Hadronic cross section measurements in e^+e^-–annihilation,
Effective fine structure constant at scale M_Z^2,
and Precise test of the Standard Model

Hadronic cross section measurements in e^+e^-–annihilation are an indispensable input for the estimation of the non–perturbative hadronic contribution to the running of the fine structure constant, one of important input parameters in the electroweak precision measurements. A number of good reviews on this subject have been written [1]. Part of the materials is summarized here. In section 1 the definition of $R_{\rm had}$, its experimental determination and the present status of $R_{\rm had}$ measurements at low energy are presented. Then the evaluations of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$,the non–perturbative hadronic contribution to the running of the fine structure constant, will be given in Section 2. Section 3 discuss the choice of the input parameters for the Standard Model, in particular, the effective fine structure constant at scale $M_{\rm Z}$.

1 Measurements of $e^+e^- \to \mathrm{hadrons}$ cross sections and R_{had} value

$R_{\rm had}(s)$, by the proper definition, is the *ratio of the total cross sections* according to following equation,

$$R_{\rm had}(s) = \frac{\sigma_{tot}(e^+e^- \to \gamma^* \to hadrons)}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} \ . \tag{1}$$

Usually, however, experiments do not determine $R_{\rm had}$ as a ratio of the total cross sections as given by Eq. 1. Rather the hadronic experimental cross section is first corrected for QED effects [2, 3, 4, 5], which include bremsstrahlung as well as vacuum polarization corrections. The latter account for the running of the fine structure constant $\alpha(s)$. After these corrections have been applied σ_{tot} is divided by the Born cross section $\sigma_0(e^+e^- \to \gamma^* \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$ so that

$$R_{rmhad}(s) = \frac{\sigma_{tot}(e^+e^- \to \gamma^* \to hadrons)^{corr}_{exp}}{\sigma_0(e^+e^- \to \gamma^* \to \mu^+\mu^-)} \ .$$

Note that, the experimental cross section $\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)$ never appears here and is used by careful groups to check how good normalization is (see e.g., [6]).

Some general comments concerning the $R_{\rm had}$ determination are in order.

- **Exclusive vs Inclusive**

  Usually, for energies below 2 GeV the cross section is measured for individual channels, while above that value the hadronic final states are treated inclusively. In the first case one can directly measure the total and differential cross sections of various exclusive reactions kinematically allowed in given energy region. Having measured the exclusive cross sections, one can determine the total cross sections and the value of R by simply summing of them. This is of course not at all trivial since one should be sure that there is neither double counting nor missing final states and even more complicated correlations between different channels are properly taken into account [7]. There is, in fact, still a systematic difference between the sum of exclusive channels and the inclusive $R_{\rm had}$ measurements in the energy range [1.4–2.1] GeV [8]. In view of the many channels in this energy region as much as possible an inclusive measurement should be pushed [9].

- **Energy scan vs Radiative return**

  The measurement of the hadronic cross section has been usually performed via the energy scan, that is, by systematically varying the e^+e^- beam energies. This traditional way of measuring of the hadronic cross section has one disadvantage - it needs dedicated experiments. On the other hand, modern particle factories, such as the Frascati ϕ-factory DAΦNE or the B-factories PEP-II and KEK-B are designed for a fixed center-of-mass energy $\sqrt{s}$. An energy scan for the measurement of hadronic cross sections is therefore not feasible and an alternative way, the radiative return method, was proposed recently. The radiative return method (For a short theory review see [10] and references therein) relies on an observation that the cross section of the reaction $e^+e^- \to \mathrm{hadrons} + \mathrm{photons}$, with photons emitted from the initial leptons, factorizes into a function H, fully calculable within QED, and the cross section of the reaction $e^+e^- \to \mathrm{hadrons}$

$$
\begin{aligned}
d\sigma(e^+e^- \to \mathrm{hadrons} + \gamma'\mathrm{s})(s, Q^2) = \\
H \cdot d\sigma(e^+e^- \to \mathrm{hadrons})(Q^2) \ ,
\end{aligned}
$$

  where Q^2 is the invariant mass of the hadronic system. Thus from the measured differential, in Q^2, cross section of the reaction $e^+e^- \to \mathrm{hadrons} + \mathrm{photons}$ one can evaluate $\sigma(e^+e^- \to \mathrm{hadrons})$ once the function H is known. As evident from the Eq.(2), the radiative return method allows for the extraction of the hadronic cross section from the production energy threshold of a given hadronic channel almost to the nominal energy of a given experiment ($\sqrt{s}$). The smaller cross section of the radiative process as compared to the process without photons emission has to be compensated by higher luminosities That requirement is met by meson factories (DAPHNE, BaBar, BELLE). All of them were built for other purposes then the hadronic cross section measurements, but their huge luminosities provide with data samples large enough for very accurate measurements of interesting hadronic channels and/or give an information on rare channels, which were not accessible in scan experiments.

  The radiative return method has been successfully applied by KLOE to measure the pion formfactor below 1 GeV [11] and by BABAR for the timelike proton-antiproton formfactor and for several exclusive final states with higher multiplicities in the mass range from threshold up to 4.5 GeV [12]. The combination of KLOE and BaBar data allows to cover the hadronic cross section in the entire mass range below ~ 4.5 GeV . For an extensive review of the recent results of both collaborations concerning the radiative return see [13] This method has the advantage of the same normalisation for each energy point, even if it requires a very solid theoretical understanding of radiative corrections, a precise determination of the angle and energy of the emitted photon, and the full control of background events, especially for events with the photon emitted in the final state (FSR). The Karlsruhe-Katowice group computed the radiative corrections up to NLO for different exclusive channels, implementing them in the event generator PHOKHARA [14, 15, 16, 17, 18]. The current precision for the $\pi^+\pi^-\gamma$ final state is 0.5%.

- **Status of $R_{\rm had}$ at low energy**

  During the last thirty years the ratio $R_{\rm had}$ has been measured by several experiments. Fig. 1 gives an updated summary of $R_{\rm had}$ measurements by different experiments and the current precision in different e^+e^- center-of-mass system (c.m.s.) energy regions by Burkhardt and Pietrzyk [19].

  - **The $\pi^+\pi^-$ threshold region.**

    Experimental data are poor below about 400 MeV because the cross section is suppressed near the threshold. The most effective way to measure the threshold in the time-like region is provided by Initial State Radiation (ISR) events, where the emission of an energetic photon allows to study the two pions at rest.

  - **The ρ peak region.**

    The $\pi^+\pi^-$ region between 0.5 and 1 GeV has been studied by different experiments. CMD-2 [20] and SND [21] performed an energy scan at the e^+e^- collider VEPP–2M ($\sqrt{s} \in$ [0.4–1.4] GeV) with $\sim 10^6$ and $\sim 4.5 \times 10^6$ events respectively, with systematic errors ranging from 0.6% to 4% in the relative cross-section, depending on the 2π energy region. The pion form factor has also been measured by KLOE using ISR, and results are also expected soon by BABAR. KLOE published a result [11] based on an integrated luminosity of 140 pb^{-1}, that led to a relative error of 1.3% in the energy region [0.6–0.97] GeV, dominated by systematics. At the moment it has already collected more than 2 fb^{-1} at the ϕ meson peak, which represents, around the ρ peak, a statistics of $\sim 2 \times 10^7$ $\pi^+\pi^-\gamma$ events. BABAR [12] has already collected more than 300 fb^{-1} at the Υ peak, and is going to collect about 1 ab^{-1} by the end of data taking.

    The results of these four experiments(CMD-2, SND, KLOE, BABAR) in the next few years will probably allow to know the $\pi^+\pi^-$ cross-section for most of the ρ shape with a relative accuracy better than 1% (even considering both statistical and systematic errors). In summary [19],

    * very minor change introduced by CMD2, KLOE and SND measurements;
    * Previous measurements in the ρ region were already relatively precise;

  - **The 1.05–2.0 GeV energy region.**

    The region [1.05–2.0 GeV] is the most poorly known. As we shall see from Fig. 2in the section 3, the $R_{\rm had}$ in this energy region contributes about 40% to the uncertainty of the total dispersion integral for $\Delta^{(5)}_{had}(m_Z^2)$ [19], It also provides most of the contribution to $a_\mu^{\rm HLO}$ above 1 GeV. New $R_{\rm had}$ measurements in this energy region therefore will be important.

  - **The high energy region**

    In the high energy region we must distinguish the J/ψ and the Υ resonances and the background inclusive measurements of the total hadronic cross section which is usually presented in term of $R_{\rm had}$ value.

For the narrow resonances ω, ϕ, the J/ψ family (6 states) and the Υ family (6 states) we can safely use the parametrization as Breit-Wigner resonances

$$\sigma_{BW}(s) = \frac{12\pi}{M_R^2}\frac{\Gamma_{ee}}{\Gamma_R}\frac{M_R^2\Gamma_R\Gamma(s)}{(s-M_R^2)^2+M_R^2\Gamma^2(s)} \tag{2}$$

or as a zero width resonance

$$\sigma_{NW}(s) = \frac{12\pi^2}{M_R}\Gamma_{ee}\,\delta(s-M_R^2) \tag{3}$$

Masses, widths and the electronic branching fractions can be taken from the Review of Particle Properties [22].

In the region from the J/ψ to the Υ the earlier R_{had}-measurements were from Mark I [23, 24], $\gamma\gamma 2$ [25], DASP [26], PLUTO [27], LENA [28], Crystal Ball (CB) [29] and MD-1 [30].

* **The** $2.0--5.0$ **GeV energy region**
  In this energy region the earlier results were with a precision of 15%-20% [24, 25, 27]. In 2001 BESII published the results of R_{had} measurements at 85 different c.m.s. energies between 2 and 4.8 GeV with an average precision of 6.6% [31], in addition to the 6 points published in 1999 [32]. This is a substantial improvement. The BESII results were used in 2001 evaluation of $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ [33, 34] and the uncertainty were reduced to 5.9%.
* **The** $5.0--7.0$ **GeV energy region**
  There is a longstanding annoyance. The values of R_{had} in the literature from MarkI [24] are surprisingly high (4.4 ± 0.4 for the energy region $5.0-7.8$ GeV) compared to theory expectations ($\sim\ 3.4$) (See Fig. 1). In 1990 the average value of R_{had} in the e^+e^- c.m.s. energy region between 5 and 7.4 GeV was reported by the Crystal Ball Collaboration [29] to be $3.44 \pm 0.03 \pm 0.018$ which is much more in line with extrapolation of perturbation QCD assuming 5 quarks and in agreement with the other experiments PLUTO, LENA and MD-1. These results were used in 1995 evaluation of $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ [35, 36]. They together with the R_{had} measurements in the energy region between 2 and 5 GeV of the BESII [31, 32] were mainly two major changes in the history of determination of $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$. Although the results of the Crystal Ball are preferred by theorists these data were never published. Thus there is room for doubt. CLEO may have ability to settle this annoyance [37].
* **The** $7.0--12.0$ **GeV energy region**
  In this region the pQCD calculations are unsure although the calculations are in good agreement with the existing data. Improved measurement of R_{had} is necessary to avoid any dependence on pQCD where it's uncertain.
  The CLEO measurement from the $\Upsilon(4S)$ continuum data ($\sqrt{s} = 10.52$ GeV), $R_{\mathrm{had}} = 3.56 \pm 0.01 \pm 0.07$ [38], is still hailed as one of the most accurate R_{had} measurement because this data point has both small statistical and systematic error.
  Recently CLEO collaboration has taken data for R at energies $7.0-11.3$ GeV ($6.96, 7.38, 8.38, 9.4, 10.0, 10.33, 11.2$ GeV). These data are under analysis and results can be expected soon [37].

* **The energy region above** 12.0 **GeV** The $R_{\rm had}$ in this region is described by the parametrization based on third order QCD [33]

As we will discuss later the hadronic cross section measurement is crucial for the accurate evaluation of the hadronic contributions to running of the electromagnetic coupling α_{QED}. It requires a more accurate knowledge of the hadronic cross section in a wide energy range from the $2m_\pi$ threshold above. An optimal exploitation of a linear collider like the ILC for precision physics requires an improvement of the precision of by something like a factor ten.

2 Hadronic Vacuum Polarization

The running of the electromagnetic coupling with momentum transfer, $\alpha(0) \to \alpha(s)$, caused by fermion-pair loop insertions in the photon propagator, is customarily written as

$$\alpha(s) \quad = \quad \frac{\alpha(0)}{1-\Delta\alpha(s)} \quad = \quad \frac{\alpha(0)}{1-\Delta\alpha_{\rm e\mu\tau}(s)-\Delta\alpha_{\rm top}(s)-\Delta\alpha^{(5)}_{\rm had}(s)}\,, \tag{4}$$

with $\alpha(0) = 1/137.036$ [39]. The contribution of leptons is calculated diagrammatically up to third order: $\Delta\alpha_{\rm e\mu\tau}(M^2_{\rm Z}) = 3149.7686 \times 10^{-5}$ with negligible uncertainty [40]. Since heavy particles decouple in QED, the top-quark contribution is small: $\Delta\alpha_{\rm top}(M^2_{\rm Z}) = -0.00007(1)$; it is calculated by TOPAZ0 and ZFITTER as a function of the pole mass of the top quark, m_t. The running electromagnetic coupling is insensitive to new particles with high masses. For light-quark loops the diagrammatic calculations are not viable as at such low energy scales perturbative QCD is not applicable. Therefore, the total contribution of the five light quark flavors to the hadronic vacuum polarization, $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$, is more accurately obtained through a dispersion integral over the measured hadronic cross-section in electron-positron annihilations at low centre-of-mass energies. The relevant vacuum polarization amplitude satisfies the convergent dispersion relation[34]

$$Re\Pi'_\gamma(s) - \Pi'_\gamma(0) = \frac{s}{\pi} Re \int_{s_0}^\infty ds' \frac{Im\Pi'_\gamma(s')}{s'(s'-s-i\varepsilon)}$$

and using the optical theorem (unitarity) one has

$$Im\Pi'_\gamma(s) = \frac{s}{e^2}\sigma_{tot}(e^+e^- \to \gamma^* \to {\rm hadrons})(s)\ .$$

In terms of the cross-section ratio

$$R(s) = \frac{\sigma_{tot}(e^+e^- \to \gamma^* \to {\rm hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}\ ,$$

where $\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$ at tree level, we finally obtain

$$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) \quad = \quad -\frac{\alpha M_Z^2}{3\pi} Re \int_{4m_\pi^2}^\infty ds \frac{R(s)}{s(s-M_Z^2-i\varepsilon)}\ .$$

The dispersion integral can be evaluated either by direct integration between measured data points or by using a parametrisation of the measured cross section of $e^+e^- \to \mathrm{hadrons}$. In the first approach one use direct integration over the experimental values of cross sections, try to rely on the experimental data as much as possible and integrate directly the data points by joining them by straight lines (trapezoidal rule). In this approach one can take into account uncertainties of separate measurements in a straightforward manner.[35]. In the second approach one make a fit of the experimental points within some model and integrate the arising parametrization of the data. This procedure inevitably leads to a model dependence and it is not clear how experimental errors especially systematic uncertainties can be taken into account[35].

Detailed evaluations of this dispersive integral from the experimental data have been carried out by many authors [8, 9, 19, 33, 34, 35, 36, 41, 42, 43].There are also several evaluations of $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ which are more theory driven [44].

An important conclusion from studies before 1989 was described in the paper [41] that the independent programs and parametrization method gave nearly identical results. Differences in central values obtained from the use of trapezoidal rule between many data points, partially smoothed functions or broad averages were negligible compared to the experimental uncertainty in the data. The uncertainty in the result obtained from the integration is, therefore. almost entirely due to the experimental errors in the determination of $R_{\mathrm{had}}(s)$.

The result of the references [35] and [36]

$$\Delta\alpha^{(5)}_{\mathrm{had}} = 2804\,(65) \times 10^{-5} \tag{5}$$

was used by The LEP Collaborations, the LEP Electroweak Working Group as the input parameter to constrain the Standard Model until summer 2000 [45].

After the BES published its consequence of substantially improved total cross section measurement between 2 and 5 GeV [32] [31] some of these analyses were updated to include the new e^+e^- data – mostly from the BES [31]as well as measurements by the CMD-2 [20] – obtaining:

$$\Delta\alpha^{(5)}_{\mathrm{had}} = 2761\,(36) \times 10^{-5} \tag{6}$$

[33], and

$$\Delta\alpha^{(5)}_{\mathrm{had}} = 2757\,(36) \times 10^{-5}$$

[34]. The reduction, by a factor of two, of the uncertainty quoted in the articles of refs. [35, 36] (70×10^{-5}), with respect to that in [33, 34] (36×10^{-5}), is mainly due to the data of BES.

The new estimates of $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ of the papers [33][34], Eq. 6, was then used as the input parameter in replace of Eq. 5 by The LEP Collaborations, the LEP Electroweak Working Group until the summer 2004.

The latest update, $\Delta\alpha^{(5)}_{\mathrm{had}} = 2758\,(35) \times 10^{-5}$ [19], includes also the measurements of KLOE [11].

Fig. 2 from ref. [46] illustrates the relative contributions from different e^+e^- c.m.s. energy regions to $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ both in magnitude and uncertainty. The region between $1.05 - 2$ GeV gives an important contribution to the uncertainty despite its small contribution to the magnitude.

Table 1: Contributions for $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)\times 10^4$ (direct integration method) and $\Delta\alpha^{(5)}_{\rm had}(-s_0)\times 10^4$ (non-perturbative part in the Adler function method), with relative (rel) and absolute (abs) error in percent.

Energy range	$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)\times 10^4$	rel [%]	abs [%]
ρ,ω ($E<2M_K$)	36.23 [13.1](0.24)	0.7	1.1
$2M_K<E<2$ GeV	21.80 [7.9](1.33)	6.1	34.9
2 GeV $<E<M_{J/\psi}$	15.73 [5.7](0.88)	5.6	15.4
$M_{J/\psi}<E<M_\Upsilon$	66.95 [24.3](0.95)	1.4	18.0
$M_\Upsilon<E<E_{\rm cut}$	19.69 [7.1](1.24)	6.3	30.4
$E_{\rm cut}<E$ pQCD	115.66 [41.9](0.11)	0.1	0.3
$E<E_{\rm cut}$ data	160.41 [58.1](2.24)	1.4	99.7
total	276.07 [100.0](2.25)	0.8	100.0

Comparison of estimates of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ performed during 90's is shown in Fig. 3 from reference [46].

Another recent update based on a compilation of the data shown in Fig. 4. yields $\Delta\alpha^{(5)}_{\rm had} = 0.027607\pm0.000225$ or $1/\alpha^{(5)}(m_{\rm Z}^2)=128.947\pm0.035$

Contributions from various energy regions and the origin of the errors in this estimate are shown in Fig. 5.

More details are given in Table 1.

In summary, to reach the high precision would require much more experimental effort to delicately measure the $\sigma(e^+e^-\to{\rm hadrons})$ cross section both at low and high energies.[9]

3 Precise test of the Standard Model: Input parameters and their uncertainties

In the last few decades the final electroweak measurements with data taken at the Z resonance have been performed by the experiments operating at the electron-positron colliders SLC and LEP. The mass and width of the Z boson, M_Z and Γ_Z, and its couplings to fermions , for example the ρ parameter and the effective electroweak mixing angle for leptons, are precisely measured:[47]

$$\begin{aligned} M_{\rm Z} &= 91.1875\pm0.0021\ {\rm GeV}\\ \Gamma_{\rm Z} &= 2.4952\pm0.0023\ {\rm GeV}\\ \rho_l &= 1.0050\pm0.0010\\ \sin^2\theta^{\rm lept}_{\rm eff} &= 0.23153\pm0.00016\ . \end{aligned}$$

Through radiative corrections evaluated in the framework of the Standard Model, the large and diverse set of precise measurements allows many relations inspired by the Standard Model to be stringently tested and the free parameters of the model to be tightly constrained . The masses of W boson and top quarks are predicted to be: $M_{\rm W}=80.363\pm0.032$ GeV and $m_t=173^{+13}_{-10}$ GeV, agreeing well with the direct measurements of these quantities at LEPII and TEVATRON[48], successfully testing the Standard Model at the level of its radiative corrections. Using in addition the direct measurements of m_t and $M_{\rm W}$, the mass of the as yet unobserved Standard Model Higgs boson is predicted with a relative uncertainty of about 50% and found to be less than 285 GeV at 95% confidence level[49].

Note that practical perturbation calculations in the Standard Model are *approximations* obtained by truncation of perturbation series. The accuracy of the finite order approximation for each Z-pole observable depends on the input parameter set chosen for Standard Model calculations. A natural choice is the QED-like parametrization in terms of

$$\alpha, \quad \alpha_s, \quad M_{\mathrm{W}}, \quad M_{\mathrm{Z}}, \quad m_{\mathrm{H}}, \quad m_f, \tag{7}$$

where M_{W} and M_{Z} are the masses of the W boson and the Z boson, m_{H} is the mass of the Higgs boson, m_f are the masses of all known fundamental fermions f, in particular, m_t is the top quark mass, and α and α_s just are two coupling constants of the electromagnetic and the strong interaction. Since loop corrections in general induce a running of the electromagnetic coupling constant α with momentum transfer (or s), The running of the strong coupling, $\alpha_s(s)$, is even larger. The Z^0 resonance is sufficiently dominant for Z-pole observables, however, that the Z-pole approximation can be taken, and the relevant coupling constants become simply $\alpha(m_Z^2)$ and $\alpha_s(m_Z^2)$.

Within the $StandardModel$, however, the mass of the W measured directly at the TEVATRON and LEPII, is related to M_{Z} and the Fermi constant G_μ through radiative corrections . A very precise value for the latter, $G_\mu = 1.16637(1) \cdot 10^{-5}$ GeV^{-2} , is derived from measurements of the muon lifetime using two-loop corrections . This 9 ppm precision on G_μ greatly exceeds the relative precision with which M_{W} can be measured in the foreseeable future. Indeed, this motivates our substitution of G_μ for M_{W} as an input parameter for $StandardModel$ calculations.

Therefore, one uses the fine-structure constant α, the Fermi coupling constant G_μ, and the mass of Z boson M_{Z} as input parameters for precise calculation of radiative corrections since they are the most precisely measured parameters. In fact,

- The fine-structure constant in the Thomson limit determined from the $e^\pm$ anomalous magnetic moment, the quantum Hall effct, and other measurements[39]

$$\begin{aligned} \alpha^{-1}(0) &= 137.03599911(46), \\ \frac{\delta\alpha}{\alpha} &= 3.6 \times 10^{-9} \end{aligned} \tag{8}$$

- The Fermi coupling constant determined from the muon lifetime formula[50][51]

$$\begin{aligned} G_\mu &= 1.16637(1), \\ \frac{\delta G_\mu}{G_\mu} &= 8.6 \times 10^{-6} \end{aligned} \tag{9}$$

- The mass of the Z boson determined from the Z-lineshape scan at LEP I[47]

$$\begin{aligned} M_{\mathrm{Z}} &= 91.1875(21), \\ \frac{\delta M_{\mathrm{Z}}}{M_{\mathrm{Z}}} &= 2.4 \times 10^{-5} \end{aligned} \tag{10}$$

- The effective fine-structure constant at the scale M_Z[19]

$$\begin{aligned} \alpha^{-1}(M_{\rm Z}) &= 128.940(48), \\ \frac{\delta\alpha(M_{\rm Z})}{\alpha(M_{\rm Z})} &= (1.6 \sim 6.8) \times 10^{-4} \end{aligned} \tag{11}$$

The relative uncertainty of $\alpha(M_{\rm Z})$ is roughly one order of magnitude worse than that of $M_{\rm Z}$, making it one of the limiting factors in the calculation of precise SM predictions.

Note that $\Delta\alpha$ enters in electroweak precision physics typically when calculating versions of the weak mixing parameter $\sin^2\Theta_i$ from α, G_μ and M_Z via[52]

$$\sin^2\Theta_i\,\cos^2\Theta_i = \frac{\pi\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\frac{1}{1-\Delta r_i} \tag{12}$$

where

$$\Delta r_i = \Delta r_i(\alpha,\,G_\mu,\,M_Z, m_H,\,m_{f\neq t},\,m_t) \tag{13}$$

includes the higher order corrections which can be calculated in the SM or in alternative models.

Δr_i depends on the definition of $\sin^2\Theta_i$. The various definitions coincide at tree level and hence only differ by quantum effects. From the weak gauge boson masses, the electroweak gauge couplings and the neutral current couplings of the charged fermions we obtain

$$\sin^2\Theta_W = 1-\frac{M_W^2}{M_Z^2} \tag{14}$$

$$\sin^2\Theta_g = e^2/g^2 = \frac{\pi\alpha}{\sqrt{2}\,G_\mu\,M_W^2} \tag{15}$$

$$\sin^2\Theta_f = \frac{1}{4|Q_f|}\left(1-\frac{v_f}{a_f}\right)\,,\quad f\neq\nu\,, \tag{16}$$

respectively. For the most important cases and the general form of Δr_i reads

$$\Delta r_i = \Delta\alpha - f_i(\sin^2\Theta_i)\,\Delta\rho + \Delta r_{i\,\rm remainder} \tag{17}$$

where

- The large term $\Delta\alpha$ is due to the photon vacuum polarization

$$\Delta\alpha = \Pi_1^{\gamma\gamma}(0) - \Pi_1^{\gamma\gamma}(M_Z^2) \tag{18}$$

  This universal term which affects the predictions for M_W, A_{LR}, A_{FB}^f, Γ_f, etc. The oder terms can be calculated safely in perturbation theory.

- $\Delta\rho$ is the famous correction to the ρ–parameter which is defined as the neutral to charged current ratio

$$\Delta\rho = \rho - 1 = \frac{G_{NC}}{G_\mu} = \frac{\Pi_1^{ZZ}(0)}{M_Z^2} - \frac{\Pi_1^{WW}(0)}{M_W^2} \tag{19}$$

$\Delta\rho$ exhibiting the leading top mass correction

$$\Delta\rho \simeq \frac{\sqrt{2}G_\mu}{16\pi^2}\, 3m_t^2 \;\; ; \quad m_t \gg m_b \tag{20}$$

which allowed LEP experiments to obtain a rather good indirect estimate of the top quark mass prior to the discovery at the TEVATRON.

Note that in (17) $f_W = c_W^2/s_W^2 \simeq 3.35$ is substantially enhanced relative to $f_f = 1$.

- The "remainder" term although sub-leading is very important for the interpretation of the precision experiments at LEP and includes part of the leading Higgs mass dependence. For a heavy Higgs particle we obtain the simple expression

$$\Delta r_i^{\mathrm{Higgs}} \simeq \frac{\sqrt{2}G_\mu M_W^2}{16\pi^2} \left\{ c_i^H \left(\ln \frac{m_H^2}{M_W^2} - \frac{5}{6}\right) \right\} \;\; ; \quad m_M \gg M_W \tag{21}$$

where $c_f^H = (1 + 9\sin^2\Theta_f)/(3\sin^2\Theta_f)$ and $c_W^H = 11/3$, for example.

The uncertainty $\delta\Delta\alpha$ implies uncertainties δM_W, $\delta\sin^2\Theta_f$ given by

$$\frac{\delta M_W}{M_W} \sim \frac{1}{2}\frac{\sin^2\Theta_W}{\cos^2\Theta_W - \sin^2\Theta_W}\,\delta\Delta\alpha \sim 0.23\,\delta\Delta\alpha \tag{22}$$

$$\frac{\delta\sin^2\Theta_f}{\sin^2\Theta_f} \sim \frac{\cos^2\Theta_f}{\cos^2\Theta_f - \sin^2\Theta_f}\,\delta\Delta\alpha \sim 1.54\,\delta\Delta\alpha \tag{23}$$

The effects of the uncertainty due to $dahz$ on the $StandardModel$ prediction for the ρ parameter and $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ can be seen in Fig 6. While the $StandardModel$ prediction for the ρ parameter is not affected by the uncertainty in $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)$ the uncertainty on the prediction of $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ within the $StandardModel$ due to the uncertainty on $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)$ is nearly as large as the accuracy of the experimental measurement of $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$. The present error in the effective electromagnetic coupling constant, $\delta\Delta\alpha(M_Z^2) = 35\times10^{-5}$ [19], dominates the uncertainty of the theoretical prediction of $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$, inducing an error $\delta(\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}) \sim 12\times10^{-5}$ which is not much smaller than the experimental value $\delta(\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}})^{\mathrm{EXP}} = 16\times10^{-5}$ determined by LEP-I and SLD [47]. This observation underlines the importance of a precise cross-section measurement of electron-positron annihilation into hadrons at low centre-of-mass energies.

Moreover, as measurements of the effective EW mixing angle at a future linear collider may improve its precision by one order of magnitude [53], a much smaller value of $\delta\Delta\alpha(M_Z^2)$ will be required. It is therefore crucial to assess all viable options to further reduce this uncertainty.

$\delta\Delta\alpha^{(5)}_{\rm had}\times 10^5$	$\delta(\sin^2\theta^{\rm lept}_{\rm eff})\times 10^5$	Request on R
35	12.5	Present
7	2.5	$\delta R/R\sim 1\%$ for $\sqrt{s}\leq M_{J/\psi}$
5	1.8	$\delta R/R\sim 1\%$ for $\sqrt{s}\leq M_{\Upsilon}$

Table 2: Values of the uncertainties $\delta\Delta\alpha^{(5)}_{\rm had}$ (first column) and the errors induced by these uncertainties on the theoretical SM prediction for $\sin^2\theta^{\rm lept}_{\rm eff}$ (second column). The third column indicates the corresponding requirements on the R measurement.

Tab. 2 (from Ref. [34][54]) shows that an uncertainty $\delta\Delta\alpha^{(5)}_{\rm had}\sim 5\times 10^{-5}$, needed for precision physics at a future linear collider, requires the measurement of the hadronic cross section with a precision of $O(1\%)$ from threshold up to the Υ peak.

In the SM the Higgs mass m_H is the only relevant unknown parameter and by confronting the calculated with the experimentally determined value of $\sin^2\Theta_i$ one obtains the important indirects constraints on the Higgs mass. The uncertainty $\delta\Delta\alpha$ thus obscure in particular the indirect bounds on the Higgs mass obtained from electroweak precision measurements. As we mentioned in section 2 the current uncertainty in $1.05--2.0$ GeV enrgy region is 15%. Improving the precision of measurements from 15% (Fig. 1) to 5% would change the total uncertainty on $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ from 0.00035 to 0.00027. The change in the fitted value of the Higgs mass would be small. However, the change $R_{\rm had}$ by $\pm 1\sigma$ in this c.m.s. energy region would shift the central value of the fitted Higgs mass by $^{+16}_{-9}$ GeV. Therefore more precise measurements in this c.m.s. energy region are important.

The importance of the external $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)=0.02758\pm 0.00035$[19] determination for the constraint on m_H is shown in Figure 7. Without the external $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ constraint, the fit results are $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)=0.0298^{+0.0010}_{-0.0017}$ and $m_H=29^{+77}_{-15}$ GeV, with a correlation of -0.88 between these two fit results.

The latest global fit of the LEP Electroweak Working Group, which employs the complete set of EW observables, leads to the value $m_H=91^{+45}_{-32}$ GeV, with a 95% confidence level upper limit of 186 GeV (see Fig. 8) [49]. This limit increases to 219 GeV when including the LEP-II direct search lower limit of 114 GeV.

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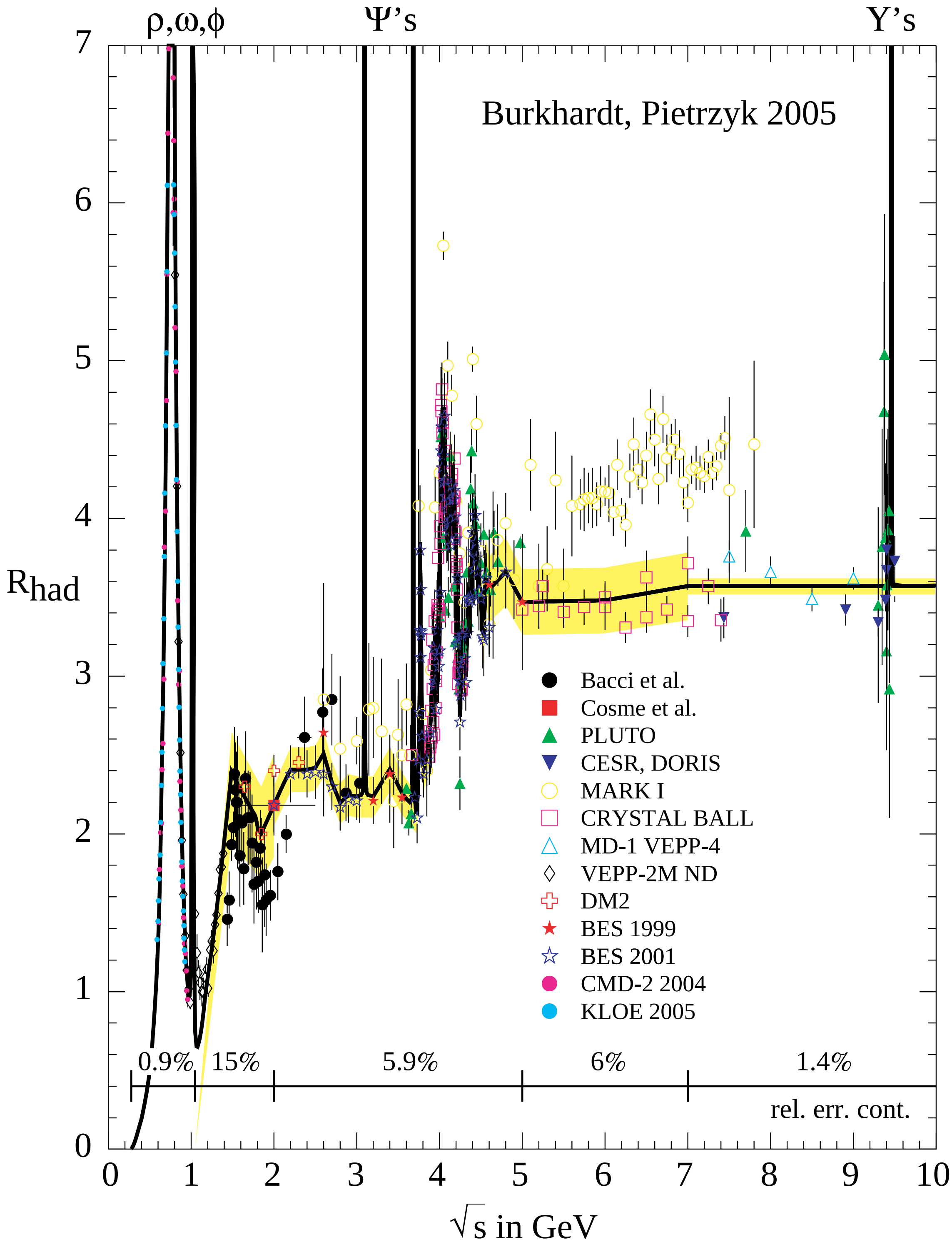

Figure 1: $R_{\rm had}$ including resonances. Measurements are shown with statistical errors. The relative uncertainty assigned to our parametrization is shown as band and given with numbers at the bottom.

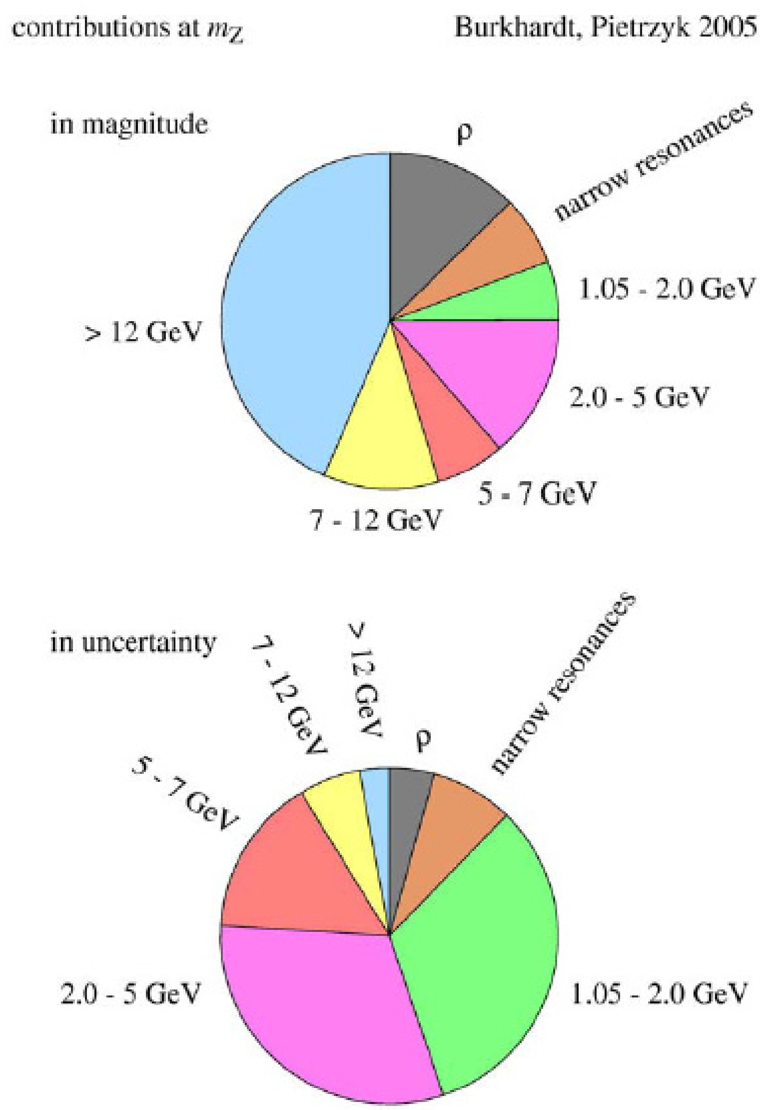

Figure 2: Relative contributions to $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ in magnitude and uncertainty from reference [46]

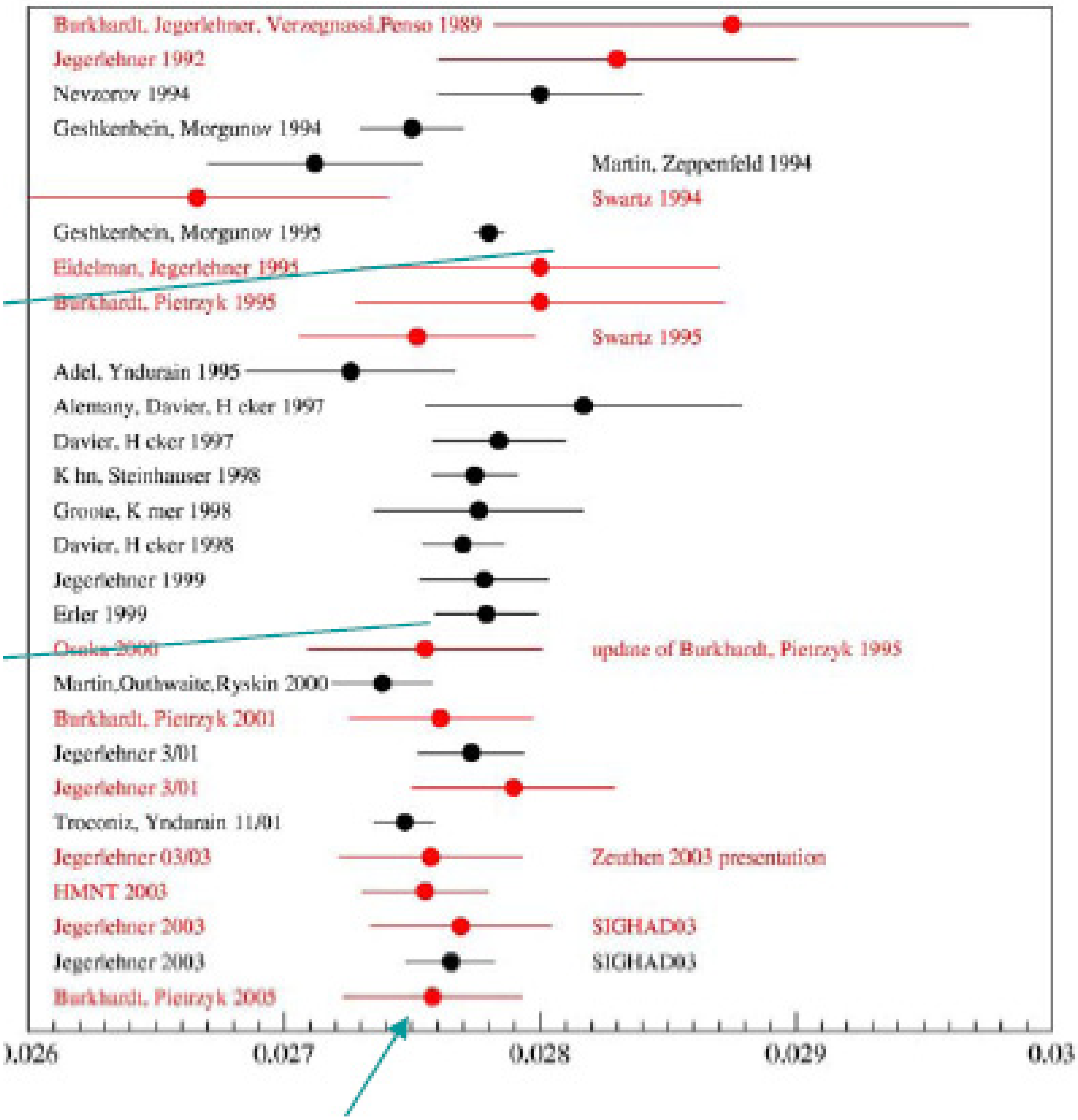

Figure 3: Comparison of some estimates of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$. Estimates based on dispersion integration of the experimental data are shown with red solid dot and estimates relying on additional theoretical assumptions shown as black solid dot.

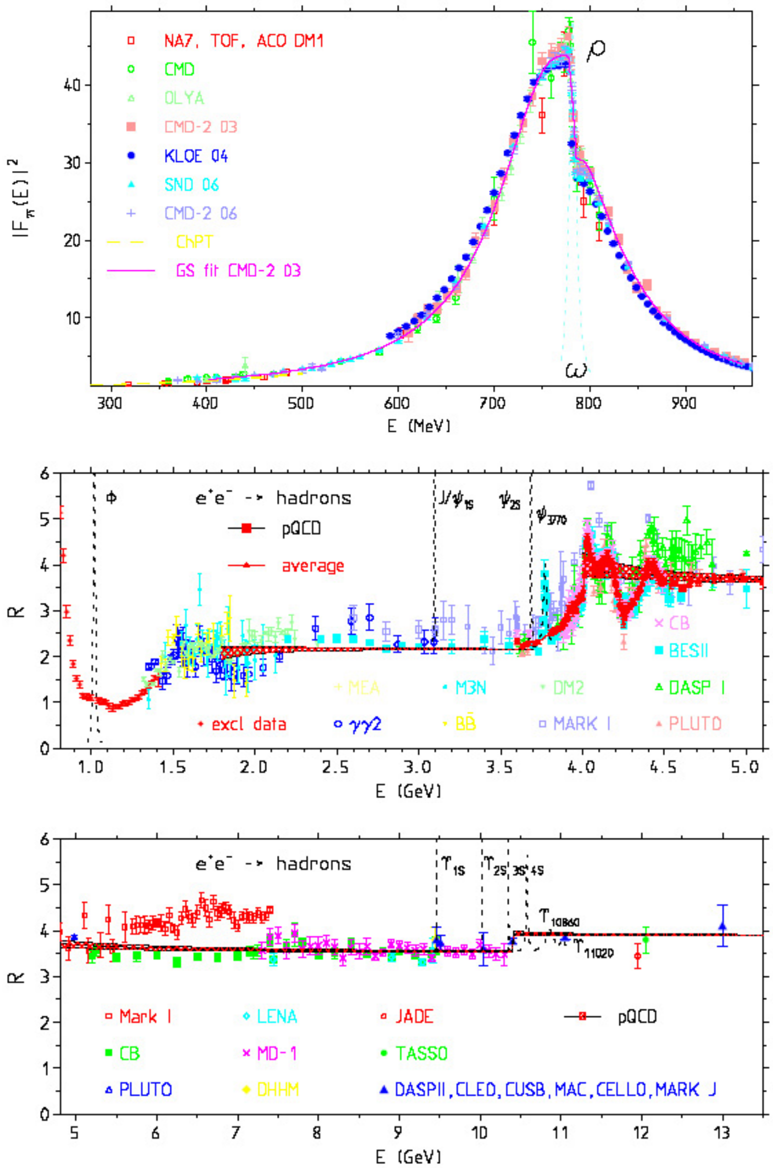

Figure 4: A compilation of the presently available experimental hadronic e^+e^-–annihilation data

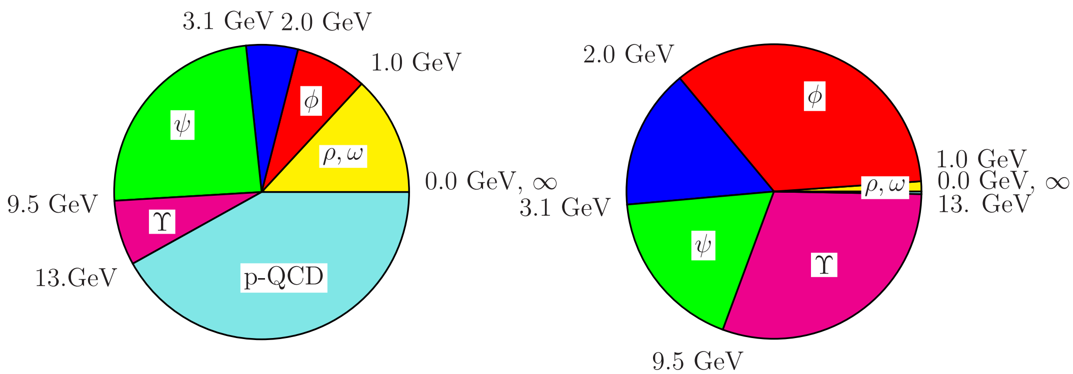

Figure 5: $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$: contributions (left) and errors^2 (right) from different regions

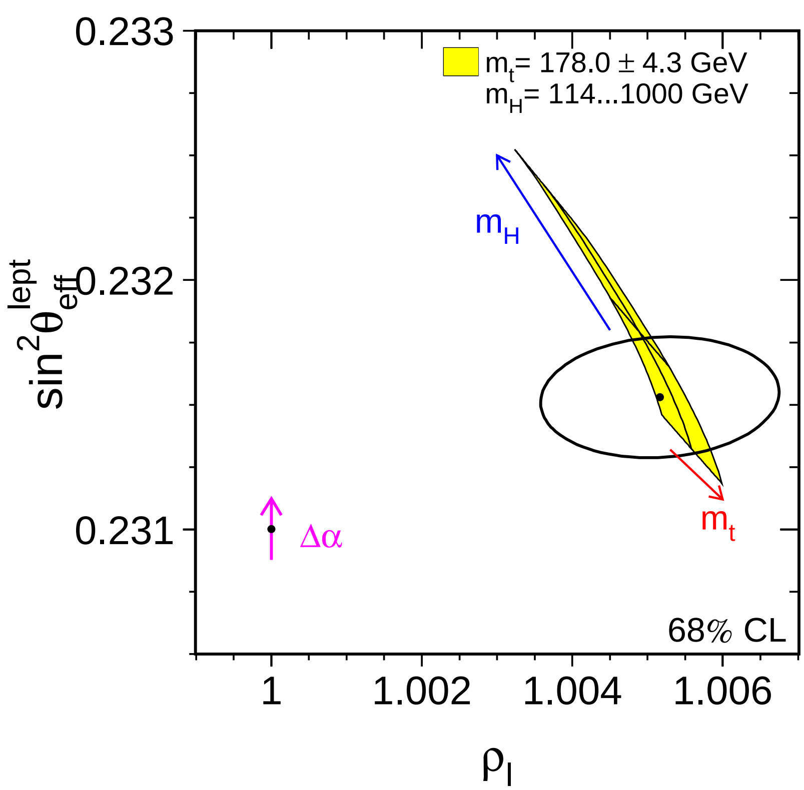

Figure 6: Contour curve of 68% probability in the (ρ_l, $\sin^2\theta_{\rm eff}^{\rm lept}$) plane. The prediction of a theory based on electroweak Born-level formulae and QED with running α is shown as the dot, with the arrow representing the uncertainty due to the hadronic vacuum polarization $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$. The same uncertainty also affects the $Standard Model$ prediction, shown as the shaded region drawn for fixed $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ while m_t and m_H are varied in the ranges indicated.

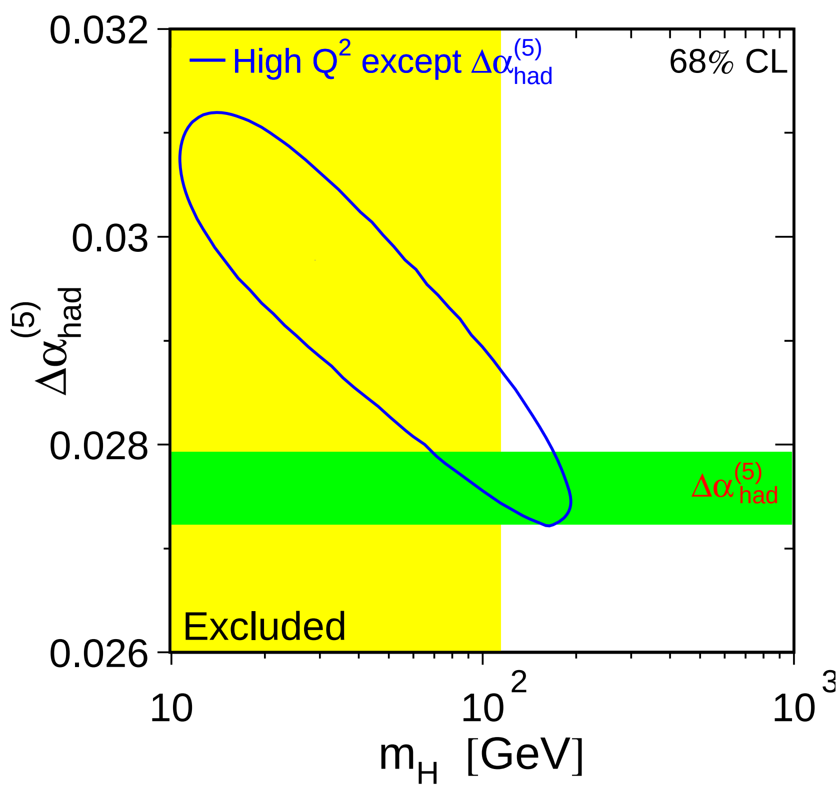

Figure 7: Contour curve of 68% probability in the $(\Delta\alpha_{\rm had}^{(5)}(M_Z^2), m_H)$ plane, based on all 18 measurements except the constraint on $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$. The direct measurements of the excluded observable is shown as the horizontal bands of width ± 1 standard deviation. The vertical band shows the 95% confidence level exclusion limit on m_H of 114.4 GeV derived from the direct search at LEPII [49].

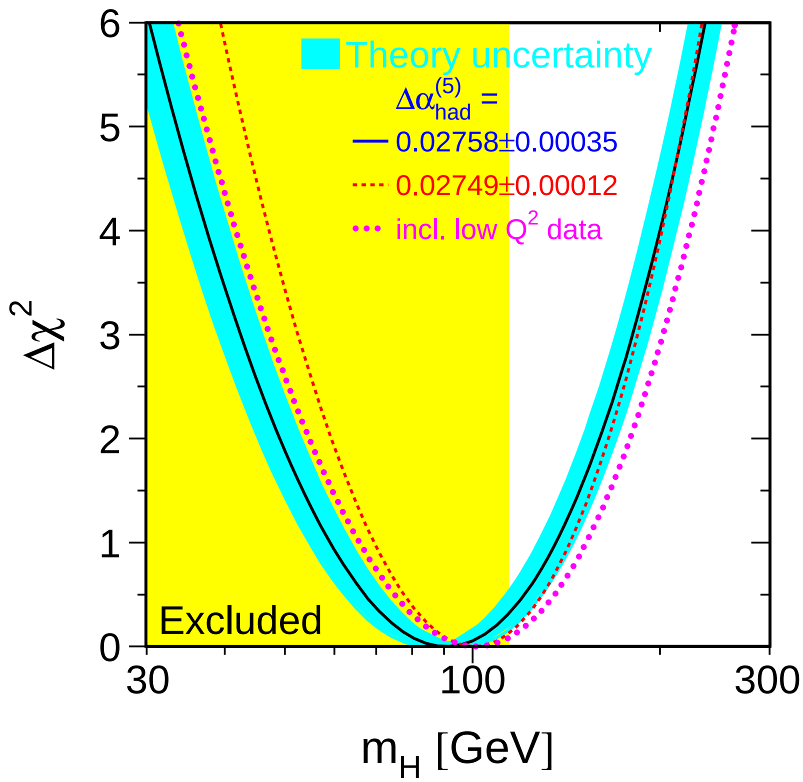

Figure 8: The line is the result of the Electroweak Working Group fit using all data [49]; the band represents an estimate of the theoretical error due to missing higher order corrections. The vertical band shows the 95% CL exclusion limit on m_H from the direct search.