of the $\pi^\pm$ is low.

- $\psi' \to \pi^+\pi^- 2\pi^0 J/\psi$: strong decays

  Similar to $\psi' \to 2(\pi^+\pi^-)J/\psi$, hard to be detected.

- $\psi' \to 4\pi^0 J/\psi$: strong decays

  Similar to $\psi' \to 2(\pi^+\pi^-)J/\psi$, detection of 8 low photons with energy around half of the $\pi 0$ mass is a challenge to the Electromagnetic Calorimeter.

- $\psi' \to \eta J/\psi$: strong decays

  Many measurements, the ratio between the rate of this mode and the isospin violation mode $\pi^0 J/\psi$ is used to measure the mass difference of the up and down quarks, and the strength of the electromagnetic decays in ψ' hadronic transition.

3. $\psi' \to \chi_{cJ} + X$

The mass difference between ψ' and χ_{c0} is 271 MeV/c^2, slightly greater than $2m_{\pi^0}$ and lower than $m_{\pi^+} + m_{\pi^-}$, considering the width of χ_{c0} is around 10 MeV/c^2, the tail of χ_{c0} can be produced in $\psi' \to \pi^+\pi^-\chi_{c0}$. The mass difference between ψ' and χ_{c1} is 176 MeV/c^2, slightly greater than m_{π^0}, and the mass difference between ψ' and χ_{c2} is 130 MeV/c^2, lower than m_{π^0}. All the possible transitions are listed below.

- $\psi' \to n\pi^0\chi_{cJ}$, $n = 1, 2$: C-violation, not allowed

- $\psi' \to \pi^+\pi^-\chi_{c0}$: G-violation, EM decays, via ρ^*, phase space very small, only low mass tail of χ_{c0} can be produced. There is no measurement and no theoretical calculation.

4. $\psi' \to h_c(^1P_1) + X$

The mass difference between ψ' and $h_c(^1P_1)$ is 160 MeV/c^2, slightly greater than m_{π^0}, the only possible transition is $\psi' \to \pi^0 h_c(^1P_1)$.

- $\psi' \to \pi^0 h_c(^1P_1)$: G-violation, EM decays

  This is the discovery mode of the $h_c(^1P_1)$ in ψ' decays by CLEO [20]. The product of the branching fraction and that of $h_c(^1P_1) \to \gamma\eta_c$ was given by the same experiment. Assuming the $E1$ transition rate of $h_c(^1P_1)$ decays, one may estimate the $\psi' \to \pi^0 h_c(^1P_1)$ branching fraction at ?? level. Theoretical estimation of this rate ranges from ?? to ??.

  More effort is needed experimentally to understand this transition, as well as study the properties of the $h_c(^1P_1)$ better.

B. η_c' decays

Since the mass of η_c' is only slightly smaller than that of ψ', so the mass difference between η_c' and many of the charmonium states are also much larger than one π mass, thus there are many possible transitions.
All the allowed transitions are summarized in Fig. 4 and explained in detail below.

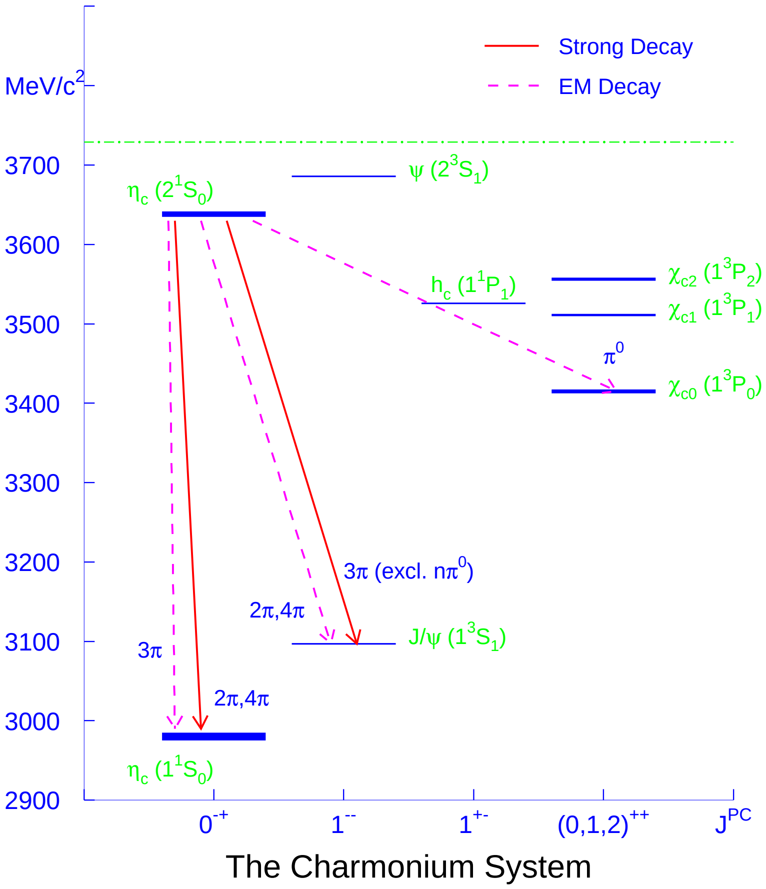

FIG. 4: Hadronic transitions of η_c' to other charmonium states.

1. $\eta_c' \to \eta_c + X$

The mass difference between η_c' and η_c is 658 MeV/c^2, greater than $4m_\pi$ and m_η, all the possible combinations are listed below.

- $\eta_c' \to \pi^0\eta_c$: P-violation, not allowed
- $\eta_c' \to \pi^+\pi^-\eta_c$: strong decays, via σ

Voloshin [27] pointed out that this decay is related to the well studied $\psi' \to \pi^+\pi^- J/\psi$, and estimated the branching fraction in η_c' decays could be around 5-10%, together with the neutral $\pi^0\pi^0$ mode. No experiment information available yet.

The study of this transition is very hard via the ψ' decays, since the η_c' is produced with very small branching fraction in ψ' radiative transition, and the photon energy is extremely low which is hard to be distinguished from the noises.

- $\eta_c' \to \pi^0\pi^0\eta_c$: strong decays, via σ

  Similar to $\eta_c' \to \pi^+\pi^-\eta_c$, the observation should be even harder due to multi-photons in the event.
- $\eta_c' \to \pi^+\pi^-\pi^0\eta_c$: G-violation, EM decays, high orbital angular momentum

  May not be detected at BESIII.
- $\eta_c' \to 3\pi^0\eta_c$: G-violation, EM decays, high orbital angular momentum

  May not be detected at BESIII.
- $\eta_c' \to 2(\pi^+\pi^-)\eta_c$: strong decays

  Phase space small, may not be detected at BESIII.
- $\eta_c' \to \pi^+\pi^- 2\pi^0\eta_c$: strong decays

  Phase space small, may not be detected at BESIII.
- $\eta_c' \to 4\pi^0\eta_c$: strong decays

  Phase space small, too many photons, may not be detected at BESIII.
- $\eta_c' \to \eta\eta_c$: P-violation, not allowed

2. $\eta_c' \to J/\psi + X$

The mass difference between η_c' and J/ψ is 541 MeV/c^2, slightly greater than $4m_{\pi^0}$ and smaller than $2(m_{\pi^+}+m_{\pi^-})$. Considering the uncertainty of η_c' mass is large and the width of η_c' is at 10 MeV/c^2 level, the high mass tail of η_c' can decay into $2(\pi^+\pi^-)$. All the possible combinations are listed below.

- $\eta_c' \to n\pi^0 J/\psi$, $n = 1, 2, 3, 4$: C-violation, not allowed
- $\eta_c' \to \pi^+\pi^- J/\psi$: G-violation, EM decays, via ρ^*
- $\eta_c' \to \pi^+\pi^-\pi^0 J/\psi$: strong decays, via ω^*
- $\eta_c' \to 2(\pi^+\pi^-)J/\psi$: G-violation, EM decays, phase space very small
- $\eta_c' \to \pi^+\pi^- 2\pi^0 J/\psi$: G-violation, EM decays, phase space very small

The detection of these above modes maybe a bit easier than the $\eta_c' \to \eta_c$ transition, since the J/ψ tag is much simpler and it is very narrow.

Naive estimation the rates for $\eta_c' \to J/\psi$ transition should be smaller than the $\eta_c' \to \eta_c$ transitions, since the former requires the flip of the quark spin. No serious theoretical effort on these estimations.

3. $\eta_c' \to \chi_{cJ} + X$

The mass difference between η_c' and χ_{c0} is 223 MeV/c^2, slightly greater than m_{π^0}; the mass difference between η_c' and χ_{c1} is 128 MeV/c^2, and that between η_c' and χ_{c2} is 82 MeV/c^2, smaller than m_{π^0}. The only allowed transition is $\eta_c' \to \pi^0\chi_{c0}$.

- $\eta_c' \to \pi^0\chi_{c0}$: G-violation, EM decays

4. $\eta_c' \to h_c(^1P_1) + X$

The mass difference between η_c' and $h_c(^1P_1)$ is 112 MeV/c^2, smaller than m_{π^0}. No allowed transition.

C. $h_c(^1P_1)$ decays

The mass difference between $h_c(^1P_1)$ and many of the charmonium states are also much larger than one π mass, thus there are many possible transitions.

All the allowed transitions are summarized in Fig. 5 and explained in detail below.

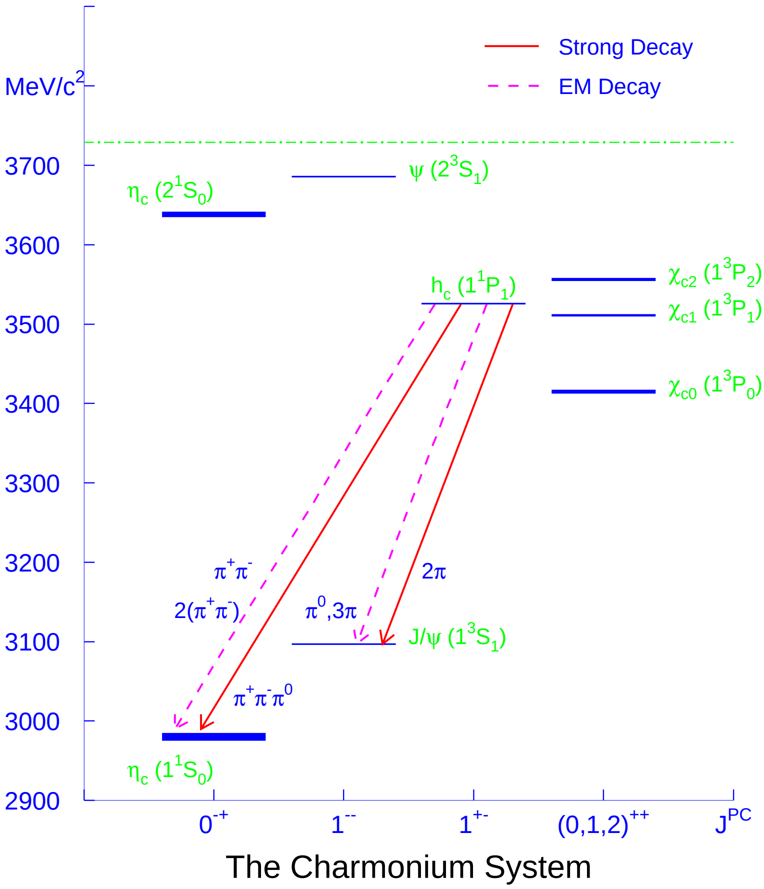

FIG. 5: Hadronic transitions of $h_c(^1P_1)$ to other charmonium states.

1. $h_c(^1P_1) \to \eta_c + X$

The mass difference between $h_c(^1P_1)$ and η_c is 546 MeV/c^2, greater than $4m_\pi$ and about the same as m_η, all the possible combinations are listed below.

- $h_c(^1P_1) \to n\pi^0\eta_c$, $n = 1, 2, 3, 4$: C-violation, not allowed
- $h_c(^1P_1) \to \pi^+\pi^-\eta_c$: G-violation, EM decays, via ρ^*
- $h_c(^1P_1) \to \pi^+\pi^-\pi^0\eta_c$: strong decays, via ω^*
- $h_c(^1P_1) \to 2(\pi^+\pi^-)\eta_c$: G-violation, EM decays
- $h_c(^1P_1) \to \eta\eta_c$: C-violation, not allowed

$h_c(^1P_1) \to \pi^+\pi^-\eta_c$ and $h_c(^1P_1) \to \pi^+\pi^-\pi^0\eta_c$ should be looked for experimentally, $h_c(^1P_1) \to 2(\pi^+\pi^-)\eta_c$ rate may be too small to be detected at BESIII.

2. $h_c(^1P_1) \to J/\psi + X$

The mass difference between $h_c(^1P_1)$ and J/ψ is 429 MeV/c^2, greater than $3m_{\pi^0}$. All the possible combinations are listed below.

There is no experimental information available, neither the theoretical calculations.

- $h_c(^1P_1) \to \pi^0 J/\psi$: G-violation, EM decays
- $h_c(^1P_1) \to \pi^+\pi^- J/\psi$: strong decays, via σ
- $h_c(^1P_1) \to \pi^0\pi^0 J/\psi$: strong decays, via σ
- $h_c(^1P_1) \to \pi^+\pi^-\pi^0 J/\psi$: G-violation, EM decays
- $h_c(^1P_1) \to 3\pi^0 J/\psi$: G-violation, EM decays

3. $h_c(^1P_1) \to \chi_{c0} + X$

The mass difference between $h_c(^1P_1)$ and χ_{c0} is 111 MeV/c^2, smaller than m_{π^0}. No allowed transition.

D. χ_{c2} decays

The mass difference between χ_{c2} and many of the charmonium states are also much larger than one π mass, thus there are many possible transitions.

All the allowed transitions are summarized in Fig. 6 and explained in detail below.

1. $\chi_{c2} \to \eta_c + X$

The mass difference between χ_{c2} and η_c is 576 MeV/c^2, greater than $4m_\pi$ and m_η, all the possible combinations are listed below.

- $\chi_{c2} \to \pi^0\eta_c$: G-violation, EM decays
- $\chi_{c2} \to \pi^+\pi^-\eta_c$: Strong decays, high orbital angular momentum
- $\chi_{c2} \to \pi^0\pi^0\eta_c$: Strong decays, high orbital angular momentum
- $\chi_{c2} \to \pi^+\pi^-\pi^0\eta_c$: G-violation, EM decays
- $\chi_{c2} \to 3\pi^0\eta_c$: G-violation, EM decays
- $\chi_{c2} \to 2(\pi^+\pi^-)\eta_c$: strong decays, phase space very small, orbital angular momentum very high
- $\chi_{c2} \to \pi^+\pi^-2\pi^0\eta_c$: strong decays, phase space very small, orbital angular momentum very high
- $\chi_{c2} \to 4\pi^0\eta_c$: strong decays, phase space very small, orbital angular momentum very high
- $\chi_{c2} \to \eta\eta_c$: strong decays, phase space small

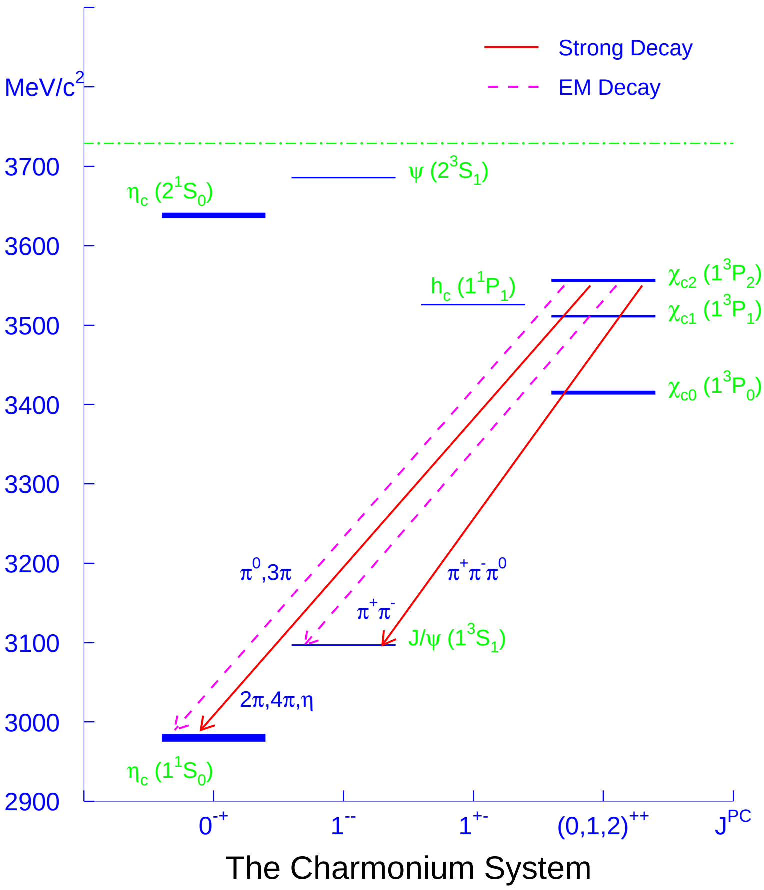

FIG. 6: Hadronic transitions of χ_{c2} to other charmonium states.

2. $\chi_{c2} \to J/\psi + X$

The mass difference between χ_{c2} and J/ψ is 459 MeV/c^2, slightly greater than $3m_\pi$. All the possible combinations are listed below.

- $\chi_{c2} \to n\pi^0 J/\psi$, $n = 1, 2, 3$: C-violation, not allowed
- $\chi_{c2} \to \pi^+\pi^- J/\psi$: G-violation, EM decays, via ρ^*
- $\chi_{c2} \to \pi^+\pi^-\pi^0 J/\psi$: strong decays, via ω^*

3. $\chi_{c2} \to \chi_{c0} + X$, $\chi_{c1} + X$

The mass difference between χ_{c2} and χ_{c0} is 141 MeV/c^2, slightly greater than m_{π^0}; the mass difference between χ_{c2} and χ_{c1} is 46 MeV/c^2, smaller than m_{π^0}. The only possible transition is $\chi_{c2} \to \pi^0\chi_{c0}$.

- $\chi_{c2} \to \pi^0\chi_{c0}$: P-violation, not allowed

4. $\chi_{c2} \to h_c(^1P_1) + X$

The mass difference between χ_{c2} and $h_c(^1P_1)$ is 30 MeV/c^2, smaller than m_{π^0}. No allowed transition.

E. χ_{c1} decays

The mass difference between χ_{c1} and many of the charmonium states are also much larger than one π mass, thus there are many possible transitions.

All the allowed transitions are summarized in Fig. 7 and explained in detail below.

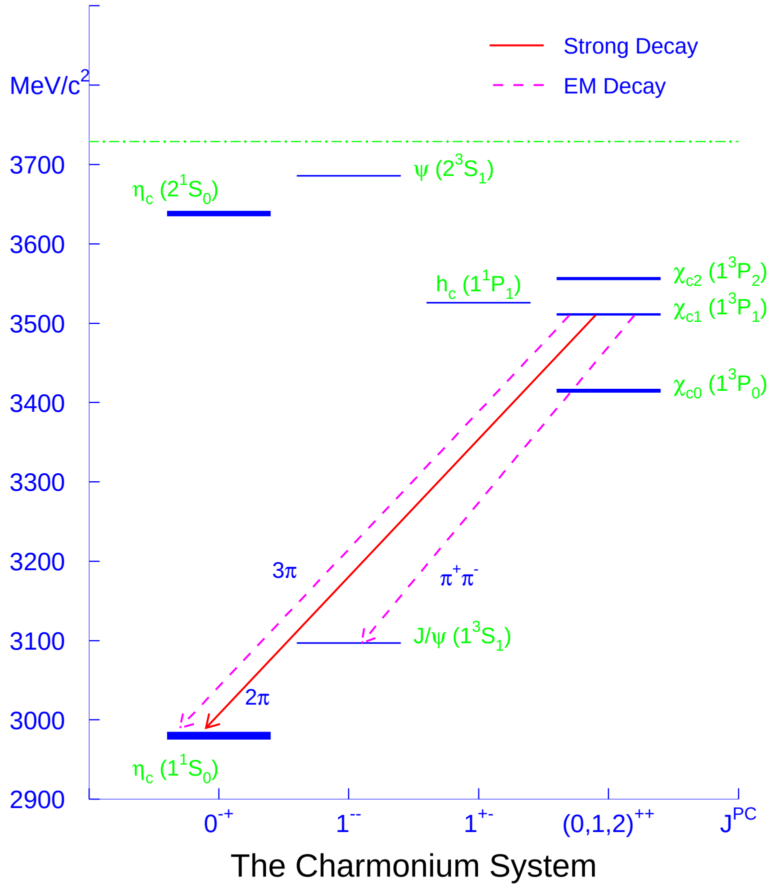

FIG. 7: Hadronic transitions of χ_{c1} to other charmonium states.

1. $\chi_{c1} \to \eta_c + X$

The mass difference between χ_{c1} and η_c is 531 MeV/c^2, greater than $3m_\pi$, all the possible combinations are listed below.

- $\chi_{c1} \to \pi^0\eta_c$: P-violation, not allowed
- $\chi_{c1} \to \pi^+\pi^-\eta_c$: strong decays, via σ

  A very rough measurement at BESII shows that it can not be observed using the BESII data sample.
- $\chi_{c1} \to \pi^0\pi^0\eta_c$: strong decays, via σ
- $\chi_{c1} \to \pi^+\pi^-\pi^0\eta_c$: G-violation, EM decays
- $\chi_{c1} \to 3\pi^0\eta_c$: G-violation, EM decays

2. $\chi_{c1} \to J/\psi + X$

The mass difference between χ_{c1} and J/ψ is 413 MeV/c^2, slightly greater than $3m_{\pi^0}$ and smaller than $m_{\pi^+} + m_{\pi^-} + m_{\pi^0}$ by 2 MeV/c^2. All the possible combinations are listed below.

- $\chi_{c1} \to n\pi^0 J/\psi$, $n = 1, 2, 3$: C-violation, not allowed
- $\chi_{c1} \to \pi^+\pi^- J/\psi$: G-violation, EM decays, via ρ^*
- $\chi_{c1} \to \pi^+\pi^-\pi^0 J/\psi$: no phase space

3. $\chi_{c1} \to \chi_{c0} + X$

The mass difference between χ_{c1} and χ_{c0} is 95 MeV/c^2, smaller than m_{π^0}. No allowed transition.

F. χ_{c0} decays

The mass difference between χ_{c0} and many of the charmonium states are also much larger than one π mass, thus there are many possible transitions.

All the allowed transitions are summarized in Fig. 8 and explained in detail below.

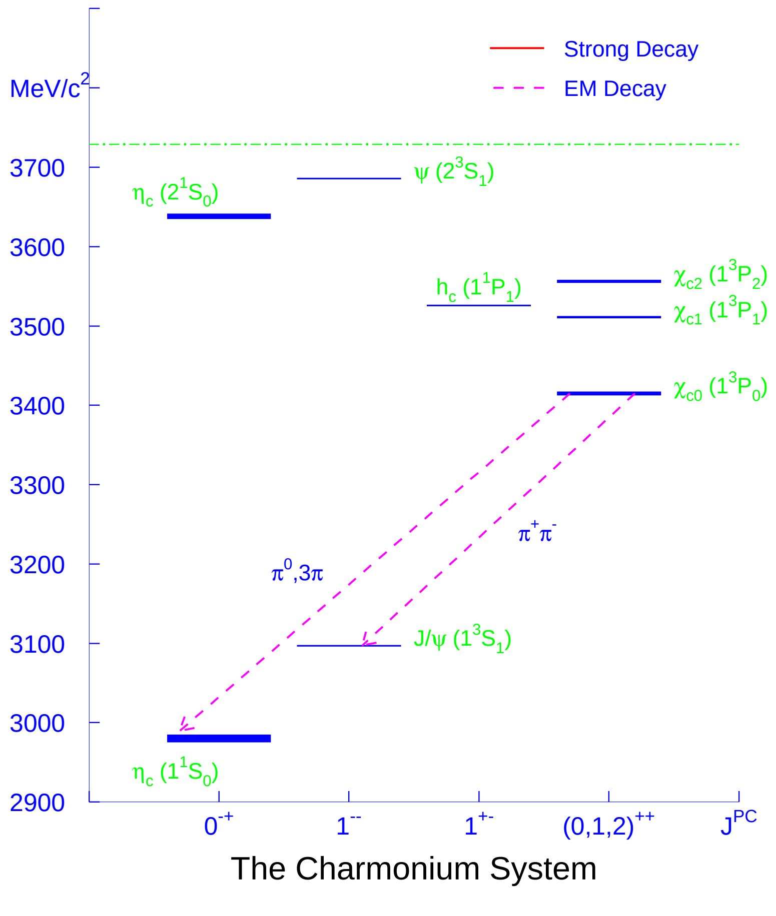

FIG. 8: Hadronic transitions of χ_{c0} to other charmonium states.

1. $\chi_{c0} \to \eta_c + X$

The mass difference between χ_{c0} and η_c is 435 MeV/c^2, greater than $3m_\pi$, all the possible combinations are listed below.

- $\chi_{c0} \to \pi^0\eta_c$: G-violation, EM decays
- $\chi_{c0} \to \pi^+\pi^-\eta_c$: P-violation, not allowed
- $\chi_{c0} \to \pi^0\pi^0\eta_c$: P-violation, not allowed
- $\chi_{c0} \to \pi^+\pi^-\pi^0\eta_c$: G-violation, EM decays
- $\chi_{c0} \to 3\pi^0\eta_c$: G-violation, EM decays

2. $\chi_{c0} \to J/\psi + X$

The mass difference between χ_{c0} and J/ψ is 318 MeV/c^2, slightly greater than $2m_\pi$. All the possible combinations are listed below.

- $\chi_{c0} \to n\pi^0 J/\psi$, $n = 1, 2$: C-violation, not allowed
- $\chi_{c0} \to \pi^+\pi^- J/\psi$: G-violation, EM decays, via ρ^*

IV. DISCUSSIONS AND CONCLUSIONS

In the above sections, we listed all the possible transitions between the known charmonium states below the charm threshold, more studies needed for a better understanding of the transitions.

It is worth to pointing out that, as the lattice QCD calculations give the lowest lying hybrid charmonium states with normal J^{PC} almost have the same mass as the charmonium states with same quantum numbers [28, 29], these states can be produced in ψ' hadronic transitions. For example, if the $J^{PC} = 1^{--}$ hybrid charmonium has mass around 3.1 GeV/c^2, the same transitions between ψ' and it are similar to that between ψ' and J/ψ. It is straightforward to list the production of the hybrids from the decays of other charmonia with the help of the channels listed in above sections.

The production rates of these hybrids should be estimated, and the experimental search for these states (new kind of hadrons) should be performed using the BESIII ψ' data sample. It is worth to have a try using the existing BESII ψ' data sample to have some feeling about the search, and simulation should be done at BESIII.

As the mass of these states are low, and the decays require the annihilation of the charm and anti-charm quark pair, the width of these states should be small, there should be no dominant decay mode(s). All these need further study.

Note added: It seems whether the $c\bar{c}g$'s calculated in the lattice QCD are the hybrid charmonium states in the quark model, or just the higher Fock states in the charmonium wave function needs to be clarified. If they are only the higher Fock states in the conventional charmonia, it will be easier to understand why their masses are the same as the charmonium states. In this case, these states will not be observed in experiment. However, the mass difference between the conventional charmonia and the hybrid charmonia for the first radial excitation states may suggest the hybrids exist experimentally. These states certainly worth to be searched for in experiment, but it can not reach at BESIII, as the maximum center of mass energy of it is only 4.2 GeV.

V. ACKNOWLEDGMENT

The author would like thank .... The work is supported by the 100-talent program of the Chinese Academy of Sciences (U-25) and the National Natural Science Foundation (10491303).

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