$\rho\pi$ puzzle in charmonium decays and non-$D\overline{D}$ measurement at $\psi(3770)$

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The explanations about $\rho\pi$ puzzle are reviewed extensively, and some relevant experimental tests are also reviewed. Some stress is layed on the S- and D-wave charmonia mixing senario, accoriding to which the large possible non-$D\overline{D}$ decay at ψ'' could be accommodate. The experimental searches for such possible existing large non-$D\overline{D}$ decays at ψ'' are recounted with some subtle experiment treatments.

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I. INTRODUCTION

Crisply defined experimental puzzles in high-energy physics always have the prospect of leading to new scientific discoveries, one prominent example is the θ-τ puzzle of 1956 which led to the parity revolution, therefore puzzles in physics often draw considerable attention to theorists. The ratio of hadronic ψ' to J/ψ decays is such a puzzle and has attracted attention since 1983.

People once expected ψ decays to hadrons via three gluon or a single direct photon. In either case, pQCD can provide a relation [1]

$$Q_h \;=\; \frac{\mathcal{B}_{\psi'\to h}}{\mathcal{B}_{J/\psi\to h}} = \frac{\mathcal{B}_{\psi'\to e^+e^-}}{\mathcal{B}_{J/\psi\to e^+e^-}} \approx 12.7\% \;. \qquad (1)$$

This relation is referred to as "12% rule" which is expected to be reasonable for exclusive decay. The "$\rho\pi$ puzzle" is that the prediction (1) is severely violated in the $\rho\pi$ and several other decay channels. The first evidence for this effect was presented by Mark-II Collaboration in 1983 [2]. Since then many theoretical explanations have been put forth to phrase this puzzle.

Recently, with great many of new experiment results from BESII and CLEOc for two-body decays, such as vector-pesudoscalar (VP), vector-tensor (VT) pesudoscalar-pesudoscalar (PP) and baryon-antibaryon ($B\overline{B}$) modes, and for multi-body decays at J/ψ, ψ' or even at ψ'', lots of predictions appeared in previous and recent papers can be tested with higher accuracy. Moreover, in theoretical explanations for $\rho\pi$ puzzle, many enlighten ideas have been put forth, which although not very successful or general, are benefit for our further understanding the charmonium dynamics. Therefore, in this paper, we collect as much as possible the information involving $\rho\pi$ puzzle (sometimes a few relevant idea are also mentioned), including theoretical speculations or explanations, and some relevant experimental measurements or results. So far as theoretical papers are concerned, some of them have connections, while some are very distinctive. However, for understanding, we try to present a classification. Simplistically thinking, since the Q-value is small, it may be caused either by increasing of denominator or decreasing of numerator, or by both cases. So we classified pertinent explanations and models into three categories:

1. J/ψ-enhancement theory, which attribute the small Q-value to the enhanced branching fraction of J/ψ decay.

2. ψ'-suppression theory, which attribute the small Q-value to the suppressed branching fraction of ψ' decay.

3. the other proposals which are excluded in the first two categories.

In the following parts of the paper, section II is the review on the theoretical suggestions, and the relevant experimental results are quoted as test or confirmation on these suggestions. Section III is devoted to the description of $2S$- and $1D$-wave mixing scenario. The numerical calculation of charmless decay at ψ'' is presented in the next section, section IV. All experimental details are contained in section V, and concrete formulas for cross section calculation are given in the appendix.

II. REVIEW ON MODELS FOR EXPLAINING THE $\rho\pi$ PUZZLE

A. J/ψ-enhancement theory

1. J/ψ-glueball admixture scheme

The glueball concept was originally introduced J/ψ particles by Freund and Nambu [3] (FN hereafter) to explain the breaking of OZI rule in vector meson decay soon after the discovery of J/ψ particles. They proposed that such breaking results from the mixing of the ω, ϕ and J/ψ mesons with an $SU(4)$-singlet vector meson $\mathcal{O}$. They found that the meson $\mathcal{O}$ should lie in the 1.4-1.8 GeV/c^2 mass range with width greater than 40 MeV/c^2, it should decay copiously into $\rho\pi$, $K^*\overline{K}$ and exhibits severe suppression of decays into $K\overline{K}$, e^+e^- and $\mu^+\mu^-$

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modes. Authors presented several quantitative predications for experimental search, two of them are

$$R_1 = \frac{\Gamma_{J/\psi \to \rho\pi}}{\Gamma_{\phi \to \rho\pi}} = 0.0115 - 0.087 \ ,$$

$$R_2 = \frac{\Gamma_{J/\psi \to K\overline{K}}}{\Gamma_{J/\psi \to \rho\pi}} < 8 \times 10^{-5} \ .$$

If we use three pions final state as a substitute for $\rho\pi$ in both ϕ [4] and J/ψ [5, 6] decays, we obtain the first ratio $R_1 \approx 0.003$, which almost one order of magnitude smaller than the predication. For second ratio, by virtue of PDG [4] value for K^+K^- and new experiment result for $K^0_S K^0_L$ [7], it is reasonable to estimate $\mathcal{B}(J/\psi \to K\overline{K}) \sim (10^{-4})$, together with the results for $\rho\pi$ [5, 6], we have $R_2 \sim 10^{-3}$ which is much larger than the theoretical predication.

The first glueball-relevant explanation for "$\rho\pi$" puzzle was proposed by Hou and Soni [21] (HS hereafter). They attributed the enhancement of $J/\psi \to K^*\overline{K}$ and $J/\psi \to \rho\pi$ decay modes to a quantum mechanical mixing of the J/ψ with a $J^{PC} = 1^{--}$ vector gluonium, designated $\mathcal{O}$ as well. The differences between FN's and HS's pictures lie in the following respects:

- Based on potential model for glueball, the mass for a low-lying three-glue state is estimated to be about 2.4 GeV [22], rather than 1.4 to 1.8 GeV in Ref. [3].
- The mixing of $\mathcal{O}$ with ψ' is taken into account, which has been ignored previously.
- Since the gauge coupling constant in QCD is momentum dependent, the mixing parameter is taken to be a function of the invariant mass q^2, which decreases rather sharply with increase in q^2. So a suppressed factor of 4 can be introduced for a glueball with mass around 2.4 GeV.

By virtue of their assumption, HS suggested the search for a vector gluonium state could be carried out using sources of three gluons occurring in certain hadronic decay of the ψ', such as $\psi' \to \pi\pi + X$, $\eta + X$, $\eta' + X$, where X decays into VP final states [21].

Based on HS's idea, Brodsky, Lepage, and Tuan [23] (BLT) provided a refined glueball explanation for "$\rho\pi$" puzzle. They assume the general validity of the perturbative QCD (pQCD) theorem that total hadron helicity is conserved in high-momentum-transfer exclusive processes, in which case the decays to $\rho\pi$ and $K^*\overline{K}$ are forbidden for both the J/ψ and ψ'. This pQCD theorem is often referred to as the rule of Hadronic Helicity Conservation (HHC) [24], which is built on the underlying assumption of short-range "pointlike" interactions among the constituents throughout. For instance, $J/\psi(c\bar{c}) \to 3g$ has a short range $\approx 1/m_c$ associated with the short time scale of interaction. Nevertheless, if subsequently the three gluons were to resonate forming an intermediate gluonium state $\mathcal{O}$ which has large transverse size covering an extended time period, then HHC would be invalid. In essence the HS model takes over in this latter stage.

Final states h which can proceed only through the intermediate gluonium state satisfy the ratio

$$Q_h = \frac{\mathcal{B}(\psi' \to e^+e^-)}{\mathcal{B}(J/\psi \to e^+e^-)} \frac{(M_{J/\psi} - M_{\mathcal{O}})^2 + \Gamma^2_{\mathcal{O}}/4}{(M_{\psi'} - M_{\mathcal{O}})^2 + \Gamma^2_{\mathcal{O}}/4} \ . \qquad (2)$$

The Q_h is small if the $\mathcal{O}$ is close in mass to J/ψ. The experimental limits at that time [2, 23, 25] imply that the $\mathcal{O}$ mass is within 80 MeV of the mass of J/ψ and its total width is less than 160 MeV. Brodsky *et al.* recommend a direct way to search for $\mathcal{O}$, that is to scan the $e^+e^- \to VP$ cross section across the J/ψ resonance.

The BES has searched for this hypothetical particle in a $\rho\pi$ scan across the J/ψ region in e^+e^- annihilations as well as in decays $\psi' \to \pi\pi\mathcal{O}$, $\mathcal{O} \to \rho\pi$, and found no evidence for its existence [9, 26]. The data constrains the mass and width of the $\mathcal{O}$ to the range $\mid M_{\mathcal{O}} - M_{J/\psi} \mid < 80$ MeV and 4 MeV $< \Gamma_{\mathcal{O}} <$ 50 MeV [27]. So far as the scan experiment is concerned, Hou [28] think the absence of distortion in BES energy scan of $J/\psi \to \rho\pi$ does not rule out $M_{\mathcal{O}} \approx M_{J/\psi}$, but serves to put a lower bound to $\Gamma_{\mathcal{O}}$. However, as indicated in Ref. [29], the experimentally constrained mass is several hundred MeV lower than QCD without dynamical quarks [30].

More recently, more experimental facts unfavorable to this glueball explanation have been reported by BES and CLEOc. One is the identification of isospin-violating VP mode $\psi' \to \omega\pi^0$ with a large branching fraction [10, 11, 16]. This contradicts with the essence of the model that the pattern of suppression is dependent on the spin-parity of the final state mesons. In addition, according to BLT's analysis, one can obtain the following relation [41]

$$\frac{\mathcal{B}(J/\psi \to \omega\pi^0)}{\mathcal{B}(J/\psi \to \rho^0\pi^0)} < 0.0037$$

which is much smaller than PDG value 0.1. The other unfavorite is the finding of suppression of ψ' decays into vector plus tensor (VT) final states [18, 19]. Since hadronic VT decays, unlike the VP decays, conserve HHC, some other mechanism must be responsible for this suppression in the model. Furthermore, it has been argued that the $\mathcal{O}$ may also explain why J/ψ decays to ϕf_0 (named previously S^*) but not to $\rho a_0(980)$ (named previously δ) , since the $\mathcal{O}$ mixes with the ϕ and enhances a mode that would be otherwise suppressed [23]. However, the observation of non-suppressed $\psi' \to \phi f_0$ [27], which implies the absence of anomalous enhancement in $J/\psi \to \phi f_0$, would rule out such an explanation. Anselmino et al. extended the idea of J/ψ-$\mathcal{O}$ mixing to the case of $\eta_c \to$VV and $p\overline{p}$ [31]. They suggested that the enhancement of these decays can be attributed to the presence of a trigluonium pseudoscalar state with a mass not far from the η_c mass. So far no experimental data has supported the existence of such a state.

In fact, as pointed out in Ref. [34], this glueball explanation raises some obvious questions as: (i) Why are only the $\rho\pi$ and $K^*\overline{K}$ channels affected and not $\mathcal{O} \to 5\pi$ etc. i.e., one must in an *ad hoc* way assume that the $\mathcal{O}$ couples predominantly to $\rho\pi$ and $K^*\overline{K}$. And (ii) if such a narrow heavy 1^{--} gluonium state exists, why have not narrow 0^{++}, 2^{++} states been seen, which should be lighter and easier to detect.

2. *intrinsic-charm-component scheme*

Brodsky and Karliner put forth an explanation for the puzzle base on the existence of intrinsic charm $|q\overline{q}c\overline{c}\rangle$ Fock components of the light vector mesons [51]. They notice the fact that quantum fluctuations in a QCD bound state wave function will inevitably produce Fock states containing heavy quark pairs. The intrinsic heavy quark pairs are multiconnected to the valence quarks of the light hadrons, and the wave functions describing these configurations will have maximal amplitude at minimal off-shellness and minimal invariant mass. In the case of the ρ meson, consider the light-cone Fock representation:

$$\rho^+ = \psi^{\rho}_{u\overline{d}}|u\overline{d}\rangle + \psi^{\rho}_{u\overline{d}c\overline{c}}|u\overline{d}c\overline{c}\rangle + \cdots .$$

Here we expect the wave function of the $c\overline{c}$ quarks to be in an S-wave configuration with no nodes in its radial dependence, in order to minimize the kinetic energy of the charm quarks and thus also minimize the total invariant mass.

The presence of the $|u\overline{d}c\overline{c}\rangle$ Fock state in the ρ will allow the $J/\psi \to \rho\pi$ decay to occur simply through rearrangement of the incoming and outgoing quark lines; in fact, the $|u\overline{d}c\overline{c}\rangle$ Fock state wave function has a good overlap with the radial and spin $|c\overline{c}\rangle$ and $|u\overline{d}\rangle$ wave functions of the J/ψ and pion. On the other hand, the overlap with the ψ' will be suppressed, since the radial wave function of the $n = 2$ quarkonium state is orthogonal to the nodeless $c\overline{c}$ in the $|u\overline{d}c\overline{c}\rangle$ state of the $\rho\pi$. Similarly, the $|u\overline{s}c\overline{c}\rangle$ Fock component of the K^* will have a favored $J/\psi K$ configuration, allowing the $J/\psi \to K^*\overline{K}$ decay to also occur by quark line rearrangement, rather than $c\overline{c}$ annihilation.

They also suggested comparing branching fractions for the η_c and η_c' as clues to the importance of η_c intrinsic charm excitations in the wavefunctions of light hadrons. However, the observations from BES and CLEOc for $\psi' \to \omega\pi^0$ would again appear to disfavor this model.

B. ψ'-suppression theory

1. *sequential-fragmentation model*

Karl and Roberts have suggested a proposal to explain the $\rho\pi$ puzzle based on the mechanism of sequential quark pair creation [32]. The main idea is that quark-antiquark pairs are produced sequentially, as a result of which the amplitude to produce two mesons in their ground state is an oscillatory function of the total energy of the system. They argue that the oscillatory fragmentation probability could have a minimum near the mass of ψ', which provide a explanation for suppressed ψ' decay. Even though their evaluations could generally accommodate the data for decays of J/ψ and ψ' to $\rho\pi$ or to $K^*\overline{K}$, it gets into trouble when one extrapolates their estimation. According to their calculation, the oscillations of probability amplitude are damped out in the region of the Υ resonances, so the $\rho\pi$ channel is present in the decay of all Υ, Υ', Υ'', $\cdots$ resonances with a common rate. This leads to a prediction $\Gamma(\Upsilon \to \rho\pi) = 0.05$ keV, or equivalently $\mathcal{B}(\Upsilon \to \rho\pi) = 9.4 \times 10^{-4}$, which is larger than the available value $\mathcal{B}(\Upsilon \to \rho\pi) < 2\times 10^{-4}$ [4]. Moreover their calculation seems also hard to explain the large branching fraction for ϕ decays to $\rho\pi$ [4] due to the fact that their fragmentation probability tends to zero as the mass of the $\rho\pi$ decaying system approaches 1GeV.

In a further analysis [33], Karl and Tuan pointed out if a suppression is observed into three-meson channels the explanation based on sequential pair creation would be undermined. Unfortunately, recently such a suppressed channel, viz. ϕKK, is found by CLEOc [20].

2. *exponential-form-factor model*

Guided by suppressed ratios of ψ' to J/ψ decays to two-body hadronic modes, Chaichian and Törnqvist suggested [34] that the hadronic form factors falling exponentially as described by the overlap of wave functions within a nonrelativistic quark model. This behavior explains the drastically suppressed two-body decay rates of ψ' compared with those of the J/ψ. Recently, report on observation of a number of ψ' VP decays [10, 11, 16] such as $\omega\eta'$, $\phi\eta'$, $\rho\eta'$ have proved the theoretically overestimated decay fraction. Moreover, the branching fraction for $\omega\pi^0$ from BES and CLEOc [35], $\mathcal{B}(\psi' \to \omega\pi^0) = (2.02 \pm 1.40) \times 10^{-5}$, is well below that predicted by this model, 1.04×10^{-4}.

Another shortcoming of the model is that it does not select out just the VP channel, the other channels, for example VT, are also estimated to have small branching fractions which are not compatible with BES measured results [19].

3. *generalized hindered M1 transition model*

A so-called generalized hindered M1 transition model is proposed by Pinsky as a solution for the puzzle [36]. It is argued that because $J/\psi \to \gamma\eta$ is an allowed M1 transition while $\psi' \to \gamma\eta'$ is hindered (in the nonrelativistic limit), using the vector-dominance model to relate $\psi' \to \gamma\eta'$ to $\psi' \to \psi\eta'$ one could find the coupling $G_{\psi'\psi\eta_c}$ is much smaller than $G_{\psi\psi\eta_c}$, and then by analogy, the coupling $G_{\omega'\rho\pi}$ would be much smaller than $G_{\omega\rho\pi}$. Here $G_{\omega\rho\pi}$ can be extracted from data by virtue of analysis using the vector-dominance model and a standard parameterization of OZI process [37]. The assuming $\psi' \to \rho\pi$ to proceed via ψ'-ω' mixing, which $J/\psi \to \rho\pi$ via J/ψ-ω mixing, one would find that $\psi' \to \rho\pi$ is much more severely suppressed than $J/\psi \to \rho\pi$. The similar estimation could be preformed for $K^*\overline{K}$ and other VP final state, and one can expect a suppressed Q:

$$\frac{\mathcal{B}(\psi' \to VP)}{\mathcal{B}(\psi \to VP)} = 1.47 \frac{\Gamma_{tot}(\psi)}{\Gamma_{tot}(\psi')} \left(\frac{G_{V'VP}}{G_{VVP}}\right)^2 \frac{F_{V'}}{F_V} = 0.06\% \ , \tag{3}$$

where $F_{V'}/F_V = 0.3$, $G_{\omega'\rho\pi}/G_{\omega\rho\pi} = 0.066$ according to Ref. [36]. This Q is much smaller than the present experimental results [10, 11, 16].

Moreover [38], in this model, the coupling $G_{\omega'\omega f_2}$ for $\omega' \to \omega f_2$ should not be suppressed because by analogy the coupling $G_{\psi'\psi\chi_{c2}}$ is not small due to the fact that the E1 transition $\psi' \to \gamma\chi_{c2}$ is not hindered. Therefore via ψ'-ω' mixing the $\psi' \to \omega' \to \omega f_2$ decay is expected to be not suppressed, which contradict BES result [19].

4. *Higher-Fock-state scheme*

Chen and Braaten (CB) proposed an explanation [29] for the $\rho\pi$ puzzle, arguing that the decay $J/\psi \to \rho\pi$ is dominated by a Fock state in which the $c\overline{c}$ pair is in a color-octet $^3S_1$ state which decays via $c\overline{c} \to q\overline{q}$, while the suppression of this decay mode for the ψ' is attributed to a dynamical effect due to the small energy gap between the mass of the ψ' and the $D\overline{D}$ threshold. Using the BES data on the branching fractions into $\rho\pi$ and $K^*\overline{K}$ as input, they predicted the branching fractions for many other VP decay modes of the ψ', as listed in Table I, from which we see most measured values fall in scope of prediction, but we also notice for $\omega\pi$ mode, the deviation from prediction is obvious.

TABLE I: Predictions and measurements for Q_{VP} in unit of 1% for all VP final states. The value for $\rho\pi$ and $K^{*0}\overline{K}^0 + c.c.$ from Ref. [52] were used as input. The theoretical parameter $x = 0.64$ is due to results of Ref. [53] and experimental results come from Ref. [10–12, 54].

VP	$x = 0.64$	Exp.
$\rho\pi$	$0 - 0.25$	0.13 ± 0.03
$K^{*0}\overline{K^0} + c.c.$	$1.2 - 3.0$	3.2 ± 0.08
$K^{*+}K^- + c.c.$	$0 - 0.36$	$0.59^{+0.27}_{-0.36}$
$\omega\eta$	$0 - 1.6$	< 2.0
$\omega\eta'$	$12 - 55$	19^{+15}_{-13}
$\phi\eta$	$0.4 - 3.0$	5.1 ± 1.9
$\phi\eta'$	$0.5 - 2.2$	9.4 ± 4.8
$\rho\eta$	$14 - 22$	$9.2^{+3.6}_{-3.3}$
$\rho\eta'$	$12 - 20$	$17.8^{+15.9}_{-11.1}$
$\omega\pi$	$11 - 17$	$4.4^{+1.9}_{-1.6}$

In addition, CB's proposal also has implications for the angular distributions for two-body decay modes. In general, the angular distribution have the form $1+\alpha\cos\theta$, with $-1 < \alpha < +1$. CB's conclusion imply that the parameter α for any two-body decay of the ψ' should be less than or equal to the α for the corresponding J/ψ decays. But it seem not support from recently BESII results for $p\overline{p}$ decay measurements.

5. *Survival-chamonia-amplitude explanation*

A model put forward by Gérald and Weyers entertains the assumption that the three-gluon annihilation amplitude and the QED amplitude add incoherently in all channels in J/ψ decays into light hadrons, while in the case of ψ' decays the dominant QCD annihilation amplitude is not into three gluons, but into a specific configuration of five gluons [56]. More precisely, they suggest that the strong annihilation of the ψ' into light hadrons is a two step process: in the first step the ψ' goes into two gluons in a 0^{++} or 0^{-+} state and an off shell $h_c(3526)$; in the second step the off shell h_c annihilate into three gluons to produce light hadrons. Their argument implies: (a) to leading order there is no strong decay amplitude for the processes $\psi' \to \rho\pi$ and $\psi' \to K^*\overline{K}$; (b) the 12 % rule should hold for hadronic processes which take place via the QED amplitude only. As far as the second implication is concerned, the present data give different ratio between ψ' and J/ψ decay for $\omega\pi^0$ and $\pi^+\pi^-$ final states, both of which are electromagnetic process. Here even form factor effect is taken into account [57], the difference between two kinds of process is still obvious. Besides the explanation for $\rho\pi$ puzzle, this model predicts a sizable $\psi' \to (\pi^+\pi^- \text{ or } \eta)h_1(1170)$ branching fraction. Indeed the BES has performed extensive analysis of decays $\psi' \to \pi\pi\rho\pi$ to look for new particles; however, it is unlikely that a conclusive signal for $h_1(1170)$ has ever been observed in the inclusive spectrum of ψ' decays to $\pi\pi\rho\pi$ [26].

In a recent paper, Artoisenet, Gérard and Weyers (AGW) update and sharpen the above idea which lead to a somewhat unconventional point of view: all non-electromagnetic hadronic decays of the ψ' goes via a transition amplitude which contain a $c\overline{c}$ pair. AGW provide two patterns for these two-step decays, the first is

$$\psi' \to (2\text{NP}g) + (3g) \ . \tag{4}$$

The physics picture is as follows: the excited $c\overline{c}$ pair in the ψ' does not annihilate directly instead, spits out two non-perturbative gluons (2NPg) and survives in a lower $c\overline{c}$ configuration (1^{--} or 1^{-+}) which then eventually annihilate into $3g$. The decays $\psi' \to (2\pi)J/\psi$ and $\psi' \to \eta J/\psi$ follow this pattern. The second pattern is

$$\psi' \to (3\text{NP}g) + (2g) \ , \tag{5}$$

where the lower $c\overline{c}$ configuration (0^{-+} or 0^{++}) annihilate into $2g$. The only on-shell channel for this type of decays is $\psi' \to (3\pi)\eta_c$, whose branching fraction is estimated as $(1-2)\%$ level. This is an important prediction worthy of experimental searching. Furthermore, the substitution of one photon for one gluon in Eqs. (4) and (5) allows

$$\psi' \to (2\mathrm{NP}g) + (2g) + \gamma \ . \tag{6}$$

This pattern corresponds to on-shell radiative decays such as $\psi' \to (\pi^+\pi^-)\eta_c\gamma$ and $\psi' \to \eta\eta_c\gamma$, which could be larger than the observed $\psi' \to \eta_c\gamma$ mode.

Besides above predication, AGW also estimate

$$\begin{aligned}\mathcal{B}(\psi' \to b_1\eta) &= (1.3 \pm 0.3) \times 10^{-3} \ , &(7)\\ \mathcal{B}(\psi' \to h_1\pi^0) &= (1.9 \pm 0.4) \times 10^{-3} \ , &(8)\\ \mathcal{B}(J/\psi \to b_1\eta) &\approx \mathcal{B}(\psi' \to b_1\eta) \approx 1\% \ . &(9)\end{aligned}$$

All these wait for further experimental tests.

6. *Charm-mixing scheme*

To explain the large Γ_{ee} of $\psi(3770)$, it is once suggested [59, 60] that the mass eigenstates ψ' and ψ'' are the mixtures of the S- and D-wave of charmonia, namely $\psi(2^3S_1)$ state and $\psi(1^3D_1)$ state. Following this thought, Rosner proposed that such mixing gives possible solution to the $\rho\pi$ puzzle [61]. By virtue of his scheme

$$\begin{aligned}\langle\rho\pi|\psi'\rangle &= \langle\rho\pi|2^3S_1\rangle\cos\theta - \langle\rho\pi|1^3D_1\rangle\sin\theta \ ,\\ \langle\rho\pi|\psi''\rangle &= \langle\rho\pi|2^3S_1\rangle\sin\theta + \langle\rho\pi|1^3D_1\rangle\cos\theta \ ,\end{aligned} \tag{10}$$

where θ is the mixing angle between pure $\psi(2^3S_1)$ and $\psi(1^3D_1)$ states. If the mixing and coupling of ψ' and ψ'' lead to complete cancellation of $\psi' \to \rho\pi$ decay ($\langle\rho\pi|\psi'\rangle = 0$), the missing $\rho\pi$ decay mode of ψ' shows up instead as decay mode of ψ'', enhanced by the factor $1/\sin^2\theta$, concrete estimation shows that [61]

$$\mathcal{B}_{\psi(3770)\to\rho\pi} = (4.1 \pm 1.4) \times 10^{-4} \quad ,$$

this corresponds to a large decay width in ψ'', and could be observed easily in ψ'' data. However, Wang, Yuan and Mo (WYM) pointed out [62] that for e^+e^- collider the continuum contribution must be taken into account carefully. In the section III, we give more detailed description about this model.

C. Other explanations

1. *final state interaction scheme*

Li, Bugg and Zou [39] pointed out that final state interactions (FSI) in J/ψ and ψ' decays give rise to effects which are of the same order as the tree level amplitudes, and may be a possible explanation for all the observed suppressed modes of ψ' decays including $\rho\pi$, $K^*\overline{K}$ and ωf_2. They thus predicted qualitatively large production rates of $a_1\rho$ and $K_1^*\overline{K}^*$ for ψ', the verification of which may give further support to their model. So far, no such measurements have been reported. Nevertheless, useful information on $a_1\rho$ and $K_1^*\overline{K}^*$ could be obtained from BES published data as shown in Refs. [17] and [19]. The lack of evidence within the invariant mass distribution plots (see Fig.3 and Fig.5 of Ref. [17] or Ref. [19]) that the $\rho\pi$ recoiled against a ρ for events of $\psi' \to \rho^0\rho^\pm\pi^\mp$ and that $\pi^\pm K^\mp$ recoiled against a K^{*0} for events of $\psi' \to \pi^+\pi^-K^+K^-$ suggests that they are unlikely to be the favored modes in ψ' decays.

2. *large phase scheme*

Suzuki supplied another version of FSI to understand J/ψ decay [41]. He performed a detailed amplitude analysis for $J/\psi \to VP$ decay to test whether or not the short-distance FSI dominates over the long-distance FSI in the J/ψ decay. His result indicates that there is a large phase between three-gluon and one-photon amplitudes. Since the large phase cannot be produced with the perturbative QCD interaction, the source of it must be in the long-distance part of strong interaction, namely, rescattering among hadrons in their inelastic energy region. Suzuki then performed the similar analysis for $J/\psi \to PP$ decay, also obtain the large phase results [42]. His analysis also shows that the exclusive decay ratio at J/ψ is in line with that of inclusive decay. This fact leads him to believe that the origin of the relative suppression of $\psi' \to VP$ to $J/\psi \to VP$ is not in J/ψ but in ψ'.

Here more words about large phase. So far as the source of large phase, besides Suzuki's explanation, in Ref. [56], Gerard argued that this phase follows from the orthogonality of three-gluon and one-photon virtual processes. In addition, there are many analyses results favorite the large phase conclusion in J/ψ decay for a variety of decay modes, such as $1^+0^-(90^\circ)$ [43], 1^-0^- $(106 \pm 10)^\circ$ [44, 45], 1^-1^- $(138 \pm 37)^\circ$ [42, 45, 78], 0^-0^- $(89.6 \pm 9.9)^\circ$ [42, 45, 78] and $N\overline{N}$ $(89 \pm 15)^\circ$ [45, 46].

Nevertheless, when extend the large phase analysis to ψ' decay, the experimental data at first seemed to favor a small phase [43], in contrary to the expectation that the decay of J/ψ and ψ' should not be much different. However, as pointed out by Wang *et al.* [49] that if the continuum one-photon process is taken into account, the phase with value around -90° could fit $\psi' \to VP$ data [49] and $\pm 90^\circ$ could fit $\psi' \to PP$ data [50] final states. Furthermore, this large phase also shows in the OZI suppressed decay modes of ψ''. In many decays modes of ψ'', the strong decay amplitudes have comparable strenght as the non-resonance continuum amplitude, the large phase around -90° leads to destructive or constructive interference. Due to destructive interference, the observed cross sections of some modes at the peak of ψ'' are smaller than the cross section measured off resonance [62]. This is demonstrated by the data from CLEO [81].

Although the present analysis supports the large phase postulation, which is important for understanding of J/ψ and ψ' decay, the relation between J/ψ and ψ' decay still remains as a puzzle.

3. mass reduction explanation

In the study [55] of radiative decays of 1^{--} quarkonimu into η and η', Ma presented a QCD-factorization approach, with which he obtained theoretical predictions in consistency with CLEOc measurement. The largest possible uncertainties in analysis are from the relativistic corrections for the value of the charm quark mass. Ma argue that the effect of the these uncertainties can be reduced by using quarkonium masses instead of using quark mass. As an example of such reduction, he provide a modified relation to the original 12% rule

$$Q_{\rho\pi} = \frac{\mathcal{B}(J/\psi \to \rho\pi)}{\mathcal{B}(\psi' \to \rho\pi)} = \frac{M^8_{J/\psi}}{M^8_{\psi'}} \frac{\mathcal{B}(J/\psi \to e^+e^-)}{\mathcal{B}(\psi' \to e^+e^-)}$$

$$= (3.6 \pm 0.6)\% \ .$$

However, this value is much larger than experimental analysis result given in Table I.

4. vector-meson-mixing model

Intending to give a comprehensive description of the ψ' two-body decay, Clavelli and Intemann (CI) proposed a vector-meson-mixing model in which the vector mesons (ρ, ω , ϕ , J/ψ) are regarded as being admixture of light- and charmed-quark-antiquark state. The coupling of J/ψ to any state of light quarks is then related to the corresponding coupling of ρ, ω, and ϕ to the same state. With few experiment inputs to determine the mixing parameters, CI calculate VP, PP, and $B\overline{B}$ decay rates for J/ψ as a function of the pseudoscalar mixing angle. Most of predictions agree with experiment results at the order of magnitude level, but discrepancy is obvious for some channels, such as $K^0_S K^0_L$ final state [7]. CI also extended their model to the hadronic decays of ψ'. Nevertheless, their evaluation for $\mathcal{B}(J/\psi \to \omega\pi^0) = 3 \times 10^{-5}$ and $\mathcal{B}(\psi' \to \omega\pi^0) = 3 \times 10^{-3}$ contradict with present results $(4.2 \pm 0.6) \times 10^{-4}$ [4] and $(1.87^{+0.68}_{-0.62} \pm 0.28) \times 10^{-4}$ [11], respectively.

Starting from effective Lagrangian whereby nonet-symmetry breaking and pseudoscalar-meson mixing can be studied, Haber and Perrier wrote down a systematic parameterization forms for decay modes of $J/\psi \to PP$ (also for $J/\psi \to VV$ or $\eta_c \to VP$), $J/\psi \to VP$ (also for $J/\psi \to VT$ or $\eta_c \to VV$), $J/\psi \to PPP$ (also for $J/\psi \to VVP$ or $\eta_c \to PPV$), and $\eta_c \to PPP$ (also for $J/\psi \to PPV$ or $\eta_c \to VVP$) [67]. The experiment data can be used to determine the phenomenological parameters which describe the properties of various mesonic decays and helpful to understand the low-energy hadron dynamics. In a further work, Seiden, Sadrozinski and Haber take the doubly Okubo-Zweig-Iizuka suppression (DOZI) effect into consideration, and presented a more general parameterization of amplitudes for $J/\psi \to PP$ decay [68]. With this form, one could easily derivative the relative decay strength between different final state.

A similar parameterization with mixing feature of the strong interaction mechanism is proposed by Feldmann and Kroll (FK) [69] for the hadron-helicity non-conserving J/ψ and ψ' decays, but with a different interpretation from those put forth in Refs. [29, 40, 53, 68]. FK assume that with a small probability, the charmonium possesses Fock components built from light quarks only. Through these Fock components the charmonium state decays by a soft mechanism which is modeled by J/ψ-ω-ϕ mixing and subsequent ω (or ϕ) decay into the VP state. In absence of the leading-twist perturbative QCD contribution, dominant mechanism is supplemented by the electromagnetic decay contribution and DOZI violating contribution. FK argue that this mechanism can probes the charmonium wave function at all quark-antiquark separations and feels the difference between a $1S$ and a $2S$ radial wave function. The node in the latter is supposed to lead to a strong suppression of the mechanism in the ψ' decays. With a few parameters, adjusted to experiment, FK obtained a numerical description of the branching fractions for many VP decay modes of the J/ψ and ψ', which agree with measured branching fractions at the order of magnitude level. Moreover, FK has extended their mixing approach to the $\eta_c \to VV$ decays and obtained a reasonable description of the branching fractions for these decays while the $\eta'_c \to VV$ decays are expected to be strongly suppressed.

D. Comment

From the above review we see that essentially none of the models are able to explain all known experimental results; not a few models appear to have more assumptions than predictions, not to mention quantitative predictions. While the current data seem to rule out convincingly some of the models, a few other models may warrant further consideration; for them both detailed theoretical analyses and additional experimental tests are demanded.

From the above review we might also notice that phenomenological models such as Feldmann and Kroll model, could usually provide more and reasonable predictions comparing with those mainly starting from general principle discussions. The reason is actually simple since (i) QCD is too general to accommodate the special calculating technique for certain decay mode; and (ii) the nonperturbative effect is too complex and too unclear to be treated strictly for the time being. Therefore, the phenomenological analysis is more welcome now because (i) it could provide an empirical understanding for present experimental results and direct for further experiment measurement or searching; and (ii) it could provide a solid foundation for further theoretical exploration.

Hereafter, we are to discuss the Rosner's mixing model in more details. So far as this model is concerned, we think, as other phenomenological model, it could provide some predictions for immediate experimental test and it also connects the J/ψ, ψ', and ψ'' together, and intend giving a universal description for charmonium decay.

III. 12% RULE AND 2S-1D MIXING

As pointed out in section I, from the pQCD, it is expected that both J/ψ and ψ' decaying into light hadrons are dominated by the annihilation of $c\bar{c}$ into three gluons, with widths proportional to the square of the wave function at the origin $|\Psi(0)|^2$ [1]. This yields the pQCD 12% rule between ψ' and J/ψ decay to the same final states, as shown in Eq. (1). The violation of this rule was first observed in $\rho\pi$ and $K^{*+}K^- + c.c.$ modes by Mark-II [2]. Since then BES-I, BES-II and CLEOc have measured many two-body decay modes of ψ' [5]-[14]. Among them, some obey the rule, like baryon-antibaryon ($B\overline{B}$) modes, while others are either suppressed in ψ' decays, like vector-pseudoscalar (VP) and vector-tensor (VT) modes, or enhanced, like $K^0_S K^0_L$. There have been many theoretical efforts trying to solve the puzzle as we reviewed in section II. Among them, the 2S-1D charmonia mixing scenario [61] predicts with little uncertainty $\mathcal{B}(\psi'' \to \rho\pi)$ which agrees with experimental data [62, 70].

In S- and D-wave charmonia mixing scheme, the mass eigenstates ψ' and ψ'' are the mixtures of the S- and D-wave charmonia, namely

$$\begin{aligned}|\psi'\rangle &= |2^3S_1\rangle \cos\theta - |1^3D_1\rangle \sin\theta \ , \\ |\psi''\rangle &= |2^3S_1\rangle \sin\theta + |1^3D_1\rangle \cos\theta \ ,\end{aligned}$$

where θ is the mixing angle between pure $\psi(2^3S_1)$ and $\psi(1^3D_1)$ states and is fitted from the leptonic widths of ψ'' and ψ' to be either $(-27\pm 2)^\circ$ or $(12\pm 2)^\circ$ [61]. The latter value of θ is consistent with the coupled channel estimates [59**?**] as well as the ratio between ψ' and ψ'' partial widths to $J/\psi\pi^+\pi^-$ [60]. Hereafter, the calculations and discussions in this paper are solely for the mixing angle $\theta = 12^\circ$ [71].

As in the discussion of Ref. [61], since both hadronic and leptonic decay rates are proportional to the square of the wave function at the origin, it is expected that if ψ' is a pure $\psi(2^3S_1)$ state, then for any hadronic final states f,

$$\Gamma(\psi' \to f) = \Gamma(J/\psi \to f)\frac{\Gamma(\psi' \to e^+e^-)}{\Gamma(J/\psi \to e^+e^-)} \ . \qquad (11)$$

The electronic partial width of J/ψ is expressed in potential model by [77]

$$\Gamma(J/\psi \to e^+e^-) = \frac{4\alpha^2 e_c^2}{M^2_{J/\psi}}\left|R_{1S}(0)\right|^2 ,$$

with α the QED fine structure constant, $e_c = 2/3$, $M_{J/\psi}$ the J/ψ mass and $R_{1S}(0)$ the radial 1^3S_1 wave function at the origin.

Since ψ' is not a pure $\psi(2^3S_1)$ state, its electronic partial width is expressed as [61]

$$\begin{aligned}\Gamma(\psi' \to e^+e^-) &= \frac{4\alpha^2 e_c^2}{M^2_{\psi'}} \\ &\times \left|\cos\theta R_{2S}(0) - \frac{5}{2\sqrt{2}m_c^2}\sin\theta R''_{1D}(0)\right|^2 ,\end{aligned}$$

with $M_{\psi'}$ the ψ' mass, m_c the c-quark mass, $R_{2S}(0)$ the radial 2^3S_1 wave function at the origin and $R''_{1D}(0)$ the second derivative of the radial 1^3D_1 wave function at the origin. In the calculations in this paper, we take $R_{2S}(0) = 0.734$ GeV$^{3/2}$ and $5R''_{1D}(0)/(2\sqrt{2}m_c^2) = 0.095$ GeV$^{3/2}$ from Refs. [61, 72].

If Eq. (11) holds for a pure 2S state, $\psi'' \to f$, $\psi' \to f$ and $J/\psi \to f$ partial widths are to be [63]

$$\begin{aligned}\Gamma(\psi'' \to f) &= \frac{C_f}{M^2_{\psi''}}\left|\sin\theta R_{2S}(0) + \eta\cos\theta\right|^2 , \\ \Gamma(\psi' \to f) &= \frac{C_f}{M^2_{\psi'}}\left|\cos\theta R_{2S}(0) - \eta\sin\theta\right|^2 , \\ \Gamma(J/\psi \to f) &= \frac{C_f}{M^2_{J/\psi}}\left|R_{1S}(0)\right|^2 \ , \qquad (12)\end{aligned}$$

where C_f is a common factor for the final state f, $M_{\psi''}$ the ψ'' mass, and $\eta = |\eta|e^{i\phi}$ is a complex parameter with ϕ being the relative phase between $\langle f|1^3D_1\rangle$ and $\langle f|2^3S_1\rangle$.

From Eq. (12), it is obvious that with $\Gamma(J/\psi \to f)$ and $\Gamma(\psi' \to f)$ known, two of the three parameters, C_f and complex η, can be fixed, thus can be used to predict $\Gamma(\psi'' \to f)$ with only one unknown parameter, say, the phase of η. Thus the S- and D-wave mixing scenario provides a mathematical scheme to calculate the partial width of ψ'' decay to any exclusive final state, with its measured partial widths in J/ψ and ψ' decays.

However, the current information concerning the ψ' decay is extremely limited, which prevents us from estimating Q_h values for most exclusive decay modes. Table II lists some hadronic final states which are measured both in J/ψ and ψ' decays, together with the calculated Q_h defined in Eq. (1). Summing up all the channels in Table II makes less than 2% of the ψ' decay through ggg annihilation.

From Table II, we notice that compared with the 12% rule, the ψ' decays to

1. the pseudoscalar-pseudoscalar (PP) mode $K^0_S K^0_L$ is enhanced;
2. the VP and VT modes are suppressed;
3. most of the $B\overline{B}$ modes are consistent with it.

The summed branching fractions and Q_h values for these three categories of decay modes are evaluated and also listed in Table II. In estimating the charmless decays of ψ'', we shall discuss these three different cases separately.

TABLE II: Branching fractions and Q_h values for some J/ψ and $\psi(2S)$ decay channels.

Modes	Channels	$\mathcal{B}_{J/\psi}(10^{-3})$	$\mathcal{B}_{\psi(2S)}(10^{-4})$	Q_h (%)	Ref.
0^-0^-	0^-0^-	0.147 ± 0.023	0.8 ± 0.5	54 ± 35	[4]
	K^+K^-	0.237 ± 0.031	1.0 ± 0.7	42 ± 30	[4]
	$K_S^0 K_L^0$	0.182 ± 0.014	0.52 ± 0.07	28.8 ± 3.7	[7, 8]
sum	PP	0.57 ± 0.07	2.32 ± 1.27	41.1 ± 22.8	
1^-0^-	$\rho\pi$	12.7 ± 0.9	0.29 ± 0.07	0.23 ± 0.6	[10, 15]
	$K^+\overline{K}^*(892)^- + c.c.$	5.0 ± 0.4	0.15 ± 0.08	0.3 ± 0.2	
	$K^0\overline{K}^*(892)^0 + c.c.$	4.2 ± 0.4	1.10 ± 0.20	2.6 ± 0.5	
	$\omega\pi^0$	0.42 ± 0.06	0.20 ± 0.06	4.8 ± 1.6	
1^-2^+	$\omega f_2(1270)$	4.3 ± 0.6	2.05 ± 0.56	4.8 ± 1.5	[19]
	ρa_2	10.9 ± 2.2	2.55 ± 0.87	2.3 ± 1.0	
	$K^*(892)^0\overline{K}_2^*(1430)^0 + c.c.$	6.7 ± 2.6	1.86 ± 0.54	2.8 ± 1.3	
	$\phi f_2'(1525)$	1.23 ± 0.21	0.44 ± 0.16	3.6 ± 1.4	
sum	VP& VT	45.45 ± 7.37	8.64 ± 2.54	1.90 ± 0.64	
$B\overline{B}$	$p\overline{p}$	2.12 ± 0.10	2.07 ± 0.31	9.8 ± 1.5	[4]
	$\Lambda\overline{\Lambda}$	1.30 ± 0.12	1.81 ± 0.34	13.9 ± 2.9	
	$\Sigma^0\overline{\Sigma}^0$	1.27 ± 0.17	1.2 ± 0.6	9.4 ± 4.9	
	$\Sigma(1385)^\pm\overline{\Sigma}(1385)^\mp$	1.03 ± 0.13	1.1 ± 0.4	10.7 ± 4.1	
	$\Xi\overline{\Xi}$	1.8 ± 0.4	$1.88 \pm 0.62\dagger$	10.4 ± 4.2	
	$\Delta^{++}\overline{\Delta}^{--}$	1.10 ± 0.29	1.28 ± 0.35	11.6 ± 4.4	
sum	$B\overline{B}$	8.62 ± 1.21	9.34 ± 2.62	10.8 ± 3.4	

Note: † simple normalization by $\Xi^-\overline{\Xi}^+ = (1/2)\Xi\overline{\Xi}$.

Since the experimental information on the exclusive decays of ψ' is rather limited, we turn to inclusive branching fractions of J/ψ and ψ' hadronic decays as an alternative. The estimation is based on the assumption that the decays of J/ψ and ψ' in the lowest order of QCD are classified into hadronic decays (ggg), electromagnetic decays (γ^*), radiative decays into light hadrons (γgg), and transition to lower mass charmonium states ($c\overline{c}X$) [73, 78]. Thus, using the relation $\mathcal{B}(ggg)+\mathcal{B}(\gamma gg)+\mathcal{B}(\gamma^*)+\mathcal{B}(c\overline{c}X)=1$, one can derive $\mathcal{B}(ggg)+\mathcal{B}(\gamma gg)$ by subtracting $\mathcal{B}(\gamma^*)$ and $\mathcal{B}(c\overline{c}X)$ from unity.

The calculated values of $\mathcal{B}(\gamma^*)$ and $\mathcal{B}(c\overline{c}X)$, together with the values used to calculate them are summarized in Table III. As regards to ψ', two final states $\gamma\eta(2S)$ and $h_c(1^1P_1)+X$ with faint branching fractions are neglected in our calculation. By deducting the contributions $\mathcal{B}(\gamma^*)$ and $\mathcal{B}(c\overline{c}X)$, we find that $\mathcal{B}(J/\psi \to ggg)+\mathcal{B}(J/\psi \to \gamma gg) = (73.5 \pm 0.6)\%$ and $\mathcal{B}(\psi' \to ggg)+\mathcal{B}(\psi' \to \gamma gg) = (19.1 \pm 2.5)\%$, then the ratio of them is

$$Q_g = \frac{\mathcal{B}(\psi' \to ggg+\gamma gg)}{\mathcal{B}(J/\psi \to ggg+\gamma gg)} = (26.0 \pm 3.5)\% \; . \qquad (13)$$

The above estimation is consistent with the previous ones [43, 73]. The relation between the decay rates of ggg and γgg is readily calculated in pQCD to the first order as [74]

$$\frac{\Gamma(J/\psi \to \gamma gg)}{\Gamma(J/\psi \to ggg)} = \frac{16}{5}\frac{\alpha}{\alpha_s(m_c)}\left(1-2.9\frac{\alpha_s}{\pi}\right).$$

Using $\alpha_s(m_c)=0.28$, one can estimate the ratio to be 0.062. A similar relation can be deduced for the ψ' decays. So we obtain $\mathcal{B}(J/\psi \to ggg) \simeq (69.2 \pm 0.6)\%$ and $\mathcal{B}(\psi' \to ggg) \simeq (18.0 \pm 2.4)\%$, while the "26.0% ratio" in Eq. (13) stands well for both ggg and γgg. Although Q_g is considerably enhanced relative to Q_h in Eq. (1), it coincides with the ratio for the $K_S^0K_L^0$ decay mode between ψ' and J/ψ, which is

$$Q_{K_S^0K_L^0} = (28.8 \pm 3.7)\% \; , \qquad (14)$$

according to the recent results from BES [7, 8]. The relation in Eq. (13) was discussed in the literature as the hadronic excess in ψ' decay [43, 73]. It implicates that while some modes are suppressed in ψ' decays, the dominant part of ψ' through ggg decays is enhanced relative to the 12% rule prediction in the light of J/ψ decays.

IV. THE CHARMLESS DECAYS OF ψ''

We define the enhancement or suppression factor as [76]

$$Q(f) \equiv \frac{\Gamma(\psi' \to f)}{\Gamma(J/\psi \to f)}\frac{\Gamma(J/\psi \to e^+e^-)}{\Gamma(\psi' \to e^+e^-)}. \qquad (15)$$

In the $2S$-$1D$ mixing scheme, for any final state f, its partial width in ψ'' decay can be related to its partial width in J/ψ and ψ' decays by Eq.(12), with an unknown parameter ϕ which is the phase of η. This unknown phase constrains the predicted $\Gamma(\psi'' \to f)$ in a finite range. We calculate

$$R_\Gamma \equiv \Gamma(\psi'' \to f)/\Gamma(J/\psi \to f) \qquad (16)$$

TABLE III: Experimental data on the branching fractions for J/ψ and ψ' decays through virtual photon and to lower mass charmonium states used in this analysis. Most of the data are taken from PDG [4], except for $\mathcal{B}(J/\psi, \psi' \to \gamma^* \to \text{hadrons})$, which are calculated by the product $R \cdot \mathcal{B}(J/\psi, \psi' \to \mu^+\mu^-)$, with $R = 2.28 \pm 0.04$ [75]. In estimating the errors of the sums, the correlations between the channels are considered.

Channel	$\mathcal{B}(J/\psi)$	$\mathcal{B}(\psi')$
$\gamma^* \to$ hadrons	(13.4±0.33)%	(1.66±0.18)%
e^+e^-	(5.93±0.10)%	(7.55±0.31)×10^{-3}
$\mu^+\mu^-$	(5.88±0.10)%	(7.3 ±0.8)×10^{-3}
$\tau^+\tau^-$		(2.8 ±0.7)×10^{-3}
$\gamma^* \to X$	(25.22±0.43)%	(3.43±0.27)%
$\gamma\eta_c$	(1.3±0.4)%	(2.8±0.6)×10^{-3}
$\pi^+\pi^-J/\psi$		(31.7±1.1)%
$\pi^0\pi^0J/\psi$		(18.8±1.2)%
$\eta J/\psi$		(3.16±0.22)%
$\pi^0 J/\psi$		(9.6±2.1)×10^{-4}
$\gamma\chi_{c0}$		(8.6±0.7)%
$\gamma\chi_{c1}$		(8.4±0.8)%
$\gamma\chi_{c2}$		(6.4±0.6)%
$c\bar{c}X$	(1.3±0.4)%	(77.4±2.5)%

as a function of $Q(f)$ and plot it in Fig. 1. In the figure, the solid contour corresponds to the solution with $\phi = 0$; the dashed one corresponds to the solution with $\phi = 180°$; and the hatched area corresponds to the solutions with ϕ taking other non-zero values.

To make it clear, we discuss the final states in three situations: $Q(f) < 1$, $Q(f) > 1$, and $Q(f) = 1$.

A. Final states with $Q(f) < 1$

If $Q(f) < 1$, the decay $\psi' \to f$ is suppressed relative to $J/\psi \to f$. The extreme situation is $Q(f) \to 0$, corresponding to the absence of the mode f in ψ' decays. This is the case which was assumed for the $\rho\pi$ mode in the original work to solve the $\rho\pi$ puzzle by the S- and D-wave mixing [61]. If $Q(f) = 0$, the solution of the second equality of Eq. (12) simply yields $\eta = R_{2S}(0)\cos\theta/\sin\theta$ which cannot have a non-zero phase, and $R_\Gamma = 9.2$.

Generally, the suppression factor could be different from zero. Even the $\rho\pi$ and other strongly suppressed VP modes are found in ψ' decays recently by BESII [15] and CLEOc [16] with $Q(VP) \sim \mathcal{O}(10^{-2})$. In this case, there are two real and positive solutions of η as shown in Fig. 1 corresponding to the maximum and minimum of their possible partial widths in ψ'' decays. The solutions with η having a non-zero phase yield the values of R_Γ between the minimum and maximum limits.

For VT final states, which are measured to have $Q(VT) \approx 1/3$, with Eq. (12), we get $2.0 \le R_\Gamma \le 21.6$, as shown in Fig. 1, where the upper and lower limits correspond to two real and positive solutions of η, the range is due to the values of η with non-zero phases.

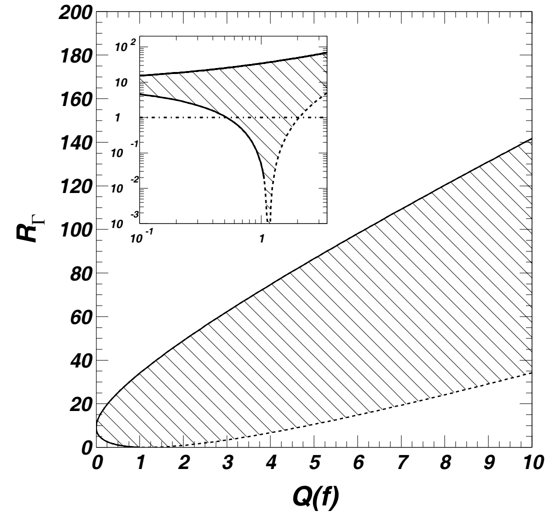

FIG. 1: R_Γ versus $Q(f)$. The solid contour corresponds to $\phi = 0$; the dashed contour corresponds to $\phi = 180°$; and the hatched area corresponds to ϕ having other non-zero values. The inset displays the variation of R_Γ in the vicinity of $Q(f) = 1$, where the dot-dashed line denotes $R_\Gamma = 1$.

B. Final states with $Q(f) > 1$

If $Q(f) > 1$, the decay $\psi' \to f$ is enhanced relative to $J/\psi \to f$. The extreme situation is $Q(f) \to \infty$, corresponding to the complete absence of the final state f in J/ψ decays. For

$$Q(f) > \left| \frac{\cos\theta R_{2S}(0)}{\cos\theta R_{2S}(0) - \frac{5}{2\sqrt{2}m_c^2}\sin\theta R''_{1D}(0)} \right|^2 = 1.06,$$

there are two real solutions of η, one is positive and the other negative. From the first equality of Eq. (12), it is seen that the positive solution leads to the larger R_Γ. i.e. larger $\Gamma(\psi'' \to f)$ (the solid contour in Fig. 1) while the negative solution leads to the smaller $\Gamma(\psi'' \to f)$ (the dashed contour in Fig. 1).

For the known enhanced mode in ψ' decays like $K_S^0K_L^0$, $Q(K_S^0K_L^0) = 2.26$, we find $1.4 \le R_\Gamma \le 52.5$, which corresponds to the ψ'' decay partial width from 0.024 to 0.87 keV.

It should be noted that for finite $\Gamma(\psi' \to f)$, $Q(f) \to \infty$ means the diminishing of $\Gamma(J/\psi \to f)$, which gives rise to $R_\Gamma \to \infty$ according to the definition of R_Γ in Eq.(16). Under such circumstance, it is more intuitive to calculate

$$R'_\Gamma \equiv \Gamma(\psi'' \to f)/\Gamma(\psi' \to f).$$

Its variation as a function of $Q(f)$ is shown in Fig. 2. As $Q(f) \to \infty$, the solid and dashed contours converge into the same point ($R'_\Gamma \to 21$). In such case S-wave state does not decay to f, but D-wave does. Its partial width in ψ' and ψ'' decays comes solely from the contribution of η, or the D-wave matrix element.

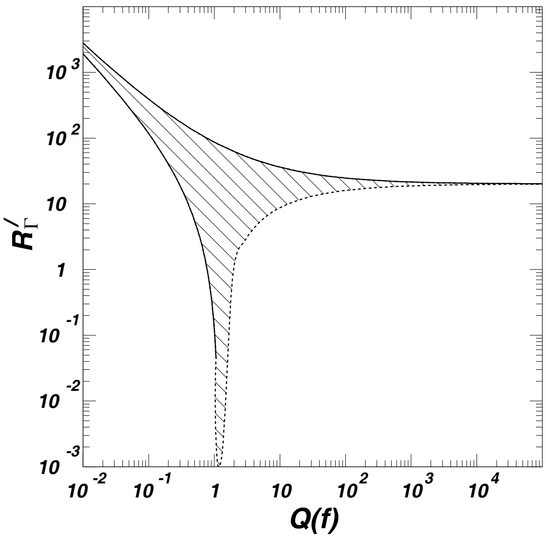

FIG. 2: R'_Γ versus $Q(f)$. The solid contour corresponds to $\phi = 0$; dashed contour corresponds to $\phi = 180°$; and the hatched area corresponds to ϕ having other non-zero values.

C. Final states with $Q(f) = 1$

These final states observe the 12% rule in J/ψ and ψ' decays. In this case, apparently one solution is identical to the pure electromagnetic decays, with $\eta = 5R''_{1D}(0)/(2\sqrt{2}m_c^2)$ and $R_\Gamma = 0.048$. As in the leptonic decays, their partial widths in ψ'' decays are small relative to the partial widths in J/ψ decays. However, there are also other solutions with overwhelming contribution from η which lead to very large partial widths in ψ'' decays. This can be seen from Fig. 1 at the point with $Q(f) = 1$. We get the maximum of R_Γ of 34.0 which corresponds to another real and positive value of η. If η has a non-zero phase, then $0.048 < R_\Gamma < 34.0$.

D. Numerical results

From Fig. 1, we see that except in the range $0.52 < Q(f) < 2.06$ and a small range of the phase, R_Γ is always greater than 1. This range excludes virtually all known decay modes except $B\overline{B}$ which has $Q(B\overline{B}) \approx 1$. Even inside this range, there are other solutions by which R_Γ (or R'_Γ) is at $\mathcal{O}(10)$. It means in this scenario, contrary to the naïve guess, the charmless decay width of ψ'' is greater than that of J/ψ or ψ'. More surprising is that R_Γ could be as large as a few tens for the final states with $Q(f) > 1$. In general, the final states which are enhanced in ψ' decays possibly have a large combined partial width in ψ'' decays, especially if the phase of η is zero or very small.

There are reasons to assume that the phase ϕ which is between the matrix elements $\langle f|2S\rangle$ and $\langle f|1D\rangle$ should be small [63]. For the decay mode like $\rho\pi$, since there is almost complete cancellation between $\cos\theta R_{2S}(0)$ and $\eta\sin\theta$ so that $\langle\rho\pi|\psi'\rangle = \cos\theta R_{2S}(0) - \eta\sin\theta \approx 0$, the phase of η must be small. If this is to be extrapolated to all final states, the physics solution will follow the solid contour of Fig. 1. Another argument comes from the universal phase between the strong and electromagnetic amplitudes of the charmonium decays. It has been known that in the two-body decays of J/ψ, the phase between the strong and electromagnetic amplitudes is universally around 90° [43, 78]. Recently, it has been found that this phase is also consistent with the experimental data of ψ' and ψ'' decays [50, 62]. Since there is no extra phase between $2S$ and $1D$ matrix elements due to electromagnetic interaction, as in the calculations of the leptonic decay rates of ψ' and ψ'', the universal phase between the strong and electromagnetic interactions implies there is no extra phase between the two matrix elements due to the strong interaction too, *i.e.* $\phi \approx 0$. This conclusion means, for the modes which are enhanced in ψ' decays, their partial widths in ψ'' decay must be greater than those in J/ψ or ψ' decays by more than an order of magnitude.

In Sect. III, we estimate that $\mathcal{B}(J/\psi \to ggg) \simeq (69.2\pm0.6)\%$ while $\mathcal{B}(\psi' \to ggg) \simeq (18.0\pm2.4)\%$. Among the final states, we know that VP and VT modes have $Q(f) < 1$. For them,

$$\begin{aligned} \sum\mathcal{B}(J/\psi \to \mathrm{VP, VT}) &\approx 4.6\% \ , \\ \sum\mathcal{B}(\psi' \to \mathrm{VP, VT}) &\approx 8.6\times10^{-4} \ . \end{aligned}$$

Furthermore, there are final states with $Q(f) \approx 1$ (such as $B\overline{B}$), for them,

$$\begin{aligned} \sum\mathcal{B}(J/\psi \to B\overline{B}) &\approx 0.9\% \ , \\ \sum\mathcal{B}(\psi' \to B\overline{B}) &\approx 9.3\times10^{-4} \ . \end{aligned}$$

After subtracting the final states which are known to have $Q(f) < 1$ and $Q(f) \approx 1$, the remaining 63.8% of J/ψ decay with a total width of $\Gamma(J/\psi \to r.f.s.) \approx 58.1$ keV, and 17.8% of ψ' decays with a total width of $\Gamma(\psi' \to r.f.s.) \approx 50.1$ keV which are gluonic either has $Q(f) > 1$ or $Q(f)$ unknown. Here $r.f.s.$ stands for the remaining final states. On the average, these final states have

$$Q(r.f.s.) \approx 2.19 \ .$$

This is roughly comparable to $Q(K_S^0 K_L^0) = 2.26$.

If there is no extra phase between $2S$ and $1D$ matrix elements as argued above, then R_Γ takes the maximum possible value with $R_\Gamma \approx 51.6$ for $Q(r.f.s.) \approx 2.19$. With this value, we find $\Gamma(\psi'' \to r.f.s.) = R_\Gamma \times \Gamma(J/\psi \to r.f.s.) \approx 3.0$ MeV for the partial width of these remaining final states in ψ'' decays, which is 13% of the total ψ'' width.

The above calculation takes the averaged $Q(f)$ for the final states with $Q(f) > 1$ and $Q(f)$ unknown, so it merely serves as a rough estimation. The exact value of the partial width should be the sum of the individual final states which in general have various $Q(f)$ values. At present a major impediment to do the accurate evaluation is the lack of experimental information. Nevertheless, if we take 13% as charmless decays (the calculations done channel by channel for VP, VT and $B\overline{B}$ modes in Table II give a summed maximum possible width of 93 keV in ψ'' decays, or 0.4% of the ψ'' total width) together with the charmonium transition contributions of 3% [76] (2.5% for radiative transition and 0.4% for $\pi\pi J/\psi$), we obtain a maximum of ψ'' non-$D\overline{D}$ decay branching fraction of 16% in the $2S$-$1D$ mixing scenario to be compared with 18% as summarized in Ref. [76].

V. MEASUREMENT NON-$D\overline{D}$ DECAYS AT ψ''

A. Introduction

The lowest charmonium resonance above the charmed particle production threshold is ψ'' which provides a rich source of $D^0\overline{D^0}$ and D^+D^- pairs, as anticipated theoretically [59]. However, non-$D\overline{D}$ decays of the ψ'' was also expected theoretically and was searched for experimentally almost two decades ago. The OZI violation mechanism [79] was utilized to understand the possibility of the non-$D\overline{D}$ decays of the ψ'' [80], and the pioneer experimental investigations of the non-$D\overline{D}$ decay modes could be found in Ref. [70].

After a period of silence, the study of the non-$D\overline{D}$ decays of the ψ'' gets renaissance as more and more data are collected at ψ'' by BES-II and CLEOc. Extensive studies have been made for exclusive non-$D\overline{D}$ decay channels [11–14, 81–83], of which the most prominent one is the hadronic transition of $\psi'' \to J/\psi\pi^+\pi^-$ once sought by Mark-III [70]. Recently both BES and CLEOc collaborations reported their measurements for this channel [84, 85], which are in marginally agreement with each other. However, except for $J/\psi\pi^+\pi^-$ final state, no statistically significant signals of the non-$D\overline{D}$ decays at ψ'' are presented up till now. One possible reason is that the existing data samples are still not large enough to search for the channels of such small branching fractions.

Besides the searches for the exclusive modes, there is the search for inclusive decays. In fact, the indication of a substantial non-$D\overline{D}$ decays of the ψ'' was originally caught attention from the comparison of the cross sections of the inclusive hadrons and $D\overline{D}$ at the ψ'' peak. Table IV summarizes the measurements of the resonance parameters and the observed cross section of the inclusive hadronic decay, and Table V summarizes the measurements of the $D\overline{D}$ cross section reported by BES-II [86] and CLEOc [87] collaborations using either double-tag or single-tag method. The simple average of the values in the two tables gives $\sigma^{obs}(\psi'') \simeq 7.75$ nb and $\sigma(D\overline{D}) \simeq 6.27$ nb, respectively, with difference of about 1.5 nb (about 19% of the total cross section of the ψ'' production), which implies the non-$D\overline{D}$ decays of the ψ'' is important. However, the existence of substantial non-$D\overline{D}$ decays is not unambiguous due to the poor statistics of the data samples and the complexity of the analysis. In addition, results from different experiments are consistent with each other only marginally. Moreover, for inclusive measurement, the contribution of the non-$D\overline{D}$ decays has been neglected in previous experiments in measuring the ψ'' resonance parameters.

Besides the experimental motivations, there are interests to look into this problem from the theoretical point of view. In Ref. [76], it is estimated that at most 600 keV ($\sim 2.5\%$) of the ψ'' total width of (23.6 ± 2.7) MeV is due to the radiative transition, and perhaps as much as another 100 keV ($\sim 0.4\%$) is due to the hadronic transition to $J/\psi\pi\pi$. All these together are far from accounting for a deficit of 19% of the total ψ'' width.

In a most recent paper [64] (refer to section IV of this paper), based on the available experimental information of J/ψ and ψ' decays, it is estimated that the charmless decay of ψ'' by virtue of the S- and D-wave charmonia mixing scheme [61] could be as large as 3.1 MeV or 13% of the total decay width of ψ''. By charmless decay, we exclude those decay modes with either open or hidden charm. If we take into account also the charmonium transition contributions of 3% [76] (2.5% for radiative transition and 0.34% for $J/\psi\pi\pi$), the total non-$D\overline{D}$ decay of ψ'' as large as 16% is conceivable in the $2S$-$1D$ mixing scenario. In addition, there are some estimations for exclusive ψ'' decay, such as Refs. [63, 94**?**]. All of these wait experimental test or confirmation. With the expected more data at ψ'' from the running CLEOc, it is feasible to search for the possible large non-$D\overline{D}$ decays. Furthermore, we notice that the accurate determination of certain exclusive final state can supply the knowledge of the phase between the S-wave and D-wave matrix elements (refer to section III of this paper). Such information can provide some clues concerning the dynamics of the OZI suppressed decays of charmonium.

In this section, we concentrate on the experimental aspects of the non-$D\overline{D}$ decays of the ψ'' in e^+e^- collider. In the following text, we discuss the exclusive, the quasi-inclusive and the inclusive methods for the non-$D\overline{D}$ searching, especially, we shall expound some of the technique details in the handling of the experimental data which were overlooked in previous measurements.

B. Exclusive method

The calculations in section IV show that those final states which are suppressed in ψ' decays relative to J/ψ, and especially those ones enhanced in ψ' decays, will show up in ψ'' decays with maximum possible partial widths more than an order of magnitude greater than their widths in J/ψ or ψ' decays. So these exclusive charmless modes should be searched for in ψ'' decays. This provides direct test of the calculations based on the $2S$-$1D$ mixing scheme. Here we discuss three typical exclusive modes: VP mode which is suppressed in ψ' decays relative to J/ψ, PP mode which is enhanced, and of particular interest, the $B\overline{B}$ and $\phi f_0(980)$ modes which observe the 12% rule.

To measure the exclusive VP mode in ψ'' decays by

TABLE IV: Resonance parameters and the total cross section of the ψ'' at the resonance peak. $\Gamma_{\psi''}$ is the full width, Γ_{ee} the partial width to eletron pairs, σ^{obs} the observed cross section, R_{flat} the value to describe the continuum contribution, σ^{Born} the Born order cross section of the ψ''. It should be noticed that in evaluating the uncertainty of σ^{Born}, the correlations between the errors are not included.

Experiment/ Accelerator	$M_{\psi''}$ (MeV/c^2)	$\Gamma_{\psi''}$ (MeV/c^2)	Γ_{ee} (eV/c^2)	σ^{obs} (nb)	R_{flat}	σ^{Born} (nb)	$\sigma^{obs}/\sigma^{Born}$
LGW/SPEAR [88]	3772 ± 6	28 ± 5	370 ± 90	10.3 ± 2.1	$\sim 2.8^a$	13.6 ± 4.1	0.75
DELCO/SPEAR [89]	3770 ± 6	24 ± 5	180 ± 60	$\sim 6.1^a$	$\sim 2.5^a$	7.7 ± 3.0	0.79
MARKII/SPEAR [90]	3764 ± 5	24 ± 5	276 ± 50	9.3 ± 1.6	2.22 ± 0.06	11.9 ± 3.3	0.78
CBAL/SPEAR [91]	3768 ± 2	34 ± 8	283 ± 70	6.7 ± 0.9	2^b	8.6 ± 2.9	0.78
BESII/BEPC [92]	3773 ± 1	26 ± 4	247 ± 35	$\sim 6.4^a$	2.44 ± 0.08	9.8 ± 2.0	0.65
BESII/BEPC [76, 93]	3772	23.2	–	7.7 ± 1.1	$\sim 2.16^a$	12.1 ± 1.9^c	0.64

[a] The value estimated from the corresponding figure provided by literature or thesis, only for reference.
[b] The R is treated as a constant in the fitting.
[c] Absent values are adopted from PDG for calculating σ^{Born}.

TABLE V: Measurement of the cross sections $\sigma(D\overline{D}) \equiv \sigma(e^+e^- \to \psi'' \to D\overline{D})$, in nb.

Collaboration	$\sqrt{s}$ (GeV)	$\sigma(D^+D^-)$	$\sigma(D^0\overline{D}^0)$	$\sigma(D\overline{D})$
BESII[86]	3.773	$2.56 \pm 0.08 \pm 0.26$	$3.58 \pm 0.09 \pm 0.31$	$6.14 \pm 0.12 \pm 0.50$
CELO [87]	3.773	$2.79 \pm 0.07^{+0.10}_{-0.04}$	$3.60 \pm 0.07^{+0.07}_{-0.05}$	$6.39 \pm 0.10^{+0.17}_{-0.08}$

e^+e^- experiments, the contribution from non-resonance virtual photon amplitude and its interference with the resonance must be treated with care. A recent study on the measurement of $\psi'' \to VP$ in e^+e^- experiments shows [62] that with the decay rate predicted by the S- and D-wave mixing, the interference between the three-gluon decay amplitude and the continuum one-photon amplitude leads to very small cross sections for some VP modes, e.g. $\rho\pi$ and $K^{*+}K^- + c.c.$, due to the destructive interference, but much larger cross sections for other VP modes, e.g. $K^{*0}\overline{K}^0 + c.c.$ due to the constructive interference. In another word, although the branching fractions of $\rho^0\pi^0$ and $K^{*0}\overline{K}^0$ differ by only a fraction due to SU(3) symmetry breaking [67], their production cross sections in e^+e^- collision differ by one to two orders of magnitude.

Among the PP modes, there is the $K^0_SK^0_L$ final state which decays only through strong interaction and does not couple to virtual photon [63, 67]. There is no complication of electromagnetic interaction and the interference between it and the resonance. So the observed $K^0_SK^0_L$ in e^+e^- experiment is completely from resonance decays. In the $2S$-$1D$ mixing scheme, with the BES recently reported $K^0_SK^0_L$ branching fractions in J/ψ [7] and ψ' [8] decays as inputs, it is estimated [63] $(1.2 \pm 0.7) \times 10^{-6} < \mathcal{B}(\psi'' \to K^0_SK^0_L) < (3.8 \pm 1.1) \times 10^{-5}$ [95]. If there is no extra phase between $\langle K^0_SK^0_L|2^3S_1\rangle$ and $\langle K^0_SK^0_L|1^3D_1\rangle$, then its branching fraction is at the upper bound. With 17.7 pb^{-1} ψ'' data, BES has set an upper limit [13], which is still beyond the sensitivity for testing the above prediction. More precise determination of this branching fraction is expected from the analysis based on larger data samples of CLEOc and BES-III.

However, for other PP modes, or more generally other final states which are enhanced in ψ' decays, in e^+e^- experiments there is still the complication from the non-resonance virtual photon amplitude and its interference with the resonance.

Of particular interest are the final states with $Q(f) \approx 1$. These are the $B\overline{B}$ modes and the vector-scalar mode $\phi f_0(980)$ [96]. As discussed in subsection IV C for $Q(f) = 1$, there are two real and positive solutions with $R_\Gamma = 0.048$ and $R_\Gamma = 34.0$. These two solutions are three orders of magnitude apart, their branching fractions are extremely sensitive to the relative phase between the $2S$ and $1D$ matrix elements. The $B\overline{B}$ branching fractions in J/ψ decays are at $\mathcal{O}(10^{-3})$, while $\phi f_0(980)$ is $(3.2 \pm 0.9) \times 10^{-4}$. If the physics solution of η is the larger one of the two real values, then the $B\overline{B}$ branching fraction in ψ'' decay would be at $\mathcal{O}(10^{-4})$ and $\phi f_0(980)$ would be 4.2×10^{-5}, which can be observed in the ψ'' data sample over 1 fb^{-1}.

A remark is in order here. As a matter of fact, many subtleties concerning the efficiency determination and Monte Carlo simulation have to be taken into consideration in order to acquire correct and accurate measurement of the exclusive decay at the ψ'' peak in e^+e^- experiments. A furthermore expound of such measurement is presented in Ref. [97].

C. Quasi-inclusive method

The exclusive method gives the branching fraction of each individual charmless decay mode, but provides no information on the total fraction of the non-$D\overline{D}$ decays. In this subsection, we develop a quasi-inclusive method, by which we can derive the total non-$D\overline{D}$ decay branching fraction from the inclusive measurement of certain particle or final state.

Certain final state f may be produced from the direct ψ'' decays, and/or from the cascade $D\overline{D}$ decays, as shown below:

$$\begin{tikzcd} \psi'' \arrow[rr] \arrow[dr] & & f \\ & D\overline{D} \arrow[ur] & \end{tikzcd}$$

The following quantities are needed to describe such a process in detail

- $\mathcal{B}(\psi'' \to f)$: the total branching fraction of final state f in ψ'' decay;
- $\mathcal{B}(\psi'' \to D\overline{D})$: the branching fraction of $D\overline{D}$ in ψ'' decays;
- $\mathcal{F}(D\overline{D} \to f)$: the branching fraction of final state f in $D\overline{D}$ decay;
- $\overline{\mathcal{B}}(D\overline{D} \to f)$: the branching fraction of final state f from non-$D\overline{D}$ ψ'' decay (direct ψ'' decays).

So we have the following relation

$$\begin{aligned} \mathcal{B}(\psi'' \to f) &= \mathcal{B}(\psi'' \to D\overline{D}) \cdot \mathcal{F}(D\overline{D} \to f) \\ &+ \overline{\mathcal{B}}(D\overline{D} \to f) \ . \end{aligned} \tag{17}$$

According to the above relation, in order to find out $\mathcal{B}(\psi'' \to D\overline{D})$, or equivalently $[1 - \mathcal{B}(\psi'' \to D\overline{D})]$, we need to know $\mathcal{B}(\psi'' \to f)$, $\mathcal{F}(D\overline{D} \to f)$ and $\overline{\mathcal{B}}(D\overline{D} \to f)$. So we first discuss how to determine these branching fractions experimentally.

1. *Determining* $\mathcal{F}(D\overline{D} \to f)$

The D meson decay branching fraction ($\mathcal{F}_i$) was originally measured through the production ($\sigma_D \cdot \mathcal{F}_i$), then converted into D decay branching fraction ($\mathcal{F}_i$) by employing the cross section σ_D at ψ'' peak [88, 99]. Unfortunately, as indicated in Table IV, there are large discrepancies among the measurements of σ_D. Furthermore, in previous measurements ψ'' was assumed solely or substantially decay into $D\overline{D}$, which is questionable. We will return to this point in detail in section V D.

Twenty years ago, a new technique was developed by MARK III group [98] to derive the D meson branching fraction without relying on the measurement of the D-production cross section. To determine the individual branching fraction ($\mathcal{F}_i$), together with the number of produced $D\overline{D}$ pairs (N), the corrected number of single tags (S_i) and double tags (D_{ij}) are employed in a χ^2 minimization fit, using the following expressions:

$$\begin{aligned} S_i &= 2N\mathcal{F}_i\epsilon_i - 2N\sum_j \mathcal{F}_i\mathcal{F}_j\alpha^i_{ij} \ ; \\ D_{ij} &= \begin{cases} 2N\mathcal{F}_i\mathcal{F}_j\epsilon_{ij} & (i \neq j) \ , \\ N\mathcal{F}_i^2\epsilon_{ii} & (i = j) \ , \end{cases} \end{aligned}$$

where ϵ_i is the efficiency for reconstructing a single tag in the ith D decay mode, ϵ_{ij} is the efficiency for reconstructing a double tag for $D\overline{D}$ decay mode i and j, and α^i_{ij} is the efficiency for reconstructing a single tag of mode i while simultaneously reconstructing the entire event as a double tag of mode i and j. The second term in the expression for S_i removes from the single-tag sample those tags which also appear in the double-tag sample. This subtraction leaves the two samples independent and eliminates the directly correlated errors. Comparing the number of observed single tag (S_i) events with double tags (D_{ij}) events yields the branching fraction of decay mode j without referring to the production cross section. In practice, the S_i serves to determine the relative branching fractions, while the D_{ij} sets the absolute scale of the branching fractions.

By virtue of the approach introduced above, we obtain $\mathcal{F}(D\overline{D} \to f)$ for a final state f without measuring production cross section. In fact, any measured results of $\mathcal{F}(D\overline{D} \to f)$ by the approach can be used for the following analysis even if the results are from different experimental groups.

2. *Determining* $\mathcal{B}(\psi'' \to f)$ *and* $\overline{\mathcal{B}}(D\overline{D} \to f)$

The $\mathcal{B}(\psi'' \to f)$ is obtained by scan experiments. Avoiding abstract, we take the inclusive K_S final state as example to explain the scan process.

Usually at least two scan curves are needed, one is the inclusive hadron final state, from which we determine the total decay width of ψ'' (Γ_t); the other is the inclusive K_S final state (K_S plus anything), from which we determine the partial decay width of this inclusive mode. The ratio of these two widths give the branching fraction $\mathcal{B}(\psi'' \to K_S + anything)$.

At the energy in the vicinity of ψ'', besides the ψ'' resonance, there are other cross sections due to the non-resonance continuum process as well as the tails of the J/ψ and ψ', which together account for a large proportion of the measured cross section at the ψ'' peak. According to the decay topology, the inclusive hadron events are divided into two categories, the $D\overline{D}$ events and the $D\overline{D}$-less final states. Here we coin a word "$D\overline{D}$-less final states" to depict all the processes which do not go through D or $\overline{D}$, including non-resonance process, tails of J/ψ and ψ', and the non-$D\overline{D}$ decays of the ψ''. By non-$D\overline{D}$ decay, we mean the ψ'' decays which do not go through D or $\overline{D}$. Here the correct Monte Carlo simulation deserves special attention. For example, the non-resonance continuum process can be simulated by Lund model [100]; while the J/ψ, ψ' tails and $D\overline{D}$ decay by the Monte Carlo which describes J/ψ, ψ' and D decays respectively [101, 102]. The synthetic hadron efficiency ϵ_{had} is expressed as

$$\epsilon_{had}^{K_S} = \frac{\epsilon_{DL}^{K_S} \cdot \sigma_{DL}^{K_S} + \epsilon_{DD}^{K_S} \cdot \sigma_{DD}^{K_S}}{\sigma_{DL}^{K_S} + \sigma_{DD}^{K_S}} \ , \tag{18}$$

where ϵ and σ denote the efficiencies and the corresponding cross sections, the subscript DL indicates the $D\overline{D}$-less decay, while DD the $D\overline{D}$ decay, the subscript K_S represents the inclusive hadron final state containing K_S particle. However, σ_{DL} and $\sigma_{DD}^{K_S}$ are to be determined from experiment. Fortunately, according to Eq. (18), what we need to know is the ratio of σ_{DL} to $\sigma_{DD}^{K_S}$, which could be acquired experimentally as explained below.

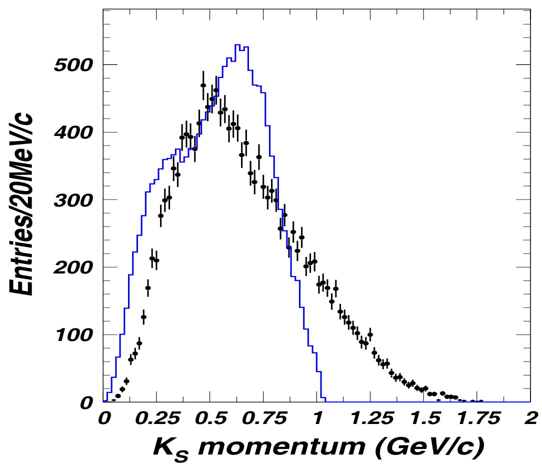

FIG. 3: K_S momentum distributions: histogram for inclusive K_S final state from $D\overline{D}$ decays; dots with error bars for inclusive K_S final state from $D\overline{D}$-less decays.

Fig. 3 shows the momentum distributions of the inclusive K_S events due to the $D\overline{D}$ (histogram) and the $D\overline{D}$-less (dots with error bar) decays, which are simulated by DDGEN and Lund generators, respectively. With the two generators, we obtain the efficiencies ϵ_{DL} and $\epsilon_{DD}^{K_S}$. For the real data sample, its momentum distribution is the synthetic one of the $D\overline{D}$ and the $D\overline{D}$-less decays with certain proportion of each. We fit the data distribution with those of Monte Carlo distributions as shown in Fig. 3, then obtain the numbers of the observed events of these two processes, which are denoted as n_{DL} and $n_{DD}^{K_S}$. Utilizing the relation

$$n = L\sigma\epsilon \ \ (L\text{: experiment luminosity}) \ ,$$

we get

$$\frac{n_{DL} \cdot \epsilon_{DD}^{K_S}}{n_{DD}^{K_S} \cdot \epsilon_{DL}} = \frac{\sigma_{DL}}{\sigma_{DD}^{K_S}} \ . \tag{19}$$

In short, we determine $\mathcal{B}(\psi'' \to f)$ by scan experiment. As to $\overline{\mathcal{B}}(D\overline{D} \to f)$, we notice that in Fig. 3, most of the momentum from $D\overline{D}$ decay is less than 1 GeV, so a requirement of the momentum greater than 1.1 GeV eliminates all the $D\overline{D}$ decay event, while leaving the events from $D\overline{D}$-less decay. Using such events, we obtain another scan curve. Fit this curve together with the curve of the total inclusive hadrons, we determine $\overline{\mathcal{B}}(D\overline{D} \to f)$. This process is similar to the determination of $\mathcal{B}(\psi'' \to f)$, but only one efficiency ϵ_{DL} is needed.

3. *Deriving* $\mathcal{B}(\psi'' \to D\overline{D})$

Since $\mathcal{B}(\psi'' \to f)$, $\mathcal{F}(D\overline{D} \to f)$ and $\overline{\mathcal{B}}(D\overline{D} \to f)$ are obtained experimentally, by solving Eq. (17), we get $\mathcal{B}(\psi'' \to D\overline{D})$, then acquire the non-$D\overline{D}$ decay branching fraction $[1-\mathcal{B}(\psi'' \to D\overline{D})]$.

Next we understand Eq. (17) from physics point of view. We introduce a new quantity defined as

$$\overline{\mathcal{F}}(D\overline{D} \to f) = \frac{\overline{\mathcal{B}}(D\overline{D} \to f)}{1-\mathcal{B}(\psi'' \to D\overline{D})} \ ,$$

which is the ratio of the branching fraction of non-$D\overline{D}$ decay for the final state f to that of the total non-$D\overline{D}$ decays. Then Eq. (17) reads

$$\begin{aligned} \mathcal{B}(\psi'' \to f) \ &= \ \mathcal{B}(\psi'' \to D\overline{D}) \cdot \mathcal{F}(D\overline{D} \to f) \\ &+ \ [1-\mathcal{B}(\psi'' \to D\overline{D})] \cdot \overline{\mathcal{F}}(D\overline{D} \to f) \ . \end{aligned} \tag{20}$$

We chose $\mathcal{B}(\psi'' \to f)$ as ordinate, and $\mathcal{B}(\psi'' \to D\overline{D})$ as abscissa, which varies from 0 to 100%. For certain final state f, $\mathcal{B}(\psi'' \to f)$ corresponds to the horizontal line, as shown in Fig. 4, where the shaded band denotes the uncertainty of $\mathcal{B}(\psi'' \to f)$. If $\mathcal{B}(\psi'' \to D\overline{D}) = 0$, then $\mathcal{B}(\psi'' \to f) = \overline{\mathcal{F}}(D\overline{D} \to f)$, which means all events of final states f coming from non-$D\overline{D}$ decay; if $\mathcal{B}(\psi'' \to D\overline{D}) = 100\%$, then $\mathcal{B}(\psi'' \to f) = \mathcal{F}(D\overline{D} \to f)$, which means all events of final states f coming from $D\overline{D}$ decay, or equivalently the complete absence of the contribution from non-$D\overline{D}$ decay. Without losing generality, we assume that $\mathcal{F}(D\overline{D} \to f) > \overline{\mathcal{F}}(D\overline{D} \to f)$, then we obtain an upward line in the coordinate, as shown in Fig. 4. Similarly, if we assume that $\mathcal{F}(D\overline{D} \to f) < \overline{\mathcal{F}}(D\overline{D} \to f)$, we obtain a downward line in the coordinate. The point of interaction gives rise to the determination of $\mathcal{B}(\psi'' \to D\overline{D})$, which is denoted by the arrow in Fig. 4. The hatched area indicates the uncertainty of $\mathcal{B}(\psi'' \to D\overline{D})$, which is due to the interaction of the askew line with the uncertain band of $\mathcal{B}(\psi'' \to f)$. Here we notice that the smaller the slope of askew line in Fig. 4, the longer is the interaction line with the uncertainty band, which means the larger uncertainty in the determination of $\mathcal{B}(\psi'' \to D\overline{D})$. On the contrary, if the slope of the askew line is larger, we obtain comparatively smaller uncertainty on $\mathcal{B}(\psi'' \to D\overline{D})$. In another word, to obtain $\mathcal{B}(\psi'' \to D\overline{D})$ as accurate as possible, we select those final states which have as large as possible the difference between $\mathcal{F}(D\overline{D} \to f)$ and $\overline{\mathcal{F}}(D\overline{D} \to f)$.

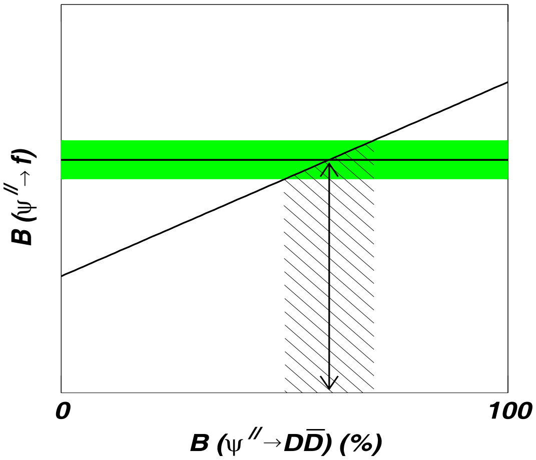

FIG. 4: Diagram for the determination of $\mathcal{B}(\psi'' \to D\overline{D})$. The horizontal line indicates a certain $\mathcal{B}(\psi'' \to f)$ with shaded band as its uncertainty; the askew line is drawn based on information of $\mathcal{B}(D\overline{D} \to f)$ and $\overline{\mathcal{B}}(D\overline{D} \to f)$; the arrow indicates the $\mathcal{B}(\psi'' \to D\overline{D})$ determined from experiment with hatched area as its uncertainty.

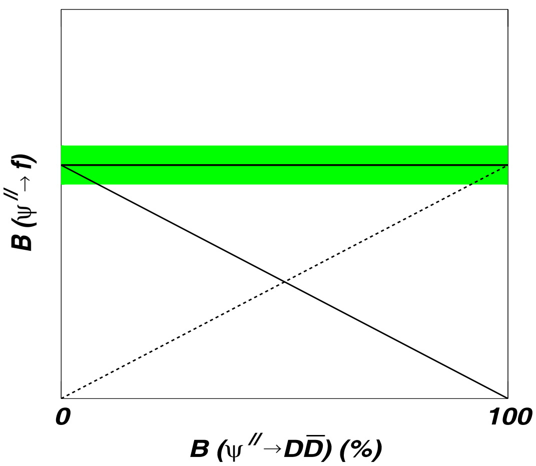

FIG. 5: Diagram for the determination of $\mathcal{B}(\psi'' \to D\overline{D})$. The horizontal line indicates a certain $\mathcal{B}(\psi'' \to f)$ with shaded band as its uncertainty; the solid askew line corresponds to $\mathcal{F}(D\overline{D} \to f) = 0$ for certain final state f while the dashed askew line corresponds to $\overline{\mathcal{F}}(D\overline{D} \to f) = 0$ for certain final state f.

One special case is that $\mathcal{F}(D\overline{D} \to f) = \overline{\mathcal{F}}(D\overline{D} \to f)$, according to Eq. (20), we have $\mathcal{B}(\psi'' \to f) = \mathcal{F}(D\overline{D} \to f) = \overline{\mathcal{F}}(D\overline{D} \to f)$. Under such circumstance, we can not get any information about the total non-$D\overline{D}$ decay. In another word, if the scan experiment obtains the $\mathcal{B}(\psi'' \to f)$, which is equal to $\mathcal{F}(D\overline{D} \to f)$, we could neither confirm nor deny the existence of non-$D\overline{D}$ decay. Another special case is $\mathcal{F}(D\overline{D} \to f) = 0$ or $\overline{\mathcal{F}}(D\overline{D} \to f) = 0$. For example, for baryon anti-baryon ($B\overline{B}$) final state, which does not come from $D\overline{D}$ decays, $\mathcal{F}(D\overline{D} \to f) = 0$. Under such circumstance Eq. (17) becomes

$$\mathcal{B}(\psi'' \to f) = [1 - \mathcal{B}(\psi'' \to D\overline{D})] \cdot \overline{\mathcal{F}}(D\overline{D} \to f) \ . \quad (21)$$

According to the above equation, $\mathcal{B}(\psi'' \to f) = \overline{\mathcal{F}}(D\overline{D} \to f)$ for $\mathcal{B}(\psi'' \to D\overline{D}) = 0$ while $\mathcal{B}(\psi'' \to f) = 0$ for $\mathcal{B}(\psi'' \to D\overline{D}) = 1$. So mathematically, when $\mathcal{B}(D\overline{D} \to f) = 0$, Eq. (21) provides a downward line in coordinate as shown in Fig. 5, where the only point of interaction is at $\mathcal{B}(\psi'' \to D\overline{D}) = 0$. Physically, at the start point of abscissa in Fig. 5, Eq. (21) merely gives a fact that all the final states f events come from non-$D\overline{D}$ decay since its decay through $D\overline{D}$ is forbidden. As for other values of $\mathcal{B}(\psi'' \to D\overline{D})$, we could not get any information because the two lines do not intersect.

If the uncertainties due to $\mathcal{F}(D\overline{D} \to f)$ and $\overline{\mathcal{F}}(D\overline{D} \to f)$ are taken into account, the askew lines in Fig. 4 and 5 become bands, just like the horizontal ones. Under such circumstance, all discussions above are valid except for the uncertainty of the determined $\mathcal{B}(\psi'' \to D\overline{D})$, which becomes larger.

D. Inclusive method

In this section, we first retrospect the previous scan experiments, and point out drawbacks of these experiments, then put forth a new method which determines the inclusive non-$D\overline{D}$ decay directly with small systematic errors.

1. *Scan experiment*

Fig. 6 draws diagrammatically the observed cross section in the vicinity of the ψ'' resonance calculated with parameters provided by PDG [4]. The total observed cross section σ^{tot} is usually expressed as

$$\sigma^{tot} = \sigma_{NR} + \sigma_{J/\psi} + \sigma_{\psi'} + \sigma_{\psi''}, \quad (22)$$

which contains four parts: the non-resonance cross section σ_{NR}, the radiative tails of J/ψ ($\sigma_{J/\psi}$) and ψ' ($\sigma_{\psi'}$), and the ψ'' resonance cross section ($\sigma_{\psi''}$). The non-resonance cross section is usually expressed in terms of R value and the μ pair cross section st Born order as $\sigma_{NR} = R \cdot \sigma(e^+e^- \to \mu^+\mu^-)$. The Breit-Wigner formula is adopted to depict the resonances of the J/ψ, ψ' and ψ'', where the total decay width of the ψ'' is energy dependent:

$$\sigma_{\psi''}(E_{c.m.}) = \frac{12\pi\Gamma_{ee}\Gamma_{\psi''}(E_{c.m.})}{(E_{c.m.}^2 - M_{\psi''}^2)^2 + \Gamma_{\psi''}^2(E_{c.m.})M_{\psi''}^2} \ ,$$

with

$$\Gamma_{\psi''}(E_{c.m.}) = C_\Gamma \left[\frac{p_{D^0}^3}{1 + (rp_{D^0})^2} + \frac{p_{D^\pm}^3}{1 + (rp_{D^\pm})^2} \right], \quad (23)$$

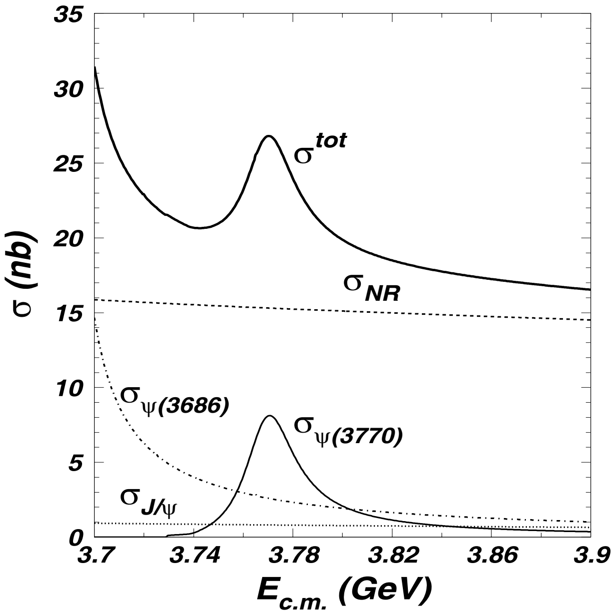

FIG. 6: The observed cross section in the vicinity of the ψ'' resonance calculated with parameters provided by PDG. The total observed cross section σ^{tot} conventionally divided into four parts: the cross section from non resonant contribution σ_{NR}, from radiative tails of J/ψ ($\sigma_{J/\psi}$) and ψ' ($\sigma_{\psi'}$), and cross section from resonance ψ'' ($\sigma_{\psi''}$).

where p is the D^0 or $D^\pm$ momentum, r is the classical interaction radius, and C_Γ is defined as follows:

$$C_\Gamma \equiv \frac{\Gamma_{\psi''}(M_{\psi''})}{\left[\frac{p_{D^0}^3}{1+(rp_{D^0})^2}+\frac{p_{D^\pm}^3}{1+(rp_{D^\pm})^2}\right]\Bigg|_{E_{c.m.}=M_{\psi''}}}.$$

Here $\Gamma_{\psi''}(M_{\psi''})$ is the ψ'' total decay width given by PDG [4]. The radiative correction scheme used by SPEAR experiment, is based on the work of Bonneau and Martin [103] and that of Jackson and Scharre [104]. The former only calculated to α^3 order which is insufficient for resonances; while the latter made some mistakes [105, 106]. The drawbacks due to the treatment of the radiative correction with these two schemes were studied for Z in Ref. [105] and for narrow resonances of ψ and Υ families in Ref. [106]; but no such study on ψ'' has been conducted so far. BES treats the radiative correction based on the structure function approach which achieves 0.1% accuracy [107]. The effect of the radiative correction can be seen from the ratio between the observed cross section σ^{obs} and the Born order cross section σ^{Born}, which is defined as

$$\sigma^{Born} = \frac{12\pi\Gamma_{ee}}{M_{\psi''}^2\Gamma_{\psi''}} \ .$$

From the last column of Table IV, we see that the treatment of the radiative correction was consistent among different experiments at SPEAR. However, the resonance parameters from different experiments differ significantly. The reason remains unknown.

Another problem in previous analyses [88–91] is that the non-$D\overline{D}$ branching ratio was neglected in the fitting of the ψ'' resonance curves. Since light hadrons have much lower thresholds than $D\overline{D}$, a larger non-$D\overline{D}$ branching ratio affects both directly the shape of the resonance curve and indirectly through the energy-dependent total width. Specially, taking into account the non-$D\overline{D}$ decays, Eq. (23) is revised by including another term, that is

$$\Gamma_{\psi''}(E_{c.m.}) = C'_\Gamma \times \left[\frac{p_{D^0}^3}{1+(rp_{D^0})^2}+\frac{p_{D^\pm}^3}{1+(rp_{D^\pm})^2}+C_{\text{non-}D\overline{D}}\right] ,$$

where $C_{\text{non-}D\overline{D}}$ is proportional to the partial width of the non-$D\overline{D}$ decays, and

$$C'_\Gamma \equiv \frac{\Gamma_{\psi''}(M_{\psi''})}{\left[\frac{p_{D^0}^3}{1+(rp_{D^0})^2}+\frac{p_{D^\pm}^3}{1+(rp_{D^\pm})^2}+C_{\text{non-}D\overline{D}}\right]\Bigg|_{E_{c.m.}=M_{\psi''}}} .$$

With the $C_{\text{non-}D\overline{D}}$ term in the expression for $\Gamma_{\psi''}$, the fitting of the resonance curve to extract the resonance parameters is done together with the fitting of the $D\overline{D}$ or the non-$D\overline{D}$ cross section. In this procedure, the non-$D\overline{D}$ decay branching fraction is extracted together with the resonance parameters.

2. *Leading particle method*

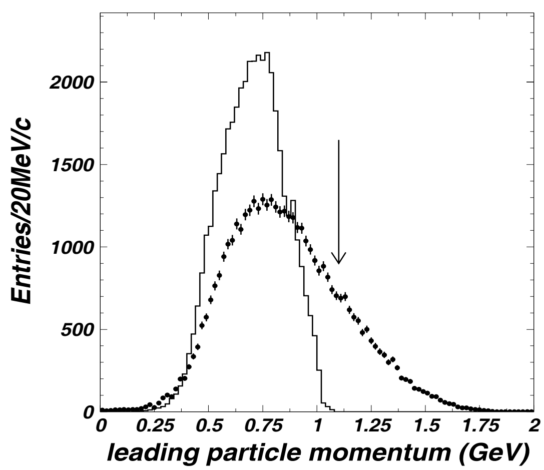

FIG. 7: Momentum distributions of the leading particle: histogram for $D\overline{D}$ events; dots with error bars for $D\overline{D}$-less events (not normalized). The arrow indicates the cut at 1.1 GeV.

At first sight, it seems easy to measure the $D\overline{D}$ cross section because the $D\overline{D}$ decay has distinctive event topology and can be selected without ambiguity. But it is impractical for the scan experiment due to its low statistics.

So we turn to the measurement of the $D\overline{D}$-less cross section, and take advantage of the salient kinetic feature of $D\overline{D}$-less decays, as mentioned in section V C 2 for final state with K_S. This kinetic feature holds for all kinds of final states, which can be seen from a rough estimation. Since the mass of the ψ'' is just above the $D\overline{D}$, the D and $\overline{D}$ are almost static. Their decay products have momentum less than 932.3MeV/c, which is half of the D^0 mass. So the particles with momentum greater than this value must come from processes other than D or $\overline{D}$ decays. With this distinction between $D\overline{D}$ decay and $D\overline{D}$-less decay, we do not need the particle identification, but merely select the particle having the largest momentum, which is called the leading particle in a hadronic event. Fig. 7 shows the Monte Carlo simulation of the momentum distributions for the leading particle from $D\overline{D}$ (denoted by histogram) and $D\overline{D}$-less decays (denoted by dot with error bar). It can be seen that the leading particles from $D\overline{D}$ decay all have momentum less then 1.1 GeV. So a cut of $p < 1.1$ GeV on momentum eliminates almost all events from $D\overline{D}$ decays, while the surviving ones must come from $D\overline{D}$-less decays. This gives a direct measurement of $D\overline{D}$-less decay without the need to tag certain particle in the final states. We refer to this method as the **leading particle method**.

In the appendix, we present the formulas for ψ'' scan. Based on these formulas, the expected cross sections in the vicinity of ψ'' resonance are depicted and drawn in Fig. 8. The upper part of the graph is the total inclusive hadron cross section; while the lower part are the curves of cross sections from $D\overline{D}$-less decays, with the assumption of non-$D\overline{D}$ decay branching fraction to be 0, 10% and 30%, respectively.

The prominent advantage of this method is the high sensitivity and good precision. Since the fraction of the non-$D\overline{D}$ decay is determined by the ratio of two curves from the same scan measurement, most of the systematic errors are canceled out, a small systematic uncertainty is expected from this method.

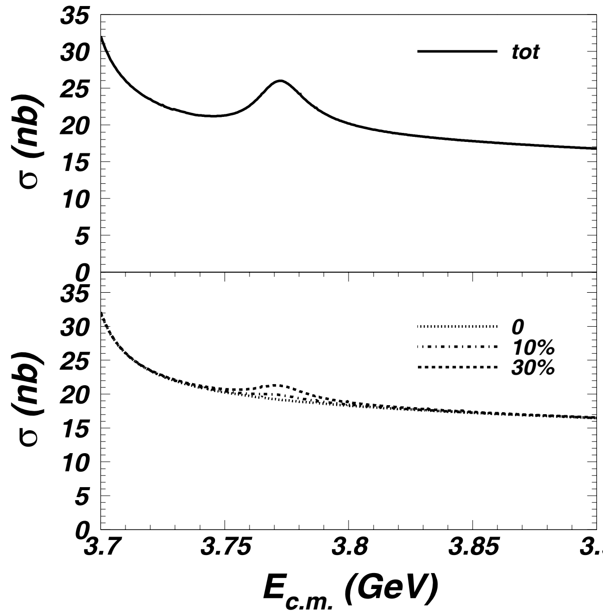

FIG. 8: The cross section in vicinity of the ψ'' resonance calculated with parameters provided by PDG. In the top graph, the curve is the total cross section. In the bottom graph, only $D\overline{D}$-less decay cross section is drawn, with assumption of non-decay fraction as 0, 10% and 30%, respectively.

3. Comments

Recently, CLEOc report a measurement of non-$D\overline{D}$ cross section [65]:

$$\sigma_{ndd} = (-0.01 \pm 0.08^{+0.41}_{-0.30})\text{nb} ,$$

which indicates the non-$D\overline{D}$ decays, even exist, would be smaller than 0.53 nb at 90% confidence level, or corresponds to an upper limit of 8.4% of the branching fraction t 90% confidence level, if take the total ψ'' cross section as $\sigma_{\psi''} = (6.38 \pm 0.08^{+0.41}_{-0.30})$nb [65].

Since non-$D\overline{D}$ searching is very important and interesting, so more accurate results are need to confirm this measurement. In addition, in the CLEOc analysis, one distinctive feature is the interference between resonance and continuum for inclusive electromagnetic processes are taken into account. It is natural to worry about other interference effects, such as that for inclusive strong processes between different resonances. Although one may argue that such kinds of interference may be small, but more meticulous studies are needed and we leave such an analysis to a work in the future. Here merely we want to point out, the interference effect which has been taken into account by CLEOc will not affect our measurement for non-$D\overline{D}$ decays by scan method as long as we set the Γ_{ee} of ψ' as a free parameter in the data fit. The reason is due to the smooth variation of the interference effect in the vicinity of ψ''.

VI. SUMMARY

Totally twelve explanations or models or schemes involving $\rho\pi$ puzzle are reviewed, and at the same time, some relevant experimental results, measurements are quoted to test or confirm the corresponding theoretical speculations or predictions. From the retrospect of previous theories, we point out development of some phenomenological models are more welcome for present situation of charm decay.

Based on the available experimental information of J/ψ and ψ' decays, we calculate the charmless decays of ψ'' by virtue of the S- and D-wave charmonia mixing scheme which was proposed to explain the large $\psi'' \to e^+e^-$ partial width and the $\rho\pi$ puzzle. We find that this leads to a possible large branching fraction, up to 13%, of the charmless final state in ψ'' decays. Although the calculation is semi-quantitative, it demonstrates that a large charmless branching fraction in ψ'' decays can well be explained.

After that, we put forth three methods for the searching for the non-$D\overline{D}$ decays of the ψ'' in e^+e^- experiments: the exclusive, the quasi-inclusive and the inclusive methods.

First, for the exclusive method, we call attention on the contribution of the non-resonance virtual photon, and more importantly, its interference with the resonate decay amplitude. Besides the confirmation of the existence of the non-$D\overline{D}$ decays of the ψ'', the measurement of the exclusive channel is very important for the interpretation of the "$\rho\pi$ puzzle" in J/ψ and ψ' decays and the determination of the phase between the strong and electromagnetic interactions in such decays.

Second, for the inclusive method, we propose a new, so-called leading particle method. This method tags a large fraction of the non-$D\overline{D}$ decays ($\sim 10\%$) without $D\overline{D}$ contamination, thus is more sensitive than the other methods in the determination of the total branching fraction of the non-$D\overline{D}$ decays of the ψ''.

As to the quasi-inclusive method, it can be used as a cross check for the measuring of the total branching fraction of the non-$D\overline{D}$ decays.

Acknowledgments

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APPENDIX: CROSS SECTION AT ψ'' MASS

The total cross section at the ψ'' peak can be expressed as the sum of all possible resonances and non-resonance contributions. Due to Initial State Radiation (ISR) and other effects, such as energy spread, what is obtained experimentally is the so-called observed cross section instead of the one at Born order.

A remark of symbol is in order here. In the follow text, we use symbol W to denote the C.M. energy of the colliding beams, which is also expressed by $E_{c.m.}$ in the literature. The half of W indicates the beam energy, often written as $E_{beam} = W/2$, while the square of W indicates the energy transform, often written as $s = W^2$ in theoretical papers.

1. ISR Correction

The ISR correction is calculated by the structure function approach [107–109], which yields the accuracy of 0.1%. In this scheme, the radiatively corrected cross section is expressed as

$$\sigma(W^2) \;=\; \int_0^{1-W_m^2/W^2} dx\; \tilde{\sigma}[W^2(1-x)]F(x,W) \quad \text{(A.1)}$$

where W_m is the cut off of the invariant mass in the event selection, and

$$\tilde{\sigma}(W^2) = \frac{\sigma^B(W)}{|1-\Pi(W^2)|^2} \;,$$

with $\sigma^B(W)$ the Born order cross section and $\Pi(W^2)$ the vacuum polarization. In Eq. (A.1)

$$F(x,W) \;=\; \beta x^{\beta-1}\delta^{V+S}(W) + \delta^H(x,W) \;, \qquad \text{(A.2)}$$

with

$$\beta \;=\; \frac{2\alpha}{\pi}\left(\ln\frac{W^2}{m_e^2} - 1\right) \;,$$

$$\delta^{V+S}(W) \;=\; 1 + \frac{3}{4}\beta + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) + \beta^2\left(\frac{9}{32} - \frac{\pi^2}{12}\right) \;,$$

$$\delta^H(x,W) \;=\; -\beta\left(1-\frac{x}{2}\right) + \frac{1}{8}\beta^2\left[4(2-x)\ln\frac{1}{x} - \frac{1+3(1-x)^2}{x}\ln(1-x) - 6 + x\right] \,.$$

Here the conversion of bremsstrahlung photons to real e^+e^- pairs is included in the cross section which is the usual experimental situation. Thus there is cancellation between the contributions of virtual and real e^+e^- pairs in the leading term [109].

The physical cross section at Born order of the process $e^+e^- \to$ Res. $\to f$ (where f denotes a certain kind of final states) is expressed by the **Breit-Wigner** formula

$$\sigma^{BW}(W) \;=\; \frac{12\pi\Gamma^0_{ee}\Gamma_f}{(W^2-M^2)^2 + \Gamma^2 M^2} \;,$$

where M and Γ are the mass and total width of the resonance; Γ^0_{ee} and Γ_f are the partial widths of the e^+e^- mode and the final state f respectively. Here Γ^0_{ee} describes the coupling strength of the resonance to e^+e^- through a virtual photon. For example, in potential model, Γ^0_{ee} is related to the wave function at the origin $\psi(0)$ in the way

$$\Gamma^0_{ee} = \frac{4\alpha^2 Q_q^2|\psi(0)|^2}{M^2} \;,$$

where Q_q is the charge carried by the quark in the quarkonium and α is the QED fine structure constant. Since the decay of a quarkonium 1^{--} state to e^+e^- pair is through a virtual photon, there is always vacuum polarization associated with this process. So the experimentally measured e^+e^- partial width, denoted explicitly as Γ^{exp}_{ee}, is related to Γ^0_{ee} by the expression

$$\Gamma^{exp}_{ee} = \frac{\Gamma^0_{ee}}{|1-\Pi(M^2)|^2} \,.$$

We follow the convention of Ref. [106] which is adopted by PDG. In this convention Γ_{ee} means Γ_{ee}^{exp}. For resonances, if they decay predominantly to light hadrons, with threshold (W_m) far less than the data taking energy (W), that is $W - W_m \gg \Gamma$, the integral of Eq.(A.1) is insensitive to W_m, because the Breit-Wigner formula behaves like a δ function. One can put the upper limit of the integration to 1, so the radiatively corrected resonance cross section is

$$\sigma(W^2) = \int_0^1 dx\ F(x,W)\sigma^{BW}[W^2(1-x)]\ , \qquad \text{(A.3)}$$

with

$$\sigma^{BW}(W) \ = \frac{12\pi\Gamma_{ee}\Gamma_f}{(W^2-M^2)^2+\Gamma^2M^2}\ . \qquad \text{(A.4)}$$

2. Observed Cross Section

The total observed cross section σ^{tot} at the ψ'' mass is usually expressed as

$$\sigma^{tot} = \sigma_{NR} + \sigma_{J/\psi} + \sigma_{\psi'} + \sigma_{\psi''}\ , \qquad \text{(A.5)}$$

which contains four parts: the cross section from non-resonant contribution σ_{NR}, from radiative tails of J/ψ ($\sigma_{J/\psi}$) and ψ' ($\sigma_{\psi'}$), and the cross section from resonance ψ'' ($\sigma_{\psi''}$).

a. *Non-resonance section*

The non-charm contribution is conventionally expressed by R value

$$\sigma_{NR} = R\cdot\sigma(e^+e^-\to\mu^+\mu^-)\ ,$$

with

$$\sigma(e^+e^-\to\mu^+\mu^-) = \frac{4\pi\alpha}{3W^2}\ .$$

Here R indicates the contribution from light quarks[110] (u, d and s).

b. *Tails due to J/ψ and ψ'*

The resonances such as J/ψ and ψ', are narrow with widths from tens to hundreds keV, while the beam energy spread of e^+e^- colliders is at the order of MeV. If the resonance width is comparable or smaller than the beam energy spread, the observed resonance cross section is the one by Eq. (A.1) folded with the beam energy spread function $G(W,W')$, which is usually taken as a Gaussian:

$$G(W,W') \ = \ \frac{1}{\sqrt{2\pi}\Delta}\exp\left[-\frac{(W-W')^2}{2\Delta^2}\right],$$

with Δ the standard deviation of the Gaussian distribution, or the beam energy spread physically. However, when the experiment energy is far from the resonance peak, the effect of the energy spread is insignificant and can be neglected[111].

As a matter of fact, Eq. (A.1) can further simplified. Notice that J/ψ and ψ' are narrow resonances, that is to say, comparing with the resonance mass M, the decay width Γ could be treated as $\Gamma\to 0$. So the Breit-Wigner formula transforms into a δ function:

$$\begin{aligned}\sigma^{BW}(W) \ &= \ \frac{12\pi\Gamma_{ee}\Gamma_f}{(W^2-M^2)^2+\Gamma^2M^2}\\ &\xrightarrow{\Gamma\to 0} \frac{12\pi^2\Gamma_{ee}B_f}{M}\cdot\delta(W^2-M^2)\ ,\end{aligned}$$

where $B_f = \Gamma_f/\Gamma$. Then the integral in Eq. (A.3) gives

$$\sigma(W^2) \cong \frac{12\pi^2\Gamma_{ee}B_f}{MW^2}\cdot F(x,W)\Big|_{x=1-\frac{M^2}{W^2}}\ ,$$

for the cross section due to the tails of the J/ψ and ψ'.

c. *Cross section of ψ''*

According to Eq. (A.3), the radiatively corrected cross section of ψ'' is expressed as

$$\begin{aligned}\sigma_{\psi''}(W) \ &= \ \int_0^1 dx\ F(x,W)\ \sigma_{ND}[W^2(1-x)]\\ &+ \int_0^{1-\frac{4m^2_{D^0}}{W^2}} dx\ F(x,W)\ \sigma_{DD}[W^2(1-x)]\ ,\end{aligned}$$

where $F(x,W)$ is given in Eq. (A.2), while

$$\sigma_{ND}(W^2) = \frac{12\pi\Gamma_{ee}\Gamma_{ND}}{(W^2-M^2_{\psi''})^2+\Gamma^2_{\psi''}(W)M^2_{\psi''}}\ , \qquad \text{(A.6)}$$

and

$$\sigma_{DD}(W^2) = \frac{12\pi\Gamma_{ee}\Gamma_{DD}(W)}{(W^2-M^2_{\psi''})^2+\Gamma^2_{\psi''}(W)M^2_{\psi''}}\ . \qquad \text{(A.7)}$$

The energy dependent total width of ψ'' is composed of two parts

$$\Gamma_{\psi''}(W) = \Gamma_{ND} + \Gamma_{DD}(W)\ , \qquad \text{(A.8)}$$

while the width listed by PDG is often defined as

$$\overline{\Gamma}_{\psi''} \equiv \Gamma_{\psi''}(W = M_{\psi''})\ . \qquad \text{(A.9)}$$

Using definition of Eq. (A.9), we further factorize the two decay widths in Eq. (A.8) as follows:

$$\Gamma_{ND} = f\cdot\overline{\Gamma}_{\psi''}\ , \qquad \text{(A.10)}$$

or

$$f = \frac{\Gamma_{ND}}{\overline{\Gamma}_{\psi''}} ,$$

that is to say, f is actually the branching fraction of the non-$D\overline{D}$ decays of ψ''. For Γ_{DD}, we have

$$\Gamma_{DD}(W) = (1-f)\cdot\overline{\Gamma}_{\psi''}\cdot\theta(W-2m_{D^0})\cdot \frac{r_D\cdot\frac{p^3_{D^0}}{1+(rp_{D^0})^2}+\theta(W-2m_{D^\pm})\cdot\frac{p^3_{D^\pm}}{1+(rp_{D^\pm})^2}}{r_D\cdot\frac{\overline{p}^3_{D^0}}{1+(r\overline{p}_{D^0})^2}+\frac{\overline{p}^3_{D^\pm}}{1+(r\overline{p}_{D^\pm})^2}} , \tag{A.11}$$

where r is the classical interaction radium; r_D, whose value is around 1.4, is the ratio of $D^0\overline{D^0}$ to D^+D^- production at ψ'' peak. p is the D^0 or $D^\pm$ momentum, reads explicitly as

$$\begin{aligned} p_{D^0} &= \frac{1}{2}\sqrt{W^2-4m^2_{D^0}} , \\ p_{D^\pm} &= \frac{1}{2}\sqrt{W^2-4m^2_{D^\pm}} ; \end{aligned}$$

and $\overline{p}$ is the D^0 or $D^\pm$ momentum at the resonance peak, viz.

$$\begin{aligned} \overline{p}_{D^0} \equiv p_{D^0}\Big|_{W=M_{\psi''}} &= \frac{1}{2}\sqrt{M^2_{\psi''}-4m^2_{D^0}} , \\ \overline{p}_{D^\pm} \equiv p_{D^\pm}\Big|_{W=M_{\psi''}} &= \frac{1}{2}\sqrt{M^2_{\psi''}-4m^2_{D^\pm}} . \end{aligned}$$

d. *Cross section of the leading particle*

The observed cross section at ψ'' mass after requiring the momentum of the leading particle within certain ranges, which is denoted by $\sigma^{l.p.}$ also contains four parts, *i.e.*

$$\sigma^{l.p.} = \sigma_{NR} + \sigma_{J/\psi} + \sigma_{\psi'} + \sigma^{l.p.}_{\psi''} .$$

Except for the last term, the other three parts are exactly the same as those of σ^{tot} in Eq.(22). As to $\sigma^{l.p.}_{\psi''}$, the radiatively corrected cross section reads

$$\sigma^{l.p.}_{\psi''}(W) = \int_0^1 dx\ F(x,W)\ \sigma_{ND}[W^2(1-x)] ,$$

where $F(x,W)$ is given in Eq. (A.2), while σ_{ND} is given by Eq. (A.6).

e. *Cross section of the $D\overline{D}$ decays*

The observed cross section of the $D\overline{D}$ decays of ψ'', denoted as $\sigma^{D\overline{D}}$, which only comes from the resonance decays, is expressed as

$$\sigma^{D\overline{D}}_{\psi''}(W) = \int_0^{1-\frac{4m^2_{D^0}}{W^2}} dx\ F(x,W)\ \sigma_{DD}[W^2(1-x)] ,$$

where $F(x,W)$ is given in Eq. (A.2), while $\sigma_{DD}(W^2)$ is by Eq. (A.7).

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[111] In our fitting program, when $W \geq M_R + 0.2$ (GeV), the effect is neglected. So for our ψ'' scan [$W \subset (3.7, 3.9)$ GeV], the effect of energy spread is taken into consideration for ψ', but not for J/ψ.