Chapter 22

Rare and forbidden charmonium decays

At present, two general trends may be distinguished in accelerator particle physics. On one hand, the very high energy accelerators, for example LHC, will permit to explore the frontier line of high energy physics. On the other hand, smaller or low energy experiments but with very high statistics and low backgrounds (B factories and BESIII for τ-Charm) would allow to perform precise tests and accurate determinations of many parameters of the Standard Model. Moreover, the scrutiny of rare processes may enlight new physics in a complementary fashion to high-energy colliders.

With huge $J/\psi$ and $\psi(2S)$ data samples, BES-III experiment will be approaching the statistics where rare $\psi$ decays can provide important tests of the standard model and possible be able to uncover deviations.

22.1 Weak Decays of Charmonium

The low lying charmonium states below the open charm threshold usually decay through intermediate photons or gluons produced by the parent $c\bar{c}$ quark pair annihilation. These OZI violating but flavor conserving decays lead to narrow widths to $J/\psi$ and $\psi(2S)$ states. In the standard model framework, the flavor changing weak decays of these states are also possible though these are expected to have rather low branching fractions. Huge $J/\psi$ data sample at BES-III would afford to examine those rare decay processes, which may become detectable. The observation of any anomalous production rate of single charmed meson in $J/\psi$ or $\psi(2S)$ decays at BES-III would be a hint of possible new physics either in the continuum via flavor-changing neutral currents [1] or in the decays of resonances due to unexpected effects of quark dynamics [2].

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1By Haibo Li, Jianping Ma and Xinmin Zhang
22.1.1 Semileptonic Decays of Charmonium

The inclusive branching fractions of $J/\psi$ weak decays via a single quark of either $c$ or $\bar{c}$ had been estimated to be $(2-4) \times 10^{-8}$ by ignoring any $W$-exchanged contribution and using the $D^0$ lifetime [3]. Such small branching ratios should make the observation of weak decays of $J/\psi$ or $\psi(2S)$ extremely difficult despite the foreseen cleanness of events. However, at BEPC-II with a peak luminosity of $10^{33} \text{cm}^{-2}\text{s}^{-1}$ at $\psi(3770)$, the expected number of $J/\psi$ is $10 \times 10^9$ per year of data taking, leading to $\approx 400$ weak decays when combined with the predicted branching ratio. The semi-leptonic decay of vector charmonium below open charm threshold, $\alpha(1^{--})$, induced by the quark level weak transition $c \to qW^*$, where $W^*$ are virtual intermediate boson. Hence, the exclusive semi-leptonic channels are:

\[ \psi(nS) \to D_q l \nu, \]  
\[ \psi(nS) \to D^*_q l \nu, \]  

where $n = 1, 2$, $q$ can be down or strange quark which corresponds to $D^\pm$ (Cabibbo-suppressed mode) or $D_s$ (Cabibbo-allowed mode) meson. Semi-leptonic weak decays of $J/\psi$ will offer several advantages over the purely hadronic ones both from the experimental and theoretical points of view: the prompt charged lepton $l = e, \mu$ can be used to tag the events, removing a large amount of conventional hadronic decays of $\psi(nS)$. Besides, the missing energy due to the escaping neutrino can be also employed experimentally to remove backgrounds. So the reconstructed charm meson in the final state should provide an unambiguous signature of the semi-leptonic weak decays of $\psi(nS)$. Meanwhile, radiative decays of the excited mesons $D^*_s$ and $D^*$ in reaction (22.2) can provide a useful extra signal, in the lab system, detectable photons are yielded in the energy interval 90 - 200 MeV for $D^*_s \to D^\pm_s \gamma$, and a soft pion is produced for the reconstruction of $D^* \to D^0 \pi^\pm$ decay mode. Those are powerful constraint in the mass reconstruction of $D_s$ and $D^0$ mesons to detect the weak decays of charmonium.

According to Heavy Quark Spin Symmetry (HQSS) [4, 5], the branching ratios for the Cabibbo-allowed modes were estimated as [3]:

\[ BR(J/\psi \to D_s l \nu) \approx 0.26 \times 10^{-8}, \]
\[ BR(J/\psi \to D^*_s l \nu) \approx 0.42 \times 10^{-8}, \]  

(22.3)

summing over both modes one gets $BR \approx 0.7 \times 10^{-8}$, which is about 20% of the expected total weak decay rate ($4 \times 10^{-8}$) as estimated in reference [3]. Taking into account the overall theoretical uncertainty ($\approx 40\%$), the expected branching ratios are within $(0.4-1.0) \times 10^{-8}$. For the Cabibbo-suppressed decay modes, one can employ the following ratio:

\[ \frac{BR(J/\psi \to D_q l \nu)}{BR(J/\psi \to D^\pm l \nu)} = \frac{BR(J/\psi \to D^*_q l \nu)}{BR(J/\psi \to D_s^* l \nu)} \approx \frac{|V_{cs}|^2}{|V_{cd}|^2} \approx 20, \]  

(22.4)

where $V_{cs} \approx 1.0$ and $V_{cd} \approx 0.22$ denote Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix elements. According to Heavy Quark Spin Symmetry (HQSS) [4, 5], the branching
ratios for the Cabibbo-allowed modes were estimated as [3]:

\[
\begin{align*}
BR(J/\psi \rightarrow D_s l \nu) & \approx 0.26 \times 10^{-8}, \\
BR(J/\psi \rightarrow D_s^* l \nu) & \approx 0.42 \times 10^{-8},
\end{align*}
\] (22.5)

summing over both modes one gets \( BR \approx 0.7 \times 10^{-8}, \) which is about 20% of the expected total weak decay rate \( (4 \times 10^{-8}) \) as estimated in reference [3]. Taking into account the overall theoretical uncertainty \( (\approx 40\%) \), the expected branching ratios are within \((0.4-1.0) \times 10^{-8}\). For the Cabibbo-suppressed decay modes, one can employ the following ratio:

\[
\frac{BR(J/\psi \rightarrow D_s l \nu)}{BR(J/\psi \rightarrow D_s^\pm l^\mp \nu)} = \frac{BR(J/\psi \rightarrow D_s^* l \nu)}{BR(J/\psi \rightarrow D_s^{\*\pm} l^{\mp} \nu)} \approx \frac{|V_{cs}|^2}{|V_{cd}|^2} \approx 20, \] (22.6)

where \( V_{cs} \approx 1.0 \) and \( V_{cd} \approx 0.22 \) denote Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix elements.

### 22.1.2 Two-body Weak Hadronic Decays of Charmonium

For the non-leptonic weak decays of Charmonium, the two-body weak hadronic decays of \( J/\psi \) and \( \psi(2S) \) in Cabibbo-allowed \( (c \rightarrow s) \) and Cabibbo-suppressed \( (c \rightarrow d) \) modes had been studied by employing the factorization scheme [6], the branching ratios for \( \psi \rightarrow PP/PV \) decays (where \( P \) and \( V \) represent pseudoscalar meson and vector meson, respectively) were predicted. Using the decay rate formula (5) in Ref. [6], one computed the branching ratios of various \( \psi \rightarrow PP \) decays which are listed in Table 22.1. Among the Cabibbo-allowed decays, one find that the dominated decay mode is \( \psi \rightarrow D_s^{+}\pi^- \), having the branching ratio

\[
BR(\psi \rightarrow D_s^{+}\pi^-) = 0.87 \times 10^{-9}, \] (22.7)

and the next in order is \( \psi \rightarrow D^0 K^0 \), whose branching ratio is

\[
BR(\psi \rightarrow D^0 K^0) = 0.28 \times 10^{-9}. \] (22.8)

The branching ratios of \( \psi \rightarrow PV \) decays in Cabibbo-allowed and Cabibbo-suppressed modes are listed in Table 22.2. For the color enhanced decay of the Cabibbo-allowed modes, one got

\[
BR(\psi \rightarrow D_s^{+}\rho^-) = 0.36 \times 10^{-8}, \] (22.9)

which is higher than the branching ratio of \( \psi \rightarrow D_s^{+}\pi^- \). Thus, the following useful relative ratio were obtained [6]

\[
\frac{BR(\psi \rightarrow D_s^{+}\rho^-)}{BR(\psi \rightarrow D_s^{+}\pi^-)} = 4.2, \] (22.10)

and therefore \( \psi \rightarrow D_s^{+}\rho^- \) may be expected to be measured at BES-III.
### Table 22.1: Branching ratios of $\psi \to PP$ decays from reference [6] in the Standard Model.
The transition mode, $\Delta C = \Delta S = +1$, corresponds to Cabibbo-allowed decay modes, while $\Delta C = +1$, $\Delta S = 0$ corresponds to Cabibbo-suppressed decay modes.

<table>
<thead>
<tr>
<th>Transition Mode</th>
<th>Decay Modes</th>
<th>Branching ratio ($\times 10^{-10}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta C = \Delta S = +1$</td>
<td>$\psi \to D_s^+ \pi^-$</td>
<td>8.74</td>
</tr>
<tr>
<td></td>
<td>$\psi \to D^0 K^0$</td>
<td>2.80</td>
</tr>
<tr>
<td>$\Delta C = +1$, $\Delta S = 0$</td>
<td>$\psi \to D_s^+ K^-$</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>$\psi \to D^+ \pi^-$</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>$\psi \to D^0 \eta$</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>$\psi \to D^0 \eta'$</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>$\psi \to D^0 \pi^0$</td>
<td>0.055</td>
</tr>
</tbody>
</table>

### Table 22.2: Branching ratios of $\psi \to PV$ decays from reference [6] in the Standard Model.
The transition mode, $\Delta C = \Delta S = +1$, corresponds to Cabibbo-allowed decay modes, while $\Delta C = +1$, $\Delta S = 0$ corresponds to Cabibbo-suppressed decay modes.

<table>
<thead>
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<th>Transition Mode</th>
<th>Decay Modes</th>
<th>Branching ratio ($\times 10^{-10}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta C = \Delta S = +1$</td>
<td>$\psi \to D_s^+ \rho^-$</td>
<td>36.30</td>
</tr>
<tr>
<td></td>
<td>$\psi \to D^0 K^{*0}$</td>
<td>10.27</td>
</tr>
<tr>
<td>$\Delta C = +1$, $\Delta S = 0$</td>
<td>$\psi \to D_s^+ K^{*-}$</td>
<td>2.12</td>
</tr>
<tr>
<td></td>
<td>$\psi \to D^+ \rho^-$</td>
<td>2.20</td>
</tr>
<tr>
<td></td>
<td>$\psi \to D^0 \rho^0$</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>$\psi \to D^0 \omega$</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>$\psi \to D^0 \phi$</td>
<td>0.65</td>
</tr>
</tbody>
</table>
22.1 Weak Decays of Charmonium

The semi-leptonic decay modes of $\psi(nS)$ can be related to the two-body hadron decay modes by applying both the spin symmetry and the non-recoil approximation to the semi-leptonic decays [3]. For $J/\psi \to D_s^+(D_s^+)^{-}\pi^-$ decay modes, one gets $q^2 = (p_1 - p_2)^2 = m_\pi^2$ ($p_1$ and $p_2$ are the four momentum of initial and final state mesons respectively) assuming factorization, as suggested by Bjorken [7] in $B$ decays. Keeping the non-recoil approximation in the hadronic transition amplitudes [8], from the equations (7) and (8) in reference [3], one arrives at a relative branching ratio:

$$\frac{BR(J/\psi \to D_s^+\pi^-)}{BR(J/\psi \to D_s^-\pi^-)} \simeq \left[ \frac{d\Gamma(J/\psi \to D_s^+l^-\nu)}{dq^2} \right] \simeq 3.5,$$  \hspace{1cm} (22.11)

if a $\rho$ substitutes the $\pi$ one gets $r \approx 1.4$. So the estimated branching ratios in Table 22.1 for $\psi(nS) \to PP$ channel can be related to $\psi(nS) \to VP$ channel if the pseudoscalar charm mesons are replaced by a vector charm mesons.

All the above estimations show an overall enhancement of final vector charm mesons with respect to the pseudoscalar ones. It suggests once more the use of $D_s^*$ or $D^{*\pm}$ as signals to search for the weak decays of $\psi(nS)$ even in non-leptonic decays.

22.1.3 Searches at BES-III

At BES-III, assuming $10^{10}$ $J/\psi$ sample to be collected in one year running time, the central value $BR \approx 0.7 \times 10^8$ leads to about 70 semi-leptonic decay events of the type $J/\psi \to D_s(D_s^*)l\nu$. Thus, the following event selection criteria could be applied on searching for such exclusive semi-leptonic channels:

- The prompt charged lepton can be used to tag the weak decay: in order to suppress cascade decay backgrounds from $J/\psi$ strong decays, the tagging lepton momentum could be required in the range from 0.5 GeV to 1.0 GeV, close to the upper kinematic limit for the decay under consideration. A high quality lepton identification from the other charged pions or kaons is needed at BES-III.

- The missing mass of the reconstructed candidates must be consistent with zero due to the undetectable neutrino.

- The mass reconstruction of a $D_s$ or $D^\pm$ meson can provide an unambiguous signature for the weak decays of $\psi(nS)$, lying below the open charm threshold. The narrow mass window of $D_s$ with good mass resolution is useful to remove the combinatorial backgrounds.

- The soft photon with the energy interval (90-200) MeV in $D_s^*$ decay and soft charged pions in $D^{*\pm}$ decay can be used further to suppress the combinatorial backgrounds: the additional constraint of an intermediate $D_s^*$ state should confirm the $D_s$ signal again.

In general, the exclusive hadronic decays are probably too tiny to look for in a particular full reconstructed channel. Thereby, it seems that an inclusive searching for $J/\psi \to D_s^* + X$ at BES-III may be available. The $\gamma$ from the excited meson should
be useful as a kinematic constraint for the mass reconstruction of $D_s$ meson as discussed in [3].

Finally, in the Standard model, the flavor changing neutral current (FCNC) in $J/\psi$ decays are predicted to be unobservable [2], so they serve as a probe of new physics. The purpose of the experimental search for the FCNC processes at BES-III is to test whether new physics can enhance the production rate sufficiently. In reference [2], a model-independent theoretical analysis had been performed, then examine the predictions of the models, such as TopColor models, minimal supersymmetric standard model (MSSM) with R-parity violation and a general two-Higgs-doublet model [2]. They found that the branching ratio of $J/\psi \rightarrow D/\overline{D}X_u$, which is mediated by $c \rightarrow u$ transition, could be as large as $10^{-6} - 10^{-5}$ in the presence of new physics.

Experimentally with BES-III data, it is very hard to look for pure $c \rightarrow u$ mediated hadronic $J/\psi \rightarrow D/\overline{D}X_u$ decays. While, at BES-III, it is easy to search for $J/\psi \rightarrow D^0/\overline{D}^0\ell^- (l = e, \mu)$ and $J/\psi \rightarrow D^0/\overline{D}^0\gamma$ decays which is dominated by FCNC processes.

### 22.2 Search for the invisible decays of Quarkonium

Invisible decays of quarkonium states such as the $J/\psi$ and the $\Upsilon$, etc., offer a window into what may lie beyond the Standard Model (SM) [9, 10]. The reason is that apart from neutrinos, the Standard Model includes no other invisible final particles that these states can decay into. It is such a window that we intend to further explore by presenting here the first experimental limits on invisible decays of the $\eta$ and $\eta'$, which complement the limit of $2.7 \times 10^{-7}$ recently established in [11] for the invisible decays of the $\pi^0$.

Theories beyond the SM generally include new physics, such as, possibly, light dark matter (LDM) particles [12]. These can have the right relic abundance to constitute the nonbaryonic dark matter of the Universe, if they are coupled to the SM through a new light gauge boson $U$ [13], or exchanges of heavy fermions. It is also possible to consider a light neutralino with coupling to the SM mediated by a light scalar singlet in the next-to-minimal supersymmetric standard model [14].

Recently, observations of a bright 511 keV $\gamma$-ray line from the galactic bulge have been reported by the SPI spectrometer on the INTEGRAL satellite [15]. The corresponding galactic positron flux, as well as the smooth symmetric morphology of the 511 keV emission, may be interpreted as originating from the annihilation of LDM particles into $e^+e^-$ pairs [12] (also constrained by [16]). It is in any case very interesting to search for such light invisible particles in collider experiments. CLEO gave an upper bound on $\Upsilon(1S) \rightarrow \gamma + \text{invisible}$, which is sensitive to dark matter candidates lighter than about 3 GeV/$c^2$ [17], and also provides an upper limit on the axial coupling of the new $U$ boson to the $b$ quark. It is crucial, in addition, to search for the invisible decays of light quarkonium ($q\overline{q}$, $q = u,d,$ or $s$ quark) states which can be used to constrain the masses of LDM particles and the couplings of the new boson to the light quarks [10].

It had been shown that the measurements on the $J/\psi$ invisible decay widths may be sensitive to constrain the physics model [18]. It is straightforward for one to calculate the branching ratio of the invisible decays of $J/\psi$ and its observed decays into electron-positron pairs [18]. Within the SM the invisible mode consists solely of decays into
three types of neutrino-antineutrino pairs. Neglecting polarization effects and taking into
account \(e^+e^-\) production through a photon only, one get \([18]\):

\[
\frac{\Gamma(J/\psi \to \nu\pi)}{\Gamma(J/\psi \to e^+e^-)} = \frac{27G^2M_{J/\psi}^4}{256\pi^2\alpha^2} \left(1 - \frac{8}{3}\sin^2(\theta_W)\right) = 4.54 \times 10^{-7},
\]

(22.12)

with \(G\) and \(\alpha\) being the Fermi and the fine structure constants respectively. \(M_{J/\psi}\) is
mass of \(J/\psi\). The uncertainty of the above formula is about 2-3\% which mainly from the
correction to \(J/\psi\) wave function, \(e^+e^-\) production through \(Z\) boson, electroweak radiative
corrections \([18]\).

At BES-III, one can then tag charmonium states which decay invisibly by looking for
a particular radiative transition such as \(\psi(2S) \to \pi^+\pi^-J/\psi\), \(\psi(2S) \to \gamma\chi_c\) and so on, the
soft \(\pi^+\pi^-\) pairs or monoenergetic radiative \(\gamma\) can be used as tags of the invisible decays
of \(J/\psi\) or \(\chi_c\) states. A detail lists of these decay chains are in Table 22.3.

<table>
<thead>
<tr>
<th>(\psi(2S)) decay mode</th>
<th>Branching fraction ((10^{-2}))</th>
<th>Number of events in 3 billions (\psi(2S)) sample</th>
<th>Invisible decay mode</th>
<th>Tagging topology</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\psi(2S) \to \pi^+\pi^-J/\psi)</td>
<td>31.7 ± 1.1</td>
<td>9.3 \times 10^8</td>
<td>(J/\psi) → invisible</td>
<td>(\pi^+\pi^-)</td>
</tr>
<tr>
<td>(\psi(2S) \to \eta J/\psi)</td>
<td>18.6 ± 0.8</td>
<td>5.6 \times 10^8</td>
<td>(J/\psi) → invisible</td>
<td>(\eta)</td>
</tr>
<tr>
<td>(\psi(2S) \to \eta J/\psi)</td>
<td>3.08 ± 0.17</td>
<td>9.3 \times 10^4</td>
<td>(J/\psi) → invisible</td>
<td>(\eta)</td>
</tr>
<tr>
<td>(\psi(2S) \to \pi^0\eta J/\psi)</td>
<td>0.123 ± 0.018</td>
<td>3.7 \times 10^6</td>
<td>(J/\psi) → invisible</td>
<td>(\eta^0)</td>
</tr>
<tr>
<td>(\psi(2S) \to \gamma\chi_{c0})</td>
<td>9.0 ± 0.4</td>
<td>2.7 \times 10^8</td>
<td>(\chi_{c0}) → invisible</td>
<td>(\gamma)</td>
</tr>
<tr>
<td>(\psi(2S) \to \gamma\chi_{c1})</td>
<td>8.7 ± 0.5</td>
<td>2.6 \times 10^8</td>
<td>(\chi_{c1}) → invisible</td>
<td>(\gamma)</td>
</tr>
<tr>
<td>(\psi(2S) \to \gamma\chi_{c2})</td>
<td>8.2 ± 0.3</td>
<td>2.5 \times 10^8</td>
<td>(\chi_{c2}) → invisible</td>
<td>(\gamma)</td>
</tr>
<tr>
<td>(\psi(2S) \to \gamma\eta_{c}(1S))</td>
<td>0.26 ± 0.04</td>
<td>7.8 \times 10^6</td>
<td>(\eta_{c}(1S)) → invisible</td>
<td>(\gamma)</td>
</tr>
<tr>
<td>(J/\psi \to \gamma\eta_{c}(1S))</td>
<td>1.3 ± 0.4</td>
<td>1.3 \times 10^8</td>
<td>(\eta_{c}(1S)) → invisible</td>
<td>(\gamma)</td>
</tr>
</tbody>
</table>

Table 22.3: \(\psi(2S)\) and \(J/\psi\) decay modes which can be used to study the invisible decays of
\(J/\psi\), \(\chi_{c0}\), \(\chi_{c1}\), \(\chi_{c2}\), \(\eta_{c}(1S)\) and \(\eta_{c}(2S)\). The branching fractions are listed from PDG \([19]\). For
each mode, a "tagging topology" is given, which is the set of visible in the detector’s
acceptance. In each case the tagging topology has a well-defined kinematics. The produced
number of events are the expected events in 3 billions \(\psi(2S)\) \((10\) billions \(J/\psi\) \) data set,
in which we did not consider the decay rate of the tagging particles.

It is also interesting to study the invisible decays of \(\eta\), \(\eta'\), \(\rho\), \(\omega\) and \(\phi\) light mesons
by using the two-body decay modes of \(J/\psi\). For example, the two-body decay modes
of \(J/\psi \to \phi\eta, \phi\eta'\) can be selected in order to study the decays of \(\eta\) and \(\eta'\) to invisible
final states, as shown in Fig. 22.1, in which the \(\phi\) signals can be reconstructed easily and
cleanly decaying into \(K^+K^-\). The reconstructed \(\phi\) particles can be used to tag the \(\eta\)
and \(\eta'\) in order to allow a search for their invisible decays. Since both \(\phi\) and \(\eta\) (\(\eta'\)) have
narrow widths, which are negligible compared with the detector resolution, the shape of
the momentum distribution of the \(\phi\) is approximately Gaussian. The mean value of the
\(\phi\) momentum distribution is 1.320 GeV/c for \(J/\psi \to \phi\eta\) and 1.192 GeV/c for \(J/\psi \to \phi\eta'\).
The missing momentum, \(P_{miss} = |\vec{P}_{miss}|\), is a powerful discriminating variable to separate
signal events from possible backgrounds, in which the missing side is not from \(\eta\) (\(\eta'\)) decay.
Here, \(\vec{P}_{miss} = -\vec{P}_{\phi}\). In addition, the \(\eta\) and \(\eta'\) decay regions are easy to define in the lab
system due to the strong boost of the \(\phi\) from \(J/\psi\) decay, as shown in Fig. 22.1.
22. Rare and forbidden charmonium decays

Figure 22.1: The demonstration of $J/\psi \rightarrow \phi \eta$ or $\phi \eta'$, the $\phi$, which is reconstructed in the $K^+K^-$ decay mode, and can be used to tag the invisible decays of the missing particles.

Figure 22.2: (a) The $m_{KK}$ distribution for candidate events. The arrows on the plot indicate the signal region of $\phi$ candidates. (b) $P_{\text{miss}}$ distribution for the events with $1.005 < m_{KK} < 1.035$ GeV/$c^2$ in (a). The means of the missing momenta for $J/\psi \rightarrow \phi \eta$ and $J/\psi \rightarrow \phi \eta'$ are located around 1.32 and 1.20 GeV/$c$, respectively, as indicated by the two arrows.
Based on 58 million \( J/\psi \) data at BES-II, in \( J/\psi \rightarrow \phi \eta \) and \( \phi \eta' \), we had searched for the invisible decays of \( \eta \) and \( \eta' \) \[20\]. An unbinned extended maximum likelihood (ML) fit is used to extract the event yield for \( J/\psi \rightarrow \phi \eta(\eta') \) [\( \phi \rightarrow K^+K^- \) and \( \eta(\eta') \rightarrow \) invisible]. In the ML fit, we use \( P_{\text{miss}} \) as the discriminating variable and require that \( 1.00 < P_{\text{miss}} < 1.45 \) GeV/c, shown in Fig. 22.2(b), where the background shape is well understood. We construct probability density functions (PDFs) for the \( P_{\text{miss}} \) distributions for \( \mathcal{F}^{\eta}_{\text{sig}} \) and \( \mathcal{F}^{\eta'}_{\text{sig}} \) signals and background \( \mathcal{F}_{\text{bkgd}} \) using detailed simulations of signal and background. The PDFs for signals are parameterized by double Gaussian distributions with common means, one relative fraction and two distinct widths, which are all fixed to the MC simulation. The PDF for background is a bifurcated Gaussian plus a first order Polynomial (\( P_1 \)). All parameters related to the background shape are floated in the fit to data. The PDFs for signals and background are combined in the likelihood function \( \mathcal{L} \), defined as a function of the free parameters \( N^{\eta}_{\text{sig}} \), \( N^{\eta'}_{\text{sig}} \), and \( N_{\text{bkgd}} \),

\[
\mathcal{L}(N^{\eta}_{\text{sig}}, N^{\eta'}_{\text{sig}}, N_{\text{bkgd}}) = \frac{e^{-(N^{\eta}_{\text{sig}} + N^{\eta'}_{\text{sig}} + N_{\text{bkgd}})}}{N!} \times \prod_{i=1}^{N} \left[ \mathcal{F}^{\eta}_{\text{sig}}(P^i_{\text{miss}}) + N_{\text{bkgd}} \mathcal{F}_{\text{bkgd}}(P^i_{\text{miss}}) \right],
\]

where \( N^{\eta}_{\text{sig}} \) and \( N^{\eta'}_{\text{sig}} \) are the number of \( J/\psi \rightarrow \phi(\rightarrow K^+K^-)\eta(\rightarrow \) invisible) and \( J/\psi \rightarrow \phi(\rightarrow K^+K^-)\eta'(\rightarrow \) invisible) signal events; \( N_{\text{bkgd}} \) is the number of background events. The fixed parameter \( N \) is the total number of selected events in the fit region, and \( P^i_{\text{miss}} \) is the value of \( P_{\text{miss}} \) for the \( i \)th event. The negative log-likelihood \( -\ln \mathcal{L} \) is then minimized with respect to \( N^{\eta}_{\text{sig}}, N^{\eta'}_{\text{sig}} \), and \( N_{\text{bkgd}} \) in the data sample. A total of 105 events are used in the fit, and the resulting fitted values of \( N^{\eta}_{\text{sig}}, N^{\eta'}_{\text{sig}} \), and \( N_{\text{bkgd}} \) are \(-2.8 \pm 1.4, 2.2 \pm 3.4, \) and \( 106 \pm 11 \), where the errors are statistical. Figure 22.3 shows the \( P_{\text{miss}} \) distribution and fitted result. No significant signal is observed for the invisible decay of either \( \eta \) or \( \eta' \). We obtain upper limits by integrating the normalized likelihood distribution over the positive values of the number of signal events. The upper limits at the 90\% confidence level are 3.56 events for \( \eta \) and 5.72 events for \( \eta' \), respectively.

The branching fraction of \( \eta(\eta') \rightarrow \gamma \gamma \) is also determined in \( J/\psi \rightarrow \phi \eta(\eta') \) decays, in order to obtain the ratio of \( B(\eta(\eta') \rightarrow \) invisible) to \( B(\eta(\eta') \rightarrow \gamma \gamma) \). The advantage of measuring \( \frac{B(\eta(\eta') \rightarrow \gamma \gamma)}{B(\eta(\eta') \rightarrow \gamma \gamma)} \) is that the uncertainties due to the total number of \( J/\psi \) events, tracking efficiency, PID, the number of the charged tracks, the cut on \( m_{KK} \), and residual noise in the BSC cancel.

The upper limit on the ratio of the \( B(\eta \rightarrow \) invisible) to \( B(\eta \rightarrow \gamma \gamma) \) is calculated with

\[
\frac{B(\eta \rightarrow \text{invisible})}{B(\eta \rightarrow \gamma \gamma)} < \frac{n^{\gamma\gamma}_{UL}}{n^{\gamma\gamma}_{\text{obs}}} \times \frac{1}{1 - \sigma_\eta}.
\]

where \( n^{\gamma\gamma}_{UL} \) is the 90\% upper limit of the number of observed events for \( J/\psi \rightarrow \phi \eta, \phi \rightarrow K^+K^-, \eta \rightarrow \) invisible decay, \( \epsilon_\eta \) is the MC determined efficiency for the signal channel, \( n^{\gamma\gamma}_{\text{obs}} \) is the number of events for the \( J/\psi \rightarrow \phi \eta, \phi \rightarrow K^+K^-, \eta \rightarrow \gamma \gamma \) decay, \( \epsilon'_\gamma/\epsilon_\eta \) is the MC determined efficiency for the decay mode, and \( \sigma_\eta = \sqrt{(\sigma_\eta^{\text{sys}})^2 + (\sigma_\eta^{\text{stat}})^2} = 8.1\% \), where \( \sigma_\eta^{\text{sys}} \) and \( \sigma_\eta^{\text{stat}} \) are the total relative systematical error for the \( \eta \) case from Table 31.1 and
the relative statistical error of \( n_{\gamma\gamma} \), respectively. For \( \eta' \), \( \sigma_{\eta'} \) is \( \sqrt{(\sigma_{\eta'}^{\text{stat}})^2 + (\sigma_{\eta'}^{\text{sys}})^2} = 21.6\% \).

The relative statistical error of the fitted yield for \( J/\psi \rightarrow 4\eta' \), \( \eta(\eta') \rightarrow \gamma\gamma \), is 2.8\% (18.5\%) according to the results from the fit to the invariant mass of \( \gamma\gamma \) in Fig. ???. We also obtain the upper limit on the ratio of the \( B(\eta' \rightarrow \text{invisible}) \) to \( B(\eta' \rightarrow \gamma\gamma) \) by replacing \( \eta \) with \( \eta' \) in Eq. (22.14).

Using the numbers in Table 31.1, the upper limit on the ratio of \( B(\eta(\eta') \rightarrow \text{invisible}) \) and \( B(\eta(\eta') \rightarrow \gamma\gamma) \) is obtained at the 90% confidence level of \( 1.65 \times 10^{-3} \) (6.69 \times 10^{-2}).

In Table 22.5, we listed possible two-body \( J/\psi \) decay modes which can be used to study the invisible decays of \( \eta, \eta', \rho, \omega \), and \( \phi \).

## 22.3 Search for C or P violating processes in \( J/\psi \) decays

With huge \( J/\psi \) and \( \psi(2S) \) data samples, BES-III experiment will be approaching the statistics where rare \( \psi \) decays can provide important tests of the standard model and possible be able to uncover deviations. One of the interesting examples is C, P or CP violating processes in \( J/\psi \) decays. An example of such modes would be \( \psi(nS) \rightarrow V^0V^0 \), where \( V^0 \) is \( J^{PC} = 1^{--} \) vector mesons (\( \phi, \omega, \rho^0 \) and \( \gamma \)). The clearest possible signal of this class of events would be \( \psi(nS) \rightarrow \phi\phi \) which would be detected in \( \psi(nS) \rightarrow K^+K^-K^+K^- \). Due to C violation the \( \psi(nS) \rightarrow V^0V^0 \) decays can only occur via \( c\bar{c} \) annihilation and W-exchanged weak decays of \( J/\psi \) as discussed in Ref. [21]. The rate of this kind of weak decay can provide a measurement of the charmonium wave function at the origin [21].

To get a rough idea, whether the weak interaction signal \( \psi(nS) \rightarrow \phi\phi \) could be expected to be seen at BES-III. Note that an \( s\pi \) must be produced from the vacuum
Table 22.4: The numbers used in the calculations of the ratios in Eq. (22.14), where $n^\eta_{UL} (n^\eta_{UL}')$ is the upper limit of the signal events at the 90% confidence level, $\epsilon_\eta (\epsilon_\eta')$ is the selection efficiency, $n^\gamma_\eta (n^\gamma_\eta')$ is the number of the events of $J/\psi \to \phi \eta (\eta')$, $\phi \to K^+ K^-$, $\eta (\eta') \to \gamma \gamma$, $\epsilon_\gamma^\eta (\epsilon_\gamma^\eta')$ is its selection efficiency. $\sigma_\eta^{\text{stat}} (\sigma_\eta'^{\text{stat}})$ is the relative statistical error and $\sigma_\eta^{\text{sys}} (\sigma_\eta'^{\text{sys}})$ is the relative statistical error of $n^\gamma_\eta (n^\gamma_\eta')$, $\sigma_\eta (\sigma_\eta')$ is the total relative error.

<table>
<thead>
<tr>
<th>quantity</th>
<th>$\eta$</th>
<th>$\eta'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n^\eta_{UL} (n^\eta_{UL}')$</td>
<td>3.56</td>
<td>5.72</td>
</tr>
<tr>
<td>$\epsilon_\eta (\epsilon_\eta')$</td>
<td>23.5%</td>
<td>23.2%</td>
</tr>
<tr>
<td>$n^\gamma_\eta (n^\gamma_\eta')$</td>
<td>1760.2 ± 49.3</td>
<td>71.6 ± 13.2</td>
</tr>
<tr>
<td>$\epsilon_\gamma^\eta (\epsilon_\gamma^\eta')$</td>
<td>17.6%</td>
<td>15.2%</td>
</tr>
<tr>
<td>$\sigma_\eta^{\text{stat}} (\sigma_\eta'^{\text{stat}})$</td>
<td>2.8%</td>
<td>18.5%</td>
</tr>
<tr>
<td>$\sigma_\eta^{\text{sys}} (\sigma_\eta'^{\text{sys}})$</td>
<td>7.7</td>
<td>11.1</td>
</tr>
<tr>
<td>$\sigma_\eta (\sigma_\eta')$</td>
<td>8.1%</td>
<td>21.6%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$J/\psi$ decay mode</th>
<th>Branching fraction ($10^{-4}$)</th>
<th>Invisible decay mode</th>
<th>Tagging topology</th>
<th>Number of events in 10 billions $J/\psi$ sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J/\psi \to \phi \eta$</td>
<td>6.5 ± 0.7</td>
<td>$\eta \to$ invisible</td>
<td>$\phi \to K^+ K^-$</td>
<td>(31.4 ± 3.4) × 10^5</td>
</tr>
<tr>
<td></td>
<td>6.5 ± 0.7</td>
<td>$\phi \to$ invisible</td>
<td>$\eta \to \gamma \gamma$</td>
<td>(25.7 ± 2.8) × 10^5</td>
</tr>
<tr>
<td>$J/\psi \to \phi \eta'$</td>
<td>3.3 ± 0.4</td>
<td>$\eta' \to$ invisible</td>
<td>$\phi \to K^+ K^-$</td>
<td>(16.2 ± 1.9) × 10^5</td>
</tr>
<tr>
<td></td>
<td>3.3 ± 0.4</td>
<td>$\phi \to$ invisible</td>
<td>$\eta' \to \gamma \rho^0$</td>
<td>(9.6 ± 1.2) × 10^5</td>
</tr>
<tr>
<td>$J/\psi \to \omega \eta$</td>
<td>15.8 ± 1.6</td>
<td>$\eta \to$ invisible</td>
<td>$\omega \to \pi^+ \pi^- \pi^0$</td>
<td>(13.9 ± 1.4) × 10^6</td>
</tr>
<tr>
<td></td>
<td>15.8 ± 1.6</td>
<td>$\omega \to$ invisible</td>
<td>$\eta \to \gamma \gamma$</td>
<td>(6.2 ± 0.6) × 10^6</td>
</tr>
<tr>
<td>$J/\psi \to \omega \eta'$</td>
<td>1.67 ± 0.25</td>
<td>$\eta' \to$ invisible</td>
<td>$\omega \to \pi^+ \pi^- \pi^0$</td>
<td>(1.5 ± 0.2) × 10^6</td>
</tr>
<tr>
<td></td>
<td>1.67 ± 0.25</td>
<td>$\omega \to$ invisible</td>
<td>$\eta' \to \gamma \rho^0$</td>
<td>(0.7 ± 0.1) × 10^6</td>
</tr>
<tr>
<td>$J/\psi \to \rho^0 \eta$</td>
<td>1.93 ± 0.23</td>
<td>$\eta \to$ invisible</td>
<td>$\rho^0 \to \pi^+ \pi^-$</td>
<td>(1.9 ± 0.2) × 10^6</td>
</tr>
<tr>
<td></td>
<td>1.93 ± 0.23</td>
<td>$\rho^0 \to$ invisible</td>
<td>$\eta \to \gamma \gamma$</td>
<td>(0.8 ± 0.09) × 10^6</td>
</tr>
<tr>
<td>$J/\psi \to \rho^0 \pi^0$</td>
<td>56 ± 7</td>
<td>$\rho^0 \to$ invisible</td>
<td>$\pi^0 \to \gamma \gamma$</td>
<td>(55.3 ± 5.8) × 10^6</td>
</tr>
</tbody>
</table>

Table 22.5: $J/\psi$ decay modes which can be used to study the invisible decays of $\eta$, $\eta'$, $\rho$, $\omega$ and $\phi$. The branching fractions are listed from PDG [19]. For each mode, a "tagging topology" is given, which is the set of visible in the detector's acceptance. In each case the tagging topology has a well-defined kinematics as outlined in section ???. The produced number of events are the expected events in 10 billions $J/\psi$ data set at BES-III, in which we already considered the decay rate of the tagging particles.
which binds with the outgoing $s\bar{s}$ from $c\bar{c}$ decays to form $\psi(nS) \to \phi\phi$ [21]. When this is done, the photon exchange piece must cancel since the final state can only occur when C (or P) violating interactions are present. In order to make a rough estimate, one can consider just the rate due to W-exchange contribution, it is straightforward to obtain [22]

$$\frac{\Gamma(J/\psi \to s\bar{s})_{\text{weak}}}{\Gamma(J/\psi \to e^+e^-)} \approx \frac{1}{2} \left( \frac{m_{J/\psi}}{m_W} \right)^4,$$

where $m_{J/\psi}$ and $m_W$ are the masses of $J/\psi$ and W boson, respectively. This leads to $BR(J/\psi \to s\bar{s})_{\text{weak}} \approx 10^{-7}$ for this weak contribution. It would lead one to expect $BR(\psi(nS) \to \phi\phi)$ below the level of $10^{-8}$ which may be even out of reach of BES-III experiment.

Experimentally, there are a few possible backgrounds which will dilute the measurement of decay of $J/\psi \to \phi\phi$. One major background is $J/\psi \to \gamma\phi\phi$ which is mainly from $J/\psi \to \gamma\eta_c(1S), \eta_c(1S) \to \phi\phi$. This kind of background can be easily removed by doing a kinematic constraint fit. A detailed calculation had been made to estimate $J/\psi \to \gamma\phi\phi$ background in Ref. [21]. Another background appears if one studies only $2(K\bar{K})$ invariant pair mass distributions. It arises from the CP-conserving reaction $J/\psi \to \phi(K\bar{K})_{s\text{-wave}}$, due to the fact that the $\phi$ mass is only two S-wave-widths away from the S-wave-mass, for example, $f_0(980) \to K\bar{K}$. Although it may be difficult to subtract in a small statistical sample, one can, in principle, remove this kind of background by either a spin-parity analysis for the $K\bar{K}$ pairs in a narrow window about $\phi$ mass, or by a subtraction normalized to an observed S-wave peak. To avoid the S-wave contribution, one can reconstruct one $\phi$ from $K^+K^-$ and another $\phi$ from $K_sK_L$ mode which is not allowed to form a S-wave. It will be easy to look for the missing mass of one $\phi$ reconstructed from $K^+K^-$, to see if there is any peak under the $\phi$ mass region by also requiring $K_S$ and $K_L$ information in the final states.

It is noted that there is possible continuum background produced through a two photon annihilation process. It is peaking background which can not be removed without considering detail angular distributions with high statistic sample. So it is very hard to deal with this kind of peaking source with small sample. The only way is to off-peak data which are taken below the $J/\psi$ mass peak. The $e^+e^- \to \gamma\gamma$ process has been investigated before [23], and it has a unique production angle ($\theta^*$) distributions, which is defined as the angle between $\phi$ and $e^-$ beam direction in the Center-mass (CM) frame. The production angle distribution for the two real photon annihilating process has the form of

$$\sigma(\cos\theta^*)_{e^+e^-\to\gamma\gamma} = \frac{1 + \cos\theta^*}{1 - \cos\theta^*},$$

while, in the process of two virtual photon into $V^0V^0$ pairs, the distribution becomes (to first order) [24]:

$$\sigma(\cos\theta^*)_{e^+e^-\to\gamma^*\gamma^*\to V^0V^0} = \frac{1 + \cos\theta^*}{k^2 - \cos\theta^*},$$

where factor $k$ will be:

$$k = \frac{2m_{V^0}^2 - S}{\sqrt{S^2 - 4S m_{V^0}^2}},$$
where $S$ is the square of CM energy. In principle, by using an angular analysis, one can remove the peaking background with high statistic data sample. To avoid the peaking background from continuum, $\psi(2S) \to \pi\pi J/\psi$ could be employed to study this kind of rare $J/\psi$ decays with 3 billions $\psi(2S)$ sample, but the statistics will be lost.

22.4 Lepton flavor violating processes in decays of $J/\psi$

Lepton flavor violating (LFV) processes are strongly suppressed in the standard model by powers of small neutrino masses [25]. Therefore, such decays can be used to probe possible new physics. At present, there are many stringent bounds for $\mu$, $\tau$ and $Z$ boson decays, such as $BR(\mu \to 3e) \leq 10^{-12}$, $BR(\mu \to e\gamma\gamma) \leq 10^{-10}$ and somewhat weaker $O(10^{-6}$ bounds on LFV $\tau$ decays [126]. There have been a lot of studies both theoretically and experimentally on testing the lepton flavor conservation law [25, 26]. With huge $J/\psi$ sample, BES-III experiment will be able to make an additional experimental searching for lepton flavor violating processes of $J/\psi \to ll'$ ($l$ and $l' = \tau, \mu, e, l \neq l'$).

To estimate the branching ratio of the lepton flavor violating $J/\psi$ decays allowed by the current experimental data, Peccei, Wang and Zhang took a model-independent approach to new physics and introduced a effective four-fermion contact interaction [25, 27, 28]:

$$\frac{4\pi}{\Lambda^2} \frac{g_\mu - g_\mu'}{4} \gamma^\mu \gamma'^\nu,$$  \hspace{1cm} (22.19)

where $\Lambda$ is the new physics cutoff. This effective operator is forbidden in the standard model, however, it will be generated in theories where lepton flavor is not conserved, such as the MSSM with and/or without R parity, models with large extra dimension [29]. Therefore, any observed signal is a direct evidence for non-standard physics and will improve our understanding of flavor dynamics, especially in the lepton sector.

There is no direct experimental limit on $\Lambda$ in equation (22.19). However, at one-loop level, attaching the neutral gauge boson $Z$ to the charm quark loop generates an effective coupling of $Z$ to $\bar{\ell}l'$. From the limits given in the particle data book on $BR(Z \to \bar{\ell}l')$ [126], one obtained the lower bounds on the branching ratio of the $J/\psi$ decay into leptons [28]:

$$\begin{align*}
BR(J/\psi \to \tau^+ e^-) &< 2.7 \times 10^{-5}; \\
BR(J/\psi \to \tau^+ \mu^-) &< 4.9 \times 10^{-5}; \\
BR(J/\psi \to \mu^+ e^-) &< 8.3 \times 10^{-6}.
\end{align*}$$

(22.20) \hspace{1cm} (22.21) \hspace{1cm} (22.22)

While, recently, Nussinov, Peccei and Zhang [25] have also examined "unitarity inspired" relations between two- and three-body lepton flavor violating decays and found that the existing strong bounds on $\mu \to 3e$ and $\mu \to e\gamma\gamma$ servery constrain two-body lepton flavor violating decays of vector mesons, like $J/\psi$, $\Upsilon(1S)'$ and $\phi$ into $\mu^\pm e^\mp$ final states. In reference [25], using $BR(\mu \to 3e) \leq 10^{-12}$ and data pertaining to the $e^+e^-$ widths of the $J/\psi$, the bounds for the two-body LFV branching ratio of $J/\psi$ decay is given

$$BR(J/\psi \to \mu^\pm e^\mp) < 4 \times 10^{-13}.$$  \hspace{1cm} (22.23)
Likewise, the generic upper bounds on LPV $\tau$ decays $BR(\tau \to l l' \bar{l'}) \leq 10^{-6}$ yields

$$BR(J/\psi \to \tau^{\pm}l^{\mp}) < 6 \times 10^{-7}. \quad (22.24)$$

with $l/l' = e/\mu$. So these inferred bounds are unlikely to be improved by future experimental data on two-body decays, especially at BES-III. However, all these bounds derived in reference [25, 28] can be avoided if there is a kinematical suppression or as a results of some cancellations [25]. Searching for lepton flavor violating decays of $J/\psi$ with huge sample from BES-III remains a worthwhile experimental challenge [28].

From 58 M $J/\psi$ data at BESII, the following upper limits had been got [30]:

$$BR(J/\psi \to \tau^{\pm}e^{\mp}) < 8.3 \times 10^{-6}; \quad (22.25)$$
$$BR(J/\psi \to \tau^{\pm}\mu^{\mp}) < 2.0 \times 10^{-6}; \quad (22.26)$$
$$BR(J/\psi \to \mu^{\pm}e^{\mp}) < 1.1 \times 10^{-6}. \quad (22.27)$$

The sensitivity of the two-body lepton flavor violating decays of $J/\psi$ could be $10^{-8} - 10^{-9}$ level at BES-III with one year luminosity at $J/\psi$ peak. It will be a significant improvement.
Bibliography


