BAD 522 V6
Based on EvtGen V00-11-07

Monte Carlo Generators for Tau-Charm-Physics at BESIII

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Abstract

This note describes two kinds of Mont-Carlo generator at BESIII. One is inherited from BESII and the other is that based on the amplitude information developed from EvtGen. We focus on the description on the model for the latter, especially for the construction of the amplitude. The models cover the hadronic decays, radiative decays and decays for investigating some physical quantities in charmonium physics. The user is suggested to refer the guide of the EvtGen for some generators applicable to both charm and B physics.

Contents

1 Introduction

There are two kinds of generator available at BESIII environment. One kind of generator is inherited from BESII environment, which adds up to 30 models available. Most of them is based on the pure phase space generator, some of them implement the angular distribution of the final state particles (eg. P2BB), and few of them using amplitude information. Another kind of generator is incorporated with EvtGen classes. In the available models (about 70) of EvtGen developed for $B-$ physics, only small fraction of models can be used in description of the charmonium decays, for example, HELAMP, PHSP, P2O3P, etc.. To provide the full requirement of analysis of charmonium decays, more models are expected to be developed for simulating charmonium decays with implement of angular distribution and amplitude information. Table 1 summarizes the generators available in BesGenModule. For details, the user refer to the manual at BESII homepage (http://bes.ihep.ac.cn/bes2/software/BES-I/MEM/EvetGen.mem).

2 EvtGen Framwork

2.1 Particle properties in EvtGen

In EvtGen, all particle property information is contained within **e**vt.pdl and is parsed at runtime. Each line in **e**vt.pdl corresponds to a particle, and has the form

```
add  p Lepton  mu-       13  0.1056584  0     0   -3  1 658654.  13
add  p Lepton  mu+      -13  0.1056584  0     0    3  1 658654.   0
add  p Meson   pi+      211  0.139570   0     0    3  0  7804.5 101
add  p Meson   pi-     -211  0.139570   0     0   -3  0  7804.5   0
add  p Meson   rho+     213  0.7685     0.151 0.4  3  2       0 121
add  p Meson   rho-    -213  0.7685     0.151 0.4 -3  2       0   0
add  p Meson   D0       421  1.86451    0     0    0  0  0.1244 105
add  p Meson   anti-D0 -421  1.86451    0     0    0  0  0.1244   0
```

The first three columns are not used in EvtGen. The fourth column corresponds to the particle name. The fifth column is the particle number according to the stdhep numbering scheme. The sixth through eighth columns contain the mass, width, and maximum allowed deviation from the mean mass in the downward direction, respectively. The nineth column contains 3 times the charge of the particle. The tenth column contains twice the spin. The eleventh column is $c\tau$ in mm. The twelvth column is the Lund-KC number. This is used for the interface to JetSet and has to match what is in lucomp.F.

2.2 Particle representation

Particles with spin up to spin 2, with the exception of spin 3/2, are handled by classes within the framework of EvtGen.[1] This section will describe how spin degrees of freedom

[1] Other particles will be added as need arises.

Table 1: Generators available in BesGenModule.

Name	Description
Bhagen	radiative Bhabha scattering up to α^3 order of QED
Ddgen	$\psi'' \to D\bar{D}$
Ddprod	$D\bar{D}$ pair with the correct angular distribution.
Dsdgen	$e^+e^- \to D^*D$
Dssgen	$e^+e^- \to D^*\bar{D}^*$
Epscat	Bhabha scattering with angular distribution.
Fff	$e^+e^- \to q\bar{q} \to$hadron according to Field-Feynman Fragmentation model.
Ffgen	$e^+e^- \to D_s^+D_s^-$
Fixpt	To use for testing, debugging, checking efficiencies, to generate specific particle in given distribution with fixed momentum.
Fsfgen	$\psi \to F^*F_2, F^* \to \gamma F_1$ decay type
Gamma2	double photon process for $e^+e^- \to P^-P^+Q^-Q^+$, with $P, Q = e, \mu, \tau$, or quarks.
Howl	pure phase space
Kk2f	$e^+e^- \to \gamma \to l^+l^-$ or $q\bar{q}$
Koralbe	$e^+e^- \to \tau^+\tau^-$, ref. CERN-Th-5885/90
Kstark	$\psi \to K^*K \to K\bar{K}\pi$
Lund	Lund model: $e^+e^- \to \psi, \psi(2S) \to$ hadrons, $e^+e^- \to \gamma^* \to hadrons$
Mugen	$e^+e^- \to \mu^+\mu^-(\gamma)$, up to α^3 order QED.
P2bb	$\psi, \psi(2S) \to B\bar{B}$ with angular distribution $1 + \alpha\cos^2\theta$
P2epem	$\psi \to e^+e^-$ with angular distribution $1 + \cos^2\theta$
P2mumu	$\psi \to \mu\mu$ with angular distribution $1 + \cos^2\theta$
Ppgen	$\psi(2S)$ exclusive decays, some with angular distribution information.
Radee	$e^+e^- \to e^+e^-\gamma$
Radgg	$e^+e^- \to \gamma\gamma\gamma$, Ref. Nucl. Phys. B186(1981), 22
Radmu	$e^+e^-\mu^+\mu^-$
Rhopi	$\psi \to 3\pi$ via $\rho\pi$ with the angular distribution $\sin^2\theta$
Sagerx	$\psi \to \gamma X, X \to PP$,$P$=pseudoscalar
Tauprd	?
Tester	A simple 4-vector generator
Twogam	2γ process, similar to Gamma2
V2llg	$V \to l^+l^-\gamma$ up to α^3 order QED

are represented, and will introduce the classes that represent particles in EvtGen. Table 2 summarizes the different types of particles currently implemented.

Class name	Rep.	J	States	Example
EvtScalarParticle	1	0	1	π, B^0
EvtDiracParticle	u_α	1/2	2	e, τ
EvtNeutrinoParticle	u_α	1/2	1	ν_e
EvtVectorParticle	ϵ^μ	1	3	ρ, J/Ψ
EvtPhotonParticle	ϵ^μ	1	2	γ
EvtTensorParticle	$T^{\mu\nu}$	2	5	D_2^*, f_2

Table 2: The different types of particles that supported by EvtGen. The spin 3/2, Rarita-Schwinger, representation has not yet been implemented.

In Table 2, u_α represents a four component Dirac spinor, defined in the Pauli-Dirac convention for the gamma matrices, as discussed in Section 2.3.4. The **E**vtDiracParticle class represents massive spin 1/2 particles that have two spin degrees of freedom. Neutrinos are also represented with a 4-component Dirac spinor by the **E**vtNeutrinoParticle class. Neutrinos are assumed to be massless and only left handed neutrinos and right handed anti-neutrinos are considered.

The complex 4-vector ϵ^μ is used to represent the spin degrees of freedom for spin 1 particles. Massive spin 1 particles, represented by the **E**vtVectorParticle class, have three degrees of freedom. The **E**vtPhotonParticle class represents massless spin 1 particles, which have only two (longitudinal) degrees of freedom.

Massive spin 2 particles are represented with a complex symmetric rank 2 tensor and are implemented in the **E**vtTensorParticle class.

For each particle initialized in EvtGen, a set of basis states is created, where the number of basis states is the same as the number of spin degrees of freedom. These basis states can be accessed through the **E**vtParticle class in either the particle's rest frame or in the parent's rest frame. For massless particles, only states in the parent's rest frame are available. As an example, consider the basis for a massive spin 1 particle in it's own rest frame

$$\epsilon_1^\mu = (0,1,0,0), \quad (1)$$
$$\epsilon_2^\mu = (0,0,1,0), \quad (2)$$
$$\epsilon_3^\mu = (0,0,0,1). \quad (3)$$

Note that these basis vectors are mutually orthogonal, and normalized. That is,

$$g_{\mu\nu}\epsilon_i^{*\mu}\epsilon_j^\nu = -\delta_{ij}. \quad (4)$$

Further, they form a complete set

$$\sum_i \epsilon_i^{*\mu}\epsilon_i^\nu = g^{\mu\nu} - p^\mu p^\nu/m^2, \quad (5)$$

where $g^{\mu\nu} - p^\mu p^\nu / m^2$ is the propagator for an on-shell spin 1 particle.

In order to write a new decay model, there is no reason to need to know the exact choice of these basis states. The code needed to describe the amplitude for a decay process is independent of these states, as long as they are complete, orthogonal and normalized.

2.3 Conventions

This section discusses the conventions we use for various physics quantities in the code.

2.3.1 Units

In EvtGen, $c = 1$, such that mass, energy and momentum are all measured in units of GeV. Similarly, time and space have units of mm.

2.3.2 Four-vectors

There are two types of four-vectors used in EvtGen, **E**vtVector4R and **E**vtVector4C, which are real and complex respectively.

A four-vector is represented by $p^\mu = (E, \vec{p})$. When a four-vector is used its components are always corresponding to raised indices. A contraction of two vectors p and k (**p***k) automatically lowers the indices on k according to the metric $g = \mathrm{diag}(1, -1, -1, -1)$ so that **p***p is the mass squared of a particle with four-momentum p.

2.3.3 Tensors

We currently only support complex second rank tensors. As in the case of vectors, tensors are always reperesented with all indices raised. The convention for the totaly antisymmetric tensor, $\epsilon_{\alpha\beta\mu\nu}$, is $\epsilon_{0123} = +1$.

2.3.4 Dirac spinors

Dirac spinors are represented as a 4 component spinor in the Dirac-Pauli representation, with initial state fermions or final state anti-fermions.

2.3.5 Gamma matrices

Dirac gamma matrices are also represented in the Dirac-Pauli representation, which has

$$\gamma^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad \gamma^1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \tag{6}$$

$$\gamma^2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, \quad \gamma^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{7}$$

This gives

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \frac{i}{4!}\epsilon_{\lambda\mu\nu\pi}\gamma^\lambda\gamma^\mu\gamma^\nu\gamma^\pi = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{8}$$

2.4 Algorithm

To illustrate how the event selection algorithm works consider the decay $B \to D^*\tau\bar{\nu}$, $D^* \to D\pi$, and $\tau \to \pi\nu$. The general case is a straight forward generalization of this example. The decay amplitude can be written as

$$A = \sum_{\lambda_{D^*}\lambda_\tau} A^{B\to D^*\tau\nu}_{\lambda_{D^*}\lambda_\tau} \times A^{D^*\to D\pi}_{\lambda_{D^*}} \times A^{\tau\to\pi\nu}_{\lambda_\tau}, \tag{9}$$

where λ_{D^*} and λ_τ label the states of spin degrees of freedom of the D^* and the τ, respectively. Thus, $A^{B\to D^*\tau\nu}_{\lambda_{D^*}\lambda_\tau}$ represents the decay amplitude for $B \to D^*\tau\nu$ for the six different combinations of D^* and τ states.

A possible implementation of Eq. 9 is to generate kinematics according to phase space for the entire decay chain and to calculate the probability, the amplitude squared, which is used in an accept-reject algorithm. This approach has two serious limitations. First, the maximum probability of the decay chain must be known. This is logicistally difficult given the large number of potential decay chains in B decays. Second, for long decay chains the accept-reject algorithm can be very inefficient as the entire chain must be regenerated if the event is rejected. We have implemented an algorithm that generates a decay chain as a sequence of sub-decays, thus avoiding both of these limitations.

First the decay of the B is considered. Kinematics are generated according to phase space and the probability is calculated

$$P_B = \sum_{\lambda_{D^*}\lambda_\tau} |A^{B\to D^*\tau\nu}_{\lambda_{D^*}\lambda_\tau}|^2. \tag{10}$$

The kinematics are regenerated until the event passes an accept-reject algorithm based on P_B. After decaying the B we form the spin density matrix

$$\rho^{D^*}_{\lambda_{D^*}\lambda'_{D^*}} = \sum_{\lambda_\tau} A^{B\to D^*\tau\nu}_{\lambda_{D^*}\lambda_\tau}[A^{B\to D^*\tau\nu}_{\lambda'_{D^*}\lambda_\tau}]^*, \tag{11}$$

which describes a D^* from the $B \to D^*\tau\nu$ decay after summing over the degrees of freedom for the τ. To generate the $D^* \to D\pi$ decay, proceed as with the B, including also ρ^{D^*}

$$P_{D^*} = \frac{1}{\mathrm{Tr}\,\rho^{D^*}} \sum_{\lambda_{D^*}\lambda'_{D^*}} \rho^{D^*}_{\lambda_{D^*}\lambda'_{D^*}} A^{D^*\to D\pi}_{\lambda_{D^*}}[A^{D^*\to D\pi}_{\lambda'_{D^*}}]^*, \tag{12}$$

where the scale factor, $1/\mathrm{Tr}\,\rho^{D^*}$, is proportional to the decay rate, and does not affect the angular distributions. This scale factor makes the maximum decay probability of each sub-decay independent of the full decay chain.

Finally, we decay the τ. We form the density matrix

$$\tilde{\rho}^{D^*}_{\lambda_{D^*}\lambda'_{D^*}} = A^{D^*\to D\pi}_{\lambda_{D^*}}[A^{D^*\to D\pi}_{\lambda'_{D^*}}]^*, \tag{13}$$

which encapsulates the information about the D^* decay needed to properly decay the τ with the full correlations between all kinematic variables in the decay. Using the $\tilde{\rho}^{D^*}$ matrix we calculate the spin density matrix of the τ

$$\rho^{\tau}_{\lambda_\tau\lambda'_\tau} = \sum_{\lambda_{D^*}\lambda'_{D^*}} \tilde{\rho}^{D^*}_{\lambda_{D^*}\lambda'_{D^*}} A^{B\to D^*\tau\nu}_{\lambda_{D^*}\lambda_\tau}[A^{B\to D^*\tau\nu}_{\lambda'_{D^*}\lambda'_\tau}]^*. \tag{14}$$

As in the other decays, kinematics are generated according to phase space and the accept-reject is based on the probability calculated as in Eq. 12, replacing D^* with τ.

The algorithm was illustrated above using an example which should convey the idea. In general consider the decay

$$A \to B_1 B_2 ... B_N \tag{15}$$

where the amplitudes are denoted by

$$A^{A\to B_1B_2...B_N}_{\lambda_A\lambda_{B_1}\lambda_{B_2}...\lambda_{B_N}}. \tag{16}$$

The probability for, P_A, for this decay is given by

$$P_A = \sum_{\lambda_A\lambda'_A\lambda_{B_1}...\lambda_{B_N}} \rho^{A}_{\lambda_A\lambda'_A} A^{A\to B_1B_2...B_N}_{\lambda_A\lambda_{B_1}\lambda_{B_2}...\lambda_{B_N}} [A^{A\to B_1B_2...B_N}_{\lambda'_A\lambda_{B_1}\lambda_{B_2}...\lambda_{B_N}}]^*. \tag{17}$$

The forward spin-density matrix ρ^{B_i}, given that B_j, $j < i$, have been decayed and have backward spin-density matrices $\hat{\rho}^{B_j}$, is given by

$$\rho^{B_i}_{\lambda_{B_i}\lambda'_{B_i}} = \sum_{\substack{\lambda_A\lambda'_A\lambda_{B_1}...\lambda_{B_N}\\ \lambda'_{B_1}...\lambda'_{B_{i-1}}}} \rho^{A}_{\lambda_A\lambda'_A}\hat{\rho}^{B_1}_{\lambda_{B_1}\lambda'_{B_1}}...\hat{\rho}^{B_{i-1}}_{\lambda_{B_{i-1}}\lambda'_{B_{i-1}}} A^{A\to B_1B_2...B_N}_{\lambda_A\lambda_{B_1}\lambda_{B_2}...\lambda_{B_N}} [A^{A\to B_1B_2...B_N}_{\lambda'_A\lambda'_{B_1}...\lambda'_{B_i}\lambda_{B_{i+1}}...\lambda_{B_N}}]^*. \tag{18}$$

After all B_i are decays the backward spin-density matrix is given by

$$\hat{\rho}^{A}_{\lambda_A\lambda'_A} = \sum_{\substack{\lambda_{B_1}...\lambda_{B_N}\\ \lambda'_{B_1}...\lambda'_{B_N}}} \hat{\rho}^{B_1}_{\lambda_{B_1}\lambda'_{B_1}}...\hat{\rho}^{B_N}_{\lambda_{B_N}\lambda'_{B_N}} A^{A\to B_1B_2...B_N}_{\lambda_A\lambda_{B_1}\lambda_{B_2}...\lambda_{B_N}} [A^{A\to B_1B_2...B_N}_{\lambda'_A\lambda'_{B_1}...\lambda'_{B_N}}]^*. \tag{19}$$

2.5 Introduction to decay models

Each decay model is a class that is derived from the base class **E**vtDecayBase, as shown in Figure 1. Models may handle many different decays, or might be specialized for one special decay. An example of a model that can handle many different decays is the **V**SS model, which decays a vector meson to a pair of scalar particles. For example, the decays $D^* \to D\pi$ and $\rho \to \pi\pi$ are handled by this model. An example of a highly specialized model is **B**TO4PICP, which describes the decay $B \to \pi^+\pi^-\pi^+\pi^-$.

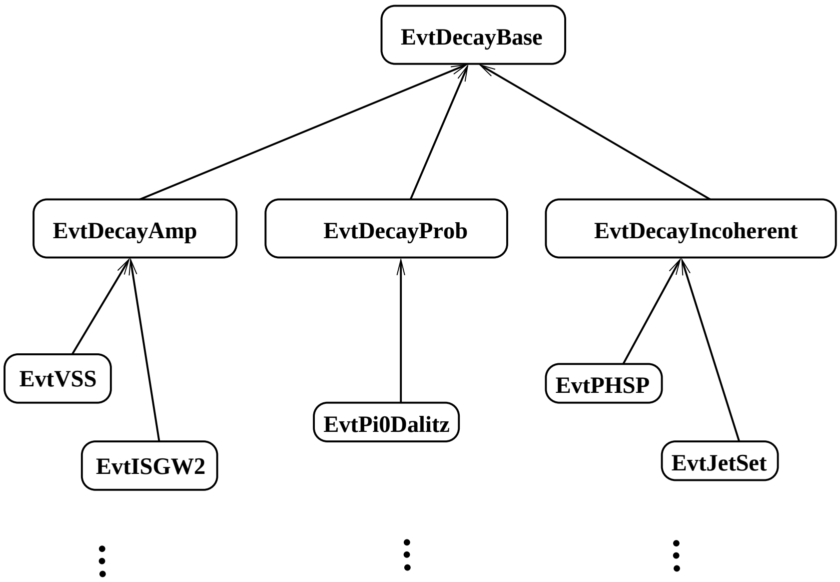

Figure 1: Diagram of the **E**vtDecayBase class and classes derived from it. **E**vtDecayAmp, **E**vtDecayTBaseProb, and **E**vtDecayIncoherent are templated classes that are used by modules that compute the full decay amplitude, that compute the decay probability, and those that return unpolarized daughters, respectively.

2.6 Creating new decay models

To simplify the writing of decay models there are three different classes that are derived from **E**vtDecayBase. These are **E**vtDecayAmp, **E**vtDecayProb, and **E**vtDecayIncoherent. When writing decay models it is most convenient to derive from one of these classes. These classes have slightly different interfaces depending on what information the decay model provides. Some models will provide the complete amplitude information where as other models might provide just a probability or a set of four-vectors. Models that don't provide the full set of amplitudes will of course not be able to simulate the complete angular distributions. Below, a short description is given to explain when you want to use each of these classes as the base class for your decay model.

- **E**vtDecayAmp allows you to specify the complete amplitudes and hence do a complete simulation of the angular distributions.
- **E**vtDecayProb allows you to calculate a probability for the decay. This probability is then used in the accept-reject method. Any spin information is lost, all produced particles are unpolarized and uncorrelated.
- **E**vtDecayIncoherent just accepts the four vectors as generated. This is most useful when interfacing to another generator, e.g. JetSet. Any spin information is lost, all produced particles are unpolarized and uncorrelated.

2.7 Available base classes in EvtGen

- **EvtAmp:**
  This class keeps track of the amplitudes computed by the decay models. It provides member functions to calculate spin-density matrices from the amplitudes.
- **EvtCPU:**
  This class contains some utilities that are useful for generating CP-violating decays from the $\Upsilon(4S)$ system.
- **EvtComplex:**
  Using the implementation of complex numbers provided by the compiler has caused constant problems with porting EvtGen to different platforms, as these implementations do not generally conform to a uniform standard. Therefore, we have implemented the **E**vtComplex class. This implemention is not complete.
- **EvtConst:**
  This class defines useful constants.
- **EvtDecayBase:**
  This class is the base class for decay models and contains the interface for the decay models to the framework.

- **EvtDiraSpinor:**
  The EvtDiracSpinor class encapsulates the properties of a Dirac spinor. It is used to reperesent spin 1/2 particles.

- **EvtGammaMatrix:**
  EvtGammaMatrix is a class for handling complex 4×4 matrices.

- **EvtGen:**
  This class provides the interface to EvtGen for an external user.

- **EvtGenKin:**
  EvtGenKine contains tools for generating kinematics such as phase space distributions.

- **EvtID:**
  The class **E**vtId is used to identify particles in EvtGen.

- **EvtKine:**
  This class provides some utility functions for calculating kinematic quantities such as decay angles.

- **EvtLineShape:**
  This class contains utilities for simulating line shapes of particles. Currently this class only provides the trivial implementation of a non-relativistic Breit-Wigner shape.

- **EvtModel:**
  This class handles the registration of decay models.

- **EvtPDL:**
  The particle information read from the **e**vt.pdl file can be accessed through member functions of the **E**vtPDL class.

- **EvtPHOTOS:**
  Provides an interface to the PHOTOS package for generation of final state radiation.

- **EvtPartProp:**
  Class to represent the particle properties of a single particle. Used by **E**vtPDL to keep the particle properties.

- **EvtParticle:**
  This is the base class for particles. It contains the common interface to particles such as the four momentum particle number, list of daughters and parent etc.

- **EvtParticleDecay:**
  Stores information for one particle decay. This class is not used yet.

- **EvtParticleDecayList:**
  Stores the list of decays of one particle. This class is not used yet

- **EvtParticleNum:**
  Defines **E**vtID for all particles.
- **EvtParser:**Used by **E**vtDecayTable to read the decay table.
- **EvtRandom:**EvtRandom provides the interface for random numbers that are used in the EvtGen package.
- **EetReadDecay:**
  This is a real mess! But it's purpose is to read in the decay table.
- **EvtReport:**
  Utility to print out mesages from EvtGen.
- **EvtResonance:**
  The EvtResonance class allows one to handle resonances as a single structure.
- **EvtSecondary:**
  Allows EvtGen not to write secondary particles to StdHep.
- **EvtSpinDensity:**
  This class represents spin-density matrices of arbitrary dimensions. (Well, this is not quite true, at the moment it is limited to dimension 5 which is the number of degrees of freedom of a spin 2 particle.)
- **EvtSpinType:**
  Defines the folowing enum for the different particle types that EvtGen handles.

  ```
  enum spintype { SCALAR,VECTOR,TENSOR,DIRAC,PHOTON,NEUTRINO,STRING };
  ```
- **EvtStdHep:**
  This class flattens out the EvtGen decay tree that is used internaly to represent the particles and stores the particles in a structure that is parallel to StdHep.
- **EvtString:**
  This class is used by EvtGen to represent character strings.
- **EvtSysTable:**
  Variables that are defined using a "Define" statement in the decay table are stored in this class.
- **EvtTemplateDummy:**
  This class was introduced just such that the EvtGen package made use of templates, this should be removed.
- **EvtTensor3C:**
  Complex rank 2 tensors in 3 dimensions.

- **EvtTensor4C:**
  This class encapsulates the properties of second rank complex tensors.
- **EvtVector3C:**
  Complex three-vectors.
- **EvtVector3R:**
  Real three-vectors.
- **EvtVector4C:**
  This is a class for reperesenting complex four-vectors.
- **EvtVector4R:**
  This is a class for representing real four vectors.

3 EvtGen developed at BESIII

3.1 Helicity amplitudes

In the models available in **EvtGen**, most of them are constructed in covariant amplitude form. However, the partial wave ratios are not usually reported by experiments. For studying the particle polarization, the helicity amplitude can be measured. For combining experimental information in the amplitude, we use the helicity amplitude form to describe the particle decays. For example, to consider the decay $C \to A + B$, the initial particle, C, is assumed to be in the state $|JM\rangle$ and the final state, $A + B$, is $|pJM\lambda_A\lambda_B\rangle$. We will assume that the interaction U that causes this transition is invariant under rotation, but is otherwise arbitrary. The matrix element can be expressed by:

$$M = \langle pJM\lambda_A\lambda_B\rangle|U|JM\rangle. \tag{20}$$

$$= \sqrt{\frac{2J+1}{4\pi}} D^{*J}_{M,\lambda_A-\lambda_B}(\phi,\theta,-\phi) H_{\lambda_A\lambda_B} \tag{21}$$

where the helicity amplitudes are defined by

$$H_{\lambda_A\lambda_B} = \langle pJM\lambda_A\lambda_B|U|JM\rangle. \tag{22}$$

For sequential decays, e.g. $A \to B + C$, $B \to D + E$, and $C \to F + G$. The initial particle, A, is in the state $|J = J_A \quad m = \lambda_A\rangle$. The amplitudes for the decay $A \to B + C$ is now given by

$$A^{A\to B+C}_{\lambda_A\lambda_B\lambda_C} = \sqrt{\frac{2J_A+1}{4\pi}} D^{*J_A}_{\lambda_A,\lambda_B-\lambda_C}(\phi_B,\theta_B,-\phi_B) H^A_{\lambda_B\lambda_C} \tag{23}$$

where θ_B and ϕ_B are the polar angles of particle B in the rest frame of particle A.

Similarly, we can write the amplitude for the decay of particle B

$$A^{B\to D+E}_{\lambda_B\lambda_D\lambda_E} = \sqrt{\frac{2J_B+1}{4\pi}} D^{*J_B}_{\lambda_B,\lambda_D-\lambda_E}(\phi_D,\theta_D,-\phi_D) H^B_{\lambda_D\lambda_E}. \tag{24}$$

The angles θ_D and ϕ_D are measured in the helicity system, i.e., the z-axis of the mass center frame is taken along the direction of the mother particle. For concrete illustration of the helicity system, Figure 2 show the helicity system for Λ and $\bar{\Lambda}$ in the sequential decay $J/\psi \to \Lambda\bar{\Lambda} \to p\bar{p}\pi^+\pi^-$.

In the same way we obtain the amplitude for the decay of particle C,

$$A^{C\to F+G}_{\lambda_C\lambda_F\lambda_G} = \sqrt{\frac{2J_C+1}{4\pi}} D^{*J_C}_{\lambda_C,\lambda_F-\lambda_G}(\phi_F,\theta_F,-\phi_F) H^C_{\lambda_F\lambda_G}. \quad (25)$$

As discussed above the coordinate system for the second particle, here particle C in the decay of particle A, is obtained by rotating the coordinates system of the first particle, B, by π around its x axis. I.e. the x-axis of particles B and C frames are parallel.

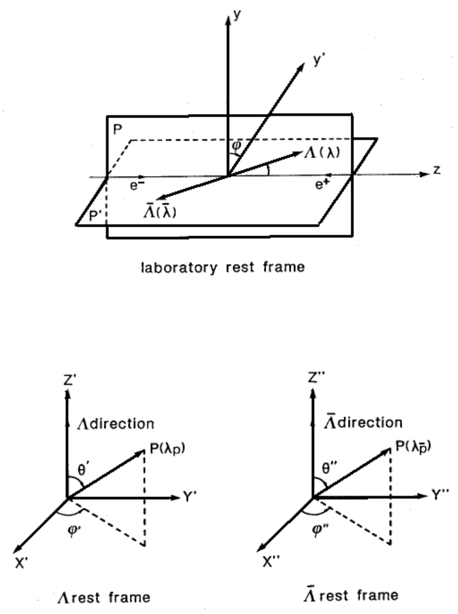

Figure 2: Helicity system of a decay defined in the sequential decay $J/\psi \to \Lambda\bar{\Lambda}, \Lambda \to p\pi^-, \bar{\Lambda} \to \bar{p}\pi^-$.

3.2 Base classes

This section describes the base classes developed at BESIII and adding to the **EvtGen** base classed.

3.2.1 EvtHelSys

This class functions as to boost the momentum of daughter particles into their mother helicity system. The daughter particle momentum are given in the laboratory system, then boosted in their mother CM system and performing a rotation to the helicity system. This class contains member functions needed for this helicity transformation.

- **friend double djmn(int j, int m, int n, double theta);**
  To define Wigner d function. The j, m and n are defined differently from the class EvtdFunction, such that $djmn(1, 0, 0, theta) = \cos(theta)$.

- **friend EvtComplex Djmn(int j, int m, int n, double phi,double theta,double gamma);**
  To define big D function based on the Wigner d function djmn(j,m,n,theta).

- **EvtVector4R Helrotate(EvtVector4R p1, double phi, double theta);**
  To rotate the 3-vector momentum of $\vec{p}_1$ to the direction $\vec{p}(|\vec{p}|, \theta, \phi)$.

- **double getHelAng(int i);**
  To get the angle (θ, ϕ) of the daughter momentum in the helicity system via the definition θ =getHelAng(1), ϕ =getHelAng(2) and $|\vec{p}|$ =getHelAng(0).

4 Decay models for charmonium physics

This section lists the different decay models that has been implemented in EvtGen only for charmonium physics developed at BESIII. The models available in EvtGen V00-11-07 are not include in this section, and they are summarized in Table 3. The user should refer to the manual of EvtGen.

4.1 JPIPI

Author: Rong-Gang Ping
Usage:
BrFr J/psi pi+ pi- JPIPI;

Explanation:

This model is constructed for the decay $\psi' \to J/\psi\pi\pi$, which is similar to the model VVPIPI in EvtGen, but the amplitude is a little different. In the model VVPIPI, the amplitude for the mass of the $\pi\pi$ system is given by $A \propto (m_{\pi\pi}^2 - 4m_\pi^2)$. In the model JPIPI, the amplitude is constructed by the chiral effective Lagrangian as $A_{\Lambda M} \propto [g(m_{\pi\pi}^2 - 2m_\pi^2) + 2g_1 E_{\pi^+} E_{\pi^-}]\epsilon_\psi^*(M) \cdot \epsilon_{\psi'}(\Lambda)$, and the parameters are taken by fit to the experimental mass spectrum of two pions [1], $g = 0.3 \pm 0.02, g_1 = -0.11 \pm 0.01$.

Example:

Decay psi(2S)
1.000 J/psi pi+ pi- JPIPI;
Enddecay

Table 3: Available decay models in EvtGen are suitable for chamonium decays, where S,P,T, L denote scalar, pseudoscalar tensor and lepton, respectively.

DECAY MODEL	DECAY MODEL
HELAMP	Any two body decay
OMEGA_DALITZ	$\omega \to 3\pi$
PARTWAVE	Any two body decay
PHSP	Any decay
PTO3P	P$\to$ 3P: $\eta_c \to \eta\pi\pi$
SINGLE	Any decay
SLN	Any decay
STS	S$\to$TS
SVP_HELAMP	S$\to$VP
SVS	S$\to$VS
SVV_HELAMP	S$\to$VV
TAULNUNU	$\tau \to e\nu_e\nu_\tau$
TAUSCALARNU	$\tau \to S + \nu_\tau$
TAUVECTORNU	$\tau \to V + \nu_\tau$
TSS	T$\to$SS
TVS_PWAVE	T$\to$VS only p-wave
VECTORISR	$e^+e^- \to V\gamma$
VLL	V$\to$LL:$J/\psi \to e^+e^-, \mu^+\mu^-, ...$
VSP_PWAVE	v$\to$SP, only p-wave
VSS	V$\to$SS:$J/\psi \to K\bar{K}, ...$
VVPIPI	V$\to$ V$\pi\pi$:$\psi' \to J/\psi\pi\pi, ...$
VVS_PWAVE	V$\to$VS:$a_0^0, J/\psi \to \rho\pi, ...$

Notes:

4.2 AngSam

Author: Rong-Gang Ping
Usage:
BrFr D1 D2 AngSam Parameter;

Explanation:

This model is constructed for the two body decay $A \to B + C$ with the daughter particle A taking the angular distribution $\frac{d\Gamma}{d\cos\theta} \propto (1 + \alpha\cos^2\theta)$ with a parameter α in the laboratory frame. The α could be negative or positive real numbers.

Example:

$\psi' \to p\bar{p}$ with the angular parameter $\alpha = 0.5$:
Decay psi(2S)
1.000 p+ anti-p- AngSam 0.5;
Enddecay

Notes:

Since this model is only based on angular distribution information in the laboratory frame, it is not suitable to the sequential decays. It is only for the decay $e^+e^- \to A \to B + C$.

4.3 P2GC0

Author: Rong-Gang Ping
Usage:
BrFr gamma chi_c0 P2GC0;

Explanation:

This model is constructed for the decay $\psi' \to \gamma\chi_{c0}$. The amplitude is constructed in the helicity amplitude given by $M \propto D^{1*}_{m,\lambda}(\theta,\phi)A_\lambda$, where the m and the λ are the helicity value of ψ' and photon, respectively. For consideration of CP invariance, the helicity amplitudes satisfy the relation $A_{-\lambda} = A_\lambda$. The angular distribution of the outgoing photon takes the form:

$$\frac{d|M|^2}{d\cos\theta} \propto (1 + \cos^2\theta).$$

Example:

$\psi(2S) \to \gamma\chi_{c0}$:

Decay psi(2S)
1.000 gamma chi_c0 P2GC0;
Enddecay

Notes:

4.4 P2GC1

Author: Rong-Gang Ping
Usage:
BrFr gamma chi_c1 P2GC1;

Explanation:

This model is constructed for the decay $\psi' \to \gamma\chi_{c1}$. The amplitude is constructed in the helicity amplitude given by $M \propto D^{1*}_{m,\lambda_1-\lambda_2}(\theta,\phi)A_{\lambda_1,\lambda_2}$, where m, λ_1 and λ_2 are the helicity values of the ψ', γ and χ_{c1}, respectively. For consideration of CP invariance, the helicity amplitudes satisfy the relation $A_{-\lambda_1,-\lambda_2} = -A_{\lambda_1,\lambda_2}$. The angular distribution of the outgoing photon takes the form $\frac{d|M|^2}{d\cos\theta} \propto (1+\alpha\cos^2\theta)$, where $\alpha = \frac{|A_{1,0}|^2-2|A_{1,1}|^2}{|A_{1,0}|^2+2|A_{1,1}|^2}$. It is believed that the $E1$ dominates this process with $\alpha = -1/3$ [2], so we take $A_{11} = A_{10} = 1$.

Example:

$\psi(2S) \to \gamma\chi_{c1}$:
Decay psi(2S)
1.000 gamma chi_c1 P2GC1;
Enddecay

Notes:

4.5 P2GC2

Author: Rong-Gang Ping
Usage:
BrFr gamma chi_c2 P2GC2;

Explanation:

This model is constructed for the decay $\psi' \to \gamma\chi_{c2}$. The amplitude is constructed in the helicity amplitude given by $M \propto D^{1*}_{m,\lambda_1-\lambda_2}(\theta,\phi)A_{\lambda_1,\lambda_2}$, where m, λ_1 and λ_2 are the helicity values of the ψ', γ and χ_{c2}, respectively. For consideration of CP invariance, the helicity amplitudes satisfy the relation $A_{-\lambda_1,-\lambda_2} = A_{\lambda_1,\lambda_2}$. The angular distribution of the outgoing photon takes the form $\frac{d|M|^2}{d\cos\theta} \propto (1+\alpha\cos^2\theta)$, where $\alpha = \frac{|A_{1,0}|^2-2|A_{1,1}|^2+|A_{1,2}|^2}{|A_{1,0}|^2+2|A_{1,1}|^2+|A_{1,2}|^2}$. Theoretically, it is believed that if the $E1$ dominates this process, the parameter $\alpha = 1/13$ [2]. Experimentally, the the helicity amplitudes have been measured by BESII [3], and they are taken as $A_{1,1}/A_{1,0} = 2.08, A_{1,2}/A_{1,0} = 3.03$.

Example:

$\psi(2S) \to \gamma\chi_{c2}$:
Decay psi(2S)
1.000 gamma chi_c2 P2GC2;
Enddecay

Notes:

4.6 AV2GV

Author: Rong-Gang Ping
Usage:
BrFr gamma J/psi AV2GV;

Explanation:

This model is constructed for the decay $\chi_{c1} \to \gamma J/\psi$. The amplitude is constructed in the helicity amplitude given by $M \propto D^{1*}_{m,\lambda_1-\lambda_2}(\theta,\phi)A_{\lambda_1,\lambda_2}$, where m, λ_1 and λ_2 are the helicity values of the χ_{c1}, γ and J/ψ, respectively. For consideration of CP invariance and angular distribution, the helicity amplitudes satisfy the relation $A_{-\lambda_1,-\lambda_2} = -A_{\lambda_1,\lambda_2}$. There are two two independent amplitudes $A_{1,1}$ and A_{10}. Theoretically, it is believed that if the $E1$ dominates this process, the polar angle of the photon relative the photon (γ_1) direction in the decay $e^+e^- \to \psi' \to \gamma_1\chi_{c1} \to \gamma_1\gamma J/\psi$ takes the form $\frac{d|M|^2}{d\cos\theta} \propto 5+\cos^2\theta$ [4]. We use this information to fix the ratio $|A_{1,0}/A_{1,1}|$.

Example:

$\chi_{c1} \to \gamma J/\psi$:
Decay chi_c1
1.000 gamma J/psi AV2GV;
Enddecay

Notes:

4.7 T2GV

Author: Rong-Gang Ping
Usage:
BrFr gamma J/psi T2GV;

Explanation:

This model is constructed for the decay $\chi_{c2} \to \gamma J/\psi$. Since no experimental information about the helicity amplitudes for this decay is available. Theoretically, it is believed that if the $E1$ dominates this process, the polar angle of the photon relative the photon (γ_1) direction in the decay $e^+e^- \to \psi' \to \gamma_1\chi_{c2} \to \gamma_1\gamma J/\psi$ takes the form $\frac{d|M|^2}{d\cos\theta} \propto 73 + 21\cos^2\theta$ [4]. The model is coded with pure phase space constrained with this angular information.

Example:

$\chi_{c2} \to \gamma J/\psi$:
Decay chi_c1
1.000 gamma J/psi T2GV;
Enddecay

Notes:

4.8 J2BB1

Author: Cai-Ying Pang, Rong-Gang Ping
Usage:
BrFr p+ anti-p- J2BB1 parameter;

Explanation:

This model is constructed for the decay ψ' and J/ψ decays into octet baryon and anti-baryon pair. The amplitude is constructed in the helicity amplitude given by $M \propto D^{1*}_{m,\lambda_1-\lambda_2}(\theta,\phi)A_{\lambda_1,\lambda_2}$, where m, λ_1 and λ_2 are the helicity values of the J/ψ, baryon and anti-baryon, respectively. For consideration of CP invariance, the helicity amplitudes satisfy the relation $A_{-\lambda_1,-\lambda_2} = A_{\lambda_1,\lambda_2}$. The angular distribution of the outgoing proton takes the form $\frac{d|M|^2}{d\cos\theta} \propto (1 + \alpha\cos^2\theta)$ with $\alpha = \frac{|A_{+-}|^2 - 2|A_{++}|^2}{|A_{+-}|^2 + 2|A_{++}|^2}$. The angular distribution parameters are the user input. Experimentally, they are give as shown in Table 4. If users do not provide parameter, the default angular distribution parameter are calculated in terms of Carimalo formula [5] $\alpha = \frac{(1+u)^2 - u(1+6u)^2}{(1+u)^2 + u(1+6u)^2}$ with $u = m_B/M$, where m_B and M are the mass of the baryon and the mother particle, respectively.

Example:

$J/\psi \to p\bar{p}$:

Table 4: Angular distribution parameter α for $J/\psi \to B\bar{B}$ decays. They are assumed to be the form of $dN/d\cos\theta \propto 1+\alpha\cos^2\theta$.

Decay mode	Measured value of α	Calculated value of α Ref. [5]
$J/\psi \to p\bar{p}$	0.68 ± 0.06[6]	0.69
$J/\psi \to \Lambda\bar{\Lambda}$	0.65 ± 0.11[7]	0.51
$J/\psi \to \Sigma^0\bar{\Sigma}^0$	-0.24 ± 0.20[7]	0.43
$J/\psi \to \Xi^-\bar{\Xi}^+$	-0.13 ± 0.59[8]	0.27
$\psi' \to p\bar{p}$	0.67 ± 0.16[9]	0.80

Decay J/psi
1.000 p+ anti-p- J2BB1 0.68;
Enddecay

Notes:

4.9 J2BB2

Author: Cai-Ying Pang, Rong-Gang Ping
Usage:
BrFr p+ anti-p- J2BB1 parameter;

Explanation:

This model is constructed for the decay ψ' and J/ψ decays into decuplet baryon and anti-baryon pair. The amplitude is constructed in the helicity amplitude given by $M \propto D^{1*}_{m,\lambda_1-\lambda_2}(\theta,\phi)A_{\lambda_1,\lambda_2}$, where m, λ_1 and λ_2 are the helicity values of the J/ψ, baryon and anti-baryon, respectively. For consideration of CP invariance, the helicity amplitudes satisfy the relation $A_{-\lambda_1,-\lambda_2} = A_{\lambda_1,\lambda_2}$. The angular distribution of the outgoing photon takes the form $\frac{d|M|^2}{d\cos\theta} \propto (1+\alpha\cos^2\theta)$ with $\alpha = \frac{|A_{\frac{1}{2}-\frac{1}{2}}|^2+2|A_{\frac{1}{2}\frac{3}{2}}|^2-2(|A_{\frac{1}{2}\frac{1}{2}}|^2+|A_{\frac{3}{2}\frac{3}{2}}|^2)}{|A_{\frac{1}{2}-\frac{1}{2}}|^2+2|A_{\frac{1}{2}\frac{3}{2}}|^2+2(|A_{\frac{1}{2}\frac{1}{2}}|^2+|A_{\frac{3}{2}\frac{3}{2}}|^2)}$. The angular distribution parameters are the user input. .

Example:

$J/\psi \to \Delta^{++}\bar{\Delta}^{--}$:
Decay J/psi
1.000 Delta++ anti-Delta– J2BB2 0.68;
Enddecay

Notes:

4.10 J2BB3

Author: Cai-Ying Pang, Rong-Gang Ping
Usage:
BrFr p+ anti-p- J2BB1 parameter;

Explanation:

This model is constructed for the decay ψ' and J/ψ decays into decuplet baryon plus octet anti-baryon pair. The amplitude is constructed in the helicity amplitude given by $M \propto D^{1*}_{m,\lambda_1-\lambda_2}(\theta,\phi)A_{\lambda_1,\lambda_2}$, where m, λ_1 and λ_2 are the helicity values of the J/ψ, baryon and anti-baryon, respectively. For consideration of CP invariance, the helicity amplitudes satisfy the relation $A_{-\lambda_1,-\lambda_2} = A_{\lambda_1,\lambda_2}$. The angular distribution of the outgoing photon takes the form $\frac{d|M|^2}{d\cos\theta} \propto (1+\alpha\cos^2\theta)$ with $\alpha = \frac{2|A_{\frac{1}{2}\frac{1}{2}}|^2+|A_{\frac{1}{2}-\frac{1}{2}}|^2+|A_{\frac{1}{2}\frac{3}{2}}|^2}{-2|A_{\frac{1}{2}\frac{1}{2}}|^2+|A_{\frac{1}{2}-\frac{1}{2}}|^2+|A_{\frac{1}{2}\frac{3}{2}}|^2}$. The angular distribution parameters are the user input.

Example:

$J/\psi \to \Sigma^+\bar{\Sigma}^{*-}$:
Decay J/psi
1.000 Sigma+ anti-Sigma*- J2BB3 0.68;
Enddecay

Notes:

4.11 DIY

Author: Rong-Gang Ping
Usage:
BrFr daughter 1 daughter 2... DIY;

Explanation:

The model Doing It Yourself (DIY) is created for user to generate event by supplying decay amplitude. The DIY model is useful for the user to generate event using partial wave analysis results, or implementing coherent effects in decays, or parameterizing the physical quantities in amplitude. The DIY generates events according the decay mode specified by user with pure phase space and passing the four-vector momentum to the amplitude and calculate the decay amplitude. The maximal value of the amplitude is singled out in 20,000 events. Then it uses the reject-accepted sampling method to generate event. There are two kinds of reference frames available to get the momentum of the final states. One is the center mass (CM) system of the mother particle and another is the helicity system with the z-axis is parallel to the outgoing direction of the mother particle in laboratory system. The user provides amplitude via a class:

```
double EvtDIY::AmplitudeSquare(){ };
```

For example, a user defines an amplitude class for the decay $J/\psi \to \rho\pi \to 3\pi$ as:

```
class rhopi{
public:
rhopi(EvtVector4R pd1, EvtVector4R pd2,EvtVector4R pd3){
_pd[0]=pd1;
_pd[1]=pd2;
_pd[2]=pd3;
}
double amps( ); //calculate amplitude
private:
EvtVector4R _pd[2];
}

///************* amplitude by user
double EvtDIY::AmplitudeSquare(){
EvtVector4R dp1=GetDaugMomLab(0);
EvtVector4R dp2=GetDaugMomLab(1),dp3=GetDaugMomLab(2);
rhopi Jpsi2rhopi(dp1,dp2,dp3);
return Jpsi2rhopi.amps();
}
```

Example:

Decay J/psi
1.000 pi+ pi- pi0 DIY;
Enddecay

Notes:

5 Test of Model

5.1 JPIPI

The model **JPIPI** is tested with the following decay table:
Decay psi(2S)
1.000 J/psi pi+ pi- JPIPI;

Enddecay
End

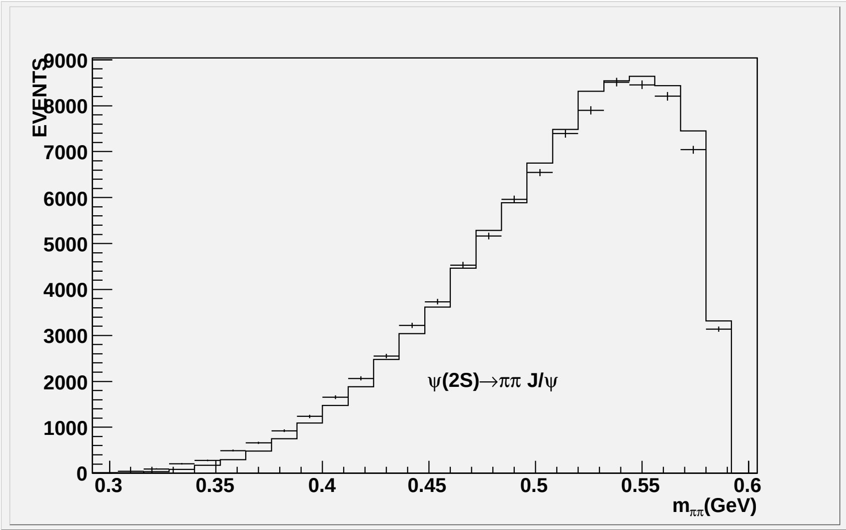

Figure 3: The comparison of the mass distribution of $m_{\pi\pi}$ in the $\psi' \to J/\psi\pi\pi$ between the model VVPIPI (error bar) and JPIPI (histogram)

5.2 AngSam

The model **AngSam** is tested by the decay $J/\psi \to p\bar{p}$ with the following decay table:
Decay J/psi
1.000 p+ anti-p- AngSam 0.67;
Enddecay
End

5.3 P2GC0

The model **P2GC0** is tested by the decay $\psi' \to \gamma\chi_{c0}$ with the following decay table:
Decay psi(2S)
1.000 gamma chi_c0 P2GC0;
Enddecay
End

5.4 P2GC1

The model **P2GC1** is tested by the decay $\psi' \to \gamma\chi_{c0}$ with the following decay table:
Decay psi(2S)

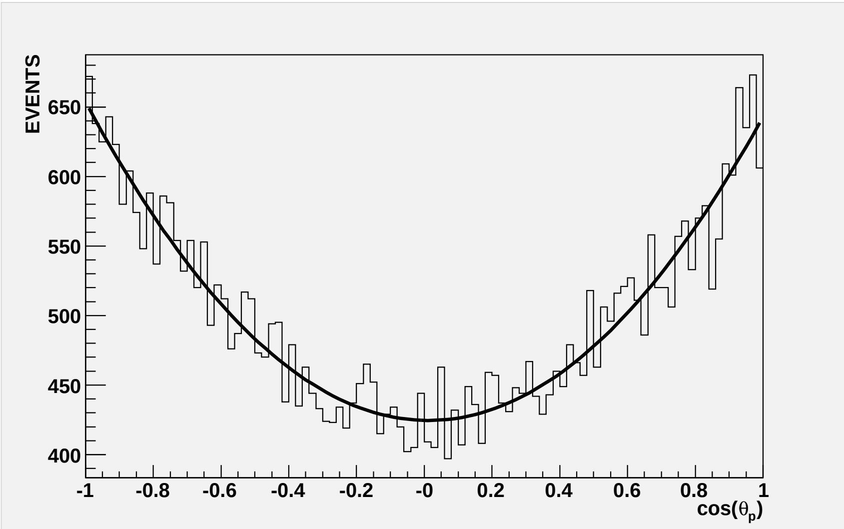

Figure 4: The test of the model AngSam with the decay $J/\psi \to p\bar{p}$ by assuming the angular distribution $d\Gamma/d\cos\theta \propto (1 + \alpha\cos^2\theta)$. The input angular distribution parameter $\alpha = 0.5$, the fit result is $\alpha' = 0.53 \pm 0.02$.

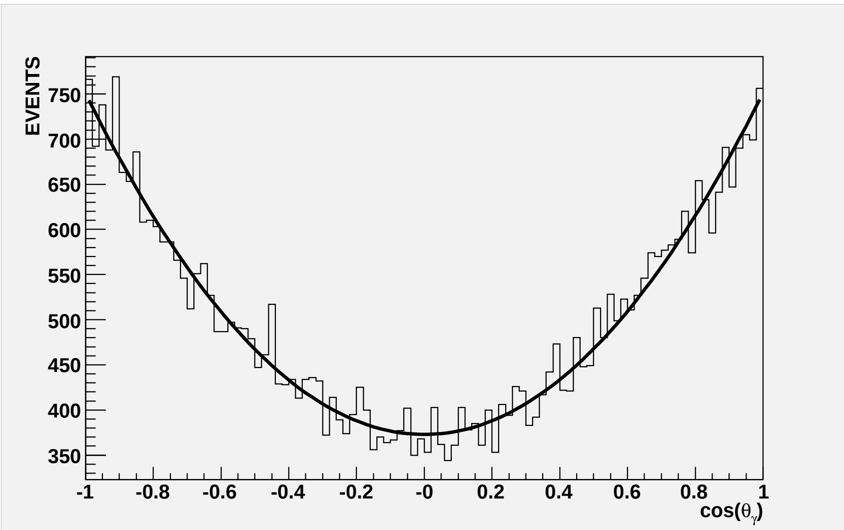

Figure 5: The model P2GC0 is tested by the angular distribution of photon in the $\psi' \to \gamma\chi_{c0}$. The input angular distribution parameter $\alpha = 1$, the fit result is $\alpha' = 1.02 \pm 0.02$.

1.000 gamma chi_c1 P2GC1;
Enddecay
End

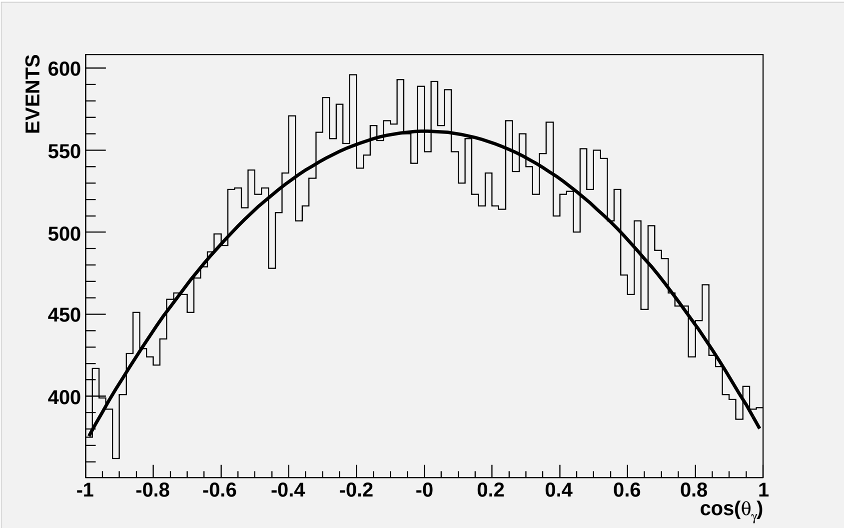

Figure 6: The model P2GC1 is tested by the angular distribution of photon in the $\psi' \rightarrow \gamma\chi_{c1}$. The input angular distribution parameter $\alpha = -1/3$, the fit result is $\alpha' = -0.33 \pm 0.02$.

5.5 P2GC2

The model **P2GC2** is tested by the decay $\psi' \rightarrow \gamma\chi_{c2}$ with the following decay table:
Decay psi(2S)
1.000 gamma chi_c2 P2GC2;
Enddecay
End

5.6 AV2GV

The model **AV2GV** is tested by the decay $\psi' \rightarrow \gamma\chi_{c1} \rightarrow \gamma\gamma J/\psi$ with the following decay table:
Decay psi(2S)
1.000 gamma chi_c1 P2GC1;
Enddecay
Decay chi_c1
1.000 gamma J/psi AV2GV;
Enddecay
End

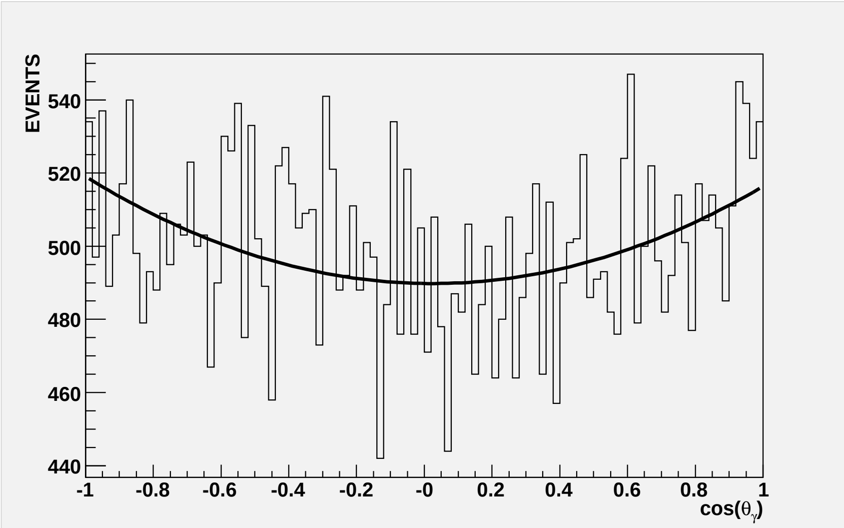

Figure 7: The model P2GC2 is tested by the angular distribution of photon in the $\psi' \to \gamma\chi_{c2}$. The input angular distribution parameter $\alpha = 1/13$, the fit result is $\alpha' = 0.057 \pm 0.015$.

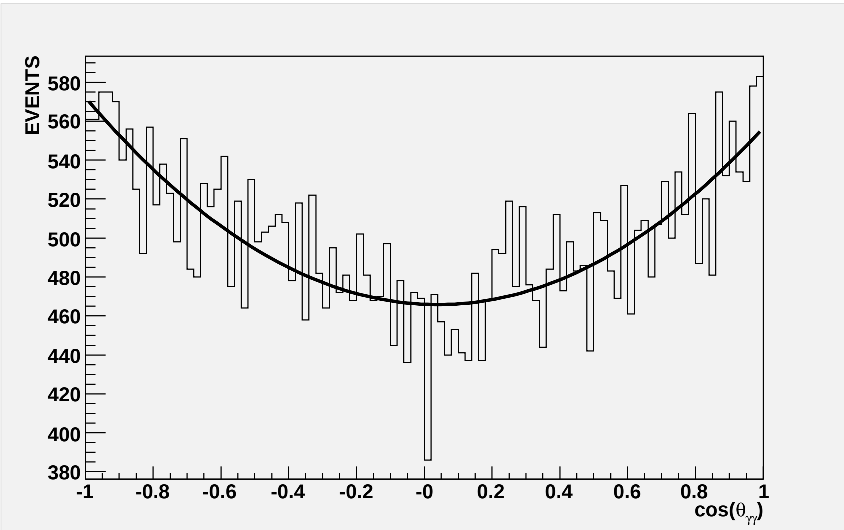

Figure 8: The model AV2GV is tested by observing the angular distribution of the photon in the $\chi_{c1} \to \gamma J/\psi$ with the reference of the photo direction in $\psi' \to \gamma\chi_{c1}$. The input angular distribution parameter $\alpha = 1/5$, the fit result is $\alpha' = 0.21 \pm 0.02$.

5.7 T2GV

The model **T2GV** is tested by the decay $\psi' \to \gamma\chi_{c2} \to \gamma\gamma J/\psi$ with the following decay table:
Decay psi(2S)
1.000 gamma chi_c2 P2GC2;
Enddecay
Decay chi_c2
1.000 gamma J/psi T2GV;
Enddecay
End

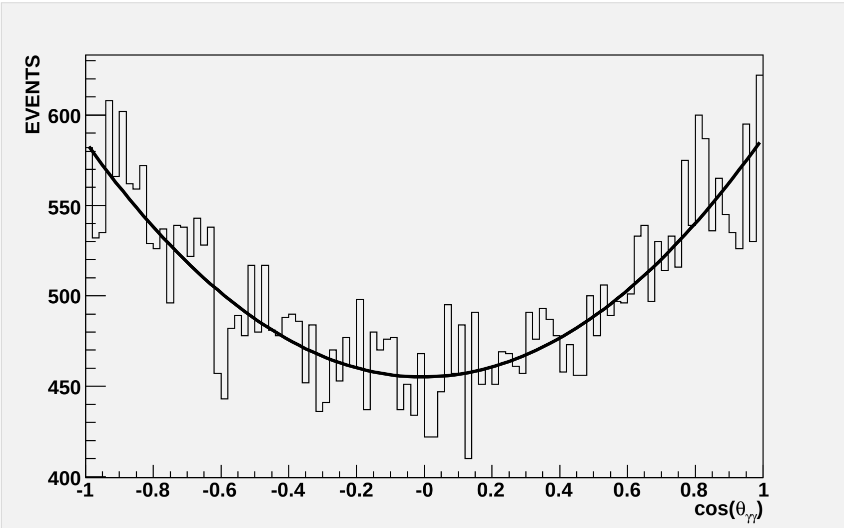

Figure 9: The model T2GV is tested by observing the angular distribution of the photon in the $\chi_{c2} \to \gamma J/\psi$ with the reference of the photo direction in $\psi' \to \gamma\chi_{c2}$. The theoretical angular distribution parameter is expected to be $\alpha = 21/73$ by adjusting the helicity amplitudes. The parameter is fixed to be $\alpha' = 0.29 \pm 0.02$. at present version.

5.8 J2BB1

The model **J2BB1** is tested by the decay $\psi' \to p\bar{p}$ with the following decay table:
Decay psi(2S)
1.000 p+ anti-p- J2BB1 0.67;
Enddecay
End

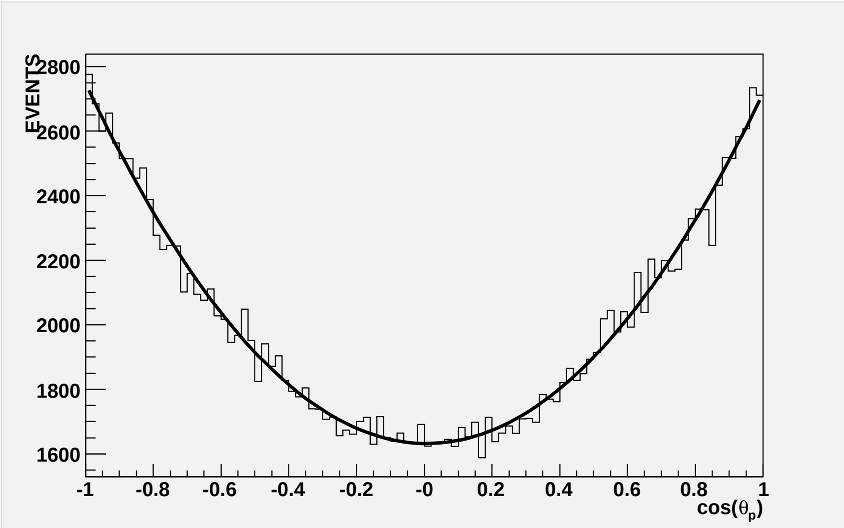

Figure 10: The model J2BB1 is tested by the $\psi' \to p\bar{p}$ with the input parameter $\alpha = 0.67$ and the simulated value is $\alpha = 0.67 \pm 0.01$.

5.9 DIY

The model **DIY** is tested by the decay $J/\psi \to \rho(770)\pi \to \pi^+(p_1)\pi^0(p_2)\pi^-(p_3)$ with the following decay table:
Decay J/psi
1.000 pi+ pi- pi0 DIY;
Enddecay
End

This model needs the user to provide the amplitudes, which is constructed as helicity amplitudes:

$$A_j^{\rho^+}(M) = \sum_{\lambda=-1,1} F^1_{\lambda,0}(r_3) D^{1*}_{M,\lambda}(\theta_3,\phi_3) BW_j(s_{12}) R^j_{0,0}(r_1^*) D^{1*}_{\lambda,0}(\theta_{12},\phi_{12}) \quad \text{for } \rho^+(770), \quad (26)$$

$$A_j^{\rho^-}(M) = \sum_{\lambda=-1,1} F^1_{\lambda,0}(r_1) D^{1*}_{M,\lambda}(\theta_1,\phi_1) BW_j(s_{23}) R^j_{0,0}(r_2^*) D^{1*}_{\lambda,0}(\theta_{23},\phi_{23}) \quad \text{for } \rho^-(770), \quad (27)$$

$$A_j^{\rho^0}(M) = \sum_{\lambda=-1,1} F^1_{\lambda,0}(r_2) D^{1*}_{M,\lambda}(\theta_2,\phi_2) BW_j(s_{12}) R^j_{0,0}(r_3^*) D^{1*}_{\lambda,0}(\theta_{13},\phi_{13}) \quad \text{for } \rho^0(770), \quad (28)$$

where r_1^*, r_2^* and r_3^* are respectively the momentum differences of the two pions' in the CM system of their mother particles ρ with quantum number $J^P = j^-$, and θ_{12} (ϕ_{12}), θ_{23} (ϕ_{23}) and θ_{13} (ϕ_{13}) are the polar (azimuthal) angles of the momentum vector of π^+, π^0 and π^- in their mother particle CM system. $BW_j(s)$ denotes the Breit-Wigner of the $\rho(j^-)$ resonance with a CM energy $\sqrt{s}$. To consider the parity conservation for $J/\psi(1^-) \to \rho(j^-)\pi(0^-)$, it leads to $F^1_{00} = 0$, thus the sum in the above equations runs over $\lambda = \pm 1$.

The helicity-coupling amplitudes $F^1_{\mu,\nu}$ and $R^j_{0,0}$ can be constructed in terms of $L-S$ coupling scheme [10], which are explicitly given by:

$$\begin{aligned} R^{\rho\to\pi\pi}_{00}(s) &= g_{10}\sqrt{s-4m^2_\pi}/\sqrt{s}, \\ F^{\psi\to\rho\pi}_{10}(s) &= g_{11}\sqrt{[m^2_\psi-(m_\pi+\sqrt{s})^2][m^2_\psi-(\sqrt{s}-m_\pi)^2]}, \end{aligned} \tag{29}$$

where g_{ls} denotes coupling constant. In this test, the coupling constat are taken as unity.

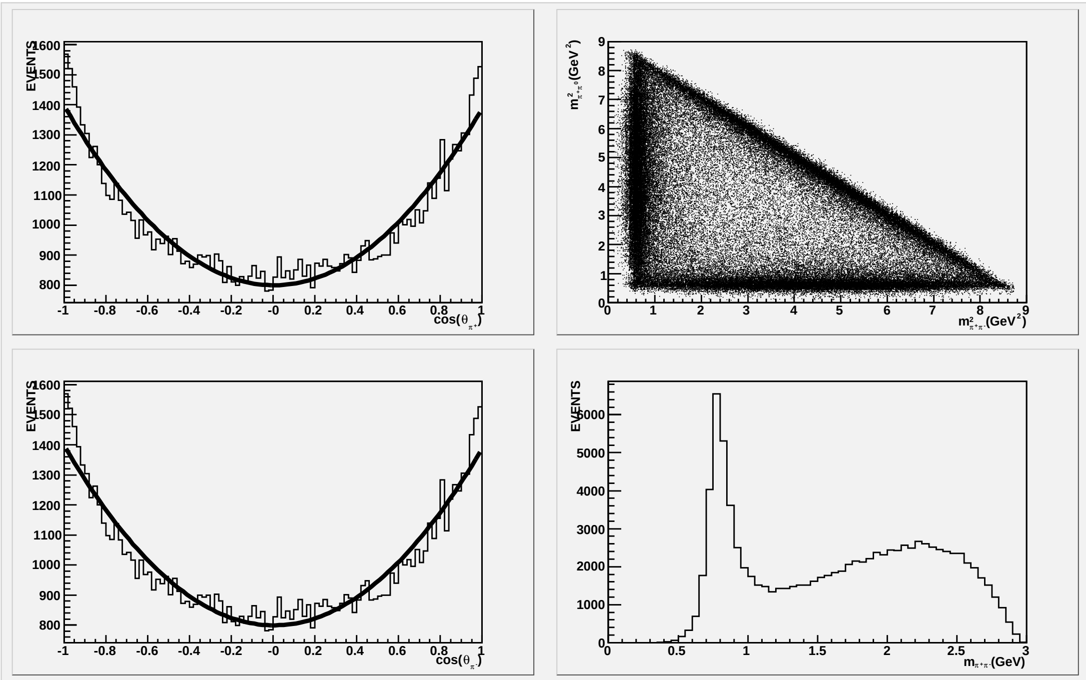

Figure 11: The model DIY is tested by using the decay $J/\psi \to \rho(770)\pi \to \pi^+\pi^-\pi^0$. The left column figures are the angular distributions for the π^+ and π^-, respectively, which take the form $c_0(1+\alpha\cos^2\theta)$ with fitted value $\alpha = 0.74 \pm 0.01$. The right column figures are the Dalitz plot and the mass distribution of $m_{\pi^+\pi^-}$.

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